Numerical study on temperature uniformity in a novel mini-channel heat sink with different flow field configurations

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A B S T R A C T

Temperature uniformity in a water-cooled mini-channel heat sink with different flow field configurations under high heating flux was numerically studied. A thermal resistance network model was established and the concept of variable-height channels was proposed to improve the temperature uniformity on the heating surface. The influence of flow field configurations on the flow distribution uniformity is firstly studied, and a circular turning constructal distributor was selected due to its best flow distribution uniformity and cooling performance. And then the effects of fluid velocity, heating area, substrate thickness and three novel channels on the temperature uniformity and thermal resistance were investigated. Result shows that for the heat sink with the circular turning distributor and variable-height channels, the temperature non-uniformity achieved on a 3 × 3 cm heating surface can be reduced to less than 1 K with the heat flux as high as 2 MW m⁻².

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1. Introduction

With a continuously increasing integration of electronic circuit, thermal management rises to be a great challenge in the design of electronic devices [1–3]. Mini-channel heat sink has been considered as an effective heat removal tool and has caught much attention during the past decades, due to its advantages including high heat transfer performance, mild pressure loss and easy fabrication [4,5]. Much research has been conducted on the heat transfer and fluid flow characteristics of mini-channels, and most of them suppose that the inlet velocities of all the channels are the same [3–7]. Garimella and Sobhan [8] presented their results of theoretical and experimental studies in mini-channels under different flow regimes, and pointed out that fluid flow and heat transfer characteristics in mini-channels with the hydraulic diameter within 0.2–3 mm are still in accordance with conventional ones. This conclusion has been confirmed by many other researchers, for example [9,10].

Temperature uniformity is highly required to obtain high performance, reliability and functionality for many electronic devices and to control critical reaction temperature for micro reactors [11]. Local high temperature and thermal stress may result in the deformation of the electronic circuit and deteriorate the efficiency of the electronic circuit [12]. Therefore, the combination of the high heat flux (2 MW m⁻²), small temperature non-uniformity (less than 1 K) and small pumping power (less than 10 W) is highly demanded for the thermal design of the electronic devices and high load radars for some special applications.

A straightforward method to improve the temperature uniformity may be to increase the dimensions of the heat sink to cover the heating surface with a larger area. However, the cost and the pumping power would also be increased. Therefore, it remains a challenge to increase the temperature uniformity on the heating surface while keeping the dimensions of the heat sink the same. Usually, uniform temperature distribution can be obtained by appropriately designing the flow field. The typical parallel heat sinks combined with headers in U-type or Z-type (see Figs. 1(a) and (b)), which have received much attention, have been studied analytically, numerically, and experimentally [13,14]. The flow uniformity in U-type header is usually better than that in Z-type header. Investigation results [15,16] have pointed out that the main factors affecting the flow distribution uniformity in parallel channels include the header stream velocity, tube diameter and inlet port location. In their studies, a punched baffle was inserted into the header to improve the flow distribution. However, the flow uniformity still could not be guaranteed if the fluid velocity is large.

Recently, optimization technology of flow configurations has received increasing interest. Many types of novel and effective flow distributors, which can provide a relatively even flow
distribution, have been proposed based on the constructal theory developed by Bejan \[17\]. Liu et al. \[18\] studied the performance of several bifurcation structure distributors (see Figs. 1(c) and (d)). Guo et al. \[12,19\] developed an arborescent distributor for a multi-channel heat exchanger reactor, which enhanced the heat exchanger performance. However, it is well recognized that even if an absolutely uniform flow distribution in parallel channels can be achieved, the corresponding temperature uniformity on the heat sink surface is still not guaranteed. For conventional parallel-channels heat sink, the typical feature of the temperature distribution is that the temperature arises along the flow direction, and the induced temperature non-uniformity would increase with the heat flux.

Cho et al. \[20\] proposed a new vascular channel with circular turning type cross-sections for the cooling of proton exchange membrane fuel cells (PEMFC). The temperature non-uniformity of the optimized cooling plate studied is 1 K. However, the heat flux is merely 155 W m\(^{-2}\), which is far smaller than the current heat flux in electronic devices. Wang et al. \[21\] investigated the asymmetric and symmetric tree-like branching networks with the heat flux 0.1 MW m\(^{-2}\), and the temperature non-uniformity is about 10 K. The cooling fluid adopted in the literature \[20,21\] is water, and the inlet temperature is 300 K. The mass flow rates are \(1.33 \times 10^{-3}\) kg s\(^{-1}\) and \(1.00 \times 10^{-3}\) kg s\(^{-1}\), respectively.

