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Performance analysis of IDEAL algorithm for three-dimensional incompressible fluid flow and heat transfer problems

Dong-Liang Sun, Zhi-Guo Qu, Ya-Ling He and Wen-Quan Tao*,†

School of Energy and Power Engineering, Xi'an Jiaotong University, Xi'an, Shaanxi 710049, People's Republic of China

SUMMARY

Recently, an efficient segregated algorithm for incompressible fluid flow and heat transfer problems, called inner doubly iterative efficient algorithm for linked equations (IDEAL), has been proposed by the present authors. In the algorithm there exist inner doubly iterative processes for pressure equation at each iteration level, which almost completely overcome two approximations in SIMPLE algorithm. Thus, the coupling between velocity and pressure is fully guaranteed, greatly enhancing the convergence rate and stability of solution process. However, validations have only been conducted for two-dimensional cases. In the present paper the performance of the IDEAL algorithm for three-dimensional incompressible fluid flow and heat transfer problems is analyzed and a systemic comparison is made between the algorithm and three other most widely used algorithms (SIMPLER, SIMPLEC and PISO). By the comparison of five application examples, it is found that the IDEAL algorithm is the most robust and the most efficient one among the four algorithms compared. For the five three-dimensional cases studied, when each algorithm works at its own optimal under-relaxation factor, the IDEAL algorithm can reduce the computation time by 12.9–52.7% over SIMPLER algorithm, by 45.3–73.4% over SIMPLEC algorithm and by 10.7–53.1% over PISO algorithm. Copyright © 2009 John Wiley & Sons, Ltd.

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KEY WORDS: segregated algorithm; IDEAL algorithm; SIMPLER algorithm; SIMPLEC algorithm; PISO algorithm; three-dimensional cases

*Correspondence to: Wen-Quan Tao, School of Energy and Power Engineering, Xi'an Jiaotong University, Xi'an, Shaanxi 710049, People's Republic of China.
†E-mail: wqtao@mail.xjtu.edu.cn

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1. INTRODUCTION

The numerical approaches for solving the Navier–Stokes equations may be broadly divided into two categories [1, 2]: density based and pressure based. The density-based approach works well for cases of high Mach number, but for low Mach number flow and heat transfer problems, it becomes unstable and its convergence rate is greatly deteriorated. The pressure-based approach or the primitive-variable approach, though originally was developed for solving incompressible fluid flows, has been successfully extended to compressible flows and widely adopted in computational fluid dynamics and numerical heat transfer. Among the pressure-based approaches, the pressure-correction method is the most widely used one because of its simplicity and straightforward in physical concept.

The first pressure-correction method is the SIMPLE algorithm, proposed by Patankar and Spalding in 1972 [3]. The major approximations made in the SIMPLE algorithm are: (1) the initial pressure field and the initial velocity field are assumed independently; hence, the interconnection between pressure and velocity is neglected, leading to some inconsistency between them and (2) the effects of the velocity corrections of the neighboring grids are arbitrarily dropped in order to simplify the solution procedure, thus making the algorithm semi-implicit. These two approximations will not affect the final solutions if the solution process converges [4]. However, they do affect the convergence rate and stability. Therefore, since the proposal of the SIMPLE algorithm, a number of variants have been proposed in order to overcome one or both of the approximations. In 1981, Patankar proposed the SIMPLER algorithm [5], which is the method for overcoming the first approximation in the SIMPLE algorithm. In the SIMPLER algorithm for overcoming the inconsistency between the initial pressure field and the initial velocity field, the initial pressure is determined by a pressure equation. In the CSIMPLER [6] and CLEAR [7, 8] algorithms, the same method is adopted to overcome the first approximation in the SIMPLE. In 1984, Van Doormaal and Raithby proposed the SIMPLEC algorithm [9], in which by changing the definition of the coefficients of the pressure-correction equation, the effects of dropping the neighboring grid velocity corrections (the second approximation in the SIMPLE algorithm) are partially compensated. Van Doormaal and Raithby also proposed the SIMPLEX algorithm [10, 11] in 1985. In the SIMPLEX algorithm, by solving a set of algebraic equations for the coefficient d in the velocity-correction equation, the effects of dropping the velocity corrections of the neighboring grids are also taken into account to some degree. However, an additional assumption is introduced: the corrections of pressure difference across every interface of the main control volume are the same. PISO algorithm [12] was proposed by Issa in 1985, which implements two correction steps of pressure correction. This makes some improvement in the completeness of pressure-correction equation of the current iteration level than that obtained by the single correction step. In the FIMOSE algorithm proposed by Latimer and Pollard [13] at one iteration level, the momentum and pressure equations are iteratively solved to reduce the effects of the second approximation in the SIMPLE algorithm. Yen and Liu [14] proposed the explicit correction step method to accelerate the convergence by making the velocity explicitly satisfy the momentum equation. In summary, more than ten variants of SIMPLE algorithm are available in the literature, but no one has completely overcome the two assumptions in the SIMPLE algorithm except the CLEAR algorithm. In the CLEAR algorithm, the update of pressure and velocity is not conducted by adding a small value of correction, rather the pressure field is re-solved based on the intermediate

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velocity, thus the effects of the neighboring grid points can be taken into account, making the algorithm fully implicit. However, the robustness of the CLEAR algorithm is somewhat deteriorated as indicated in [15] where a modified algorithm, named by CLEARER was proposed. However, by re-introduction of the correction terms into the algorithm, the fully implicit character has been destroyed in the CLEARER algorithm. In order to retain the fully implicit feature, while further enhance the robustness and convergence characteristics, on the basis of the CLEAR algorithm [7, 8], the inner doubly iterative efficient algorithm for linked equations (IDEAL) [16, 17] was proposed by the present authors. In the algorithm there exist inner doubly iterative processes for pressure equation at each iteration level, which almost completely overcome the two approximations in the SIMPLE algorithm. Thus, the coupling between velocity and pressure is fully guaranteed, greatly enhancing the convergence rate and stability of solution process.

With the development of different coupling algorithms between pressure and velocity, the comparisons between different algorithms have also been extensively conducted. These include: the comparison between the FIMOSE, SIMPLER and SIMPLEC algorithms by Latimer and Pollard [13]; the comparison of the PISO, SIMPLER and SIMPLEC algorithms by Jang *et al.* [18]; the comparison of the PISO and SIMPLE algorithms for steady turbulent flow problems by Wanik and Schnell [19]; the comparison of the SMAC, PISO and ITA schemes for unsteady flows by Kim and Benson [20]; the comparison of the SIMPLE with PISO for transient flows by Barton [21]; the comparison study of the convergence characteristics and robustness for the SIMPLE, SIMPLER, SIMPLEC and SIMPLEX algorithms at fine grids by Zeng and Tao [22], etc. From the above comparisons, it can be concluded that, globally speaking, the SIMPLER, SIMPLEC and PISO algorithms are relatively better. In [8, 17] comparisons were also conducted between the SIMPLER, CLEAR and IDEAL algorithms for incompressible fluid flow and heat transfer problems.

Numerical simulation of complex fluid flow and heat transfer problems has become an effective tool in scientific research and engineering design and its application range has been widely extended in recent years. One important extension is from two-dimensional flow to three-dimensional case. All of the algorithm comparisons mentioned above are conducted only for two-dimensional fluid flow and heat transfer problems. There is very little information concerning the performance comparisons of different algorithms for three-dimensional fluid flow and heat transfer problems in the literature. The extension of the dimensionality in the simulation of fluid flow problems not only cause to a significant increase in computational effort, but also may drastically change the numerical characteristics of algorithms. It is the authors' experience that almost all of the above-mentioned algorithms make no appreciable difference when three-dimensional problems are solved. Thus, it is a very challenging task and an urgent need to develop an efficient and robust algorithm for solving three-dimensional fluid flow and heat transfer problems. The major purpose of the present paper is to adopt the IDEAL algorithm for three-dimensional incompressible fluid flow and heat transfer problems and make a systemic comparison between the IDEAL and the algorithms of SIMPLER, SIMPLEC and PISO, which are probably the three most widely used algorithms in the literature.

In the following, the major solution procedure of the IDEAL algorithm is first briefly reviewed. Then the comparison conditions and the convergence criterion are described, followed by a systemic comparison of the robustness and convergence rate among the four algorithms for five three-dimensional application examples. Finally, some conclusions are drawn.

2. BRIEF REVIEW OF THE IDEAL ALGORITHM

First, the formulation of the governing equations and a brief description of the discretized results will be presented. For simplicity of presentation, we take incompressible laminar steady flow in Cartesian coordinates as an example. The governing equations are as follows:

Continuity equation:

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0 \tag{1}$$

Momentum equation:

$$\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho vu)}{\partial y}+\frac{\partial(\rho wu)}{\partial z}=-\frac{\partial p}{\partial x}+\eta\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+S_u \tag{2}$$

$$\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(\rho vv)}{\partial y}+\frac{\partial(\rho wv)}{\partial z}=-\frac{\partial p}{\partial y}+\eta\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+S_v \tag{3}$$

$$\frac{\partial(\rho uw)}{\partial x}+\frac{\partial(\rho vw)}{\partial y}+\frac{\partial(\rho ww)}{\partial z}=-\frac{\partial p}{\partial z}+\eta\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+S_w \tag{4}$$

In this paper, the finite volume method is applied to discretize the continuity and momentum equations on a staggered grid system.

