An under-relaxation factor control method for accelerating the iteration convergence of flow field simulation

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Abstract

Purpose – This paper aims to accelerate the iteration convergence for elliptic fluid flow problems, so that an under-relaxation factor control method is developed.

Design/methodology/approach – There should be an optimal under-relaxation factor that can result in the equivalence of the global residual norms of momentum equation u and momentum equation v. The two residual norms of the momentum equations will be equivalent through controlling the velocity under-relaxation factors, and then the iteration convergence can be accelerated. Two expressions ($\alpha = (\alpha^0)^{\beta^\gamma}$ and $\alpha = (\alpha^0)^{(1/\beta)^\gamma}$) are proposed to adjust the values of under-relaxation factors for every n iterations.

Findings – From the five preliminary computations it is found that the value of γ can be larger than 1 and of n can be less than 5 for an open system, and the value of γ should be less than 1 and that of n should be larger than 10 for a closed system. These two pairs of parameters are then used in another five examples. It is found that the saving in CPU times is at least 43.9 percent for the closed system and 67.5 percent for the open system.

Research limitations/implications – When the *Re* or *Ra* of the two-dimensional problems are low, this control method is feasible. More research work is needed in order to apply it in three-dimensional or high *Re* or *Ra* problems.

Originality/value – This method is helpful for the acceleration of iteration convergence in simple problems, and is a preparation for the advanced research in complicated problems.

Keywords Simulation, Flow, Iterative methods

Paper type Research paper

Nomenclature

A	= area of control-volume face
a_e, a_{nb}	= coefficients in the discretized conservation equations
b	= source term in discretized conservation equations
c	= constant
D	= characteristic length of channel or cavity
DIV	= divergence
G	= global residual norm
H	= height
L	= length
n	= iteration number for the control method
p	= pressure
p^*	= intermediate pressure

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p'	= pressure correction	ε	= pre-specified small value to control convergence
R	= residual	ρ	= density
Ra	= Rayleigh number		
Re	= Reynolds number	*Subscripts*	
r	= radial coordinate	c	= lowest temperature
T	= temperature	E, N, P	= grid points
u, v	= velocity components in x and y direction	e, n, p	= cell faces
u^*, v^*	= intermediate velocity	h	= highest temperature
$u'\ v'$	= velocity correction	in	= inlet
x, y	= spatial coordinates	nb	= neighbor points
α	= under-relaxation factor	p	= pressure
β	= exponent in equations (15)–(18)	u, v	= velocity components in x and y direction
γ	= exponent in equations (15)–(18)		

1. Introduction

In numerical analysis of fluid flow and heat transfer problems, iterative methods are frequently adopted in which velocity components are solved in segregated manner and the linkage between velocity and pressure is ensured by the SIMPLE-series algorithm. Since, the leading iterative approach SIMPLE was proposed (Patankar and Spalding, 1972), it has been widely applied in the fields of computational fluid dynamics (CFD) and numerical heat transfer (NHT). Over the past three decades, many variants such as SIMPLER, SIMPLEC, SIMPLEX and so on were developed, which consist the so-called SIMPLE-family solution algorithms. During the development of the SIMPLE-family algorithms, how to accelerate the iteration convergence is one of the key problems for enhancing the solution algorithm.

In SIMPLE-family algorithms, the iteration convergence can be accelerated by three methods (Tao, 2000). First, an explicit correction step for the velocities was suggested (Yen and Liu, 1993). By applying this explicit correction step to the SIMPLE, SIMPLEC and PISO algorithms, significant reductions in the number of iterations and CPU time to achieve convergence were demonstrated. The second method is to choose appropriate values of the under-relaxation factors. Patankar (1980) pointed out that for the SIMPLE algorithm the velocity under-relaxation factor of 0.5 and the pressure under-relaxation of 0.8 were found to be satisfactory in a large number of fluid-flow computations. However, it is recommended that if the computational grid is not severely nonorthogonal, the relation:

$$\alpha_u + \alpha_p = c \tag{1}$$

gives almost the optimum result, where the constant c is 1 (Demirdzic *et al.*, 1987) or 1.1 (Peric, 1990). Later a pressure under-relaxation factor based on the minimization of the global residual norm of the momentum equations was proposed (Chatwani and Turan, 1991). The procedure was applied to SIMPLE and SIMPLEC algorithms to automatically select the pressure under-relaxation factor to minimize the global residual norm of the momentum equations at each iteration level, but a notable increase in convergence was not achieved. Some other researchers (Latimer and Pollard, 1985; Macarthur and Patankar, 1989; Marek and Straub, 1993) all stated the need for a method of automatically optimizing the relaxation factors. Some new methods are developed in recent years to accelerate the iteration convergence. For example, fuzzy mathematics has been developed as a new branch of mathematics in the last 20 years. It is extensively applied in many fields of science and technology such as the control of convergence in CFD simulations (Ryoo *et al.*, 1999; Dragojlovic *et al.*, 2001). Later, this method was improved (Liu *et al.*, 2002). On the other hand, a new algorithm SOAR based on the comparison between SIMPLE algorithm and the Newton-Raphson method for automatically determining the optimum values of relaxation factors for SIMPLE algorithm was developed (Morii and Vierow, 2000). Recently, the SOAR algorithm and the SIMPLE algorithm for computing performance in collocated grid are compared (Morii, 2005). The results showed that the SOAR had better computing performance than that of SIMPLE as the grid is refined.

The other methods for different special cases are grouped into the third method. For example, the SIMPLE-family algorithms will converge slowly when there is rapidly varying pressure in flow field, such as flow through a blunt sampler and dust-laden fluid flow through a paper filter. A method to improve the rate of the convergence for the SIMPLE-family algorithms for such cases is proposed (Wen and Ingham, 1993, 1994).

This paper gives a simpler method for the under-relaxation factor control based on a new idea for elliptic fluid flow simulation, which is different from the above-mentioned methods. The discussion will focus on the 2D recirculating flows. By using the proposed method the two momentum equations can be iterated consistently through controlling the velocity under-relaxation factor, and the iteration convergence can be significantly accelerated. The feasibility of the proposed method is validated by ten typical examples.

2. Under-relaxation factor control method

2.1 Criteria of iteration convergence

Commonly, for an open flow system which has both inflow and outflow boundaries, the criterion for terminating the iteration of nonlinear computation is that the relative residual norm of the momentum equations is less than a pre-specified small value. For example, this pre-specified small value was $\varepsilon \leq 1 \times 10^{-3} \sim 1 \times 10^{-5}$ (Latimer and Pollard, 1985). That is for momentum equation u, if following condition is satisfied:

$$G_u = \frac{\left(\sum_{\text{node}}\left\{a_e u_e - \left[\sum_{\text{nb}} a_{\text{nb}} u_{\text{nb}} + b + A_e(p_P - p_E)\right]\right\}^2\right)^{1/2}}{\rho u_{in}^2 \leq \varepsilon} \tag{2}$$

the convergence of u momentum equation is reached. In this paper, this pre-specified value is taken as small as 1×10^{-8}.

For a closed system, a numerical integration can be made for the momentum equation along any section in the field to obtain the reference momentum shown in the denominator of equation (2).

For a two-dimensional elliptic flow field, there are two norms of momentum equations of x direction and y direction, respectively, which are denoted by G_u and G_v. Therefore, the criteria for iteration convergence is:

$$\max(G_u, G_v) \leq \varepsilon \tag{3}$$

It usually takes much time to make the maximum of G_u and G_v to be less than the same pre-specified small value since the two values of G_u and G_v are not always in the same order.