Based on the above review, it can be found that research results concerning the temperature uniformity of heat sinks for cooling

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**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>channel cross-section, m(^2)</td>
</tr>
<tr>
<td>(c_p)</td>
<td>specific heat capacity, J kg(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>(D_h)</td>
<td>hydraulic diameter, m</td>
</tr>
<tr>
<td>(D_i)</td>
<td>width of pre-duct, m</td>
</tr>
<tr>
<td>(H)</td>
<td>height, m</td>
</tr>
<tr>
<td>(h)</td>
<td>heat transfer coefficient, W m(^{-2}) K(^{-1})</td>
</tr>
<tr>
<td>(h(x))</td>
<td>local heat transfer coefficient, W m(^{-2}) K(^{-1})</td>
</tr>
<tr>
<td>(k)</td>
<td>slope of channels</td>
</tr>
<tr>
<td>(L)</td>
<td>channel length, m</td>
</tr>
<tr>
<td>(l_i)</td>
<td>length of pre-duct, m</td>
</tr>
<tr>
<td>(m)</td>
<td>mass flow rate, kg s(^{-1})</td>
</tr>
<tr>
<td>(Nu)</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>(P)</td>
<td>pressure, Pa</td>
</tr>
<tr>
<td>(Pr)</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>(R)</td>
<td>thermal resistance, m(^2) K W(^{-1})</td>
</tr>
<tr>
<td>(Re)</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>(Q)</td>
<td>total heat transfer rate, W</td>
</tr>
<tr>
<td>(q)</td>
<td>heat flux, W m(^2)</td>
</tr>
<tr>
<td>(T)</td>
<td>temperature, K</td>
</tr>
<tr>
<td>(\Delta T)</td>
<td>temperature non-uniformity, K</td>
</tr>
<tr>
<td>(u)</td>
<td>velocity, m s(^{-1})</td>
</tr>
<tr>
<td>(W)</td>
<td>width, m</td>
</tr>
<tr>
<td>(W_{pump})</td>
<td>pumping power, W</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>control coefficient</td>
</tr>
<tr>
<td>(\beta)</td>
<td>1/2 angle of two bifurcations</td>
</tr>
<tr>
<td>(\eta)</td>
<td>fin efficiency</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>thermal conductivity, W m(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>relative mass flow rate variation</td>
</tr>
<tr>
<td>(\rho)</td>
<td>density, kg m(^{-3})</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>relative mass flow variation variance</td>
</tr>
</tbody>
</table>

**Superscript**

- mean

**Subscript**

- 1 D one dimension/conductive
- \(b\) substrate
- \(ch\) channel
- conv convective
- eff effective
- \(f\) fluid
- \(i\) number of each channel
- \(in\) inlet
- \(s\) solid
- \(sp\) spreading
- \(w\) fin

---

**Fig. 1.** Different channel head configurations, (a) U-type; (b) Z-type; (c) circular turning type; (d) tree-type.
The objective of the present study is to optimize the flow distributors and channel geometries to improve the flow distribution uniformity and temperature uniformity. The second objective is to investigate the effects of different parameters such as the substrate thickness, heating area and fluid velocity on the temperature uniformity. The ultimate goal is to develop a mini-channel heat sink of which the temperature non-uniformity remains within 1 K for a heat flux of 2 MW m⁻² as required by certain IT equipment. Based on the analysis of the heat transfer process and the thermal resistances, a new idea to reach good temperature uniformity is proposed. By combining CFD simulation and thermal resistance network model analysis, three different channels are designed which are expected to meet the requirements.

The present paper is organized as follows. In Section 2 the thermal resistance network is established, and based on which, three novel channels are schematically proposed. In Section 3 the computational domain and the numerical methods are introduced. Then, in Section 4 flow uniformity for different flow distributors is depicted. Effects of channel height, heating area, substrate thickness, fluid velocity and the influence of novel and conventional channels on the temperature uniformity and thermal resistance are investigated. Finally, some conclusions are drawn in Section 5.

2. Thermal resistance network analysis

A schematic view of the mini-channel heat sink studied in this paper is outlined in Fig. 2(a). In this section a preliminary analysis of appropriate streamwise channel shape for obtaining uniform temperature distribution is conducted by thermal resistance network. To simplify the analysis, a single unit of the heat sink is used as the representative of the whole device as depicted in Fig. 2(b). The heat generated in the heating surface conducts through the substrate, and one part dissipates to the liquid through the bottom of the channel, and the rest transfers into the fin and is brought away by the cooling fluid. Since the mean temperature of the fluid increases along the flow direction, a decreasing thermal resistance should be achieved to obtain a relatively uniform temperature on the bottom of the substrate. Therefore, to improve the temperature uniformity we propose that the channel height \( H_{ch} \) (or the thickness of the substrate) should be varied step-wisely or linearly along the flow direction. For the heat transfer in the single channel, a thermal resistance network is established as illustrated in Fig. 2(c), and the total thermal resistance for the small element bodies can be expressed as follows [22,23]

\[
R_{\text{total}}(x) = R_{1-D}(x) + R_{\text{conv}}(x)
\]