Discretized continuity equation:

$$(\rho u)_e A_e-(\rho u)_w A_w+(\rho v)_n A_n-(\rho v)_s A_s+(\rho w)_t A_t-(\rho w)_b A_b=0 \tag{5}$$

Discretized momentum equation:

$$\frac{a_e}{\alpha_u}u_e=\sum a_{nb}u_{nb}+b_e+d_e(p_P-p_E) \tag{6}$$

$$\frac{a_n}{\alpha_v}v_n=\sum a_{nb}v_{nb}+b_n+d_n(p_P-p_N) \tag{7}$$

$$\frac{a_t}{\alpha_w}w_t=\sum a_{nb}w_{nb}+b_t+d_t(p_P-p_T) \tag{8}$$

where under relaxation is incorporated into the solution process of the algebraic equations and the terms $(1-\alpha_u)a_e u_e^0/\alpha_u$, $(1-\alpha_v)a_n v_n^0/\alpha_v$ and $(1-\alpha_w)a_t w_t^0/\alpha_w$ have been, respectively, incorporated into the source terms b_e, b_n and b_t. The expressions of the coefficients a and source terms b depend on the discretized schemes, and have been well documented in the literatures [4, 23, 24]. For the simplicity of presentation, they are not shown here.

In the following, the IDEAL algorithm will be presented. In [16] the IDEAL algorithm has been proposed for incompressible fluid flow and heat transfer problems, and in [17] comparisons have been made for two-dimensional cases. In the present paper the algorithm is conducted on

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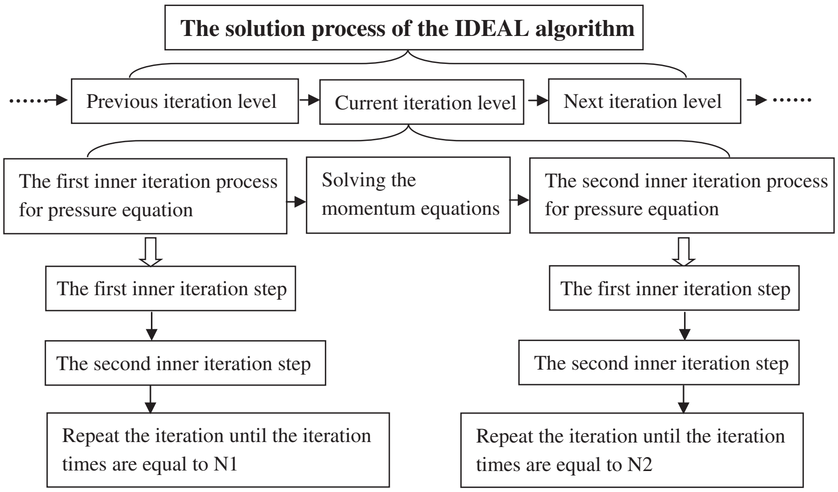

Figure 1. The framework of the solution process of the IDEAL algorithm.

a staggered system in three-dimensional Cartesian coordinates. For the convenience of further presentation, the major points of the IDEAL algorithm are reviewed here.

Figure 1 shows the framework of the solution process of the IDEAL algorithm in detail. The pressure-based solution method is iterative in nature. In the following we will often use the term 'iteration level'. By one iteration level we mean that all the computations are completed at the same values of the coefficients of the discretized momentum equations. In the IDEAL algorithm at each iteration level there exist two inner iteration processes, or inner doubly iterative processes for pressure field solution. The first inner iteration process for pressure equation almost completely overcomes the first approximation in the SIMPLE algorithm. The second inner iteration process almost completely overcomes the second approximation in the SIMPLE algorithm. The solution procedure of the IDEAL algorithm is presented as follows:

Step 1: Assume an initial velocity field u^0, v^0 and w^0.

Step 2: Calculate the coefficients a and source terms b of the discretized momentum equations (6), (7) and (8) by the initial velocity field.

The first inner iteration process for pressure equation:

Step 3: Calculate the pseudo-velocities $\tilde{u}^0$, $\tilde{v}^0$ and $\tilde{w}^0$ defined in the following equations:

$$u_e^{\text{Temp}} = \frac{\sum a_{nb} u_{nb}^0 + b_e}{a_e/\alpha_u} + d_e(p_P^{\text{Temp}} - p_E^{\text{Temp}}) = \tilde{u}_e^0 + d_e(p_P^{\text{Temp}} - p_E^{\text{Temp}}) \tag{9}$$

$$v_n^{\text{Temp}} = \frac{\sum a_{nb} v_{nb}^0 + b_n}{a_n/\alpha_v} + d_n(p_P^{\text{Temp}} - p_N^{\text{Temp}}) = \tilde{v}_n^0 + d_n(p_P^{\text{Temp}} - p_N^{\text{Temp}}) \tag{10}$$

$$w_t^{\text{Temp}} = \frac{\sum a_{nb} w_{nb}^0 + b_t}{a_t/\alpha_w} + d_t(p_P^{\text{Temp}} - p_T^{\text{Temp}}) = \tilde{w}_t^0 + d_t(p_P^{\text{Temp}} - p_T^{\text{Temp}}) \tag{11}$$

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Step 4: Solve the pressure equation (12) and obtain the temporary pressure p^{Temp}

$$\begin{aligned}
\frac{a_P}{\alpha_p} p_P^{\text{Temp}} &= \sum a_{nb} p_{nb}^{\text{Temp}} + b \\
a_P &= a_E + a_W + a_N + a_S + a_T + a_B \\
a_E &= (\rho A d)_e, \quad a_W = (\rho A d)_w, \quad a_N = (\rho A d)_n \\
a_S &= (\rho A d)_s, \quad a_T = (\rho A d)_t, \quad a_B = (\rho A d)_b \\
b &= (\rho \tilde{u}^0 A)_w - (\rho \tilde{u}^0 A)_e + (\rho \tilde{v}^0 A)_s - (\rho \tilde{v}^0 A)_n + (\rho \tilde{w}^0 A)_b \\
&\quad - (\rho \tilde{w}^0 A)_t + (1-\alpha_p)\frac{a_P}{\alpha_p} p_P^{\text{PTemp}}
\end{aligned} \tag{12}$$

Equation (12) is obtained by substituting Equations (9), (10) and (11) into the discretized continuity equation (5). In the first inner iteration process for pressure equation, the pressure under-relaxation factor α_p is incorporated into the pressure equation (12). The under-relaxation factor is used to make the solution process more stable for some very complicated cases. Generally speaking, the solution process of the IDEAL algorithm is stable enough, hence for most cases the pressure in Equation (12) need not be under relaxed and the pressure under-relaxation factor α_p is set as 1.

Step 5: Calculate the temporary velocities u^{Temp}, v^{Temp} and w^{Temp} from Equations (9), (10) and (11) by the temporary pressure p^{Temp}. Then one inner iteration step is finished and the next inner iteration step will be started.

Step 6: Regard u^{Temp}, v^{Temp}, w^{Temp} and p^{Temp} calculated in Steps 4 and 5 as the temporary velocity and pressure of the previous inner iteration step, denoted by u^{PTemp}, v^{PTemp}, w^{PTemp} and p^{PTemp}. Return to Step 3, and then all the superscripts 0 in Steps 3 and 4 are replaced by PTemp, and the values of $\tilde{u}^0$, $\tilde{v}^0$ and $\tilde{w}^0$ are updated. Then pressure equation (12) is re-solved. Repeat such iteration process composed of Steps 3, 4 and 5 until the iteration times are equal to the pre-specified times $N1$.

After the first inner iteration process for pressure equation is finished, the final temporary pressure p^{Temp} is regarded as the initial pressure p^*.

Step 7: Solve the discretized momentum equations (6), (7) and (8), by the initial velocity and pressure p^*, and obtain the intermediate velocities u^*, v^* and w^*.

The second inner iteration process for pressure equation:

Step 8: Calculate the pseudo-velocities $\tilde{u}^*$, $\tilde{v}^*$ and $\tilde{w}^*$ defined in the following equations:

$$u_e^{\text{Temp}} = \frac{\sum a_{nb} u_{nb}^* + b_e}{a_e/\alpha_u} + d_e(p_P^{\text{Temp}} - p_E^{\text{Temp}}) = \tilde{u}_e^* + d_e(p_P^{\text{Temp}} - p_E^{\text{Temp}}) \tag{13}$$

$$v_n^{\text{Temp}} = \frac{\sum a_{nb} v_{nb}^* + b_n}{a_n/\alpha_v} + d_n(p_P^{\text{Temp}} - p_N^{\text{Temp}}) = \tilde{v}_n^* + d_n(p_P^{\text{Temp}} - p_N^{\text{Temp}}) \tag{14}$$

$$w_t^{\text{Temp}} = \frac{\sum a_{nb} w_{nb}^* + b_t}{a_t/\alpha_w} + d_t(p_P^{\text{Temp}} - p_T^{\text{Temp}}) = \tilde{w}_t^* + d_t(p_P^{\text{Temp}} - p_T^{\text{Temp}}) \tag{15}$$

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Step 9: Solve the pressure equation (16) and obtain the temporary pressure p^{Temp}

$$
\begin{aligned}
a_P p_P^{\text{Temp}} &= \sum a_{nb} p_{nb}^{\text{Temp}} + b \\
a_P &= a_E + a_W + a_N + a_S + a_T + a_B \\
a_E &= (\rho A d)_e, \quad a_W = (\rho A d)_w, \quad a_N = (\rho A d)_n \\
a_S &= (\rho A d)_s, \quad a_T = (\rho A d)_t, \quad a_B = (\rho A d)_b \\
b &= (\rho \tilde{u}^* A)_w - (\rho \tilde{u}^* A)_e + (\rho \tilde{v}^* A)_s - (\rho \tilde{v}^* A)_n + (\rho \tilde{w}^* A)_b - (\rho \tilde{w}^* A)_t
\end{aligned} \tag{16}
$$

It should be noted that in the second inner iteration process for pressure equation, the pressure need not be under relaxed.