2.2 Analysis of the existence of an optimum velocity under-relaxation factor

For the SIMPLE algorithm, the revised velocities in the correction step are:

$$u = u^* + u' \quad v = v^* + v' \quad p = p^* + p' \tag{4}$$

For momentum equation u, the residual is given by:

$$\begin{aligned} R_u &= a_e u_e - \sum a_{\text{nb}} u_{\text{nb}} - b - A_e(p_P - p_E) \\ &= a_e u_e - \sum a_{\text{nb}} u_{\text{nb}} - b - A_e[(p_P^* + \alpha_p p'_P) - (p_E^* + \alpha_p p'_E)] \\ &= R_{u,0} - \alpha_p A_e(p'_P - p'_E) \end{aligned} \tag{5}$$

where:

$$R_{u,0} = a_e u_e - \sum a_{\text{nb}} u_{\text{nb}} - b - A_e(p_P^* - p_E^*) \tag{6}$$

A similar equation can be written for the v momentum equation. The global residual norm is given by, therefore:

$$G = \sum (R_u^2 + R_v^2) \tag{7}$$

Chatwani and Turan (1991) considered that G could be minimized with respect to the under-relaxation factor for the pressure correction α_p by setting the derivative to be zero. This leads to following equation for α_p:

$$\alpha_p = \frac{\sum \left[R_{u,0} A_e(p'_P - p'_E) + R_{v,0} A_n(p'_P - p'_N)\right]}{\sum \left[A_e^2(p'_P - p'_E)^2 + A_n^2(p'_P - p'_N)^2\right]} \tag{8}$$

Assuming that the difference of the pressure corrections between two adjacent points is everywhere identical and is denoted by $\Delta p'$. The above equation for α_p can be rewritten as:

$$\alpha_p = \frac{\sum (R_{u,0} A_e + R_{v,0} A_n)}{\sum (A_e^2 \Delta p' + A_n^2 \Delta p')} \tag{9}$$

Combining the two residuals of the momentum equations with equation (8) the following relations can be derived:

$$R_u = \frac{R_{u,0} \sum A_e^2 - A_e \sum (R_{u,0} A_e) + R_{u,0} \sum A_n^2 - A_e \sum (R_{v,0} A_n)}{\sum (A_e^2 + A_n^2)} \tag{10}$$

$$R_v = \frac{R_{v,0}\sum A_n^2 - A_n\sum(R_{v,0}A_n) + R_{v,0}\sum A_e^2 - A_n\sum(R_{u,0}A_e)}{\sum(A_e^2 + A_n^2)} \tag{11}$$

For a grid system with $A_e = A_n$, assuming that the value of $R_{u,0}$ on all points are identical, and the value of $R_{v,0}$ on all points are identical, too, we can get the following relation:

$$R_u = -R_v = \frac{\sum(R_{u,0}A_n^2) - \sum(R_{v,0}A_eA_n)}{\sum(A_e^2 + A_n^2)} \tag{12}$$

That is:

$$G_u = G_v \tag{13}$$

Above analysis shows that the optimal pressure under-relaxation factor proposed by Chatwani and Turan (1991) can lead to the equivalence between the two global residual norms of momentum equations u and v under the above four assumptions ($\Delta p'$ identical, $A_e = A_n$, $R_{u,0}$ identical and $R_{v,0}$ identical). And it is the authors' believe that the equivalence of the two residuals will greatly help to improve the convergence. Such special case, of course, can hardly occur in practical numerical simulations. However, if we can somehow make the two residuals approximately the same, the iteration convergence rate will also be accelerated. Since, the velocity under-relaxation α_u, α_v and the pressure under-relaxation factor are inherently related, say equation (1) (Demirdzic *et al.*, 1987), we have tried to reach such a situation by controlling the velocity underrelaxation factor. In the following our numerical findings will be presented.

2.3 Velocity under-relaxation factor control method

The above analysis illustrates that there may be an optimal velocity under-relaxation factor which can make the two residual norm of the two momentum equations strictly the same. And it is the authors' believe that when such an under-relaxation factor is used, the convergence will be accelerated. As how to find such a relaxation factor, it is a matter of experiences. Through a great number of numerical practices we developed a velocity under-relaxation factor control method, which can approximately realize the above purpose. The velocity under-relaxation factor control method is now described as follows.

At first, the ratio between G_u and G_v is denoted by β, i.e.:

$$\beta = \frac{G_v}{G_u} \tag{14}$$

The optimal velocity under-relaxation factor can be adjusted for three different categories as follows:

(1) β is less than 1 when the velocity under-relaxation factors are small, or β is larger than 1 when the velocity under-relaxation factors are large. The controlling relation is defined as:

$$\alpha = (\alpha^0)^\beta \tag{15}$$

where α is the velocity under-relaxation factor used for the present level, α^0 the under-relaxation factor used in the previous iteration.

(2) β is larger than 1 when the under-relaxation factors are small, and β is less than 1 when the under-relaxation factors are large. The controlling relation is defined as:

$$\alpha = (\alpha^0)^{1/\beta} \tag{16}$$

(3) When the adoption of one of the above rules leads to a severe change in the velocity under-relaxation factor, the iteration process may be deteriorated and even leads to diverge. If this happens, i.e. the change of the velocity under-relaxation factor is over the given range (the upper limit of the velocity under-relaxation factor is taken as 0.98) with one of the two relations, the velocity under-relaxation factor should be re-adjusted. This re-adjustment is very simple: use the other relation instead of the one used before. That is, when use of the first relation leads to an acute variation of the velocity under-relaxation factor, then use the second relation to get a new value, and vice versa.

In order to further optimize the under-relaxation factor through the above two relations, two more parameters, γ and n, are introduced. The two relations, equations (15), (16) can be rewritten as:

$$\alpha = (\alpha^0)^{\beta^\gamma} \tag{17}$$

$$\alpha = (\alpha^0)^{(1/\beta)^\gamma} \tag{18}$$

The value of γ is always larger than zero. It is used to adjust the value of the exponent in the two relations and enhance or alleviate the variation of the under-relaxation factor value between two iteration levels. For instance, the iteration may diverge with an acute variation of the under-relaxation factor, and then reducing the value of γ is helpful to alleviate this phenomenon. The value of β may change severely if we adjust the under-relaxation factor each iteration, especially at the beginning of the iteration, which may decelerate the iteration convergence or even leads to the divergence of the iteration. So, we should adopt the method after several iterations from the beginning of the computation, then adjust the under-relaxation factor every n iterations. Additionally, to avoid the severe change of β, the range of β is given from 0.2 to 5.0. For a common example, the method can be adopted after 200 iterations from the beginning of the computation. Therefore, n is regarded as another parameter to enhance the convergence. From the following examples, we can see that an appropriate value of n is very important to the iteration convergence and some suggested value of n will be presented.

In the following presentation, the results of preliminary numerical tests will first be presented, from which the values of γ and n will be obtained through detail numerical practices for five selected problems. Then these values are used in another five more examples directly, without any try and error computations. If the suggested values can still lead to a significant saving of computational times for the five additional examples, then the feasibility of the proposed method can be considered justified in some extent. Fortunately, it is the case.