(1)

\[
R_{\text{conv}}(x) = \frac{1}{h_f(x) dx(W_{ch} + W_w)}
\]

(2)

\[
R_{1-D}(x) = \frac{H_{ch}(x)}{\lambda dx(W_{ch} + W_w)}
\]

(3)

where \( R_{\text{conv}}(x) \) and \( R_{1-D}(x) \) are the local convective resistance and one-dimensional conduction thermal resistance of substrate, respectively. It is worth noting that the spreading thermal resistance equals zero since the bottom of the heat sink fully coincides with the active heating surface, hence, it is not considered here. Other variables are listed in the nomenclature.

For the analysis of convective thermal resistance, the Gnielinski correlation [24] is adopted to obtain the Nusselt number \( Nu(x) \) of the channel, and thus the local convective heat transfer coefficient \( h(x) \) can be obtained as:

\[
h(x) = \frac{Nu(x) \cdot \lambda_f}{D_h(x)}
\]

(4)

where \( \lambda_f \) and \( D_h(x) \) are the conductivity of fluid and hydraulic diameter of the channel, respectively. Assuming a one-dimensional fin model, the local fin efficiency and local effective heat transfer coefficient can be given by [24]:

\[
\eta(x) = \frac{\tanh(m(x)H_{ch}(x))}{m(x)H_{ch}(x)}
\]

(5)

\[
h_{\text{eff}}(x) = \frac{h(x)(2\eta(x)H_{ch}(x) + W_{ch})}{W_w + W_{ch}}
\]

(6)

Here \( m(x) = \left( \frac{2\lambda_f}{\rho C_p u x} \right)^{0.5} \). The energy balance for small element shown in Fig. 2(b) can be expressed as:

\[
\rho C_p A u (T_f(x + dx) - T_f(x)) = q \cdot (W_w + W_{ch}) \cdot dx
\]

(7)

where \( T_f(x), q, A, \) and \( u \) are the local mean fluid temperature, heat flux, channel cross-section area and average fluid velocity, respectively. The mean fluid temperature along the channel can be integrated as:
\[ T_I(x) = \frac{q(W_w + W_{ch})x}{\rho u A c_p} + T_m \]  

Assuming that the temperature of the heating area \( T_w \) is uniform, the following equations can be drawn according to the definition of thermal resistance of unit length at the position of \( x \) and \( x + dx \):

\[ T_w - T_I(x) = q(W_w + W_{ch}) R_{total}(x) dx \]

\[ T_w - T_I(x + dx) = q(W_w + W_{ch}) R_{total}(x + dx) dx \]

Subtracting Eq. (9) from Eq. (10), we have

\[ -T_I(x + dx) + T_I(x) = q(W_w + W_{ch}) (R_{total}(x + dx) - R_{total}(x)) dx \]

Substituting Eqs. (1)-(3), (6) and (8) into Eq. (11) acquires

\[ -dx/\rho c_p u A = \frac{dH_b(x)}{\lambda_t(W_w + W_{ch})} \]

\[ + \frac{1}{h(x)[W_{ch} + 2\eta(x + dx)H_{ch}(x + dx)]} - \frac{1}{h(x)[W_{ch} + 2\eta(x)H_{ch}(x)]} \]

Combining Eq. (5) and (12) yields the following equation

\[ -\rho c_p u A/dx = \frac{dH_b(x)}{\lambda_t(W_w + W_{ch})} \]

\[ + \frac{W_{ch}[h(x) - h(x + dx)] + 2/h(x)[\tanh(m(x)H_{ch}(x)) - h(x + dx)\tanh(m(x + dx)H_{ch}(x + dx))] - h(x)[h(x + dx)W_{ch} + 2\eta(x + dx)H_{ch}(x + dx)]}{h(x)[h(x + dx)W_{ch} + 2\eta(x + dx)H_{ch}(x + dx)]} \]

Taylor series expansion for \( \tanh(t) \) is as follows

\[ \tanh(t) = \sum_{n=1}^{\infty} \frac{(2n-1)}{(2n)!} t^{2n-1} (-1)^{n+1} \]

The term \( m(x)H_{ch}(x) \) in Eq. (13) is quite small, nearly equals to zero. Therefore, the third order Taylor series expansion for \( \tanh(t) \) is adopted

\[ \tanh(t) = t - \frac{t^3}{3} + O(t^5) \]

