International Journal of Heat and Mass Transfer 53 (2010) 2877–2884

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Effectiveness–thermal resistance method for heat exchanger design and analysis

Z.Y. Guo [a,*], X.B. Liu [a], W.Q. Tao [b], R.K. Shah [c]

[a] Department of Mechanics, Tsinghua University, Beijing 100084, PR China
[b] School of Power Energy Engineering, Xi'an Jiaotong University, Shaanxi 710049, PR China
[c] Department of Energy Science and Engineering, Indian Institute of Technology Bombay, Mumbai 400 076, India

ARTICLE INFO

Article history:
Received 27 July 2009
Received in revised form 19 January 2010
Accepted 22 January 2010
Available online 6 March 2010

Keywords:
Heat exchanger
Thermal resistance
Entransy dissipation
Entropy generation

ABSTRACT

The equivalent thermal resistance of a heat exchanger is defined based on the concept of the entransy dissipation rate, which measures the irreversibility of heat transfer for the purpose of object heating or cooling, rather than from the heat to work conversion. The relationships between the heat exchanger effectiveness and the thermal resistance (or conductance) are developed, which do not depend on its flow arrangement, and hence useful for the performance comparison among heat exchangers with different flow arrangements. In addition, such relationships bridge a gap between the heat exchanger irreversibility and its effectiveness. The monotonic decrease of the effectiveness with increasing the thermal resistance shows that the heat exchanger irreversibility can be described by its thermal resistance when evaluated from the transport process viewpoint, while the so-called entropy generation paradox occurs, if the irreversibility is measured by the entropy generation number for a heat exchanger.

1. Introduction

Since heat exchangers are used in a number of applications in various industries, improving the performance of heat exchangers plays an important role in the efficient energy utilization. Heat exchanger design and analysis can be conducted commonly by the LMTD (logarithmic mean temperature difference) [1,2] method or the effectiveness–number of transfer units (P—NTU) method. For the LMTD method, heat transfer equation can be written as:

$$\dot{Q} = UA\Delta T_{\rm M} = UAF\Delta T_{\rm LM} \tag{1}$$

where U, overall heat transfer coefficient; A, surface area for heat transfer; $\Delta T_{\rm M}$, true mean temperature difference between two fluids of the heat exchanger, and is related to log-mean temperature difference by a correction factor F. Note that F is dependent on the flow arrangement of two fluids in addition to the temperature effectiveness P and heat capacity rate ratio R. For single-pass (parallel or counterflow) heat exchangers:

$$\Delta T_{\rm M} = \Delta T_{\rm LM} = \frac{\Delta T_{\rm i} - \Delta T_{\rm o}}{\ln(\Delta T_{\rm i}/\Delta T_{\rm o})} \tag{2}$$

where $\Delta T_{\rm i}$ and $\Delta T_{\rm o}$ are the temperature differences of the two fluids at inlet and outlet ends of either fluid in the exchanger. The LMTD method can be extended to complex flow arrangements, such as multiple-pass shell-and-tube exchangers by introduction of a correction factor F noted above. Studies by Bowman et al. [3] showed that the correct mean temperature difference for a complex geometry can be obtained by the LMTD multiplied by the correction factor F. Bowman et al. [3] and TEMA [4] derived expressions and/or provided charts for the correction factor, which are different for different heat exchanger flow arrangements. If the fluid terminal temperatures and overall heat transfer coefficient are known, we can straightforward obtain the total heat transfer area A required for specified heat transfer rate in a sizing problem. Nevertheless, iterations are needed by the LMTD method for the rating problem to determine the outlet temperatures for the case of fixed U and A. A more appropriate method to solve the rating problem is the effectiveness–NTU method devised by Kays and London [5]. They defined a dimensionless parameter called the heat exchanger effectiveness P which is the ratio of actual heat transfer rate to the maximum possible heat transfer rate. For the single-pass parallelflow and counterflow heat exchangers, P—NTU expressions are:

$$P_{\rm para} = \frac{1 - \exp[-{\rm NTU}(1 + C^*)]}{1 + C^*} \tag{3a}$$

$$P_{\rm coun} = \frac{1 - \exp[-{\rm NTU}(1 - C^*)]}{1 - C^* \exp[-{\rm NTU}(1 - C^*)]} \tag{3b}$$

where ${\rm NTU} = UA/C_{\min}$ is the number of transfer units, and $C^* = C_{\min}/C_{\max}$ is the ratio of heat capacity rate of the fluid with the smaller heat capacity (hereafter, the minimum fluid) to that of the fluid with the larger one (hereafter, the maximum fluid). Kays and London [6], Shah and Sekulic [7] and Shah and Pignotti [8] have presented effectiveness–NTU formulas for over 100 different heat exchanger flow arrangements in the form of charts, tables and analytical closed-form P—NTU formulas. The effectiveness–NTU

* Corresponding author. Tel.: +86 10 6278 2660; fax: +86 10 6278 3771.
E-mail address: demgzy@tsinghua.edu.cn (Z.Y. Guo).

0017-9310/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2010.02.008

Nomenclature

A	surface area for heat transfer (m^2)
c	specific heat (J/kg K)
C_{max}	the larger heat capacity rate of two fluids (W/K)
C_{min}	the smaller heat capacity rate of two fluids (W/K)
C^*	heat capacity rate ratio
EEIN	enthalpy exchange irreversibility norm
F	correction factor
$\dot{G}$	entransy flow rate (W K)
$\dot{G}_\phi$	entransy dissipation rate (W K)
G_{vh}	entransy (J K)
G_1	the resistance factor of non-counterflow arrangement
G_2	the resistance factor of unbalanced flow
K	thermal conductivity (W/m K)
l	length (m)
LMTD	logarithmic mean temperature difference (K)
$\dot{m}$	mass flow (kg/s)
M	mass (kg)
N^*	dimensionless thermal conductance
N_S	entropy generation number
NTU	number of transfer unit
P	heat exchanger or temperature effectiveness
$\dot{Q}$	heat transfer rate (W)
$\dot{q}$	heat flux (W/m^2)
$\dot{Q}_l$	heat transfer rate per unit length (W/m)
Q_{vh}	internal energy (J)
R	thermal resistance (K/W)
R^*	dimensionless equivalent thermal resistance
R_{ex}	equivalent thermal resistance (K/W)
$\dot{S}$	entropy generation rate (W/K)
T	temperature (K)
t	time (s)
U	overall heat transfer coefficient (W/m^2 K)
V	control volume (m^3)
x	coordinate (m)

Greek symbols

δ	wall thickness in 1D heat conduction (m)
$\dot{\varepsilon}$	entransy flux (W K/m^2)
$\dot{\varepsilon}_\phi$	local entransy dissipation rate (W K/m^3)
ε	entransy density (J K/m^3)
ρ	density (kg/m^3)

Subscripts

AM	arithmetic mean
b	bulk
c	cold fluid
cond	heat conduction
conv	heat convection
coun	counterflow arrangement
ex	equivalent
h	hot fluid
i	inlet
LM	logarithmic mean
M	mean
o	outlet
para	parallel flow arrangement
tube	tube flow
w	wall

method also offers advantages for the performance comparison between various types of heat exchangers, that is from given value of NTU the goodness of the heat exchanger can be easily identified from its value of P.