3. Preliminary test examples

Five flow and heat transfer problems (lid-driven cavity, flow in a 2D axisymmetric sudden expansion, flow over a backward-facing step, flow in annulus with the inner wall rotating about the axis and natural convection in a square cavity) are used to validate the performance of the method and to find out the suggested values of γ and n. The governing equations are discretized by the control volume method. The SIMPLEC algorithm is adopted to deal with the coupling between velocity and pressure, where the pressure correction under-relaxation factor is fixed at 1 (van Doormaal and Raithby, 1984; Tao, 2001). And the velocity under-relaxation factor is dealt with by the proposed control method. The pre-specified small value ε is 1×10^{-8}. Computations are also conducted for $\alpha_u = \alpha_v = 0.5$ and the results by using these two under-relaxation factors will be compared with the results obtained by adopting the proposed control method. We have tried different constants of α_u and α_v. The results show that the computational CPU time is different under different initial values of α_u and α_v. For different cases, the optimal values of α_u and α_v are also different. Hence, the saving in CPU time are different by our method when different initial α_u and α_v are used. However, since for most cases people usually try to use $\alpha_u = \alpha_v = 0.5$ as suggested by Patankar, therefore, such practice is adopted in this paper. To compare the CPU time between the velocity under-relaxation factor control method and $\alpha_u = \alpha_v = 0.5$ the relative time is adopted, i.e. for an example, the CPU time under $\alpha_u = \alpha_v = 0.5$ is divided by the CPU time under different cases.

3.1 Lid-driven cavity

The schematic of lid-driven cavity flow is shown in Figure 1. Computation is conducted for $Re = 100$ ($Re = UD/v$), and the grid system used is 52×52. Table I shows the CPU time under different values of γ and n. It can be observed that the saving in CPU time is

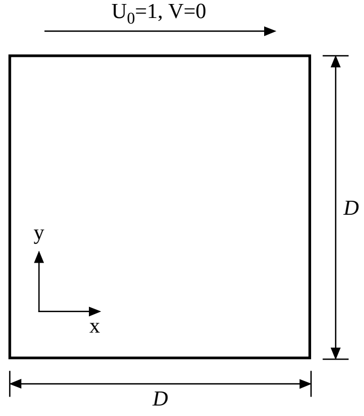


Figure 1. Lid-driven cavity ($Re = 100$)

Table I. CPU time of lid–driven cavity (s)

n	1	2	3	4	5	10	15	20	30	50	$\alpha_u = 0.5$
$\gamma = 0.5$	0.307	0.276	0.210	0.194	0.200	0.250	0.290	0.334	0.398	0.483	1
$\gamma = 0.8$	0.301	0.328	0.276	0.231	0.201	0.213	0.244	0.274	0.328	0.403	
$\gamma = 1.0$	0.293	0.324	0.295	0.256	0.212	0.201	0.224	0.255	0.298	0.363	
$\gamma = 1.2$	0.326	0.325	0.307	0.269	0.237	0.191	0.209	0.238	0.282	0.339	
$\gamma = 1.5$	0.271	0.323	0.306	0.336	1.027	0.195	0.203	0.221	0.257	0.302	

80.9 percent with the appropriate value of γ and n compares to $\alpha_u = 0.5$. The iteration can converge under different value of n with the smaller value of γ. When the value of γ becomes larger, the iteration is easy to be diverged. Additionally, Figure 2 shows the relationship between iteration times and under-relaxation factor with $n = 10$. It can be observed that when the value γ is smaller, the variation of the under-relaxation factor is not significant. When the value of γ becomes larger, the variation of the under-relaxation factor becomes acute. Therefore, for this closed system, n is about 10 and γ can be larger than 1.

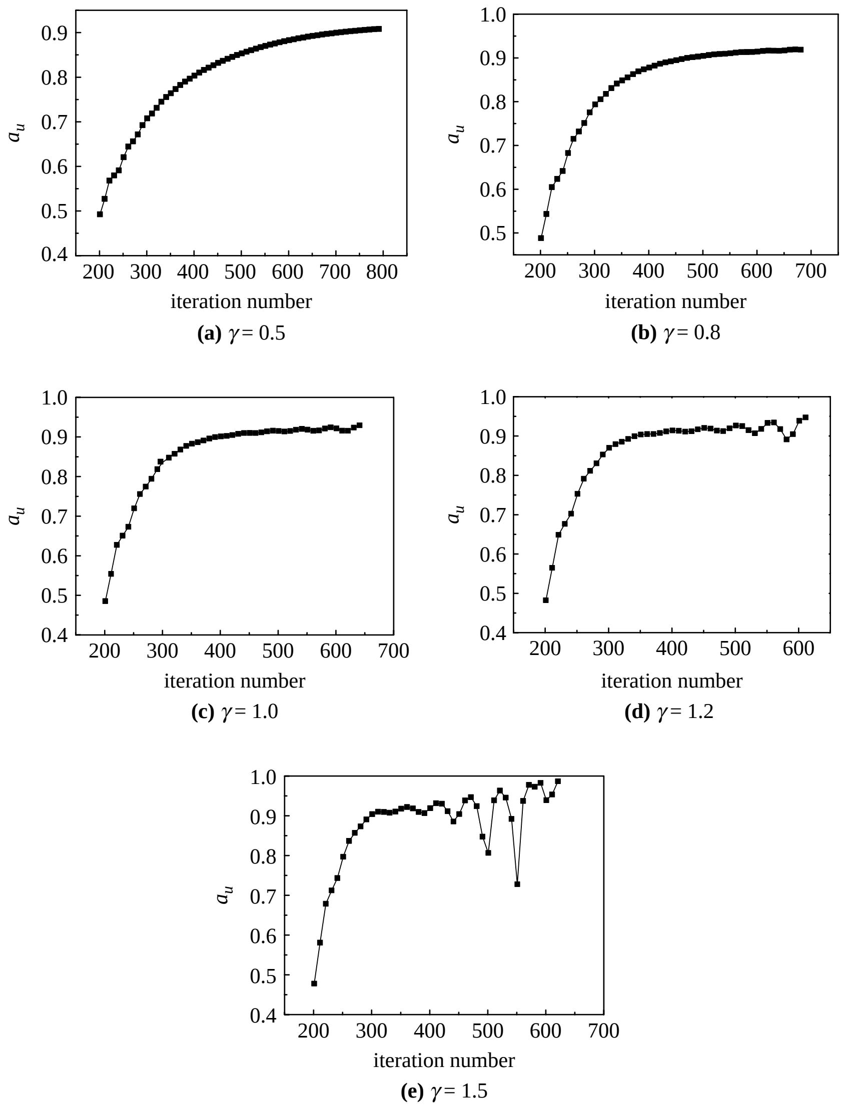


Figure 2.
The relationship between iteration times and under-relaxation factor

3.2 Flow in a 2D axisymmetric sudden expansion

Figure 3 shows the schematic of flow in a 2D axisymmetric sudden expansion. The sizes in computational region are $L_x/D_{\text{in}} = 60$, $L_{\text{in}}/D_{\text{in}} = 10$, $D/D_{\text{in}} = 2$. $Re = 150$ ($Re = UD/v$). The grid system used is 102 × 22. Table II shows the CPU time under different values of γ and n. It can be observed that the saving in CPU time is 70.0 percent with the appropriate value of γ and n compares to $\alpha_u = 0.5$. As shown in Figure 2 and 4, shows that the variation of the under-relaxation factor becomes acute when the value of γ becomes larger and $n = 5$. Additionally, we also find that the under-relaxation factor was over the given range in this example and it fell into the third category described in the second section. So the under-relaxation factor was re-adjusted. It is finally found that a larger value of γ can accelerate the iteration convergence. Thus, for this open system, γ can be larger than 1, and n can be given by a relative small value.

