Numerical studies on the inherent interrelationship between field synergy principle and entransy dissipation extreme principle for enhancing convective heat transfer

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Abstract

In 1998 Guo et al. integrated the boundary-layer energy equation along the thermal boundary layer thickness, and noted that at outside boundary the temperature gradient is zero and the convection term is actually the inner production of vector velocity and temperature gradient, they found that for a fixed flow rate and temperature difference, the smaller the intersection angle between velocity and temperature gradient the larger the heat transfer rate. This idea is called field synergy principle (FSP). Later it has been shown that FSP can unify all mechanisms for enhancing single phase heat transfer. In 2007 Guo and his co-workers proposed a new concept: entransy to describe the potential of a body to transfer thermal energy and the entransy dissipation extreme principle (EDEP). It is indicated that for any heat transfer process the entransy of the system is always dissipated, which can be regarded as the indication of the irreversibility of the transport process. For a heat transfer process with given boundary temperature condition the best one has the maximum entransy dissipation, while for that with given boundary heat flux condition the best one has minimum entransy dissipation. The combination of the two cases is called the entransy dissipation extremum principle.

The purpose of this paper is to reveal the inherent interrelationship between the ideas of field synergy principle and the entransy extremum principle. Numerical simulations are conducted for five examples of convective heat transfer. All the numerical results demonstrate the inherent consistency between FSP and EDEP.

1. Introduction to FSP

Although the basic principles of heat transfer theory have been built up at least for more than half-century, its development is still one of the hottest topics in the field of the applied thermal science. Among the three modes of heat transfer the focus of the present work is concentrated on the convective heat transfer. Generally speaking, at preliminary stage (i.e., approximately before 1960s), most studies focused on revealing the fundamental mechanism of convective heat transfer and establishing correlations between Nusselt number and Reynolds number, and there was almost no such a term as “heat transfer enhancement/ augmentation” in the open literature and textbooks [1–9]. Later, the energy crisis in 1970s broke this situation. The dilemma greatly shocked the global economy and forced people to reduce the excessive energy consumption and efficiently utilize the available energy sources. It is estimated that among the all kinds of energy sources existing in the world, about 80% will go through the thermal energy form before they are transformed into electricity. Therefore thermal energy transformation or transition is a very important process in the energy utilization. The thermal energy transmission by convective heat transfer needs some power to drive the fluid. Thus seeking methods to enhance heat transfer in a certain process with minimal energy consumption is of significant importance in reducing energy consumption. Since then, heat transfer enhancement has become one of the hottest research subjects in the field of heat transfer. To the authors knowledge the terminology of Enhancement of Heat Transfer/Augmentation of Heat Transfer was first put forward in open literature by Bergles in [10]. After 1990s, the technology of heat transfer enhancement has evolved from the so-called second-generation technology to the third-generation technology [11–13] and significant achievements have been achieved. In 2002, the fourth-generation concept of heat transfer enhancement technology was proposed in [14].
During the last few decades, great achievements on convective heat transfer enhancement have been obtained and various kinds of heat transfer enhancement have been adopted for single-phase convective heat transfer, i.e., (1) mixing the main flow and/or the flow in the wall region by using rough surface, insert, vortex generators, etc., (2) reducing the boundary layer thickness by using interrupted fins or jet impingement, etc., (3) creating velocity gradient at wall, etc. Many such techniques are presented in [15–17].

However, the essence of the convective heat transfer enhancement was still unclear in the nineties of the last century, even for the single phase convective heat transfer. Although some explanations can account for the mechanism of the heat transfer enhancement in some special cases, they were no unified principle or theory to explain the physical mechanism for the enhancement of single-phase convective heat transfer process till the end of the last century.