Because heat transfer is an irreversible process, one tries to establish the ties between the effectiveness and the irreversibility associated with the heat exchanger and to understand why heat exchangers with different flow arrangements differ in heat transfer performance under the same given design conditions. Bejan [9] introduced the concept of irreversibility due to finite temperature difference and fluid flow friction in heat transfer process based on the second law of the thermodynamics. To evaluate quantitatively the entropy generation in counterflow heat exchangers, he introduced an entropy generation number N_S defined as the entropy generation rate scaled by the heat capacity rate of the minimum fluid, i.e., $\dot{S}/C_{min}$. He provided the formula for N_S for a counterflow heat exchanger and showed that N_S has a maximum from $\mathrm{NTU} = 0 (P = 0)$ to $\mathrm{NTU} = \infty (P = 1)$. Since the relation between N_S and P before $N_{S,max}$ conflicts with one's expectation that heat transfer irreversibility and the consequent N_S would decrease as the NTU increases, Bejan [10,11] called it "the entropy generation paradox". Sekulic [12] conducted an irreversibility analysis for heat exchangers in terms of enthalpy exchange irreversibility norm (EEIN), defined by the ratio of the actual entropy generation to the maximum possible entropy generation of an adiabatic mixing of the streams. In the counterflow case, the EEIN has a maximum too from $\mathrm{NTU} = 0$ to $\mathrm{NTU} = \infty$. An attempt was made by Hesselgreaves [13] to resolve the entropy generation paradox through non-dimensionalising the entropy generation rate by $\dot{Q}/T_1$. Ogiso [14] and Xu et al. [15] demonstrated that the Bejan's paradox can be removed by using the entropy generation index $\dot{S}/UA$. Based on the irreversibility analysis for 18 heat exchanger flow arrangements, Shah and Skiepko [16] showed that the heat exchanger effectiveness can be maximum, having an intermediate value or minimum at the maximum irreversibility operating point. Therefore they concluded that the concept of minimum irreversibility associated with the maximum efficiency for energy conversion process in thermal systems is not quite applicable to the heat exchanger analysis. Guo et al. [17] introduced the concepts of entransy and entransy dissipation in terms of the analogy between heat and electric conduction. Their study demonstrated that when evaluated from the viewpoint of energy transport process, the entransy dissipation rather than the entropy generation is a measure of the irreversibility of a heat transfer process. This concept has been applied in the optimization of heat conduction [18] and convection [19].

In the following presentation, the concepts entransy and entransy dissipation are first briefly reviewed for the readers' convenience (Guo et al. [17]). Then the irreversibility analysis for heat exchangers is carried out and the thermal resistance of a heat exchanger is defined based on the concept of entransy dissipation rate. The relations between the heat exchanger effectiveness and the thermal resistance/thermal conductance are derived for the performance analysis and comparison between various flow arrangements. Finally some conclusions are made.

2. Entransy dissipation and equivalent thermal resistance of heat exchanger

2.1. Heat conduction irreversibility and resistance [17]

As well known, the heat conduction and the electric conduction are analogous with Fourier's law for 1D conduction being analogous to Ohm's law for electric circuits. In the analogy, many quantities in two systems have many one-to-one correspondences, as

Table 1
Analogies between electrical and thermal parameters.

Electrical charge stored in a capacitor Q_{ve}	Electrical current (charge flux) I	Electrical resistance R_e	Capacitance $C_e = Q_{ve}/U_e$
Heat stored in a body $Q_{vh} = McT$	Heat flow $\dot{Q}_h$	Thermal resistance R_h	Heat capacity $C_h = Q_{vh}/T$
Electrical potential U_e	Electrical current density $\dot{q}_e$	Ohm's law $\dot{q}_e = -K_e \frac{dU_e}{dn}$	Electrical potential energy in a capacitor $E_e = \frac{1}{2}Q_{ve}U_e$
Thermal potential (temperature) $U_h = T^{\dagger}$	Heat flux density $\dot{q}_h$	Fourier's law $\dot{q}_h = -K_h \frac{dT}{dn}$	*Entransy* [17] $G_{vh} = \frac{1}{2}Q_{vh}T$

† T is actually $\Delta T = T - 0$ (absolute zero).

listed in Table 1, except for the electric potential energy in a capacitor. In view of this fact, an appropriate quantity, G_{vh} has been defined [17] as :

$$G_{vh} = \frac{1}{2}Q_{vh}T \tag{4}$$

where $Q_{vh} = McT$ is the internal energy or the heat stored in an object relative to a reference system with absolute zero temperature. This quantity, which corresponds to the electric potential energy in the electric system, is referred to as *Entransy* because it possesses both the nature of "energy" (from which the prefix "en-" is borrowed) and the heat transfer (from which the postfix "-trans" is copied) ability, i.e., ability for transferring thermal energy which, in turn, is resulted from both energy and temperature level.

For heat conduction problems without a heat source, the thermal energy conservation equation is:

$$\rho c \frac{\partial T}{\partial t} = -\nabla \cdot \dot{q} = \nabla \cdot (K\nabla T) \tag{5}$$

The above equation multiplied by T leads to the entransy balance equation as follows:

$$\rho c T \frac{\partial T}{\partial t} = -\nabla \cdot (\dot{q}T) + \dot{q} \cdot \nabla T \tag{6a}$$

Eq. (6a) can be rewritten as:

$$\frac{\partial \varepsilon}{\partial t} = -\nabla \dot{\varepsilon} - \dot{\varepsilon}_{\phi} \tag{6b}$$

where $\varepsilon = \frac{1}{2}\rho c T^2$, $\dot{\varepsilon} = \dot{q}T$ and $\dot{\varepsilon}_{\phi} = -\dot{q} \cdot \nabla T$.