3.3 Flow over a backward-facing step

The schematic of flow over a backward-facing step is shown in Figure 5. The sizes in computational region are $L_x/H_1 = 60$, $L_{\text{in}}/H_1 = 10$, $H_2/H_1 = 2$. $Re = 100$ ($Re = UD/v$). The grid system used is 102 × 32. Table III shows the CPU time under different values of γ and n. It can be observed that the saving in CPU time is 74.3 percent with the appropriate value of γ and n compared to $\alpha_u = 0.5$. Figure 6 shows the relationship between iteration times and under-relaxation factor with $n = 5$. The computational results of this example are similar to the computational results of flow in a 2D axisymmetric sudden expansion.

3.4 Flow in annulus with the inner wall rotating about the axis

The schematic of flow in annulus with the inner wall rotating about the axis is shown in Figure 7. $Re = Ur_i/v = 100$. The grid system used is 52 × 52. Table IV shows the CPU time under different values of γ and n. It can be observed that the saving in CPU time is

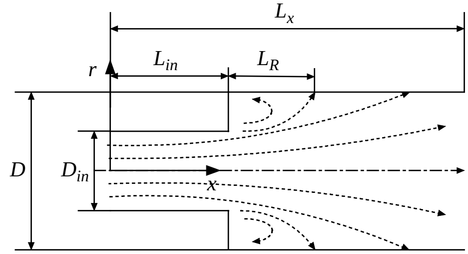


Figure 3. Schematic of flow in a 2D axisymmetric sudden expansion ($Re = 150$)

n	1	2	3	4	5	10	15	20	30	50	$\alpha_u = 0.5$
$\gamma = 0.5$	0.309	0.325	0.362	0.382	0.395	0.403	0.409	0.405	0.428	0.429	1
$\gamma = 0.8$	0.314	0.307	0.332	0.353	0.361	0.392	0.402	0.397	0.397	0.409	
$\gamma = 1.0$	DIV	0.306	0.320	0.330	0.341	0.391	0.397	0.394	0.384	0.426	
$\gamma = 1.2$	DIV	0.300	0.307	0.312	0.325	0.383	0.399	0.367	0.387	0.396	
$\gamma = 1.5$	DIV	DIV	DIV	0.310	0.321	0.372	0.351	0.342	0.391	0.407	

Note: DIV – divergence

Table II. CPU time of flow in a 2D axisymmetric sudden expansion (s)

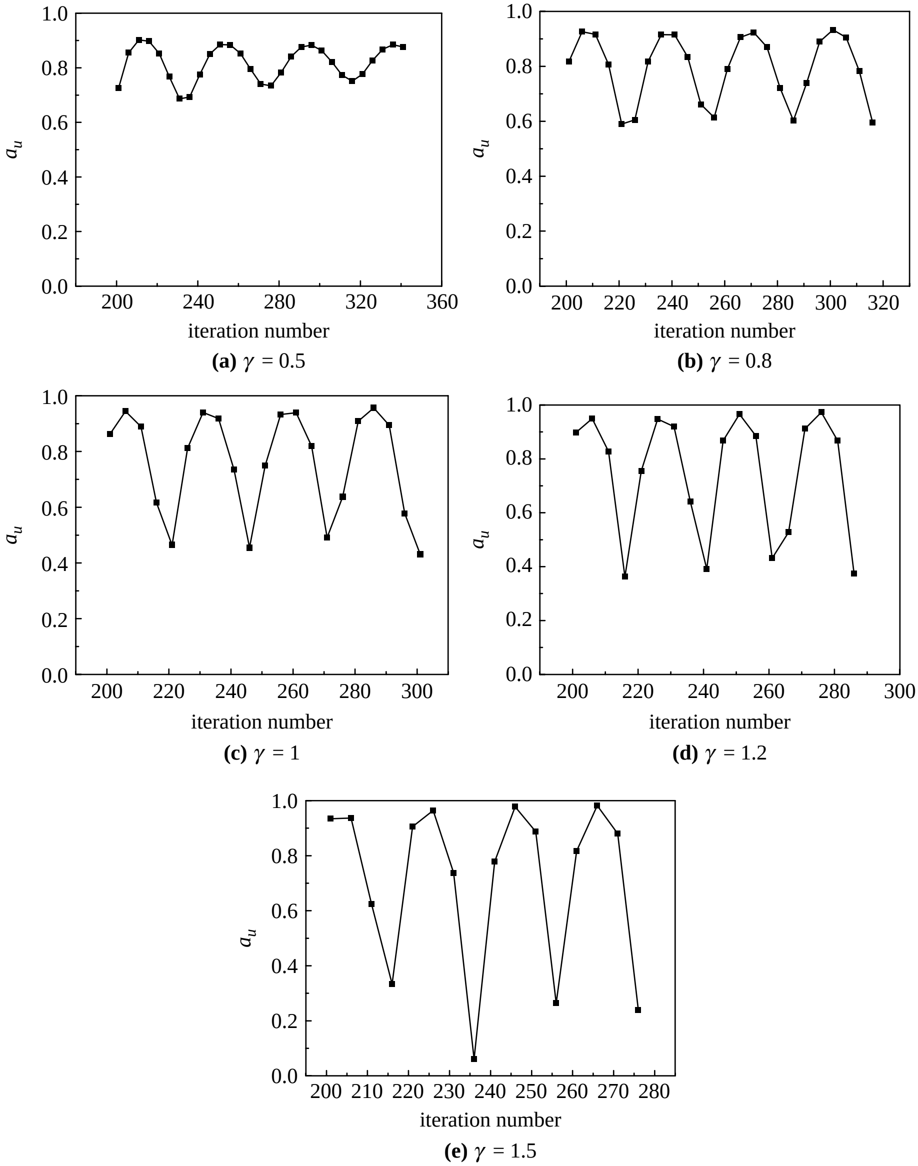


Figure 4.
The relationship between iteration times and under-relaxation factor

43.9 percent with the appropriate value of γ and n compares to $\alpha_u = 0.5$. A smaller value of γ is good for accelerating the iteration convergence. Figure 8 shows the relationship between iteration times and under-relaxation factor with $n = 10$. When the value of γ is larger than 1, the variation of the value of the under-relaxation factor is acute. So, for this closed system, the value of γ should be less than 1, and n should be given by a relative large value.

3.5 Natural convection in a square cavity

The schematic of natural convection in a square cavity is shown in Figure 9. Computation is conducted for $Ra = 10^4$ ($Ra = \rho^2 g\beta\Delta T D^3 Pr/\mu^2$), and the grid system used is 42 × 42. Table V shows the CPU time under different values of γ and n. It can be observed that the saving in CPU time is 53.4 percent with the appropriate value of γ and n compares to $\alpha_u = 0.5$. A smaller value of γ is good for accelerating the iteration convergence. Figure 10 shows the relationship between iteration times and under-relaxation factor with $n = 10$. It can be found that when the value of γ is small, the value of under-relaxation factor changes from small to large, and when the value of γ becomes larger, the value of the under-relaxation factor becomes larger too. So, for this closed system, the value of γ should be less than 1, and n should be given by a relative large value.