Based on Eq. (13) and (15), we get

\[ -dx/\rho c_p u A = \frac{dH_b(x)}{\lambda_t(W_w + W_{ch})} \]

\[ + \frac{W_{ch}[h(x) - h(x + dx)]}{h(x)[h(x + dx)W_{ch} + 2\eta(x + dx)H_{ch}(x + dx)]} \]

\[ - 2/h(x)[\tanh(m(x)H_{ch}(x)) - h(x + dx)\tanh(m(x + dx)H_{ch}(x + dx))] \]

Note that the variation of channel height is contrary to the variation of substrate thickness and the local heat transfer coefficient is assumed to be the same, namely, \( h(x) = h(0) \), Eq. (16) can be simplified as

\[ -dx/\rho c_p u A = \frac{dH_b(x)}{\lambda_t(W_w + W_{ch})} \]

\[ + \frac{2dH_b(x) - \frac{dm(x)H_{ch}(x)H_{ch}(x + dx)}{dH_b(x)} \frac{H_{ch}(x + dx)}{h(0)[W_{ch} + 2\eta(x + dx)H_{ch}(x + dx)]}}{h(0)[W_{ch} + 2\eta(x + dx)H_{ch}(x + dx)]} \]

For simplification, we assume that the local channel height \( H_{ch}(x) \) equals to \( H_{ch}(0) \). Based on Eq. (5), \( \eta(x) \) equals to \( \eta(0) \). Eq. (17) can be further derived as

\[ k = \frac{dH_b(x)}{dx} \]

\[ = -x(\rho c_p u A)^{-1} \left( \frac{1}{\lambda_t(W_w + W_{ch})} + \frac{2 - 4h(0)H_{ch}(0)^2}{h(0)[W_{ch} + 2\eta(0)H_{ch}(0)^2]} \right) \]

In order to have all the assumptions made during the deviation of Eq. (18) considered, a control coefficient \( x \) is adopted to adjust the slope (see in Fig. 3), and which is ranging from 0 to 2.5.

It can be seen from Eq. (18) that the slope of the substrate thickness is determined by two factors, one is the heat capacity multiplying the mass flow rate of the cooling fluid, and the other represents the relative variation of the local thermal resistance.

According to convective heat transfer theory, the increase in mass flow rate and fluid heat capacity decreases the temperature difference between the fluid and heating surface, which benefits the temperature uniformity of the heating surface. Therefore, the slope of substrate thickness is inversely related to the heat capacity multiplying mass flow rate of fluid. If the second term in the right hand of Eq. (18) is positive, it is obvious that increasing the channel height can be helpful to achieve a uniform temperature. In the present study, three different novel channels are designed, as illustrated in Figs. 3(b)-(d) for which the second term is always positive.

3. Physical and mathematical model

3.1. Description of channel header geometries

A mini-channel heat sink is usually composed of three parts: an inlet flow distributor (inlet header), mini-channels and a collector. The cooling fluid entering the distributor is distributed into channels, removes the heat away and flows out of the collector.
The structure of the distributor is extremely important to achieve a relatively uniform flow distribution among channels. In the following paragraphs the structure of the inlet header is firstly described.

Four kinds of distribution headers attached to the mini-channels are shown in Fig. 1, namely, U-type, Z-type, circular turning type and tree type, where \( \beta \) is one half of the angle between two bifurcations. Three sets of tree type distribution header are studied with \( \beta \) of 30, 45, 60 degree, respectively. As shown in this figure, 4 levels of bifurcations are designed in the tree-type and circular turning type; a total 16 channels of the last level are obtained as shown in Figs. 1(c), 1(d) and 4(a). The length of pre-duct before each level is denoted by \( l_i \). The minimum Reynolds number studied in this paper for different width and length of each bifurcation level in are listed in Table 1. The circular turning type and tree type distributors are referred to as constructal distributors.

The mini-channel heat sink with its size of \( L \times L \times H \) is composed by a cover and a plate body, where channels are fabricated on as shown in Fig. 4. A uniform heat flux is imposed on the bottom surface of the substrate, with its size of \( L \times L \). The values of geometric parameters adopted in this study are presented in Table 2.