In 1998, Guo and his co-workers [18–21] proposed the concept of enhancing single-phase convective heat transfer for the parabolic fluid flow situation by transforming the convective term of the energy equation into the form of dot product of velocity vector and the temperature gradient, and integrating the energy equation over the thermal boundary layer. Consider a 2-D boundary steady-state flow over a cold flat plate at zero incident angle the energy equation is as follows:

\[ \rho C_p \left( \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) \]  

(1)

The integration of Eq. (1) over the thermal boundary layer yields:

\[ \int_0^h \rho C_p \left( \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) dy = -\lambda \frac{\partial T}{\partial y} \bigg|_w = q_w \]  

(2)

That is:

\[ \rho C_p \int_0^h (\vec{U} \cdot \nabla T) dy = -\lambda \frac{\partial T}{\partial y} \bigg|_w = q_w \]  

(3)

The product of the velocity vector and the temperature gradient can be given by

\[ \vec{U} \cdot \nabla T = |\vec{U}| \cdot |\nabla T| \cos \theta \]  

(4)

with \( \theta \) denoting the intersection angle between the velocity vector and the temperature gradient.

From Eqs. (3) and (4) it can be seen that the convective heat transfer performance can be effectively improved by reducing the intersection angle between the velocity vector and the temperature gradient. According to the Webster Dictionary [22] “synergy” means combined or cooperative action or force. Hence this idea is called field synergy principle (FSP), and the intersection angle synergy angle. Later, Tao et al. [23, 24] extended the FSP to the case of elliptic flow and tested its applicability via many numerical examples. Their work shows that the FSP gives a general mechanism for enhancing single phase convective heat transfer, and the three existing explanations mentioned above can be unified by FSP. In [25] Guo et al. further described the meanings of synergy. It is pointed out that the synergy between the velocity vector and the temperature gradient means: (a) the intersection angle between the velocity and the temperature gradient should be as small as possible; (b) the local values of the three scalar fields should all be simultaneously large; (c) the velocity and temperature profiles at each cross section should be as uniform as possible for internal flows. This is the complete understanding of the terminology “synergy”. From then on, extensive works have been done to apply it for the development of heat transfer enhancement technology.

Intrinsically, the strength of the convective heat transfer relies on the synergy between the velocity and temperature fields. The question is how to characterize the synergy degree between two fields. The most useful application of the FSP is to reveal for the entire flow field where the synergy is bad and hence it is there enhancement technique should be adopted. Because enhancement technique usually will result in an increase in fluid pressure drop. Only those local areas in the flow domain where synergy are bad the adoption of enhancing technique may lead to increase heat transfer appreciably with a mild or small pressure drop increase. In this regard, the local synergy angle between velocity and the temperature gradient is the most suitable one.

The local synergy angle between the velocity vector and the temperature gradient is defined as

\[ \theta = \cos^{-1} \left( \frac{\vec{U} \cdot \nabla T}{|\vec{U}| |\nabla T|} \right) \]  

(5)

With the local field synergy angle, many studies were conducted to obtain a general index to describe the field synergy degree in the entire flow system. The question is how to appropriately average the local synergy angle. Zhou [26] proposed five different ways for averaging synergy angles. Those are defined, respectively, by (1) simple arithmetic mean, (2) volume-weighted mean, (3) vector module-weighted mean, (4) vector dot product-weighted mean and (5) domain integration mean. It is found that except the simple arithmetic mean method, the rest are in accordance with each other qualitatively. For the case of air flowing across a certain finned tube, the variations of the mean synergy angles of different definitions with fluid velocity are plotted in Fig. 1. Clearly, there are no great qualitative differences between the variation trends of the different averaged field synergy angles. As it is the variation trend of field synergy angle that is used to guide practical problems, it is safe to adopt any one of them to qualitatively explain the reason/mechanism of the heat transfer enhancement. Usually, the average synergy angle based on the volume-weighted mean, and domain integration mean are employed, which can be written as

Volume – weighted mean \( \theta_m = \frac{\sum \theta_i dV_i}{\sum dV_i} \)  

(6a)

Domain integration mean \( \theta_m = \arccos \frac{\sum |\vec{U}| |\nabla T| \cos \theta_i dV}{\sum |\vec{U}| |\nabla T| dV} \)  

(6b)

It should be noted that the definition of Eq. (6b) is the most agreeable to the complete understanding of the concept of synergy described in [25].