Step 10: Calculate the temporary velocities u^{Temp}, v^{Temp} and w^{Temp} from Equations (13), (14) and (15) by the temporary pressure p^{Temp}. Then one inner iteration step is finished and the next inner iteration step will be started.

Step 11: Regard u^{Temp}, v^{Temp}, w^{Temp} and p^{Temp} calculated in Steps 9 and 10 as the temporary velocity and pressure of the previous inner iteration step, denoted by u^{PTemp}, v^{PTemp}, w^{PTemp} and p^{PTemp}. Return to Step 8, and then all the superscripts $*$ in Steps 8 and 9 are replaced by PTemp, and the values of $\tilde{u}^*$, $\tilde{v}^*$ and $\tilde{w}^*$are updated. Then pressure equation (16) is re-solved. Repeat the iteration composed of Steps 8, 9 and 10 until the iteration times are equal to the pre-specified times $N2$.

After the second inner iteration process for pressure equation is finished, the final temporary velocities u^{Temp}, v^{Temp} and w^{Temp} are regarded as the final velocities u, v and w of the current iteration level.

Step 12: Solve the discretization equations of the other scalar variables if necessary.

Step 13: Regard the final velocities u, v and w as the initial velocities u^0, v^0 and w^0 of the next iteration level, then return to Step 2 of the next iteration level. Repeat such iterative procedure until convergence is reached.

It is interesting to note that in the IDEAL algorithm, as in the algorithm of SIMPLER and CLEAR, the pressure field used to solve the momentum equations, i.e. p^*, is solved by the pressure equation. Since the algebraic equation is solved iteratively, an initial pressure field is required, and the goodness of this initial field has a profound effect on the solution convergence. The numerical practice provided in [6] revealed this important effect. Our numerical practices show that if the pressure results of the first inner iteration are taken as the initial field for the next level solution, the total solution procedure can be somewhat enhanced.

In the IDEAL algorithm the first inner iteration times $N1$ and the second inner iteration times $N2$ (hereafter $N1$ and $N2$) can be adjusted. $N1$ and $N2$ should be increased with the increase of the velocity under-relaxation factor. At a larger velocity under-relaxation factor the solution process may become very unstable; therefore, the inner iteration times need to be increased to ensure the convergence of solution process and to enhance the robustness.

3. COMPARISON CONDITIONS AND CONVERGENCE CRITERION

For making meaningful comparisons of the four algorithms, numerical comparison conditions and convergence criterion should be specified. In our study the comparison conditions and convergence

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criterion include:

(1) *Hardware and codes*: All the calculations in this paper are performed on the computer of CPU 2.01 GHz and RAM 2.0 GB along with FORTRAN 77 compiler. For the justness of comparison, the codes of SIMPLER, SIMPLEC, PISO and IDEAL algorithms are compiled under the same program structure. In order to reduce the truncated errors, double precision digital is adopted to implement computation in our codes.

(2) *Discretization scheme*: In order to guarantee the stability and accuracy of the numerical solution, the SGSD scheme [25] is adopted, which is at least of second-order accuracy and absolutely stable. For stability of the solution process, the deferred-correction method is adopted, which was proposed in [26] and latter enhanced in [27].

(3) *Solution method of the algebraic equations*: The algebraic equations are solved by the alternative direction implicit method.

(4) *Under-relaxation factor*: In the SIMPLER and IDEAL algorithms, the pressure under-relaxation factor is set as 1.0. In the SIMPLEC and PISO algorithms the pressure need not be under relaxed at all [9, 12]. For the four algorithms, the same value is adopted for the velocity and temperature under-relaxation factors. For the convenience of presentation, the time step multiple E is used in the following presentation, which relates to the under-relaxation factor α by the following equation [9]:

$$E=\frac{\alpha}{1-\alpha} \quad (0<\alpha<1) \tag{17}$$

Some correspondence between α and E is presented in Table I. It can be seen that with the time step multiple, we have a much wider range to show the performance of the algorithm in the high-value region of the under-relaxation factor.

(5) *Grid system*: For each problem the same uniform grid system is used for execution of the four algorithms. The details of each grid system will be presented individually.

(6) *Convergence criterion*: The adopted convergence criterion requires that both the relative maximum mass and the relative maximum u, v, w-component momentum residuals are less than some pre-specified small values.

The relative maximum mass residual is expressed as

$$\mathrm{Rs}_{\mathrm{Mass}}=\frac{\mathrm{MAX}[|(\rho u^*A)_w-(\rho u^*A)_e+(\rho v^*A)_s-(\rho v^*A)_n+(\rho w^*A)_b-(\rho w^*A)_t|]}{q_m} \tag{18}$$

where u^*, v^* and w^* are the intermediate velocities of each iteration level and q_m is the reference mass flow rate. For the open system, we take the inlet mass flow rate as the reference mass flow rate. For the closed system, we make a numerical integration for the mass flow rate along any section in the field to obtain the reference mass flow rate [23].

Table I. Some correspondence between α and E.

α	0.1	0.5	0.9	0.95	0.96	0.97	0.98	0.99	1
E	0.111	1	9	19	24	32.3	49	99	Infinite

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The relative maximum u, v, w-component momentum residuals are expressed as

$$\mathrm{Rs}_{U\mathrm{Mom}} = \frac{\mathrm{MAX}\{|a_e u_e^0 - [\sum_{nb} a_{nb} u_{nb}^0 + b + A_e(p_P - p_E)]|\}}{\rho u_m^2} \tag{19}$$

$$\mathrm{Rs}_{V\mathrm{Mom}} = \frac{\mathrm{MAX}\{|a_n v_n^0 - [\sum_{nb} a_{nb} v_{nb}^0 + b + A_n(p_P - p_N)]|\}}{\rho u_m^2} \tag{20}$$

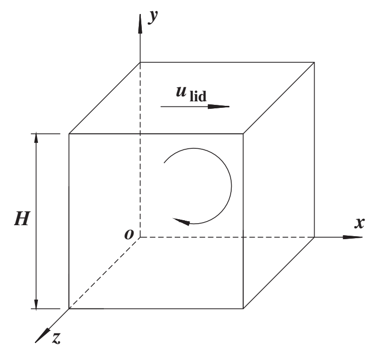

Figure 2. Flow configuration of lid-driven cavity flow in a cubic cavity.

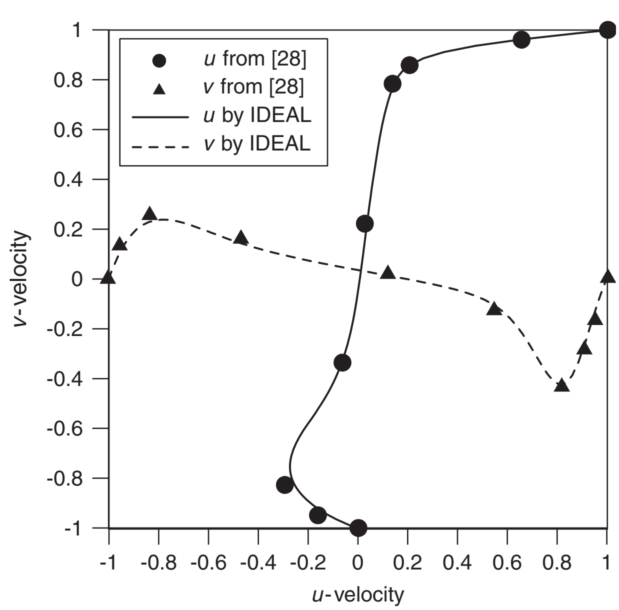

Figure 3. Comparison of velocity profiles u and v along the central axes on plane $z = 0.5H$ for $Re = 1000$.

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$$\mathrm{Rs}_{W\mathrm{Mom}} = \frac{\mathrm{MAX}\{|a_t w_t^0 - [\sum_{nb} a_{nb} w_{nb}^0 + b + A_t(p_P - p_T)]|\}}{\rho u_m^2} \tag{21}$$

where u^0, v^0 and w^0 are the initial velocities of each iteration level and ρu_m^2 is the reference momentum. For the open system, we take the inlet momentum as the reference one. For the closed system, we make a numerical integration for the momentum along any section in the field to obtain the reference momentum [23].

(7) *Double precision computations*: Even though our preliminary study in the single precision has also obtained quantitatively the same results, in order to reduce the possible effects of the

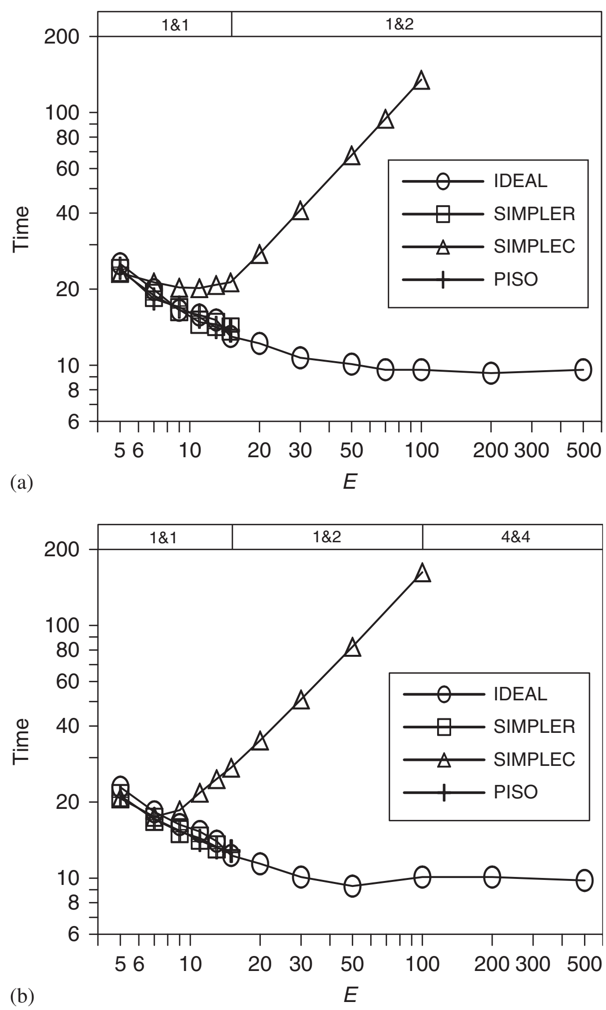

Figure 4. Comparison of computation time and robustness of IDEAL, SIMPLER, SIMPLEC and PISO algorithms for: (a) $Re=100$ and (b) $Re=300$ with grid number $=32\times 32\times 32$ of problem 1.