4. Application results and discussion

As discussed above, if the two residual modules of the momentum equations are in the same order namely the value of β is nearly to be 1, the iteration will be easy to be converged. Then the key issue is whether we can propose some values of γ and n which can make β around 1 and, more important, are the proposed values common to a series of flow problems, so that one can directly adopt them in the flow computation without any preliminary test. We choose the lid-driven cavity flow and flow in a 2D axisymmetric sudden expansion as the representatives of an open system and a closed system, respectively. The results of the effects of γ on β for the two selected representative problems are shown in Figures 11 and 12, respectively. It can be observed that for the two systems, the value of β is easy to be 1 when the value of γ is small, while with increasing γ the approach of 1 for the value of β gradually

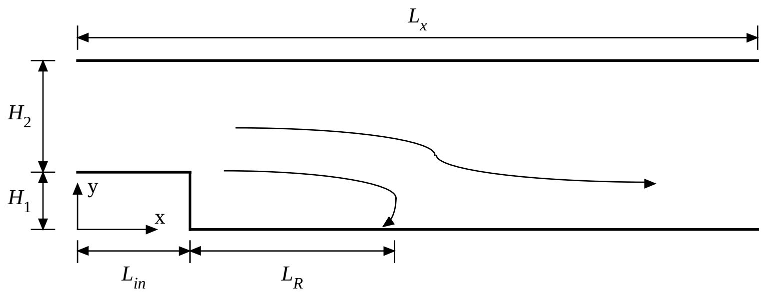


Figure 5. Schematic of flow over a backward-facing step ($Re = 100$)

Table III. CPU time of flow over a backward-facing step (s)

n	1	2	3	4	5	10	15	20	30	50	$\alpha_u = 0.5$
$\gamma = 0.5$	DIV	0.257	0.278	0.295	0.303	0.324	0.335	0.334	0.347	0.342	1
$\gamma = 0.8$	DIV	DIV	0.262	0.262	0.279	0.304	0.325	0.314	0.331	0.317	
$\gamma = 1.0$	DIV	DIV	DIV	0.257	0.262	0.295	0.311	0.310	0.317	0.334	
$\gamma = 1.2$	DIV	DIV	DIV	DIV	0.257	0.289	0.307	0.304	0.305	0.317	
$\gamma = 1.5$	DIV	DIV	DIV	DIV	0.257	0.282	0.282	0.278	0.281	0.318	

Note: DIV – divergence

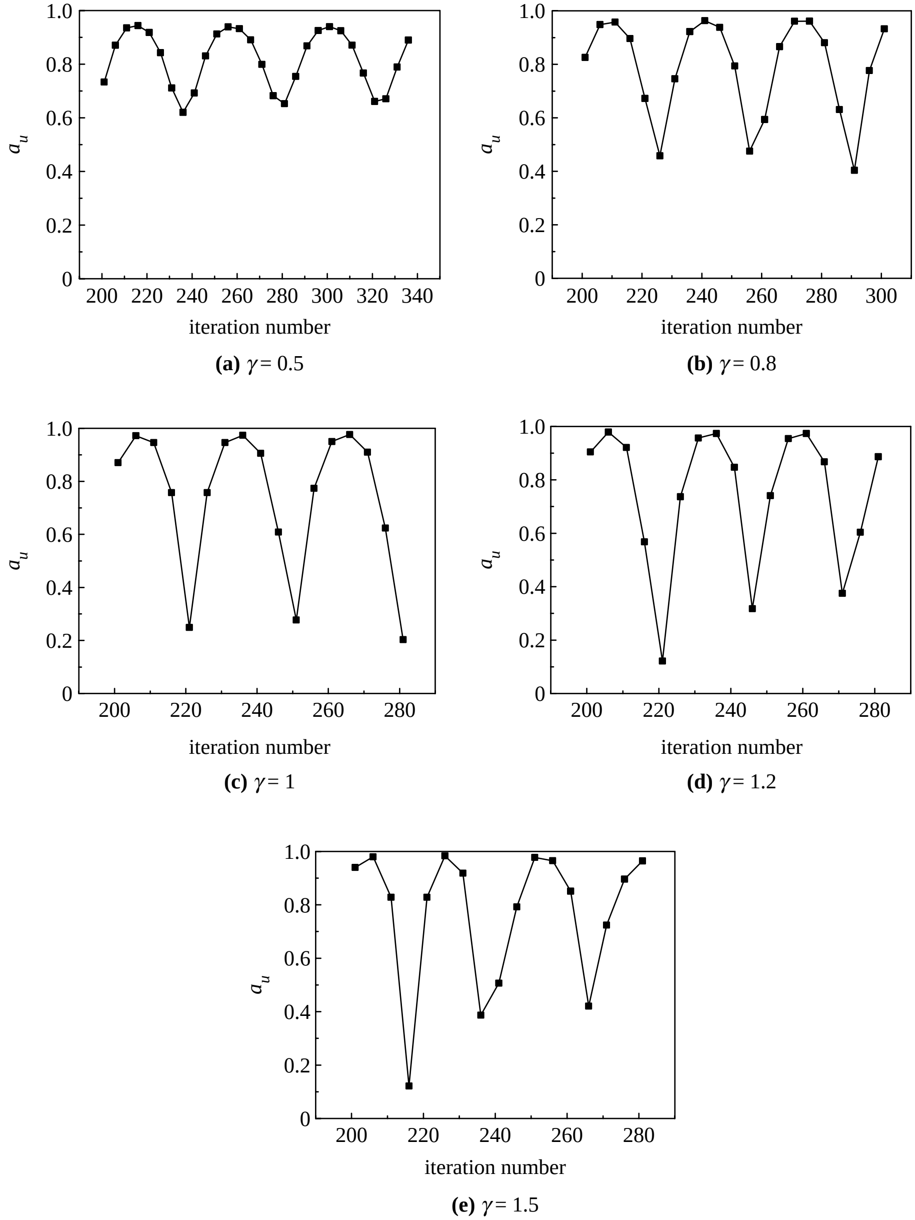


Figure 6.
The relationship between iteration times and under-relaxation factor

becomes difficult. For lid-driven cavity, for example, the value of β is oscillating when γ is equal to 1.5. It elucidates that the small value of γ is good for iteration convergence. On the other hand, the large value of γ can accelerate the convergence for an open system. Combined the results above, we can draw the conclusions that the value of

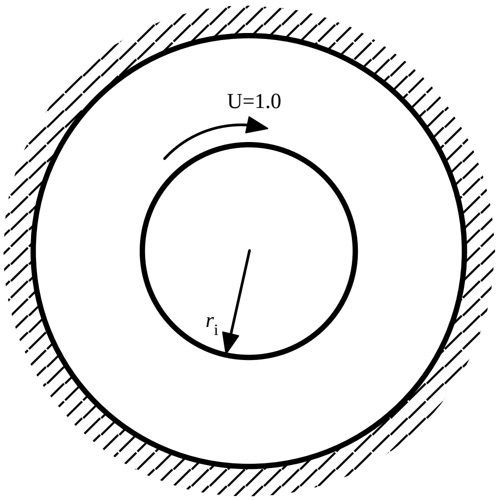


Figure 7. Schematic of flow in annulus with the inner wall rotating about the axis ($Re = 100$)

Table IV. CPU time of flow in annulus with the inner wall rotating about the axis (s)

n	1	2	3	4	5	10	15	20	30	50	$\alpha_u = 0.5$
$\gamma = 0.5$	0.810	0.635	0.596	0.594	0.586	0.611	0.629	0.657	0.874	0.816	1
$\gamma = 0.8$	1.702	0.696	0.626	0.619	0.589	0.579	0.609	0.609	0.853	0.756	0
$\gamma = 1.0$	4.613	0.737	0.678	0.632	0.628	0.561	0.617	0.585	0.858	0.734	0
$\gamma = 1.2$	DIV	0.972	0.696	0.671	0.670	0.637	0.591	0.599	0.880	0.692	0
$\gamma = 1.5$	1.203	1.488	0.792	0.688	0.773	0.682	0.577	0.667	0.903	0.663	0

Note: DIV – divergence

γ can be larger than 1 and of n can be less than 5 for an open system, and the value of γ should be less than 1 and of n should be larger than 10 for a closed system.