Fig. 1. Variations of the mean synergy angle with different definition.
A lot of papers have been published to validate or to apply the FSP to develop the enhancement techniques for convective heat transfer [27–48], among whom reference [30] is especially worth mentioning. In that paper a special experimental system was designed, fabricated, and installed to demonstrate that when velocity vector is perpendicular to the fluid temperature gradient, flow velocity has no effect on the heat transfer.

2. Introduction to EDEP

2.1. Definition of entransy

In 2007 Guo and his co-workers proposed another new concept: entransy to describe the potential of a body to transfer thermal energy and the entransy dissipation extremum principle (EDEP) for the optimization of heat transfer process [49]. The physical meaning of entransy is the ability of a body to transfer its thermal energy (heat) to the environment.

The basic definition of the entransy of a body is:

\[ E = \frac{1}{2}[(\rho c_v T) \times V] \times T \]  

(7)

In the above equation, the term in the square brackets is the thermal energy stored in body (relative to the reference temperature of zero degree), and the temperature standing by the brackets is the temperature that this energy is attached. It is this temperature shows the ability of transferring heat, hence it can be regarded as the potential of this amount of energy.

To understand why the coefficient of 1/2 is needed in Eq. (7), the reversible heating process of an object with temperature, \( T \), and specific heat at constant volume, \( c_v \), can be used to show the necessity [49]. For a reversible heat transfer process, the temperature difference between the object and the heat source and the heat added are infinitesimal. Continuous heating of the object implies an infinite number of heat sources that heat the object successively. The temperature of these heat sources increases infinitesi- mally with each source giving an infinitesimal amount of heat to the object. The temperature represents the potential of the heat. Hence the “potential energy” of the thermal energy increases in parallel with the increasing thermal energy (thermal charge) when heat is added. When an infinitesimal amount of heat is added to an object, the increment in “potential energy” of the thermal energy can be written as the product of the thermal charge and the thermal potential (temperature) differential

\[ dE = QdT \]  

(8)

where \( Q = M c_v T \), with \( M \) being the mass of the body. If absolute zero is taken as the zero temperature potential, then the “potential energy” of the thermal energy in the object at temperature \( T \) is

\[ E = \int_T QdT \]  

(9)

The word “potential energy” is quoted because its unit is J K, not Joules. For a constant specific heat

\[ E = \int_T QdT = \int_T M c_v T dT = \frac{1}{2} M c_v T^2 \]  

(10)

2.2. Entransy dissipation and its extremum principle

In the heat transfer process, the amount of the energy is conserved, while the ability of transferring heat is reduced because of the thermal resistance. That is to say, there is entransy dissipation in the heat transfer process. For any heat transfer process between two substances the one with a higher temperature loses entransy, while the other at lower temperature gains the entransy. However, as a whole the lost part is always larger than the gained part, leading to the dissipation of entransy. The entransy dissipation reflects the loss of heat transfer ability caused by the irreversible character in the heat transfer process. Thus entransy dissipation is the indicator of the irreversibility of a heat transfer process [49].

The optimization of a heat transfer process should make the process entransy dissipation extremum. According to Guo et al. [49,50], the entransy dissipation extremum principle (EDEP) for convection transfer processes can be described as follows: For a given temperature condition the best heat transfer process of a fluid–solid system has the maximum dissipation of its entransy, which leads to the maximum heat transfer rate; While for a given heat flux boundary condition, the best heat transfer process of the fluid–solid system has the minimum entransy dissipation, which leads to the minimum temperature difference.