### 3.2. Governing equations

The thermal and flow characteristics of the above heat sinks are numerically simulated by using commercial CFD software FLUENT. Following assumptions are made:

#### Table 1
Minimum Reynolds number in each bifurcation level with \( U_{in} = 3 \text{ m s}^{-1} \).

<table>
<thead>
<tr>
<th>Bifurcations</th>
<th>( l_i/\text{mm} )</th>
<th>( D_i/\text{mm} )</th>
<th>( Re )</th>
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<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>8</td>
<td>19,300</td>
</tr>
<tr>
<td>1</td>
<td>3.5</td>
<td>3.55</td>
<td>14,150</td>
</tr>
<tr>
<td>2</td>
<td>2.5</td>
<td>2.55</td>
<td>7930</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>1.5</td>
<td>3530</td>
</tr>
<tr>
<td>4</td>
<td>–</td>
<td>1</td>
<td>2410</td>
</tr>
</tbody>
</table>

### 3.3. Boundary conditions

Symmetry boundary condition is imposed in the middle of constructal distributors. No-slip condition is employed for all velocities on the solid walls, \( u = U_{in} \) for the entrance of the distributor and local one way condition for the exit [25,26], \( T_{in} = 293 \text{ K} \) for the entrance temperature, \( q = 2 \text{ MW m}^{-2} \) at the substrate bottom surface.

### Table 2
Model parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( W_a )</th>
<th>( W_w )</th>
<th>( W_{ch} )</th>
<th>( H_b )</th>
<th>( L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value (mm)</td>
<td>1</td>
<td>0.9</td>
<td>4</td>
<td>4</td>
<td>30</td>
</tr>
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</table>

### Table 3
Constants properties vs. temperature correlations.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( a_1 )</th>
<th>( b_1 )</th>
<th>( c_1 )</th>
<th>( d_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>999.84</td>
<td>8958.9</td>
<td>–0.58166</td>
<td>2.414 \times 10^{-5}</td>
</tr>
<tr>
<td>1</td>
<td>18.225</td>
<td>–40.535</td>
<td>6.3556 \times 10^{-3}</td>
<td>247.8</td>
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<td>2</td>
<td>–7.92 \times 10^{-3}</td>
<td>0.11243</td>
<td>–7.964 \times 10^{-6}</td>
<td>140.0</td>
</tr>
<tr>
<td>3</td>
<td>–5.545 \times 10^{-5}</td>
<td>–1.014 \times 10^{-4}</td>
<td></td>
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</tr>
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<td>4</td>
<td>1.498 \times 10^{-7}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>–3.913 \times 10^{-10}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.816 \times 10^{-6}</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(1) Natural convection within the channels is neglected.
(2) The flow and heat transfer are steady and in turbulent regime.
(3) All the lateral walls are adiabatic.

Fig. 4. Schematical diagram of heat sink (a) top view; (b) A–A view.
is chosen in the comparison study. Comparison of the velocities in channels under different Reynolds numbers is shown in Fig. 5. The result of present study is in good agreement with that in Ref. [18], showing the feasibility of the grid system adopted in this study.

4. Results and discussion

The thermal performance, such as the temperature uniformity and thermal resistance, can be greatly influenced by the flow distribution uniformity. Those channels with relatively higher mass flow rate may function well, while those with lower mass flow rate may create undesirable hot spots which can directly induce higher thermal resistance and temperature non-uniformity. Thus, the flow uniformity of different channels is evaluated with the mass flow rate variance $\sigma$ and relative variation $\xi$ in all channels

$$\sigma = \frac{\sum_{i=1}^{n} (m_i - \bar{m})^2}{\bar{m}}$$

$$\xi = \frac{m_{\text{max}} - m_{\text{min}}}{\bar{m}}$$

where $m_i$, $\bar{m}$ and $n$ are the mass flow rate of one channel, the average mass flow rate of all channels and the number of channels, respectively.

The pumping power consumption required should also be considered, which can be obtained as:

$$W_{\text{pump}} = \Delta P \bar{V}$$

where $\Delta P$ and $\bar{V}$ are the pressure loss and volumetric flow rate of the fluid in the heat sink, respectively.

The overall thermal resistance $\theta$, which indicates the ability of cooling plates to decrease the peaks of the maximum temperature of the heating surface, is always adopted to evaluate a heat sink’s cooling performance. It is expressed as [4,5,7]

$$\theta = \frac{T_{\text{max}} - T_{\text{in}}}{Q}$$

where $T_{\text{max}}$ and $Q$ are the maximum temperature of the heating surface and the total heat imposed on the heating surface, respectively.

The maximum temperature difference on the heating surface is approximately 10.6°C, which indicates the ability of cooling plates to decrease the peaks of the maximum temperature of the heating surface, is always adopted to evaluate a heat sink’s cooling performance. It is expressed as [4,5,7]

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In the following sections, flow and thermal performance with different distributors will be discussed firstly to select the distributor with the best performance. Then, parametric studies on the thermal and flow characteristics are performed. Finally, the results of comparison between novel channels and conventional ones are presented.