The LHS of Eq. (6b) is the time variation of entransy density. The first and second terms on the right of Eq. (6b) are the entransy flux and the local entransy dissipation rate, respectively. This indicates that during heat transfer process the entransy is not conserved due to dissipation caused by thermal resistance, while the thermal energy is conserved. The local entransy dissipation rate $\dot{\varepsilon}_{\phi} = -\dot{q} \cdot \nabla T = K(\nabla T)^2$ resembles the local electric energy dissipation rate in the electric system and the mechanical energy dissipation rate in viscous fluid flow.

For simplicity, consider one-dimensional steady heat conduction in a plate with thickness δ as shown in Fig. 1, where the output heat flux is the same as the input one ($\dot{q}_o = \dot{q}_i$), while the output entransy flux is less than the input one ($\dot{\varepsilon}_o < \dot{\varepsilon}_i$) due to the entransy dissipation inside the plate:

$$\dot{G}_i - \dot{G}_o = l\int_0^{\delta}\left(-\dot{q} \cdot \frac{dT}{dx}\right)dx = l\dot{q}(T_i - T_o) = \dot{Q}(T_i - T_o) \tag{7}$$

where $\dot{G}_i = l\dot{\varepsilon}_i$ and $\dot{G}_o = l\dot{\varepsilon}_o$ stand for the input and the output entransy flow rates.

The thermal resistance is commonly defined as the ratio of the temperature difference to the heat flux. This definition will induce some arbitrariness for multi-dimensional problems of heat conduction where a so-called equivalent mean temperature difference must be defined, otherwise this definition is valid locally for a differential point. However, the definition of the equivalent mean temperature difference is not unique. Thus the value of the equivalent thermal resistance may differ from different averaging method for the same problem. In order to avoid this drawback, the equivalent thermal resistance for multi-dimensional problems based on the entransy dissipation rate has been [17] defined as follows:

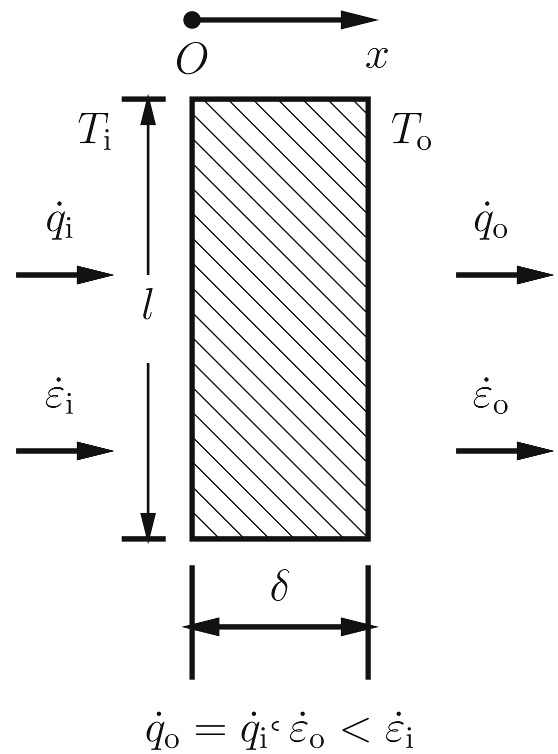

Fig. 1. One-dimensional steady heat conduction.

$$R_{cond} = \frac{\dot{G}_{\phi}}{\dot{Q}^2} \tag{8}$$

where $\dot{G}_{\phi} = \int_V K(\nabla T)^2 dV$ is the entransy dissipation rate over the whole heat transfer area, and $\dot{Q}$ is the total heat flow. On the contrary to the other methods of the definition, the definition of the equivalent thermal resistance based on the entransy dissipation rate has no arbitrariness because it is uniquely determined as long as the temperature distribution is known.

For the one-dimensional, steady heat conduction without an internal source, Eq. (8) reduces to the conventional expression of the thermal resistance as follows:

$$R_{cond} = \frac{\dot{Q}\Delta T}{\dot{Q}^2} = \frac{\Delta T}{\dot{Q}} \tag{9}$$

2.2. Convection irreversibility and resistance for tube flow heat transfer

For heat convection in a circular tube with cold fluid at a constant temperature (e.g., evaporation) as shown in Fig. 2a, the thermal energy balance equation in terms of the fluid bulk temperature gives:

$$-\dot{m}c\frac{\mathrm{d}T_{\mathrm{b}}(x)}{\mathrm{d}x} = \dot{Q}_{\mathrm{l}}(x) \tag{10}$$

where $\dot{m}$ is the mass flow rate, $\dot{m}c$ is the heat capacity rate, T_{b} is the fluid bulk mean temperature. The LHS in Eq. (10) is the variation of enthalpy flow rate per unit tube length, while the RHS, $\dot{Q}_{\mathrm{l}}(x)$, is the local heat transfer rate per unit length between fluid and the wall.

Multiplying the both sides of Eq. (10) by $T_{\mathrm{b}}(x)$ yields:

$$-\dot{m}cT_{\mathrm{b}}(x)\frac{\mathrm{d}T_{\mathrm{b}}(x)}{\mathrm{d}x} = T_{\mathrm{b}}(x)\dot{Q}_{\mathrm{l}}(x) \tag{11}$$

The LHS in Eq. (11) is the variation of the entransy flow rate per unit tube length, and the right one is the local entransy leaving from the fluid associated with heat transfer at T_{b} in the radial direction shown in Fig. 2b.

By integrating Eq. (11) over the tube length gives:

$$\dot{G}_{\mathrm{i}} - \dot{G}_{\mathrm{o}} = \frac{1}{2}\dot{m}cT_{\mathrm{bi}}^2 - \frac{1}{2}\dot{m}cT_{\mathrm{bo}}^2 = \int_0^l T_{\mathrm{b}}(x)\dot{Q}_{\mathrm{l}}(x)\mathrm{d}x \tag{12a}$$

where $\dot{G}_{\mathrm{i}}, \dot{G}_{\mathrm{o}}$ are the entransy flow rates in and out from the tube.