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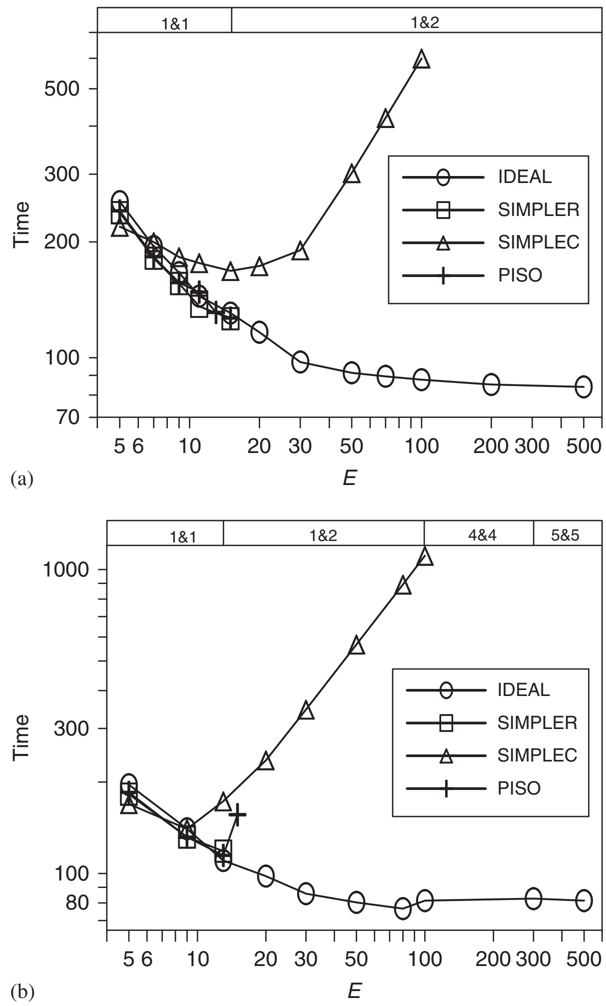

Figure 5. Comparison of computation time and robustness of IDEAL, SIMPLER, SIMPLEC and PISO algorithms for: (a) $Re=100$ and (b) $Re=500$ with grid number $=52\times52\times52$ of problem 1.

truncation error and the numerical noise, the double precision is adopted in the comparison computation.

4. NUMERICAL COMPARISONS

In the following comprehensive comparisons are made among the SIMPLER, SIMPLEC, PISO and IDEAL algorithms for five three-dimensional problems of fluid flow and heat transfer,

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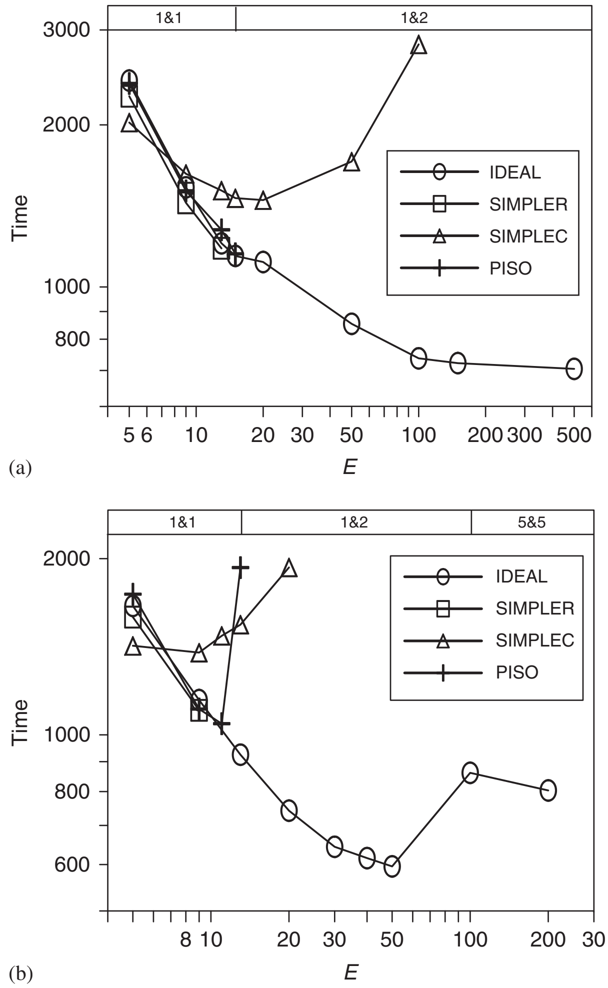

Figure 6. Comparison of computation time and robustness of IDEAL, SIMPLER, SIMPLEC and PISO algorithms for: (a) $Re=100$ and (b) $Re=1000$ with grid number $=82\times82\times82$ of problem 1.

which are:

(1) lid-driven cavity flow in a cubic cavity (problem 1);
(2) lid-driven cavity flow in a cubic cavity with complicated structure (problem 2);
(3) laminar fluid flow over a backward-facing step (problem 3);
(4) laminar fluid flow through a duct with complicated structure (problem 4);
(5) natural convection in a cubic cavity (problem 5).

Problems 1–4 are fluid flow problems. Among these four problems, problems 1 and 2 belong to closed system, problems 3 and 4 belong to open system. Problem 5 is a velocity–temperature coupling heat transfer problem. All of the five problems are based on the following

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Table II. Reduced ratio of computation time of IDEAL algorithm over SIMPLER, SIMPLEC and PISO algorithms at their own optimal time step multiples in problem 1.

	Grid number					
	$32\times32\times32$		$52\times52\times52$		$82\times82\times82$	
Re	100	300	100	500	100	1000
Computation time of IDEAL (s)	9.3	9.3	84.0	76.7	704.5	695.8
Reducing ratio over SIMPLER (%)	33.1	30.1	33.5	35.3	40.3	45.9
Reducing ratio over SIMPLEC (%)	54.0	46.9	50.0	45.3	51.4	56.9
Reducing ratio over PISO (%)	32.1	27.3	33.8	33.2	38.8	43.0

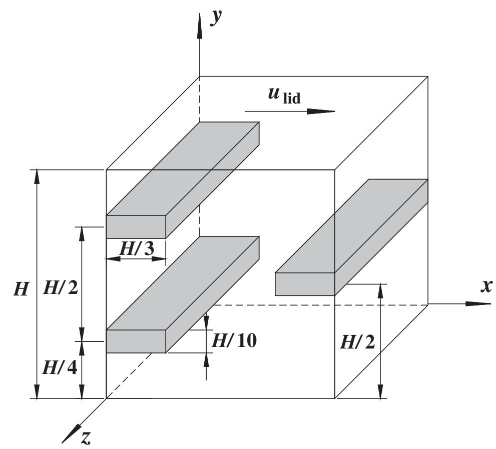

Figure 7. Flow configuration of lid-driven cavity flow in a cubic cavity with complicated structure.

assumptions: laminar, incompressible, steady state and constant fluid property. For the fifth problem, the Boussinesq assumption was adopted [28].

4.1. Fluid flow problems

4.1.1. Problems of closed system. Problem 1: *Lid-driven cavity flow in a cubic cavity*. Lid-driven cavity flow in a cubic cavity has served in CFD/NHT as a benchmark problem for testing numerical procedures for three-dimensional fluid flows [29–31]. The flow configuration is shown in Figure 2. Calculations are conducted for $Re=100$–1000 with grid numbers $=32\times32\times32$–$82\times82\times82$, and the allowed residuals $\mathrm{Rs}_{\mathrm{Mass}}, \mathrm{Rs}_{U\mathrm{Mom}}, \mathrm{Rs}_{V\mathrm{Mom}}$ and $\mathrm{Rs}_{W\mathrm{Mom}}$ should be all less than 10^{-8}. The Reynolds number is defined by

$$Re=\frac{u_{\mathrm{lid}}H}{v} \tag{22}$$

In Figure 3 the velocity profiles along the central lines on the plane $z=0.5H$ are presented. As shown in this figure, the results calculated by the IDEAL algorithm are in excellent agreement with those reported by Tang *et al.* [28]. This comparison gives some support to the reliability of

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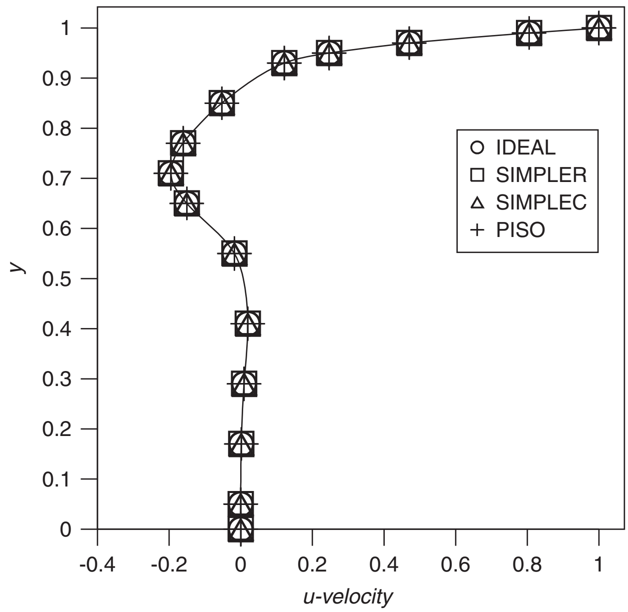

Figure 8. Comparison of velocity profiles u along the central axes y on plane $z=0.5H$ for $Re=500$.

the proposed three-dimensional IDEAL algorithm and the developed code. From the following comparisons with other well-documented algorithms (SIMPLER, SIMPLEC and PISO) further strong support to the present algorithm and code will be provided.