4.1. Effect of flow distributors

Figure 6 shows the relative velocity distribution characteristics with different flow distributors when the inlet velocity $U_{in}$ equals $4 \text{ m/s}$. It is observed that the constructal distributors can provide a relatively more uniform flow distribution in channels than the conventional Z-type and U-type configurations. The Z-type flow profile in channels qualitatively agrees well with that in Ref. [14], and the U-type flow profile qualitatively accords well with the results of Ref. [13].

Figure 7 shows the relative mass flow rate variation $\xi$ and variance $\sigma^2$ in channels with different distributors under different inlet velocities. It can be easily seen that the circular turning type distributor achieves the best flow uniformity. The tree-type distributor performs better with an increase in $\beta$. The Z-type configuration performs the worst in terms of its mass flow rate variance and variation being the highest. For example, when the inlet velocity is $4 \text{ m/s}$, the relative mass flow rate variations of circular turning type, U-type, Z-type and tree-type with $\beta$ equaling 60, 45, and 30 degrees distributors are 0.05, 0.34, 0.68, 0.06, 0.07 and 0.15, respectively. The mass flow rate variances are 0.14, 0.98, 2.27, 0.18, 0.23 and 0.43, respectively. The flow distribution performance turns bad with an increase in the distributor inlet velocity as can be expected.

The effects of the inlet velocity on the temperature non-uniformity and pumping power are illustrated in Fig. 8. It is shown that the pumping power of circular turning distributor is mild and its temperature uniformity is the best compared with the other configurations. When the inlet velocity is $4 \text{ m/s}$, the pumping powers of circular turning distributor, U-type, Z-type and tree-type distributor with $\beta$ equaling 45 degree are 4.3, 2.1, 2.7 and 5.9 W; the temperature non-uniformities are 4.7, 13.2, 21.1 and 6.5 K, respectively. Unlike the channel smoothness of circular type distributor, the tree-type distributors when $\beta$ equals 45 or 60 degrees cause more pumping power. The pumping powers with U-type and Z-type configurations are lower than those with constructal distributors since there is no distributor before the fluid entering the channels, and the temperature non-uniformities are higher due to the uneven flow uniformity. Therefore, it can be concluded that a better flow uniformity can produce a better temperature uniformity, and the circular turning type distributor performs the best which is consistent with the results of Ref. [18].

It can also be found from the Fig. 8 that the temperature non-uniformity is highly affected by the inlet velocity. The temperature non-uniformity in circular turning distributors and tree-type distributors with $\beta$ equaling 30 or 45 degrees firstly decreases with the fluid velocity, reaches its minimum, and then increases with the velocity and reaches a relatively stable value. This can be explained as follows. Even though a good flow distribution uniformity and a small temperature uniformity can be achieved with a low fluid velocity, the cooling performance is relatively poor due to the high convective thermal resistance. When the velocity is high, the distribution performance turns bad, and which could be dominant in determining a higher temperature non-uniformity on the heating surface. When the velocity is high enough, the net effect of a decreasing flow distribution performance and an increasing cooling performance makes the temperature non-uniformity increase rather slowly with inlet velocity and even reaches its minimum. It is noted that even though the distributing performance of tree-type distributors with $\beta$ equaling 30 degrees is worst among all the constructal distributors (see Fig. 7), the mass flow rate distribution in channels is rather different. For example, the mass flow rate in channel with its number being 1 or 16 is the lowest (see Fig. 6), which means more fluid is distributed in the middle of the heat sink. This kind of distribution benefits its cooling performance.
Considering the satisfying flow performance and the lowest temperature non-uniformity of heat sink with circular turning distributors, it is therefore chosen for the following further investigation.

### 4.2. Effect of aspect ratio

Much research [28, 29] has been conducted on the effect of aspect ratio on the thermal resistance. Results reveal that the aspect ratio is beneficial to the cooling performance since it can improve the local Nusselt number along the flow direction. However, little attention was devoted to the temperature uniformity. In this study, the aspect ratio is changed only by altering the channel height while fixing channel width and the thickness of substrate. As shown in Fig. 9, the temperature non-uniformity reduces with aspect ratio, indicating that better temperature uniformity can be achieved by increasing the aspect ratio. For example, the temperature non-uniformity varies from 7.0 K to 3.8 K when the aspect ratio ranging from 2 to 7 at constant inlet velocity 4 m s⁻¹. It can also be observed that the thermal resistance has the same variation trend. This can be explained as follows. With the increase of channel height, the effective dissipative area and convective heat transfer rate both increase, leading to a decrease in convective thermal resistance and better temperature uniformity. When the channel height is too high, the heat transferring to the fins becomes less effective due to an increasing conduction thermal resistance of the side walls. Therefore, when the aspect ratio is higher than 4, the slope of the temperature non-uniformity and thermal resistance decreases with aspect ratio mildly.