It should be noted that all the heat transfer rate between fluid and wall is leaving the wall at temperature T_{w}, thus the entransy is transferred in two ways. One way is through the fluid flow, the other through the wall. Since the entransy flow rate leaving from the tube wall at T_{w} is $\int_0^l T_{\mathrm{w}}\dot{Q}_{\mathrm{l}}(x)\mathrm{d}x$ (see Fig. 2b), we have the entransy balance equation for the whole tube:

$$\dot{G}_{\phi} = \dot{G}_{\mathrm{i}} - \dot{G}_{\mathrm{o}} - \int_0^l T_{\mathrm{w}}\dot{Q}_{\mathrm{l}}(x)\mathrm{d}x = \int_0^l \dot{Q}_{\mathrm{l}}(x)(T_{\mathrm{b}} - T_{\mathrm{w}})\mathrm{d}x \tag{12b}$$

where $\dot{G}_{\phi}$ is the entransy dissipation rate over the whole tube flow.

Thus the equivalent thermal resistance and the temperature difference of tube flow can be defined, respectively, as:

$$R_{\mathrm{tube}} = \frac{\dot{G}_{\phi}}{\dot{Q}^2} \tag{13}$$

and

$$\Delta T_{\mathrm{M,tube}} = \frac{\dot{G}_{\phi}}{\dot{Q}} = R_{\mathrm{tube}}\dot{Q} \tag{14}$$

where the heat flow rate $\dot{Q} = \int_0^l \dot{Q}_{\mathrm{l}}(x)\mathrm{d}x = \dot{m}c(T_{\mathrm{bi}} - T_{\mathrm{bo}})$.

Substituting Eq. (12b) into Eq. (14) gives:

$$\Delta T_{\mathrm{M,tube}} = \frac{\frac{1}{2}\dot{m}cT_{\mathrm{bi}}^2 - \frac{1}{2}\dot{m}cT_{\mathrm{bo}}^2 - \dot{Q}T_{\mathrm{w}}}{\dot{m}c(T_{\mathrm{bi}} - T_{\mathrm{bo}})} = \frac{T_{\mathrm{bi}} + T_{\mathrm{bo}}}{2} - T_{\mathrm{w}} = \Delta T_{\mathrm{AM}} \tag{15}$$

Hence the true mean temperature difference for tube flows with constant wall temperature is in fact the arithmetic mean temperature difference of fluid terminal temperatures. For the constant wall temperature case studied, the convective heat transfer rate is $\dot{Q} = UA\Delta T_{\mathrm{LM}}$ [20] and $1/UA$ is regarded as the convection thermal resistance, we have the relation between the tube flow thermal resistance and the convection thermal resistance:

$$R_{\mathrm{tube}} = \frac{\Delta T_{\mathrm{M,\,tube}}}{\dot{Q}} = \frac{1}{UA}\frac{\Delta T_{\mathrm{AM}}}{\Delta T_{\mathrm{LM}}} = R_{\mathrm{conv}} \cdot \frac{\Delta T_{\mathrm{AM}}}{\Delta T_{\mathrm{LM}}} \tag{16}$$

It can be found that the tube flow thermal resistance is always larger than the convection resistance because the arithmetic mean temperature difference between the fluid and the wall is always larger than that of the logarithmic mean.

Like the heat exchanger effectiveness, we can define the tube flow heat transfer effectiveness as:

$$P_{\mathrm{tube}} = \frac{\text{actual heat transfer}}{\text{maximum possible heat transfer}} = \frac{T_{\mathrm{bi}} - T_{\mathrm{bo}}}{T_{\mathrm{bi}} - T_{\mathrm{w}}} \tag{17}$$

Substituting ΔT_{LM} in Eq. (16) by $\dot{Q}/UA = \Delta T_{\mathrm{LM}}$ and $\dot{Q} = \dot{m}c(T_{\mathrm{bi}} - T_{\mathrm{bo}})$ leads to the relation between the effectiveness and the resistance:

$$\begin{aligned} R_{\mathrm{tube}} &= \frac{1}{UA}\frac{\Delta T_{\mathrm{AM}}}{\Delta T_{\mathrm{LM}}} = \frac{1}{UA}\frac{UA}{\dot{m}c(T_{\mathrm{bi}} - T_{\mathrm{bo}})}\left[\frac{T_{\mathrm{bi}} + T_{\mathrm{bo}}}{2} - T_{\mathrm{w}}\right] \\ &= \frac{1}{\dot{m}c(T_{\mathrm{bi}} - T_{\mathrm{bo}})}\left[T_{\mathrm{bi}} - T_{\mathrm{w}} - \frac{1}{2}(T_{\mathrm{bi}} - T_{\mathrm{bo}})\right] \\ &= \frac{1}{\dot{m}c}\left(\frac{1}{P_{\mathrm{tube}}} - \frac{1}{2}\right) \end{aligned} \tag{18}$$

and

$$R_{\mathrm{tube}}^* = R_{\mathrm{tube}}\dot{m}c = \frac{1}{P_{\mathrm{tube}}} - \frac{1}{2} \tag{19a}$$

By introducing fluid heat capacity rate $C = \dot{m}c$, then we have:

$$R_{\mathrm{tube}}^* = R_{\mathrm{tube}}C = \frac{1}{P_{\mathrm{tube}}} - \frac{1}{2} \tag{19b}$$

or

$$P_{\mathrm{tube}} = \frac{2}{2R_{\mathrm{tube}}^* + 1} \tag{20}$$

where R_{tube}^* is the dimensionless thermal resistance for the tube flow heat transfer. It can be obviously seen that the smaller the