Figures 4, 5 and 6 show the computation time and robustness of the IDEAL, SIMPLER, SIMPLEC and PISO algorithms for different grid numbers and different Reynolds numbers of problem 1. The inner iteration times $N1$ and $N2$ in the IDEAL algorithm are displayed at the top of these figures. For example 1 & 1 and 1 & 2 at the top of Figure 4(a) show that in the two ranges of E the two inner iterative times are 1 and 1 and 1 and 2, respectively. From the three figures, following three features may be noted. First, $N1$ and $N2$ increase with the increase of time step multiple, i.e. with the under-relaxation factor. Second, among the four algorithms compared, the IDEAL algorithm is far more robust than SIMPLER, SIMPLEC and PISO algorithms, and it can converge almost at any time step multiple for any case in problem 1. The SIMPLER and PISO algorithms have the worst robustness and the SIMPLEC algorithm is something in between. Third, for the consumed computation time the SIMPLEC algorithm needs the largest, and the SIMPLER and PISO algorithms come next. The IDEAL algorithm needs the least.

Table II shows the reduced ratio of computation time of IDEAL algorithm over SIMPLER, SIMPLEC and PISO algorithms at their own optimal time step multiples for different cases of problem 1. Our computations were conducted in a personal computer with memory of 1 G and 2.01 GHz frequencies. The CPU computational times of the IDEAL algorithm are listed for different number of grids, while for other algorithms only the relative saving in CPU times. When each method works at its own optimal time step multiple, the IDEAL algorithm can reduce the computation time by 30.1–45.9% over the SIMPLER algorithm, by 45.3–56.9% over the SIMPLEC algorithm and by 27.3–43.0% over the PISO algorithm for problem 1.

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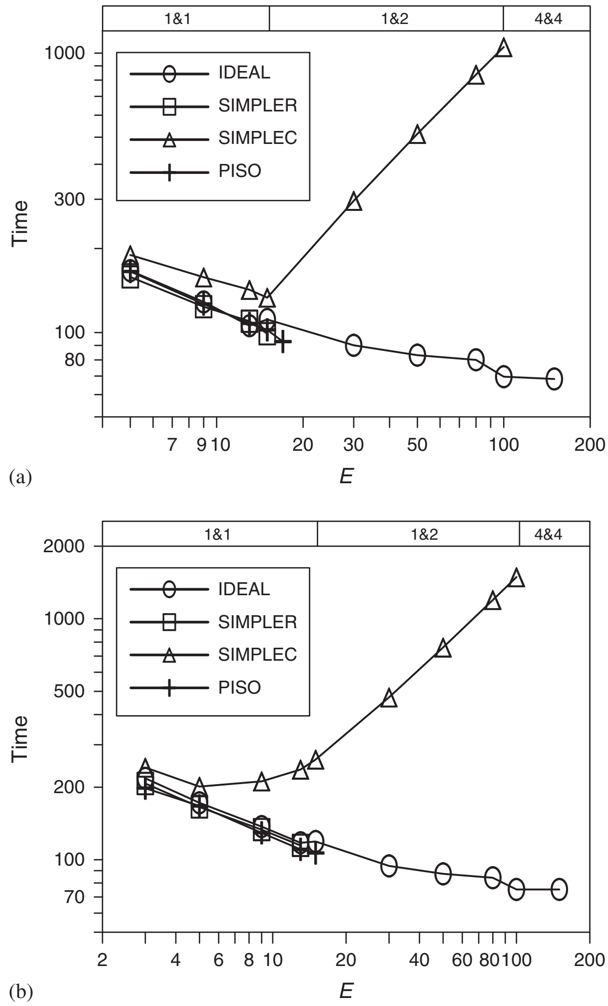

Figure 9. Comparison of computation time and robustness of IDEAL, SIMPLER, SIMPLEC and PISO algorithms for: (a) $Re=100$ and (b) $Re=500$ with grid number $=52\times 52\times 52$ of problem 2.

Problem 2: *Lid-driven cavity flow in a cubic cavity with complicated structure*. Problem 1 belongs to the simple closed system. The IDEAL algorithm shows its significant advantages over the SIMPLER, SIMPLEC and PISO algorithms for this simple closed system. In order to show the better performance of the IDEAL algorithm superior to the other three algorithms for a complicated closed system, problem 2 is especially designed. The flow configuration of problem 2 is shown in Figure 7. Three blocks of baffle plates are inserted into the cubic cavity to make the flow configuration more complicated. The domain extension method [23] is applied for this irregular computation domain, i.e. the three blocks are supposed to be the fluids with very large viscosity and computations are conducted for the entire cubic.

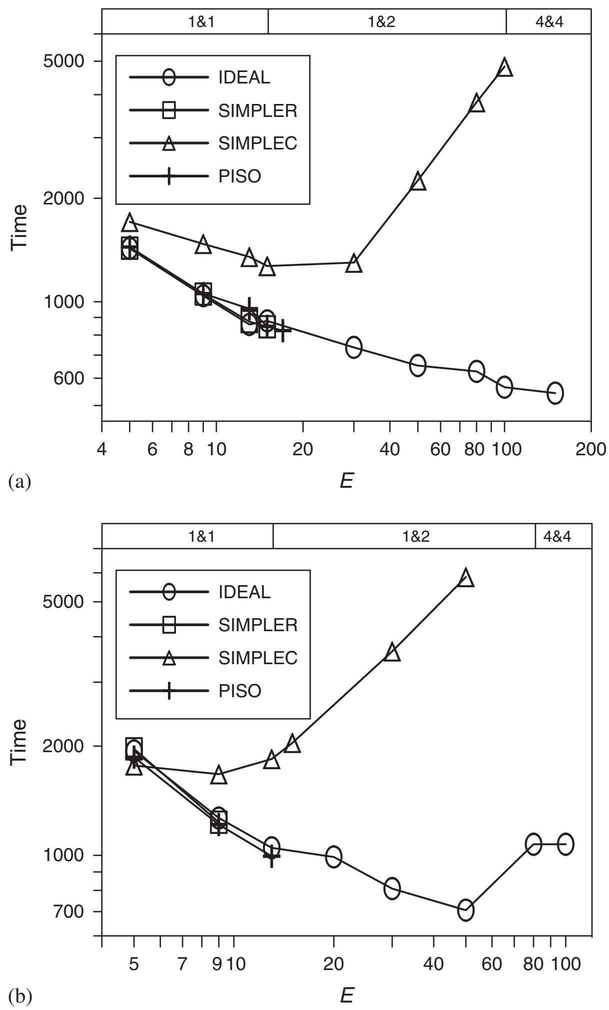

Figure 10. Comparison of computation time and robustness of IDEAL, SIMPLER, SIMPLEC and PISO algorithms for: (a) $Re=100$ and (b) $Re=800$ with grid number $=82\times82\times82$ of problem 2.

Calculations are conducted for $Re=100$–800 and grid numbers $=52\times52\times52$–$82\times82\times82$. The allowed residuals $\mathrm{Rs}_{\mathrm{Mass}}$, $\mathrm{Rs}_{U\mathrm{Mom}}$, $\mathrm{Rs}_{V\mathrm{Mom}}$ and $\mathrm{Rs}_{W\mathrm{Mom}}$ should be all less than 10^{-8}. The Reynolds number is defined by

$$Re=\frac{u_{\mathrm{lid}}H}{\nu} \tag{23}$$

In Figure 8 the velocity profiles u along the central line y on the plane $z=0.5H$ from the four algorithms are presented. The results calculated by the IDEAL algorithm are in excellent agreement with those calculated by the other three algorithms. Figures 9 and 10 show the computation

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Table III. Reduced ratio of computation time of IDEAL algorithm over SIMPLER, SIMPLEC and PISO algorithms at their own optimal time step multiples in problem 2.

	Grid number			
	$52\times52\times52$		$82\times82\times82$	
Re	100	500	100	800
Computation time of IDEAL (s)	68.3	75.2	537.0	706.3
Reducing ratio over SIMPLER (%)	31.1	34.4	36.5	42.8
Reducing ratio over SIMPLEC (%)	48.8	62.6	57.7	57.8
Reducing ratio over PISO (%)	26.5	29.5	34.9	28.9

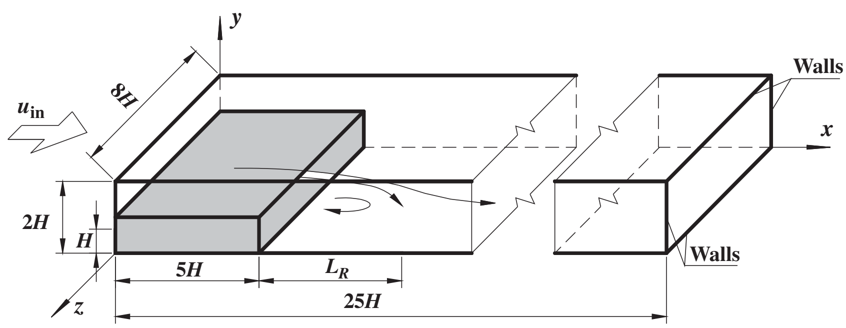

Figure 11. Flow configuration of laminar fluid flow over a backward-facing step.