It is worth noting that with a fixed volumetric flow rate at the distributor inlet the increase of aspect ratio will lead to a decrease in the temperature non-uniformity. For example, when the aspect ratio is 4 and the inlet velocity is 6 m s⁻¹, the temperature non-uniformity is 5.5 K; while when the aspect ratio is 6 and the inlet velocity is 4 m s⁻¹, the temperature non-uniformity is 4.1 K. The major reason is that the effective dissipative area has linear effect on the heat transfer rate while the effect of decreasing velocity on heat transfer coefficient is non-linear, usually $h \sim u^{-0.33-0.8}$ [24]. Therefore, the net effect of increasing aspect ratio on heat transfer rate is positive under the same volumetric flow rate.

It can also be found that the pumping power increases with the aspect ratio. With a constant inlet velocity, the pressure loss decreases and the volumetric flow rate increases with an increase in aspect ratio. The net effect of increasing aspect ratio could increase the pumping power. Taking all factors mentioned above into consideration, the thermal performance can be improved with the increase of aspect ratio. However, the compactness is also important, and a trade-off between compactness and thermal performance should be made. Besides, when the aspect ratio is higher than 4, the thermal resistance and temperature non-uniformity drops rather mildly. Thus, the aspect ratio equaling 4 is selected and following results are based on this selection.

**Fig. 10.** Variations of temperature non-uniformity and thermal resistance with heating area.

**Fig. 11.** Temperature distribution of the bottom surface: (a) $L_q = 5$ mm; (b) $L_q = 10$ mm; (c) $L_q = 20$ mm; (d) $L_q = 30$ mm.
4.3. Effect of heating area

The effect of heating area on the temperature uniformity and thermal resistance under different velocities is illustrated in Fig. 10. The dimension of the heating area is denoted by $L_q$, and the heat flux is $2\,\text{MW}\,\text{m}^{-2}$. As can be seen in this figure, the temperature non-uniformity increases firstly and then decreases with the increase of $L_q$. Physically this can be explained as follows. The spreading thermal resistance turns zero when $L_q$ approaches zero or the dimension of the entire cooling surface [22,23]. Therefore, the temperature non-uniformity has its maximum value at some middle dimension of the heating area. The thermal resistance decreases monotonically with $L_q$. The major reason is as follows. Even though the temperature difference of $T_{\text{max}}$ and $T_{\text{in}}$ increases with $L_q$, the total heat $Q$ dissipated by the liquid is quadratic with $L_q$. Therefore, the net effect of the increase in $L_q$ decreases the thermal resistance according to Eq. (25).

Figure 10 also shows that the inlet flow velocity has little effects on both thermal resistance and temperature non-uniformity. This can be understood as follows. With the inlet velocity ranging from 4 to 6 m s$^{-1}$, the convective thermal resistance occupies a small portion of the total thermal resistance, hence, the total thermal resistance is not sensitive to the inlet velocity; and even though the absolute wall temperature is reduced with the increasing inlet velocity, its variation over the surface is mainly dependent on the heat flux, hence the temperature uniformity received little effect from the inlet velocity.

Figure 11 shows temperature distribution on the bottom surface of the substrate. When the heating area $L_q \times L_q$ is smaller than the bottom area $L \times L$, the highest temperature of the heating surface is at the center and decreases outwards. When the cooling area is fully occupied by the heating source the temperature distributes approximately linearly along the flow direction. Therefore, in contrast with making the dimension of the heat sink much larger than the heating area, a better temperature uniformity and lower thermal resistance can be acquired when the dimension of the heat sink equals heat source and with its surrounding thermal insulated. Thus, $L_q$ equaling 30 mm is selected in the following study.

4.4. Effect of substrate thickness

The effect of substrate thickness $H_b$ on the temperature non-uniformity and thermal resistance is depicted in Fig. 12. The temperature non-uniformity sharply decreases as the substrate thickness increases, and the thermal resistance linearly increases with the substrate thickness. The increase of substrate thickness at fixed inlet velocity implies the relative increase of conduction resistance in the total thermal resistance, and the total resistance also increases. Thus, poor cooling performance may be induced due to an increase in the total thermal resistance. A compromise of the $T_{\text{max}}$ (less than 353 K), the temperature non-uniformity (less than 5 K) and thermal resistance (less than 0.03 W K$^{-1}$) is made, and therefore, $H_b$ equaling 4 mm is selected in the following study.
4.5. Comparison of novel channels and conventional channel

Based on the previous study, the thermal resistance and temperature uniformity of heat sink with conventional and three novel channels are studied. The cross sections of the four channels are shown in Fig. 3. When the distributor inlet velocity equals 4 m s\(^{-1}\), the temperature distribution on the heating surface with three novel channels with the control coefficient \(z\) equaling 1 is shown in Fig. 13. The temperature non-uniformity with the conventional channel under the same operating conditions is 4.75 K (see Fig. 11(d)). It can be easily found that the novel channels can greatly improve the temperature non-uniformity on the heating surface with their temperature non-uniformities being equal to 1.4 K, 0.97 K and 1.4 K, respectively.