The method proposed by Chatwani and Turan (1991) accompanied with equation (13) is also adopted for the above five examples. It is found that by using this method both pressure under-relaxation and velocity under-relaxation are very unstable, which may even results in the iteration divergence. Thus, it seems that the method is not common for most other cases although it has a good character for some special cases presented in that paper.

In order to evaluate the feasibility of the proposed method, another five examples, such as natural convection in an annual cavity ($Ra = \rho^2 g\beta\Delta T(r_o - r_i)^3\Pr/\mu^2 = 10^4$), natural convection in a vertical annual pipe ($Ra = \rho^2 g\beta\Delta T(r_o - r_i)^3\Pr/\mu^2 = 10^4$), isolated island problem ($Ra = \rho^2 g\beta\Delta TD^3\Pr/\mu^2 = 10^5$), flow in a 2D axisymmetric finned tube ($Re = UD/v = 200$, where D is the diameter of the tube) and flow over a parallel finned channel ($Re = UD/v = 200$, where D is the distance between the two plate) are used to validate the method under the values of γ and n basically obtained from the preliminary tests, and are fixed at 1.2 and 5, respectively, for the open system, and 1.0 and 10, respectively, for closed system. The schematics of the five examples are shown in Figure 13 and the computational results are shown in Table VI. In the table, the CUP times required when $\alpha_u = 0.5$ are also shown.

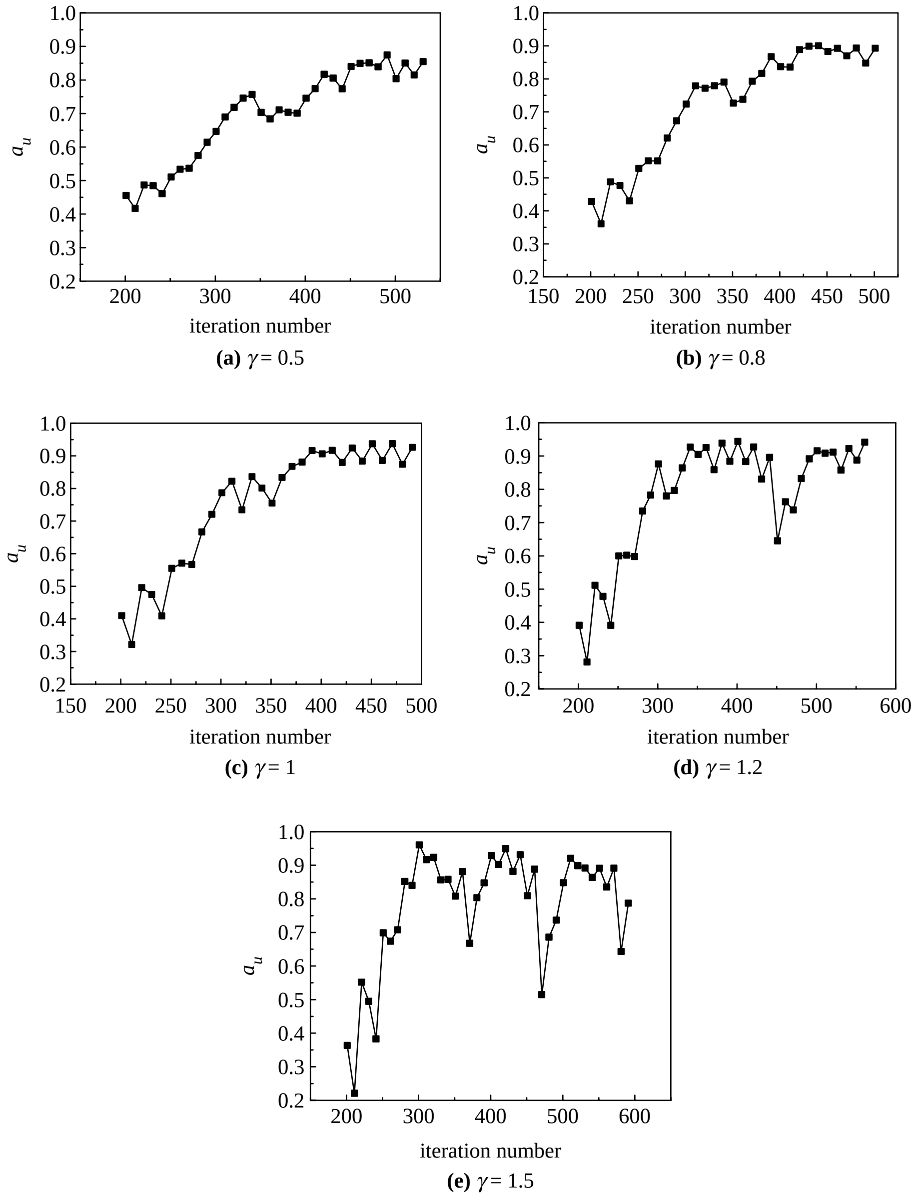


Figure 8.
The relationship between iteration times and under-relaxation factor

Table VI also shows the good performance of proposed method for the under-relaxation factor control. Based on the above examples it may be recommended that for the proposed method the values of γ and n are 1.2 and 5, respectively, for an open system and 1.0 and 10, respectively, for a closed system.

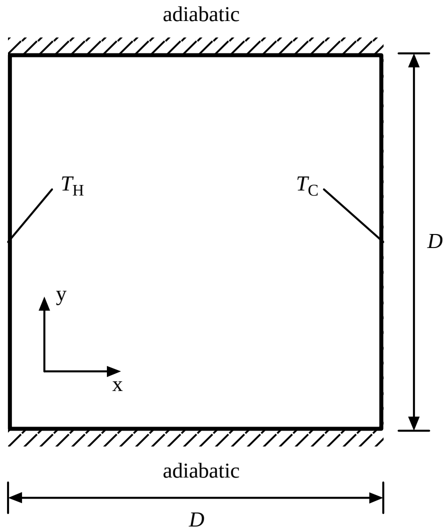


Figure 9. Schematic of natural convection in a square cavity ($Ra = 10^4$)

Table V. CPU time of natural convection in a square cavity (s)

n	1	2	3	4	5	10	15	20	30	50	$\alpha_u = 0.5$
$\gamma = 0.5$	DIV	0.475	0.466	0.472	0.485	0.571	0.614	0.687	0.660	0.846	1
$\gamma = 0.8$	DIV	DIV	DIV	0.468	0.474	0.527	0.547	0.627	0.550	0.775	
$\gamma = 1.0$	DIV	DIV	DIV	0.471	0.481	0.527	0.543	0.581	0.531	0.756	
$\gamma = 1.2$	DIV	DIV	0.513	DIV	0.474	0.493	0.523	0.550	0.535	0.718	
$\gamma = 1.5$	DIV	DIV	0.514	0.833	DIV	0.466	0.535	0.528	0.510	0.685	

Note: DIV – divergence

Finally, the implementation of the proposed control methods is described here. In the program, the initial value of the velocity under-relaxation factor is 0.5, the given upper limit of velocity under-relaxation factor is 0.98, and the given range of β is 0.2 to 5.0. After 200 levels from the beginning of the computation with SIMPLEC algorithm, it computes the value of β, and one of the relations (17) and (18) is adopted first to update the velocity under-relaxation factor. If the velocity under-relaxation factor is over the given upper limit, the other relation is automatically used at the next update. In the subsequent computation the velocity under-relaxation factor is updated for every n iterations until the convergence is reached.