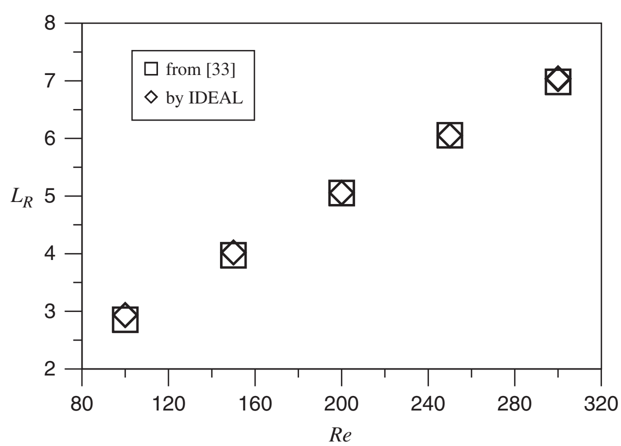

Figure 12. The predicted reattached lengths, L_R, on plane $z=4H$ obtained by IDEAL algorithm and from Reference [33].

time and robustness of the IDEAL, SIMPLER, SIMPLEC and PISO algorithms for different grid numbers and different Reynolds numbers of problem 2. From these two figures, we can find that the relative performances of different algorithms in the complicated closed system are almost the

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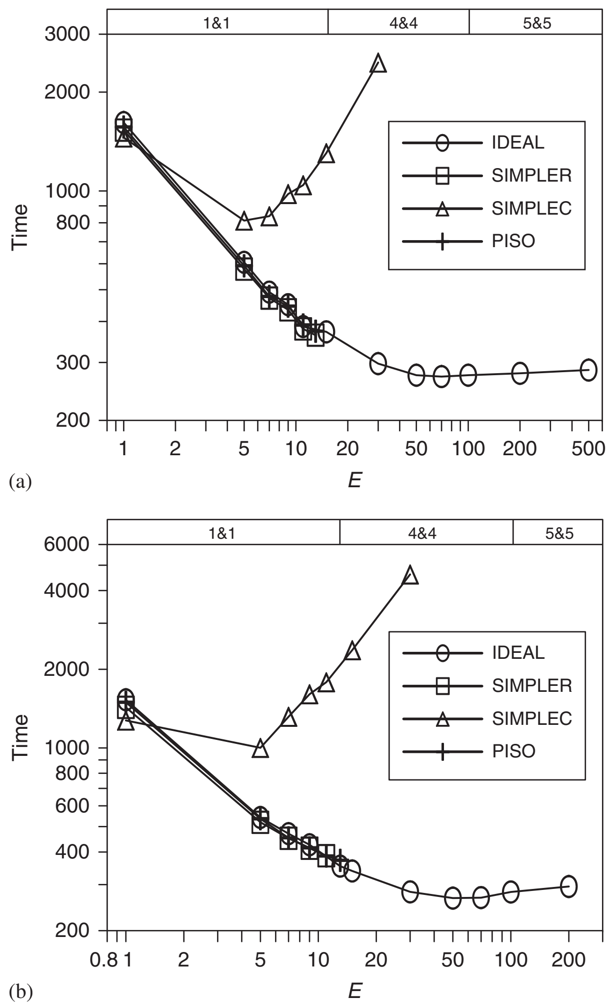

Figure 13. Comparison of computation time and robustness of IDEAL, SIMPLER, SIMPLEC and PISO algorithms for: (a) $Re = 100$ and (b) $Re = 300$ with grid number $= 127 \times 32 \times 63$ of problem 3.

same as those in the simple closed system. Thus, the IDEAL algorithm also shows its advantages for complicated closed systems.

Table III shows the reduced ratio of computation time of the IDEAL algorithm over the SIMPLER, SIMPLEC and PISO algorithms at their own optimal time step multiples for different cases of problem 2. When each method works at its own optimal time step multiple, the IDEAL algorithm can reduce the computation time by 31.1–42.8% over the SIMPLER algorithm, by 48.8–62.6% over the SIMPLEC algorithm and by 26.5–34.9% over the PISO algorithm for problem 2.

4.1.2. Problems of open system. Problem 3: Laminar fluid flow over a backward-facing step. Laminar fluid flow over a backward-facing step shown in Figure 11 belongs to simple open system.

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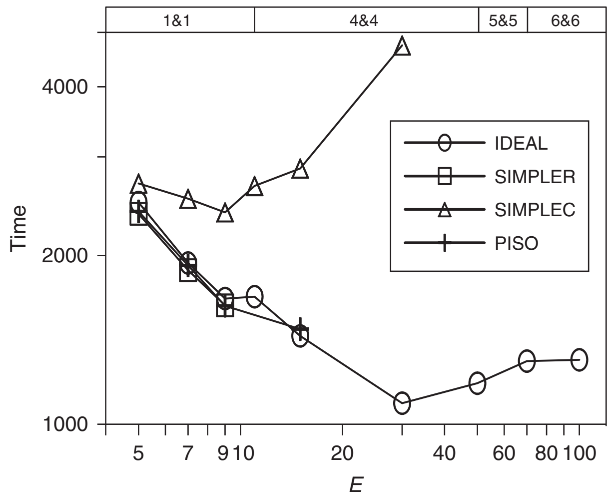

Figure 14. Comparison of computation time and robustness of IDEAL, SIMPLER, SIMPLEC and PISO algorithms for $Re=100$ with grid number $=202\times 52\times 63$ of problem 3.

Table IV. Reduced ratio of computation time of IDEAL algorithm over SIMPLER, SIMPLEC and PISO algorithms at their own optimal time step multiples in problem 3.

	Grid number		
	$127\times 32\times 63$		$202\times 52\times 63$
Re	100	300	100
Computation time of IDEAL (s)	271.8	266.3	1090.2
Reducing ratio over SIMPLER (%)	25.2	31.2	33.1
Reducing ratio over SIMPLEC (%)	66.5	73.4	54.4
Reducing ratio over PISO (%)	27.3	28.3	26.2

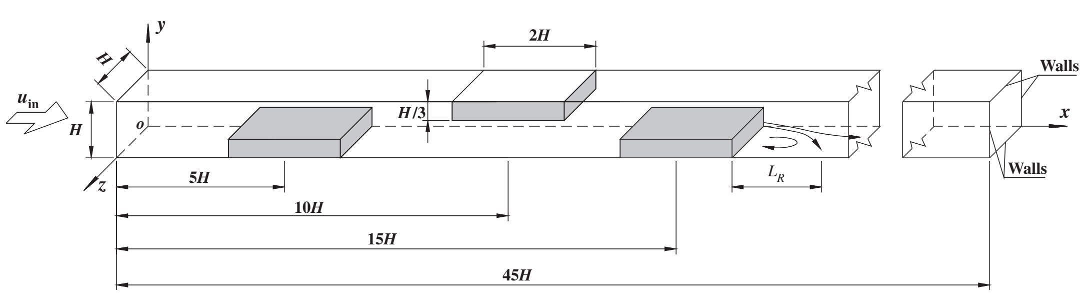

Figure 15. Flow configuration of laminar fluid flow through a duct with complicated structure.

It is another typical configuration widely adopted in computational fluid dynamics study. Again the domain extension method is used to deal with the solid step and solutions are performed for the entire region with $2H\times 8H\times 25H$.

Table V. Predicted reattachment lengths on plane $z=0.5H$ in problem 4.

Re	IDEAL	SIMPLER	SIMPLEC	PISO
100	0.9725	0.9728	0.9730	0.9725
300	2.1099	2.1095	2.1100	2.1095
500	3.2426	3.2495	3.2525	3.2423

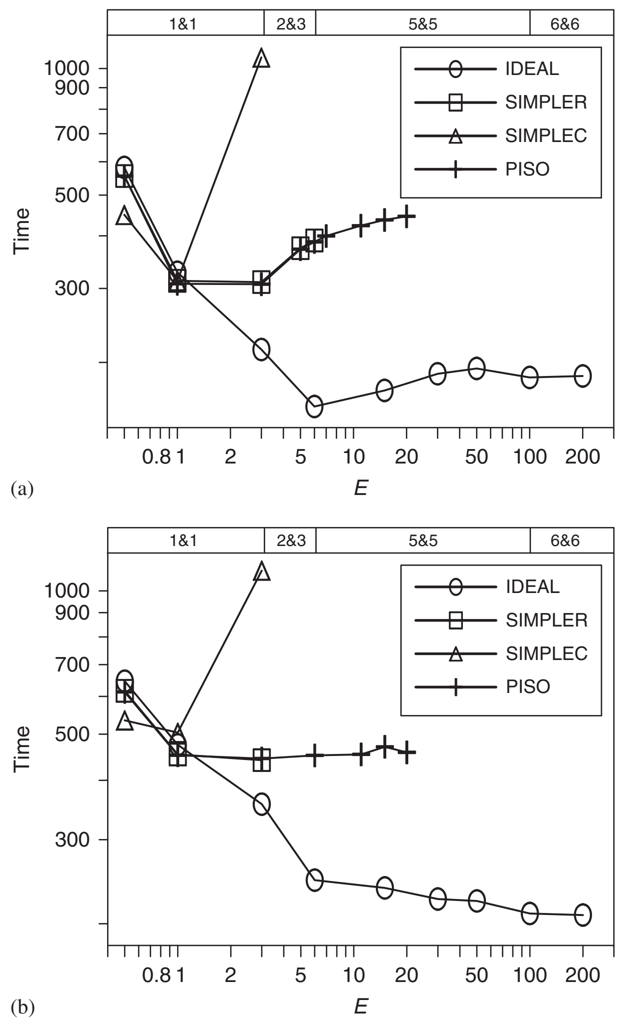

Figure 16. Comparison of computation time and robustness of IDEAL, SIMPLER, SIMPLEC and PISO algorithms for: (a) $Re=100$ and (b) $Re=300$ with grid number$=150\times20\times20$ of problem 4.