Since the proposal of the entransy concept in [49], a large number of papers have been published showing the feasibility of the concept and its various applications in different fields. To name a few, Refs. [51–61] may be consulted. Very recently Chen et al. [51] and Li and Guo [62] made a comprehensive review over the entransy theory and its wide applications. The two papers are very useful for interesting readers to consult.

3. Inherent interrelationship between FSP and EDEP and numerical validation

3.1. Intuitive consideration

From FSP, the best heat transfer process has the largest convective heat transfer rate at the same flow rate and the same temperature difference between fluid and wall. Suppose the wall keep a uniform temperature (\( T \)), then from EDEP, the best heat transfer process has the maximum entransy dissipation. If the wall keeps uniform heat flux (\( q \)) then the best heat transfer process should have the minimum entransy dissipation. That is, the better the synergy, the larger (or the smaller) the entransy dissipation for given temperature (or given heat flux) condition, respectively. This means that synergy between velocity and fluid temperature gradient should have inherent consistency with the dissipation of entransy. The above intuitive consideration for FSP and EDEP is very meaningful and understandable. However, so far there is no direct demonstration in the open literature for the inherent relationship between FSP and EDEP. In this section, five numerical examples will be provided to show such inherent consistency between FSP and EDEP.

3.2. Entransy balance equation for convective heat transfer

To proceed, the entransy balance formulation for convective heat transfer between a wall and a fluid will be provided. For the convective heat transfer when fluid is cooled, as shown in Fig. 2(a), we have

\[ \frac{1}{2} c_v q m T_{in}^2 = \frac{1}{2} c_v q m T_{out}^2 + c_v q m (T_{in} - T_{out}) T_w + \Delta E \]  

(11)

where \( q_m \) is the mass flow rate. The term at the left hand side is the entransy flow-in carried by the fluid, while the first term at the right hand side is the entransy flow-out carried by the fluid, the second term at the right hand side is the entransy flows to the wall, and the last term is the entransy dissipated during this heat transfer process.

If fluid is heated, Fig. 2(b), the entransy balance equation is:

\[ \frac{1}{2} c_v q m T_{out}^2 = \frac{1}{2} c_v q m T_{in}^2 + c_v q m (T_{out} - T_{in}) T_w - \Delta E \]  

(12)
The meanings of the four terms can be clearly understood from the comparison with Eq. (11).

3.3. Numerical validation

In the following presentation five numerical examples will be provided to validate the consistency between FSP and EDEP. All the physical problems simulated take the following assumptions:

1. Flow and heat transfer are in steady state.
2. Fluid thermo-physical properties are constant (except Example 5).
3. For Examples 1–4 wall temperature is given, while for Example 5 wall heat flux is given.
4. Fluids are incompressible and the energy dissipation due to the shear stress is neglected.

FVM is used to discretize the governing equations. SIMPLE-family algorithms [63] are adopted to deal with the linkage between velocity and pressure. Numerical solutions are conducted by using the software FLUENT 6.26. After the converged solutions are obtained, the domain averaged synergy angle is determined by a UDF incorporated into FLUENT.

3.3.1. Example 1. Turbulent gas flow cooled by an H-type finned tube (RNG k-epsilon model)

As shown in Fig. 3(a) the gas flow and heat transfer over H-type finned tube surface is studied at the periodically fully developed flow region. The H-type fin is rectangle in shape with a slot in its center part, making its appearance like an English capital letter H. There are four rectangular vortex generators in the fin. The grid system generated by GAMBIT is presented in Fig. 3(b), with total 580,000 grids. Comparison is made for the H-type fin without vortex generators with total 340,000 grids. From the numerical results of temperature and velocity fields, the domain averaged synergy angle, $\theta$, the domain averaged Nusselt number, $Nu$, and the entransy dissipation of the process $\Delta E$ were determined. The variations of the three parameters with Reynolds number are shown in Fig. 4. It can be seen that for this given wall temperature case, for the surface with vortex generators its Nusselt number is higher than that without vortex generators, its synergy angle is lower than that without vortex generators, and its entransy dissipation is higher than that without vortex generators. That is the FSP is fully consistent with EDEP.
3.3.2. Example 2. Laminar air flow cooled/heated by continuously finned tubes with/without vortex generators