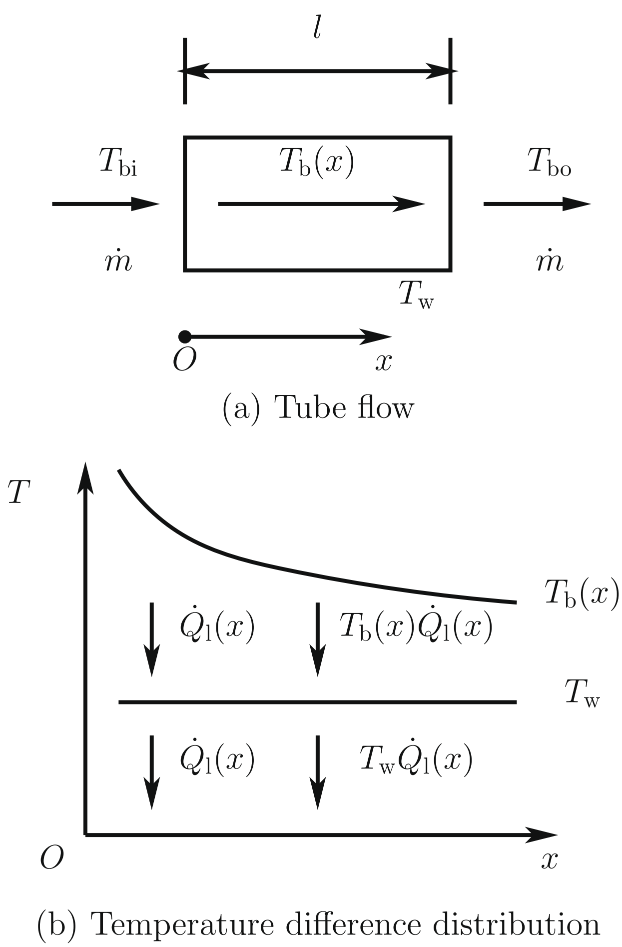

Fig. 2. Heat convection in a circular tube.

thermal resistance and hence smaller the consequent irreversibility is, the higher is the effectiveness of the tube flow heat transfer.

2.3. Heat exchanger irreversibility and resistance

Consider a heat exchanger with arbitrary flow arrangement, as shown in Fig. 3. The one-dimensional thermal energy conservation equations for hot and cold fluid in each passage are:

$$-C_h \frac{dT_h(x)}{dx} = \dot{Q}_l(x) \tag{21}$$

and

$$C_c \frac{dT_c(x)}{dx} = \dot{Q}_l(x) \tag{22}$$

where C_h, C_c are heat capacity rates of hot and cold fluid, $\dot{Q}_l(x)$ is the local heat transfer between two fluids per unit tube length.

Multiplying the both sides of Eqs. (21) and (22) by T_h and T_c, respectively, gives:

$$-C_h T_h(x_h) \frac{dT_h(x_h)}{dx_h} = \dot{Q}_l(x_h) T_h(x_h) \tag{23}$$

$$C_c T_c(x_c) \frac{dT_c(x_c)}{dx_c} = \dot{Q}_l(x_c) T_c(x_c) \tag{24}$$

The LHS in Eqs. (23) and (24) are the variation of entransy flow rate of hot and cold fluid, respectively. The RHS in Eq. (23) is the entransy output from the hot fluid and the RHS in Eq. (24) is the entransy input to the cold fluid.

Integrating Eqs. (23) and (24) over the tube length, we have:

$$\dot{G}_{hi} - \dot{G}_{ho} = \frac{1}{2} C_h T_{hi}^2 - \frac{1}{2} C_h T_{ho}^2 \tag{25}$$

and

$$\dot{G}_{co} - \dot{G}_{ci} = \frac{1}{2} C_c T_{co}^2 - \frac{1}{2} C_c T_{ci}^2 \tag{26}$$

where $\dot{G}_{hi}, \dot{G}_{ci}$ are the entransy flow rates of hot and cold fluid at inlet, and $\dot{G}_{ho}, \dot{G}_{co}$ are the entransy flow rate of hot and cold fluid at outlet, respectively.

Thus, the entransy dissipation rate in the heat exchanger is:

$$\begin{aligned} \dot{G}_\phi &= \left(\dot{G}_{hi} + \dot{G}_{ci}\right) - \left(\dot{G}_{ho} + \dot{G}_{co}\right) \\ &= \left(\frac{1}{2} C_h T_{hi}^2 + \frac{1}{2} C_c T_{ci}^2\right) - \left(\frac{1}{2} C_h T_{ho}^2 + \frac{1}{2} C_c T_{co}^2\right) \end{aligned} \tag{27}$$

We can then define the equivalent thermal resistance of heat exchanger as:

$$R_{ex} = \frac{\dot{G}_\phi}{\dot{Q}^2} \tag{28}$$

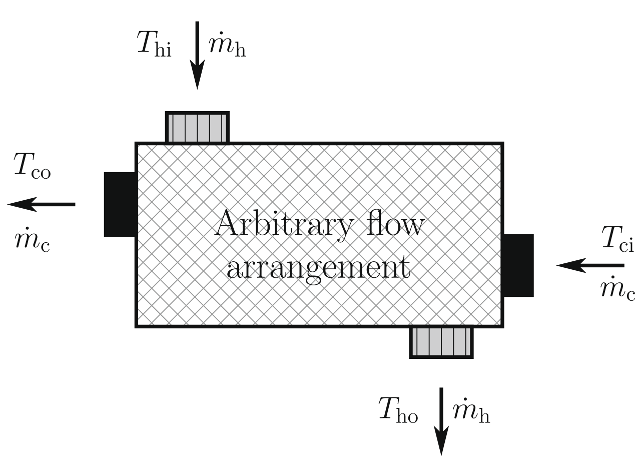

Fig. 3. Two fluids heat exchanger with arbitrary flow arrangement.

and the equivalent temperature difference of heat exchanger as:

$$\Delta T_{ex} = \frac{\dot{G}_\phi}{\dot{Q}} = R_{ex} \dot{Q} \tag{29}$$

3. Effectiveness–thermal resistance method

The reciprocal of UA, which is usually referred to as the thermal resistance of a heat exchanger, can be regarded as the thermal resistance of convective heat transfer since the tube wall thermal resistance is usually negligible or at least comparatively small:

$$R_{conv} = \frac{1}{UA} = \frac{\Delta T_M}{\dot{Q}} \tag{30}$$

Substituting Eq. (27) to Eq. (29), we have the equivalent temperature difference for the heat exchanger with any flow arrangement:

$$\begin{aligned} \Delta T_{ex} &= \frac{\frac{1}{2}\left[C_h\left(T_{hi}^2 - T_{ho}^2\right) - C_c\left(T_{co}^2 - T_{ci}^2\right)\right]}{C_h(T_{hi} - T_{ho})} \\ &= \frac{T_{hi} + T_{ho}}{2} - \frac{T_{co} + T_{ci}}{2} = \Delta T_{AM} \end{aligned} \tag{31}$$

where ΔT_{AM} is the arithmetic mean temperature difference. In the derivation of Eq. (31), the energy balance condition for a heat exchanger, $C_h(T_{hi} - T_{ho}) = C_c(T_{co} - T_{ci})$, has been used.