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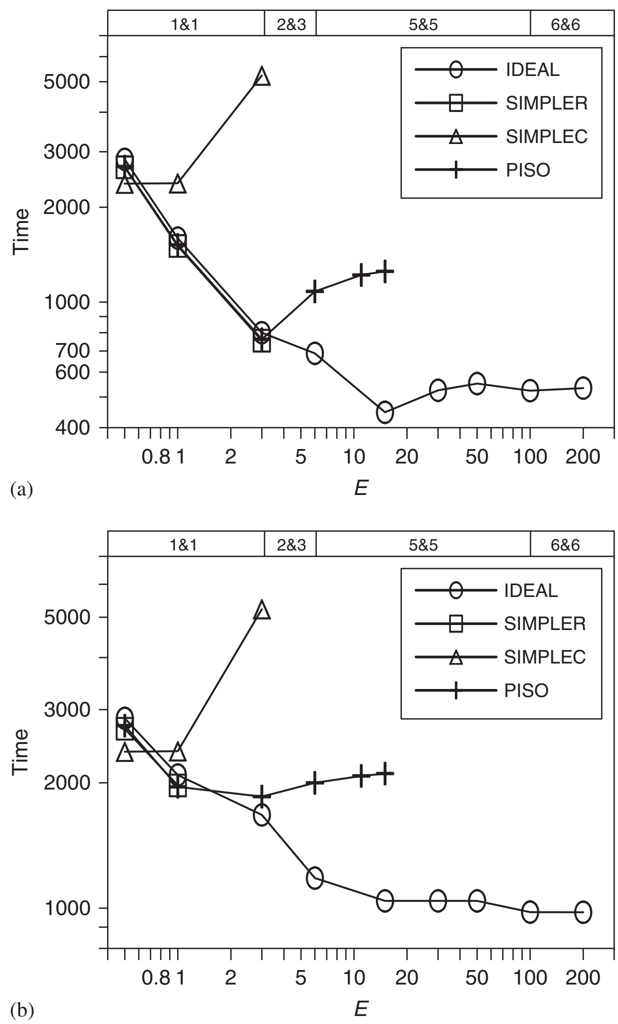

Figure 17. Comparison of computation time and robustness of IDEAL, SIMPLER, SIMPLEC and PISO algorithms for: (a) $Re = 100$ and (b) $Re = 500$ with grid number $= 190 \times 29 \times 29$ of problem 4.

Calculations are conducted for $Re = 100$–300 and grid numbers $= 127 \times 32 \times 63$–$202 \times 52 \times 63$. The inflow velocity distribution is taken from Shah and London [32], and the fully developed boundary condition is used at the outflow boundary. The residuals Rs_{Mass}, Rs_{UMom}, Rs_{VMom} and Rs_{WMom} are all set to be less than 10^{-7}. The Reynolds number is defined by

$$Re = \frac{u_{in} H}{v} \tag{24}$$

Figure 12 shows the predicted reattachment lengths, L_R, on plane $z = 4H$ obtained, respectively, by IDEAL algorithm and from Reference [33]. The results calculated by the IDEAL algorithm

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Table VI. Reduced ratio of computation time of IDEAL algorithm over SIMPLER, SIMPLEC and PISO algorithms at their own optimal time step multiples in problem 4.

	Grid number			
	150×20×20		190×29×29	
Re	100	300	100	500
Computation time of IDEAL (s)	157.1	208.5	447.2	976.9
Reducing ratio over SIMPLER (%)	49.4	52.7	40.7	50.5
Reducing ratio over SIMPLEC (%)	49.7	58.6	67.0	58.9
Reducing ratio over PISO (%)	48.8	53.1	41.4	47.3

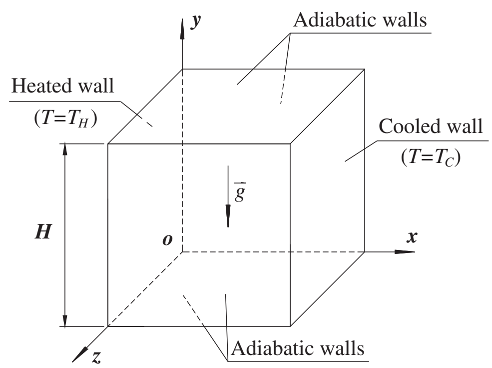

Figure 18. Flow configuration of natural convection in a cubic cavity.

Table VII. Comparison of Nu_m solutions with previous works for different *Ra* values of problem 5.

Ra	10^4	10^5	10^6
Fusegi *et al.* [34]	2.1000	4.3610	8.770
Wakashima and Saitoh [35]	2.0814	4.4309	8.8681
IDEAL	2.0842	4.4048	8.8005

agree very well with those from [33]. Figures 13 and 14 show the computation time and robustness of the IDEAL, SIMPLER, SIMPLEC and PISO algorithms for different grid numbers and different Reynolds numbers of problem 3. As shown in these two figures, the SIMPLER algorithm has the worst robustness, and the robustness of the PISO and SIMPLEC algorithms is a bit better. The IDEAL algorithm is the best. From Figures 13 and 14 we can find that the IDEAL algorithm can converge almost at any time step multiple for any case of problem 3. As far as the consumed computation time is concerned, the SIMPLEC algorithm needs the largest, and the SIMPLER and PISO algorithms come next. The IDEAL algorithm needs the least.

Table IV shows the reduced ratio of computation time of IDEAL algorithm over the SIMPLER, SIMPLEC and PISO algorithms at their own optimal time step multiples for different cases of problem 3. When each method works at its own optimal time step multiple, the IDEAL algorithm

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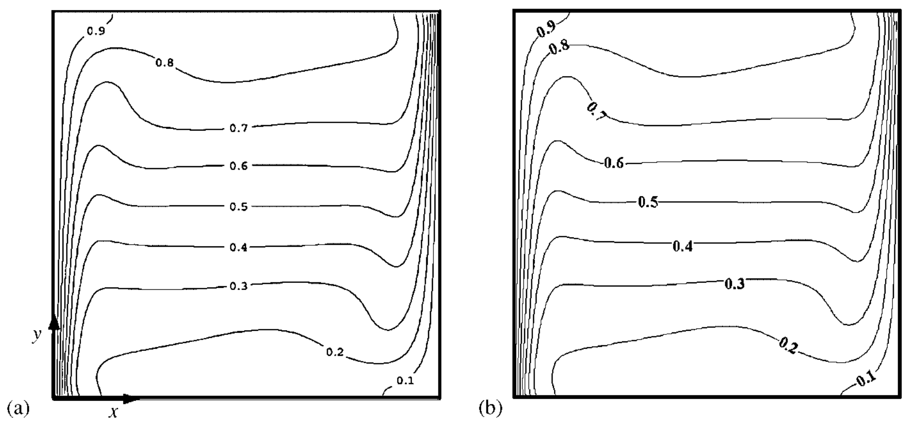

Figure 19. Temperatures at the plane $z=0.5H$ for $Ra=10^6$, obtained (a) from Reference [35] and (b) by IDEAL algorithm.

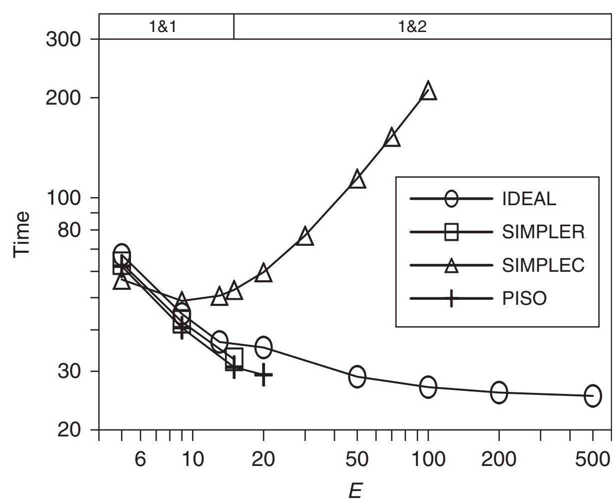

Figure 20. Comparison of computation time and robustness of IDEAL, SIMPLER, SIMPLEC and PISO algorithms for $Ra=10^4$ with grid number$=30\times30\times30$ of problem 5.

can reduce the computation time by 25.2–33.1% over the SIMPLER algorithm, by 54.4–73.4% over the SIMPLEC algorithm and by 26.2–28.3% over the PISO algorithm for problem 3.

Problem 4: *Laminar fluid flow through a duct with complicated structure.* Laminar fluid flow through a duct with complicated structure belongs to complicated open system. This problem is adopted to examine whether the IDEAL algorithm is still superior to the SIMPLER, SIMPLEC and PISO algorithms in a complicated open system. The flow configuration of problem 4 is shown in Figure 15. Three blocks of baffle plates are inserted into the duct to make the flow configuration more complicated. The three solid blocks are treated by the domain extension method.