In Fig. 5(a), the computational domain for two-row tubes with continuous fin is presented, where dashed lines are the computational boundaries. Here the inlet boundary was set 1.5 times of the streamwise fin length ahead of the fin and the outflow boundary was set 5 times of the streamwise fin length behind the fin region. In such a case for the inflow boundary uniform inlet velocity and temperature may be assumed and for the outflow boundary the one-way coordinate assumption [63] may be used. The number of total girds are 770,000. Fig. 5(b) is the top view of the grid system. The same simulations are also performed for two-row tubes with continuous fin without vortex generators. The variations of the above-mentioned three parameters with Reynolds number are shown in Fig. 6. It is interesting to note that the simulations are conducted for both fluid heated case and fluid cooled case, and both cases obtained the same results.

3.3.3. Example 3. Laminar air flow cooled by five-row finned tubes

In Fig. 7 two kinds of six-row fin-and-tube surfaces are presented. The final grid system of the surface with vortex generators was 1,936,288, which is not shown for the simplicity. The comparisons of the three parameters for the surfaces with and without vortex generators are provided in Fig. 8. Obviously the results agree with both FSP and EDEP very well.
3.3.4. Example 4. Fully developed turbulent flow in tubes of constant temperature with/without dimples by k-epsilon model

The geometries of two tubes simulated are shown in Fig. 9(a), and the grid systems are presented in Fig. 9(b). The numerical results for the three parameters are provided in Fig. 10. The comparisons presented in Fig. 10 once again demonstrate the inherent consistency between the FSP and EDEP.

3.3.5. Example 5. Turbulent heat transfer of heated air flowing through composite porous structure with uniform heat flux condition

Air flow and heat transfer through two porous structures (Fig. 11(a)) are simulated with uniform heat flux boundary condition at the porous material surfaces. A unit structure of the porous material is shown in Fig. 11(b), which has 14 surfaces. The diameter of the rod in the unit, ds, has two values: one is larger and the other is smaller, being called as dense and sparse, respectively. Numerical results of the comparisons of the three parameters are presented in Fig. 12. It can be observed that the combination of s–s has the highest Nusselt number, the smallest averaged synergy angle and the smallest entransy dissipation. These results are in good accordance with the FSP and EDEP. The inherent consistency for the given boundary heat flux condition is also demonstrated.
4. A unique formulation of EDEP

As presented above, the present formulation of entransy dissipation extremum principle is, in some extent, not very convenient to the users, because its description is dependent on the boundary conditions: for given wall temperature boundary \((T\text{-boundary})\) the entransy dissipation is the maximum while for given wall heat flux boundary \((q\text{-boundary})\) the entransy dissipation is the minimum. It is our expectation that since the dissipation of the entransy is taken as the indicator of irreversibility, the optimized situation should have the minimum entransy dissipation based on some undiscovered unit. From the performance evaluation of heat transfer enhancement techniques \([64]\) we may get some hint: in the performance evaluation of heat transfer enhancement techniques, we compare the heat transfer based on the same pumping power, why we do not compare the entransy dissipation in the same way: i.e., we should compare the entransy dissipation based on the same amount of heat transferred, that is an optimized heat transfer process should have the minimum entransy dissipation per unit energy transferred. For the given heat flux case, above statement can be easily derived from existing formulation of EDEP; The key issue is to verify this idea for the isothermal boundary condition for which according to the present formulation of EDEP the best situation has the maximum entransy dissipation. Before we present numerical demonstration of this unique formulation, we first discuss this concept from point of view of dimensional analysis. The physical meaning of the entransy dissipation per unit energy transferred can be well understood from its dimension:

\[
\frac{\text{energy} \times \text{temperature}}{\text{energy}} = \text{[temperature]}
\]

That is the physical meaning of the entransy dissipation per unit energy transferred is the temperature, or more appropriately, the temperature difference. Since temperature difference is the most essential driving force of heat transfer, the best heat transfer at any condition should have the minimum temperature difference, hence the minimum entransy dissipation per unit energy transferred.