Combining Eqs. (29) and (30) yields:

$$R_{ex} = R_{conv} \frac{\Delta T_{AM}}{\Delta T_M} = R_{conv} \frac{\Delta T_{LM}}{\Delta T_M} \frac{\Delta T_{AM}}{\Delta T_{LM}} = R_{conv} G_1 G_2 \tag{32}$$

where $G_1 = \Delta T_{LM}/\Delta T_M = 1/F$ is the resistance factor of non-counterflow arrangement and $G_2 = \Delta T_{AM}/\Delta T_{LM}$ is the resistance factor of unbalanced flow (i.e., $C_h \neq C_c$). F is the correction factor.

Substituting $\Delta T_{AM}, \Delta T_M$ into Eq. (32):

$$\begin{aligned} R_{ex} &= \left(\frac{1}{UA}\right) \frac{\Delta T_{AM}}{\Delta T_M} \\ &= \frac{1}{UA} \frac{UA}{C_{min}(T_{co} - T_{ci})} \left[\frac{T_{hi} + T_{ho}}{2} - \frac{T_{ci} + T_{co}}{2}\right] \\ &= \frac{(T_{hi} - T_{ci}) - \left(\frac{T_{hi} - T_{ho}}{2} + \frac{T_{co} - T_{ci}}{2}\right)}{C_{min}(T_{co} - T_{ci})} \\ &= \frac{1}{C_{min}} \left[\frac{1}{P} - \frac{1}{2}\left(\frac{C_{min}}{C_{max}} + 1\right)\right] \end{aligned} \tag{33}$$

Then, the dimensionless thermal resistance and conductance of a heat exchanger are, respectively, as follows:

$$R^* = \frac{R_{ex}}{(C_{min})^{-1}} = \frac{1}{P} - \frac{1}{2}\left(\frac{C_{min}}{C_{max}} + 1\right) \tag{34}$$

$$N^* = \left[\frac{1}{P} - \frac{1}{2}\left(\frac{C_{min}}{C_{max}} + 1\right)\right]^{-1} \tag{35a}$$

The dimensionless thermal conductance can be further expressed as:

$$N^* = \frac{1}{R^*} = \frac{UA}{C_{min}} \frac{\Delta T_M}{\Delta T_{AM}} = \mathrm{NTU} \frac{\Delta T_M}{\Delta T_{AM}} = NTU \frac{F}{G_2} \tag{35b}$$

The relationship between the effectiveness and dimensionless thermal resistance or conductance can be rewritten as:

$$P = \frac{2}{2R^* + (1 + C^*)} \tag{36}$$

$$P = \frac{2N^*}{2 + N^*(1 + C^*)} \tag{37}$$

where $C^* = C_{min}/C_{max}$ is the heat capacity ratio.

Unlike the relation between the effectiveness and the number of heat transfer unit, which differs for different flow arrangements, Eqs. (36) and (37) hold for any heat exchanger geometries although N^* is dependent on the flow arrangement. Hence, this effectiveness–thermal resistance method can be regarded as a mixed method of log-mean temperature difference and P—NTU.

In the following some typical cases will be discussed.

(a) For the balanced counterflow heat exchanger, $C^* = 1, N^* = \mathrm{NTU}$ (Eq. (35b)), Eq. (37) reduces to the expression related to the effectiveness and the number heat transfer unit:

$$P = \frac{2N^*}{2+2N^*} = \frac{\mathrm{NTU}}{1+\mathrm{NTU}} \tag{38}$$

(b) For the balanced parallel flow heat exchanger with infinite number of heat transfer units, $C^* = 1, \mathrm{NTU} \to \infty$.

$$N^* = \mathrm{NTU}\frac{\Delta T_M}{\Delta T_{AM}} = 1 \tag{39}$$

$$P = \frac{N^*}{1+N^*} = 0.5 \tag{40}$$

This result agrees with that from P—NTU method.

(c) For the counterflow and parallel flow heat exchanger with $C^* \to 0, \mathrm{NTU} \to \infty, \Delta T_M \to 0$.

$$N^* = \mathrm{NTU}\frac{\Delta T_M}{\Delta T_{AM}} = 2 \tag{41}$$

$$P = \frac{2N^*}{2+N^*} = 1 \tag{42}$$

Eq. (37) is illustrated in Fig. 4, where point *a* on the curve for $C^* = 1$ represents the maximum effectiveness $P_{max} = 0.5$ for the parallel flow, because its N^* cannot be greater that unity. Point *b* on the curve for $C^* = 0.5$ corresponds to the maximum effectiveness $P_{max} = 0.667$ for the parallel flow. Point *c* on the curve for $C^* = 0$ holds for the balanced counterflow only, because for other flow arrangements, $N^* < 2$ and hence $P_{max} < 1$.

4. About the entropy generation paradox

Based on the second law of thermodynamics, Bejan [10,11] obtained the relation between the heat transfer induced entropy generation number for the balanced counterflow heat exchanger:

$$N_S = \frac{\dot{S}}{C_{min}} = \ln\left\{\left[1 - P\left(1 - \frac{T_{hi}}{T_{ci}}\right)\right]\left[1 + P\left(\frac{T_{ci}}{T_{hi}} - 1\right)\right]\right\} \tag{43}$$

where $\dot{S}$ is entropy generation rate, and T_{hi}/T_{ci} is the ratio of inlet temperatures.

The behavior of N_S at constant T_{hi}/T_{ci} is illustrated in Fig. 5.

Bejan [11] expected any heat transfer irreversibility to increase monotonically as the heat exchanger area (or NTU) decreases. The behavior in the range $P \in [0, 0.5]$, which does not agree with the expectation (Fig. 5), is then called the entropy generation paradox [11]. Shah [16] indicated, however, that this so-called paradox for the standalone heat exchanger is an intrinsic behavior of the temperature difference irreversibility function and it can never be removed without violating the second law. He calculated the variation of dimensionless entropy generation number $\dot{S}/\dot{S}_{max}$ and temperature effectiveness P with respect to NTU of a 2 pass-2 pass plate heat exchanger with overall parallelflow and individual passes in counterflow (Fig. 6), a 1-2 TEMA G exchanger with overall counterflow (Fig. 7) and a 1-2 TEMA J exchanger (Fig. 8). He [16] showed that the dimensionless entropy generation can be maximum (Fig. 6), having an intermediate value (Fig. 7) or local minimum (Fig. 8) at the maximum temperature effectiveness point P_{max}.