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Calculations are conducted for $Re = 100$–500, grid numbers $= 150\times 20\times 20$–$190\times 29\times 29$. The inflow velocity is uniform, and the fully developed boundary condition is used at the outflow. The residuals Rs_{Mass}, Rs_{UMom}, Rs_{VMom} and Rs_{WMom} are all set to be less than 10^{-7}. The Reynolds number is defined by

$$Re = \frac{u_{in} H}{v} \tag{25}$$

Table V shows the predicted reattachment lengths, L_R, on plane $z = 0.5H$ obtained by the four different algorithms. The results computed by the IDEAL algorithm are almost the same as those by the other three algorithms. Figures 16 and 17 show the computation time and robustness of the IDEAL, SIMPLER, SIMPLEC and PISO algorithms for different grid numbers and different Reynolds numbers of problem 4. From these two figures, we can find that the SIMPLEC algorithm in the complicated open system becomes less robust and less effective than in the simple open system, the performances of the SIMPLER, SIMPLEC are in the middle and the IDEAL algorithm is the most robust and efficient.

Table VI shows the reduced ratio of computation time of the IDEAL algorithm over the SIMPLER, SIMPLEC and PISO algorithms at their own optimal time step multiples for different cases of problem 4. When each method uses its own optimal time step multiple, the IDEAL algorithm can reduce the computation time by 40.7–52.7% over the SIMPLER algorithm, by 49.7–67.0% over the SIMPLEC algorithm and by 41.4–53.1% over the PISO algorithm for problem 4.

4.2. Velocity–temperature coupling problems

Problem 5: Natural convection in a cubic cavity. Natural convection in a cubic cavity is a velocity–temperature coupling problem, which is a classical fluid flow and heat transfer problem widely adopted in computational heat transfer community [34, 35]. The flow configuration of problem 5

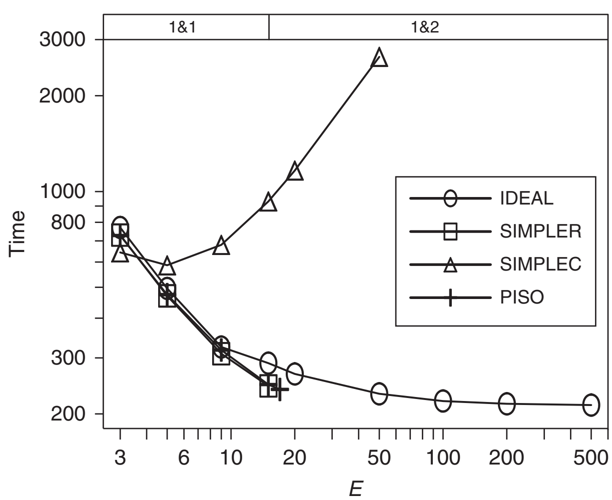

Figure 21. Comparison of computation time and robustness of IDEAL, SIMPLER, SIMPLEC and PISO algorithms for $Ra = 105$ with grid number $= 50\times 50\times 50$ of problem 5.

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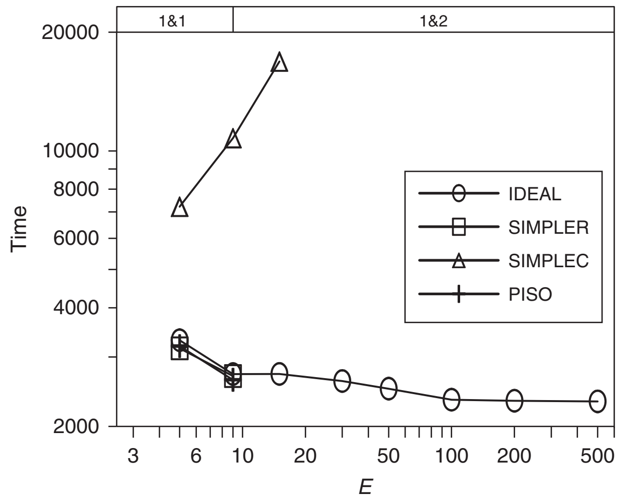

Figure 22. Comparison of computation time and robustness of IDEAL, SIMPLER, SIMPLEC and PISO algorithms for $Ra = 106$ with grid number $= 80\times 80\times 80$ of problem 5.

Table VIII. Reduced ratio of computation time of IDEAL algorithm over SIMPLER, SIMPLEC and PISO algorithms at their own optimal time step multiples in problem 5.

	Grid number		
	$30\times 30\times 30$	$50\times 50\times 50$	$80\times 80\times 80$
Ra	10^4	10^5	10^6
Computation time of IDEAL (s)	25.3	213.2	2314.7
Reducing ratio over SIMPLER (%)	22.4	12.9	13.7
Reducing ratio over SIMPLEC (%)	48.3	63.7	67.9
Reducing ratio over PISO (%)	13.7	10.7	11.9

is shown in Figure 18. The cubic cavity has four adiabatic walls with two vertical walls being maintained at constant but different temperatures.

Calculations are conducted for $Ra = 10^4$–10^6 and grid numbers $= 30\times 30\times 30$–$80\times 80\times 80$ with the residuals $\mathrm{Rs}_{\mathrm{Mass}}$, $\mathrm{Rs}_{U\mathrm{Mom}}$, $\mathrm{Rs}_{V\mathrm{Mom}}$ and $\mathrm{Rs}_{W\mathrm{Mom}}$ being all less than 10^{-7}. The Rayleigh number is defined by

$$Ra = \frac{\rho g \beta H^3 (T_H - T_C)}{a\eta} \tag{26}$$

In Table VII, a comparison is given between the solutions from the IDEAL algorithm and the results from [34, 35]. The comparison concerns the mean Nusselt, Nu_m, which is defined as

$$Nu_m = \frac{\int\int (Nu_{\mathrm{Local}}(y,z)|_{x=0} + Nu_{\mathrm{Local}}(y,z)|_{x=H})\,\mathrm{d}y\,\mathrm{d}z}{2H^2} \tag{27}$$

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where

$$Nu_{\mathrm{Local}}(y,z)|_{x=0 \text{ or } x=\mathrm{H}} = \frac{-(\partial T/\partial x)H}{T_H - T_C} \tag{28}$$

The present results agree very well with the solutions reported by Fusegi *et al.* [34] and Wakashima and Saitoh [35]. In Figure 19, the temperatures at the plane $z=0.5H$ for $Ra=10^6$ are shown. From the figure, we can find that the temperature field calculated by IDEAL algorithm agrees very well with that from the Reference [35].

Figures 20, 21 and 22 show the computation time and robustness of the IDEAL, SIMPLER, SIMPLEC and PISO algorithms for different grid numbers and different Rayleigh numbers of problem 5. From these figures, we can find that the performances of different algorithms in velocity–temperature coupling problems are almost the same as those in decoupled fluid flow problems, for example, problems 1 and 2. The IDEAL algorithm is the most robust and most efficient one among the four algorithms compared.

Table VIII shows the reduced ratio of computation time of the IDEAL algorithm over the SIMPLER, SIMPLEC and PISO algorithms at their own optimal time step multiples for different cases of problem 5. When each method uses its own optimal time step multiple, the IDEAL algorithm can reduce the computation time by 12.9–22.4% over the SIMPLER algorithm, by 48.3–67.9% over the SIMPLEC algorithm and by 10.7–13.7% over the PISO algorithm for problem 5.

5. CONCLUSIONS

In the present paper, the performance of the IDEAL algorithm for three-dimensional incompressible fluid flow and heat transfer problems has been analyzed by a systemic comparison with three other most widely used algorithms (SIMPLER, SIMPLEC and PISO). The main conclusions are as follows:

(1) The IDEAL algorithm is the most robust and most efficient one among the four algorithms compared.
(2) The IDEAL algorithm can converge almost at any time step multiple for the five problems studied.
(3) When each algorithm works at its own optimal time step multiple, the IDEAL algorithm can reduce the computation time by 12.9–52.7% over the SIMPLER algorithm, by 45.3–73.4% over the SIMPLEC algorithm and by 10.7–53.1% over the PISO algorithm.

Because of the significant superiority of the IDEAL algorithm, it is expected that the proposed IDEAL algorithm will be widely adopted in the computations of three-dimensional incompressible fluid flow and heat transfer problems.

The extensions of the three-dimensional IDEAL algorithm to non-orthogonal curvilinear systems and to unstructured grid systems are now underway in the authors' group and will be reported elsewhere.

NOMENCLATURE

a	coefficient in the discretized equation
A	surface area
b	constant term in the discretized equation
d	coefficient in the velocity-correction equation
E	time step multiple
g	gravitational acceleration
$N1$, $N2$	inner doubly-iterative times
p	pressure
q_m	reference mass flow rate
Ra	Rayleigh number
Re	Reynolds number
$\mathrm{Rs}_{\mathrm{Mass}}$	relative maximum mass residual
$\mathrm{Rs}_{U\mathrm{Mom}}, \mathrm{Rs}_{V\mathrm{Mom}}, \mathrm{Rs}_{W\mathrm{Mom}}$	relative maximum u, v, w-component momentum residuals
S	source term
T	temperature
u, v, w	velocity component in x, y, z directions
$\tilde{u}, \tilde{v}, \tilde{w}$	pseudo-velocity
x, y, z	coordinates
α	under-relaxation factor
β	expansion coefficient
η	dynamic viscosity
ν	kinematic viscosity
ρ	density

Subscripts

e, w, n, s, b, t	cell surface
in	inlet
P, E, N, S, W, B, T	grid point
m	mean
nb	neighboring grid points
u, v, w	referring to u, v, w momentum equations

Superscripts

PTemp	temporary value in previous inner iteration step
Temp	temporary value in current inner iteration step
0	initial value
$*$	intermediate value

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