Figure 14 shows the effect of control coefficient \(z\) in Eq. (18) on the thermal resistance for the three proposed channels. It can be seen that for the one-staged channel the thermal resistance decreases firstly and then increases with the control coefficient, while for the two-staged and linear variation channels, the thermal resistance reduces with the control coefficient. The thermal resistance for conventional channel is 0.03 W K\(^{-1}\), which can be improved to 0.028 W K\(^{-1}\) with the proposed channels. For a constant inlet velocity, the convective thermal resistance increases with the control coefficient due to a lower velocity. However, the conductive thermal resistance drops with the control coefficient due to a thinner substrate thickness. Based on Eq. (18), when the control coefficient is small, the variation of the decreasing conductive thermal resistance is larger than that of the increasing convective thermal resistance, leading to a smaller total thermal resistance with a small control coefficient. When the control coefficient is large, the total thermal resistance may improve because of the predominant increase of convective resistance. This explain the variation trend of the one-staged channel. For the two-stage and linear cases similar explains can be given.

The effect of control coefficient \(z\) on the temperature non-uniformity of the three different channels is depicted in Fig. 15. Note that the control coefficient equals zero corresponding to the conventional channels. When \(z\) ranges from 1 to 1.5, the temperature non-uniformity reaches the lowest value under different velocities. Take the linear variation channels for example, the thermal resistance drops relatively heavily when the control coefficient is less than 1.5 and then it drops smoothly when the control coefficient is higher than 1.5. When the control coefficient is high, the local thermal resistance along the flow direction varies heavily, which would induce a higher temperature non-uniformity. For example, when the inlet velocity is 4 m s\(^{-1}\) and the channel height decreases linearly, the temperature non-uniformity drops from 4.75 K to 1.29 K for the control coefficient \(z\) ranging from 0 to 1.5, and then increases from 1.29 K to 3.43 K for the control coefficient \(z\) ranging from 1.5 to 2.5.

As shown in Fig. 14, the temperature non-uniformity of heat sink with conventional channel is 4.75 K when inlet velocity equals 4 m s\(^{-1}\), while those of the novel channels are quite smaller with the value of 0.92–1.44 K for the case of \(z = 1\). The related pumping power and pressure drop are 4.28 W and 32 kPa, respectively, which are acceptable. Therefore the three novel heat sinks with constructal distributor and variable-height channel can be applicable to such special cases in which the high heat flux (2 MW m\(^{-2}\)) and small temperature non-uniformity (less than 1 K) are demanded.

5. Conclusions

In this paper how to improve the temperature uniformity on heating surface of heat sink is numerically studied. Based on the thermal resistance network, three novel channels with variable-channel height along flow direction are designed. The heat sinks with different configurations and conventional and novel channels are parametrically simulated. The major conclusions can be drawn as follows:

1. The temperature non-uniformity and thermal resistance of the heating surface are highly influenced by the uniformity of flow distribution. Compared with the tree-type, U-type and Z-type configurations, the circular turning one performs the best.

2. The temperature non-uniformity and thermal resistance of the heating surface decrease with the aspect ratio. The temperature non-uniformity drops from 7.0 K to 3.8 K with the aspect ratio ranging from 2 to 7 when the inlet velocity equals 4 m s\(^{-1}\).

3. With the cooling efficiency and temperature non-uniformity considered, the dimensions of the heat sink should be the same as the heating surface.

4. Mini-channel heat sink with circular turning type distributor and variable-height channel could be adopted for those high-demand special cases. The temperature non-uniformity of the conventional channel can be improved from 4.7 K to 0.97 K with our proposed channels, and the thermal resistance can also be decreased from 0.03 W K\(^{-1}\) to 0.028 W K\(^{-1}\). Moreover, the required pressure drop or pumping power are acceptable.
(5) The proper slope of channel height along the flow direction can be obtained with

\[ K = \frac{dH_s(x)}{dx} \]

\[ = -\frac{\alpha (\rho c_p u A)}{q_s \left[ W_w + W_{ch} \right]} + \frac{2 - 4h(0)H_{ch}^2(0)/\gamma W_{ch}^3}{h(0)[W_{ch}^2 + 2h(0)H_{ch}^3(0)/\gamma]^2} \]

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References