In the following the above-mentioned 5 examples will be reconsidered from the view point of entransy dissipation per unit energy transferred.

Fig. 11. Laminar air flow through composite porous structure with constant heat flux of Example 5 (a) geometry of composite porous structure (b) unit of porous structure.

Fig. 12. Variation of \(\text{Nu, } \theta\) and \(\Delta E\) with \(Re\) of Example 5 \((q = 100,000 \text{ W/m}\text{\(^2\)})\) (a) \(\text{Nu vs. } Re\) (b) \(\theta\) vs. \(Re\) (c) entransy dissipation vs. \(Re\).
Fig. 13. Comparisons of variation trends of $h$ and $\Delta E/Q$ with $Re$ for five examples (a) $h$ and $\Delta E/Q$ vs. $Re$ (Example 1) (b) $h$ and $\Delta E/Q$ vs. $Re$ (Example 2) (c) $h$ and $\Delta E/Q$ vs. $Re$ (Example 3) (d) $\theta$ and $\Delta E/Q$ vs. $Re$ (Example 4) (e) $\theta$ and $\Delta E/Q$ vs. $Re$ (Example 5).
transferred. Numerical results are presented in Fig. 13, where the variations of the averaged synergy angle with $Re$ are shown in figure (a) and the variations of entransy dissipation per unit energy transferred with $Re$ are shown in figure (b). From the figures the proposed unique formation of EDEP is verified obviously.

In the study of heat transfer enhancement, one of our major goals is to develop high-efficiency and low-resistance heat transfer element. Low-resistance is related to pressure field, hence, it is our expectation that for the enhancement technique possessing high heat transfer efficiency and low resistance, the three fields, i.e., velocity, temperature gradient and pressure gradient should have, at least in some sense, a better synergy between them. And with this unique formulation of EDEP, we may expect that the high-efficiency and low-resistance heat transfer element should has the minimum of entransy dissipation and the minimum of pumping power consumption based on unit heat transferred. Further more research work is needed in order to verify the expectation and our group is going on this way.

5. Conclusion

From above presentation following three conclusions may be made:

1. For the convective heat transfer the field synergy principle and the entransy dissipation extremum principle are inherently consistent in that a better synergy corresponds to a less entransy dissipation for unit energy transferred.

2. For the convective heat transfer the extremum principle may be reformulated by following unique expression: an optimized heat transfer process with any boundary condition dissipates the minimum entransy for unit energy transferred. The dimension of the entransy dissipation per unit energy transferred is temperature (unit of K), hence entransy dissipation per unit energy may be regarded as an equivalent temperature difference of the heat transfer.

3. The best convective heat transfer enhancing technique at given condition may be obtained by satisfying following object function: the minimum of entransy dissipation and the minimum pumping power consumption per transferred unit energy.

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References

[26] J. Zhou, Mechanisms of heat transfer enhancement and energy saving of the compact heat exchanger surfaces and study on their optimization (PhD thesis), Xi’an Jiaotong University, Xi’an, 2006.
[33] Z.Y. Li, Study on the turbulent fluid flow and heat transfer in rotating ducts (PhD thesis), Xi’an Jiaotong University, Xi’an, 2001.
[41] Y.G. Lei, Theoretical and experimental investigation on heat transfer augmentation of internal and external flow with high efficiency (PhD thesis), Xi’an Jiaotong University, Xi’an, 2009.