Finally, we would like to make some further discussion related to the initial field assumption and the application of the present method to 3D cases. First, it is well-known that the initial fields strongly affect the iteration convergence (Patankar, 1980; Tao, 2001). A good initial field can improve the iteration convergence. In the present work, the inlet condition is used as initial field for open systems, and for closed systems the zero velocities in the computational domain are adopted as the initial fields. To the authors' knowledge, such initial fields can be simply carried out and adopted easily by all researchers and no any specific techniques are required. And for

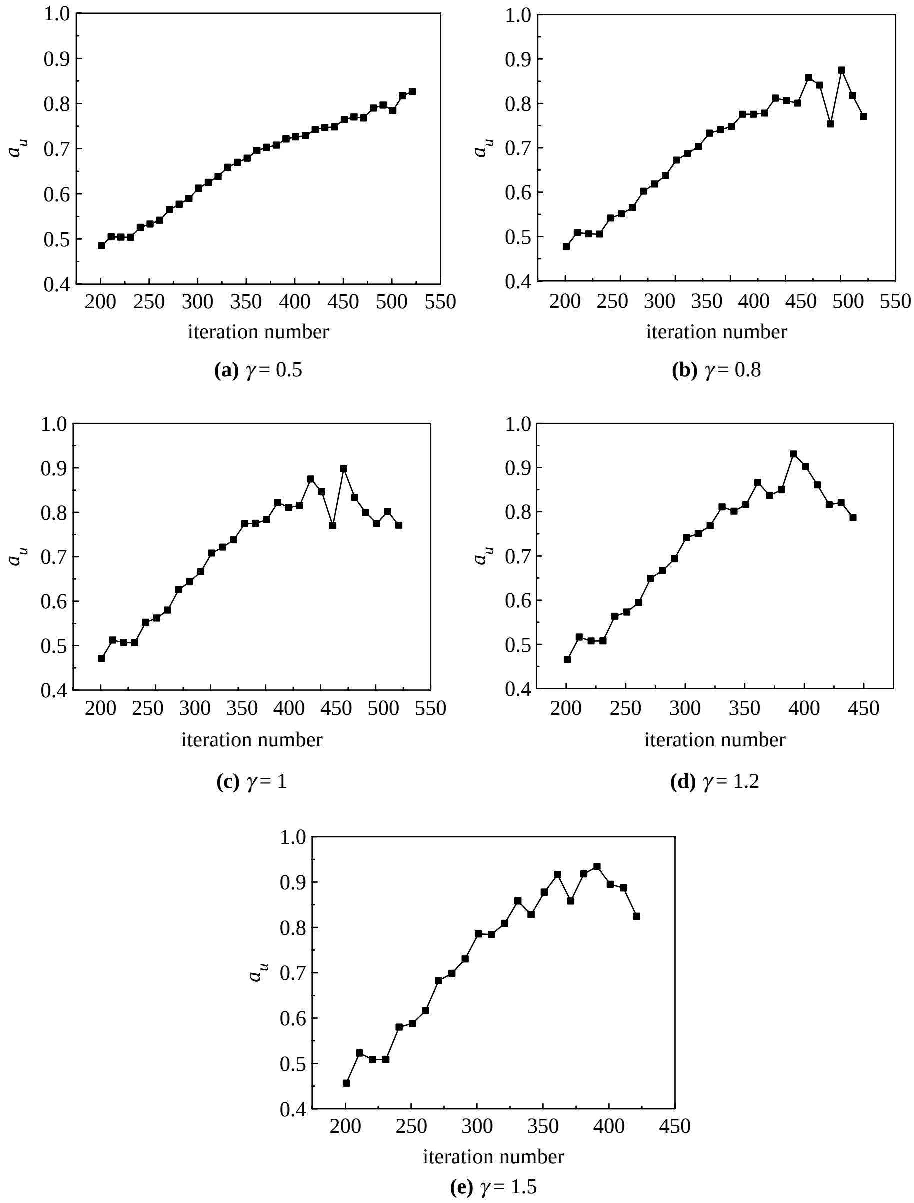


Figure 10.
The relationship between iteration times and under-relaxation factor

such initial fields, our method works well. Thus, we believe that our method is useful since it can effectively converge the iteration by the very simple and straightforward initial field assumptions. As far as the three dimensional cases are concerned, there are three global residual norms for 3D case. Whether the convergence will be accelerated if the residual norms of the three momentum equations are more or less equal and how to

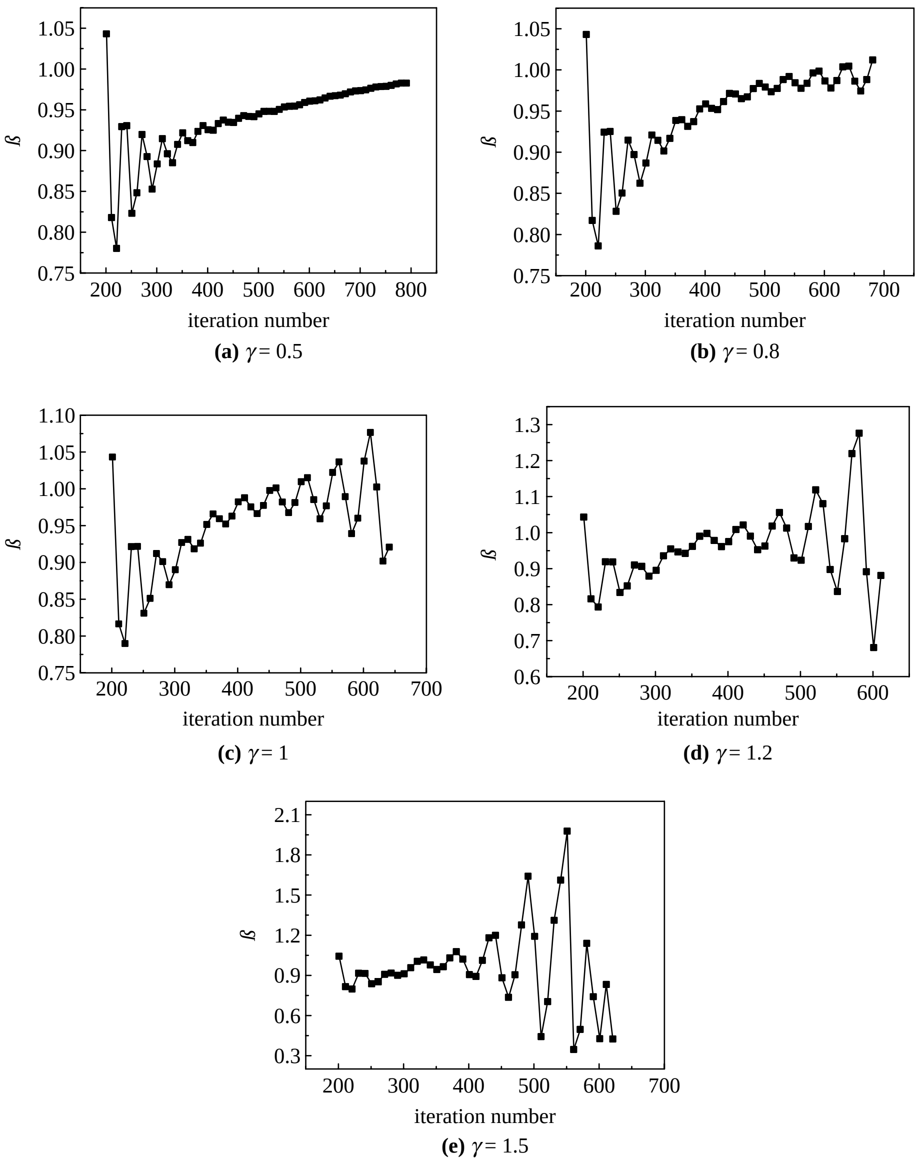


Figure 11. The value of β in lid-driven cavity

apply the method for high *Re* or *Ra* cases, more research work is needed. And the work is underway in the authors' group.