According to the forgoing analysis, the heat exchanger irreversibility should be measured by the entransy dissipation based dimensionless thermal resistance, rather than the entropy genera-

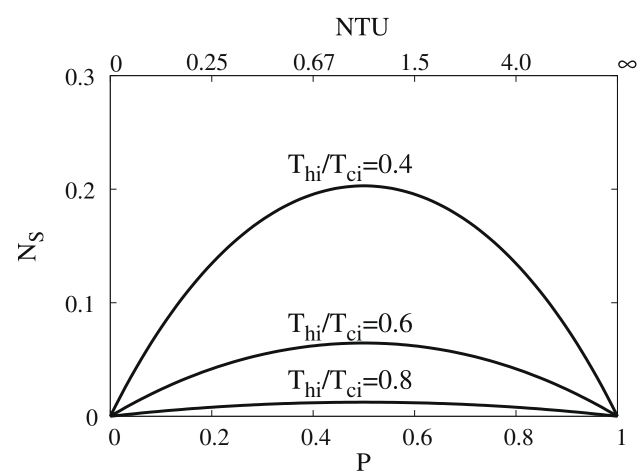

Fig. 5. Entropy generation rate in a balanced counterflow heat exchanger with zero pressure drop irreversibility [10].

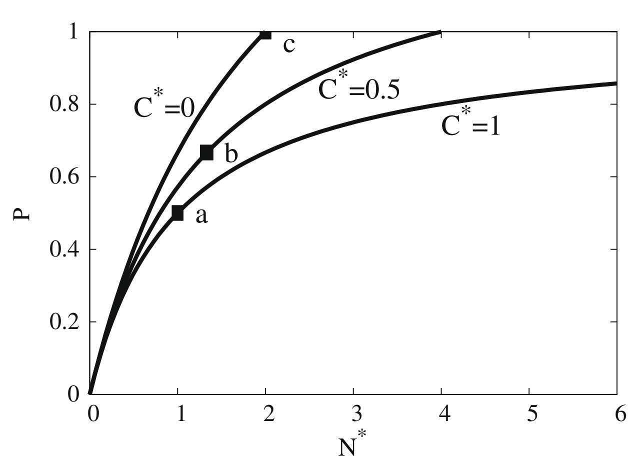

Fig. 4. Dimensionless thermal conductance versus the effectiveness with the heat capacity ratio as a parameter.

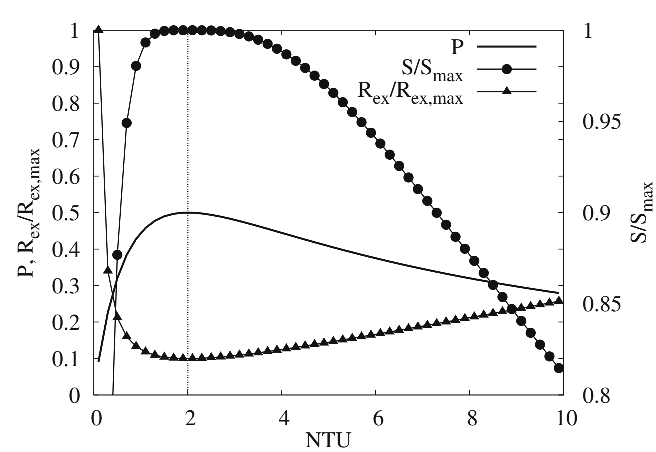

Fig. 6. $\dot{S}/\dot{S}_{max}$, $R_{ex}/R_{ex,max}$ and P as a function of NTU for a 2 pass-2 pass plate heat exchanger with overall parallelflow and individual passes in counterflow for $T_{hi}/T_{ci} = 2.0$.

tion number. Fig. 9 gives the relation between the dimensionless thermal resistance and the effectiveness with heat capacity ratio as a parameter. It can be seen in Fig. 9 that the effectiveness decreases monotonically with increasing the dimensionless thermal resistance. Furthermore, Figs. 6–8 show that the dimensionless equivalent thermal resistance is always the minimum at the maximum temperature effectiveness point, that is, no paradox occurs.

5. Concluding remarks

1. The equivalent thermal resistance of heat exchanger is defined based on the concept of entransy dissipation rate, which equals to the thermal resistance of convective heat transfer multiplied by the ratio of the arithmetic temperature difference to the mean temperature difference. This temperature difference ratio reflects the thermal resistance increase due to the deviation of the heat exchanger geometry from the balanced counterflow.
2. The relationship between the effectiveness and the thermal resistance (or thermal conductance) is derived, which, unlike the relationship between the effectiveness and the number of heat transfer unit, holds for heat exchangers with any flow arrangement, and is useful for the performance comparison among heat exchangers with different flow arrangement.

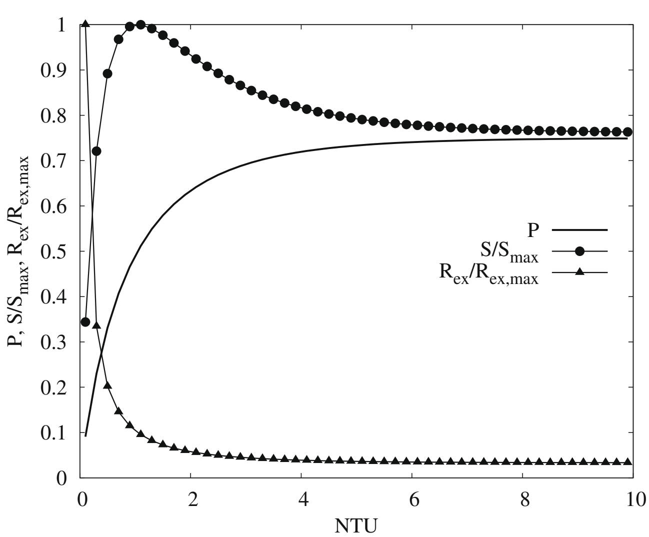

Fig. 7. $\dot{S}/\dot{S}_{max}, R_{ex}/R_{ex,max}$ and P as a function of NTU for a 1-2 TEMA G exchanger with overall counterflow for $T_{hi}/T_{ci} = 2.0$.

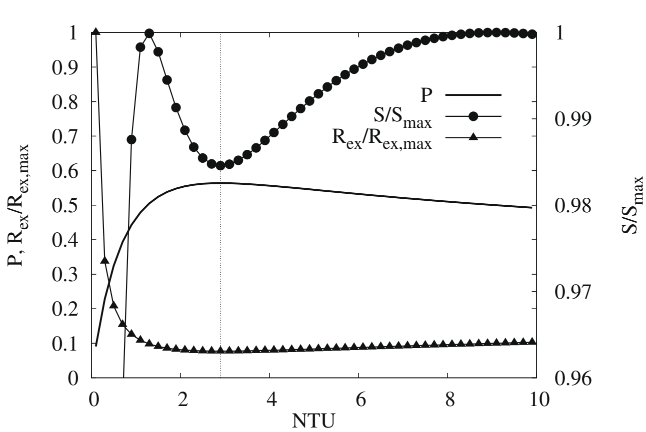

Fig. 8. $\dot{S}/\dot{S}_{max}, R_{ex}/R_{ex,max}$ and P as a function of NTU for a 1-2 TEMA J exchanger for $T_{hi}/T_{ci} = 2.0$.

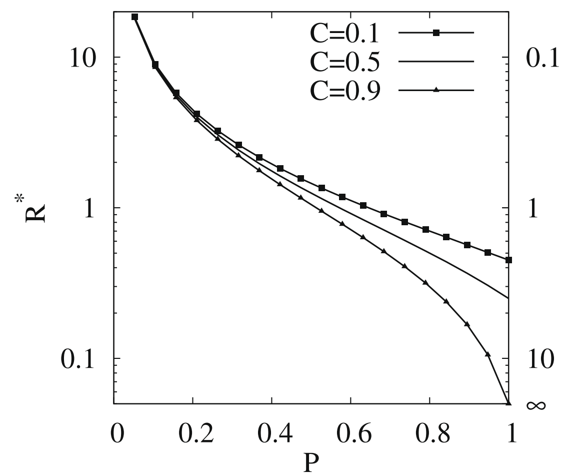

Fig. 9. The dimensionless thermal resistance versus the effectiveness.

3. The irreversibility of heat transfer for the purpose of heat-work conversion is well known measured by the entropy generation rate, while the irreversibility of heat transfer for the purpose of object heating or cooling should be measured by the entransy dissipation rate. Hence, the thermal resistance based on the entransy dissipation represents the heat exchanger irreversibility and the effectiveness decreases monotonically with increasing the thermal resistance of a heat exchanger. The so-called entropy generation paradox, i.e., the non-monotonic variation of the effectiveness with the entropy generation number – is due to the fact that the entropy generation number does not reflect the heat exchanger irreversibility evaluated from transport process rather than from heat-energy conversion.

Acknowledgment

This work was supported by a grant from the Major State Basic Research Development Program of China (973 Program, No. 2007CB206901).

References

[1] R.K. Shah, Heat exchangers, in: W.M. Rohsenow, J.P. Hartnett, Y.I. Cho (Eds.), Handbook of Heat Transfer, McGraw-Hill, New York, 1998 (Chapter 17).
[2] J.P. Holman, Heat Transfer, ninth ed., McGraw-Hill, New York, 2002.
[3] R.A. Bowman, A.C. Mueller, W.M. Nagel, Mean temperature difference in design, Trans. ASME 62 (1940) 283–294.
[4] TEMA, Standards of tubular exchanger manufacturers, 1978.
[5] W.M. Kays, A.L. London, Compact Heat Exchangers, first ed., McGraw-Hill, New York, 1955.
[6] W.M. Kays, A.L. London, Compact Heat Exchangers, second ed., McGraw-Hill, New York, 1964.
[7] R.K. Shah, D.P. Sekulic, Fundamentals of Heat Exchanger Design, John Wiley and Sons, New Jersey, 2003.
[8] A. Pignotti, R.K. Shah, Effectiveness–number of transfer units relationships for heat exchanger complex flow arrangements, Int. J. Heat Mass Transfer 35 (5) (1992) 1275–1291.
[9] A. Bejan, The concept of irreversibility in heat exchanger design: counterflow heat exchangers for gas-to-gas applications, J. Heat Transfer Trans. ASME 99 (3) (1977) 374–380.
[10] A. Bejan, Second law analysis in heat transfer, Energy 5 (8–9) (1980) 720–732.
[11] A. Bejan, Entropy Generation Minimization: The Method of Thermodynamic Optimization of Finite-size Systems and Finite-time Processes, CRC Press, Boca Raton, 1996.
[12] D.P. Sekulic, Entropy generation in a heat exchanger, Heat Transfer Eng. 7 (1) (1986) 83–88.
[13] J.E. Hesselgreaves, Rationalisation of second law analysis of heat exchangers, Int. J. Heat Mass Transfer 43 (22) (2000) 4189–4204.
[14] K. Ogiso, Duality of heat exchanger performance in balanced counter-flow systems, J. Heat Transfer Trans. ASME 125 (3) (2003) 530–532.
[15] Z. Xu, S. Yang, Z. Chen, A modified entropy generation number for heat exchangers, J. Therm. Sci. 5 (4) (1996) 257–263.

[16] R.K. Shah, T. Skiepko, Entropy generation extrema and their relationship with heat exchanger effectiveness–number of transfer unit behavior for complex flow arrangements, J. Heat Transfer Trans. ASME 126 (6) (2004) 994–1002.
[17] Z.-Y. Guo, H.-Y. Zhu, X.-G. Liang, Entransy – a physical quantity describing heat transfer ability, Int. J. Heat Mass Transfer 50 (13–14) (2007) 2545–2556.
[18] L.G. Chen, S.H. Wei, F.R. Sun, Constructal entransy dissipation minimization for 'volume-point' heat conduction, J. Phys. D Appl. Phys. 41 (19) (2008) 195506.
[19] Q. Chen, J.X. Ren, Generalized thermal resistance for convective heat transfer and its relation to entransy dissipation, Chin. Sci. Bull. 53 (23) (2008) 3753–3761.
[20] F.P. Incropera, D.P. DeWitt, Fundamentals of Heat and Mass Transfer, fifth ed., Wiley, New York, 2002.