5. Conclusions

An under-relaxation factor control method is developed to accelerate iteration convergence of flow field computation. The good performance of this method is

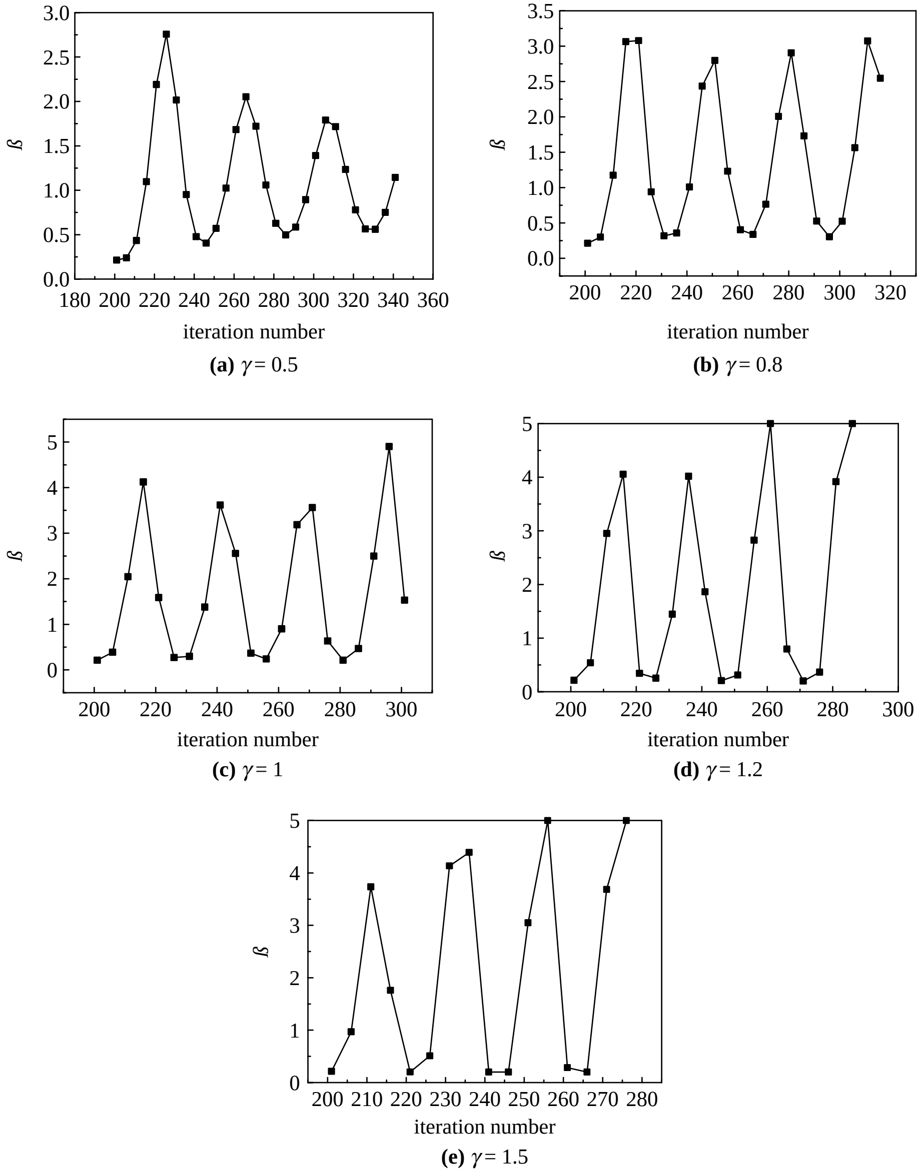
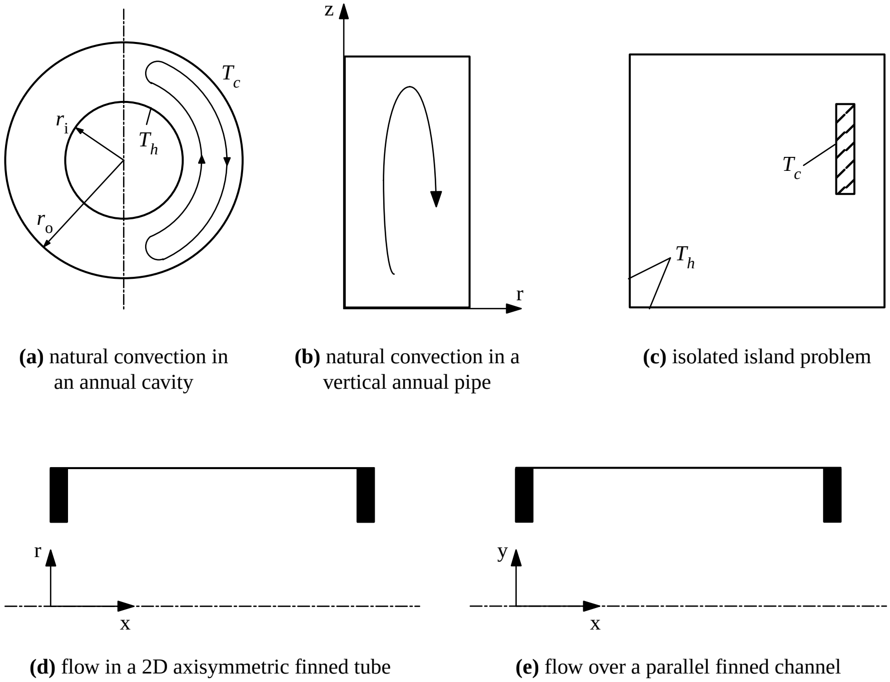


Figure 12.
The value of β in flow in a 2D axisymmetric sudden expansion

validated by ten typical 2D flow and heat transfer examples. The following conclusions are drawn:

- For 2D recirculating flows when the residuals of the two momentum equations are nearly the same, the iteration will soon converge. The ratio of the two residuals, β, is the indication of the consistency of the two residuals.



(a) natural convection in an annual cavity
(b) natural convection in a vertical annual pipe
(c) isolated island problem
(d) flow in a 2D axisymmetric finned tube
(e) flow over a parallel finned channel

Figure 13. Schematic of another five examples

Table VI. CPU time of the five examples (s)

Example	γ	n	CPU time of the proposed method	$\alpha_u = 0.5$	CPU time saved (percent)
Natural convection in an annual cavity	1.0	10	0.362	1	63.8
Natural convection in a vertical annual tube	1.0	10	0.435	1	56.5
Isolated island problem	1.0	10	0.455	1	54.5
Flow in a 2D axisymmetric finned tube	1.2	5	0.186	1	81.4
Flow over a parallel finned channel	1.2	5	0.146	1	85.4

- In order to make β being around 1, different relations are given for adjusting the velocity under-relaxation factor. Two parameters γ and β, are introduced in the re-determination of the under-relaxation factor by equations (15) and (16). The adjustment of α_u should be conducted every n iterations.
- Preliminary tests show that the value of γ can be larger than 1 and of n can be less than 5 for an open system, and the value of γ should be less than 1 and of n should be larger than 10 for a closed system. The two pairs of recommended values are: γ and n equal 1.2 and 5, respectively, for an open system and 1.0 and 10, respectively, for a closed system.
- Five flow and heat transfer problems are used to validate the proposed method. When the proposed values of the two parameters are used, compared with

$\alpha_u = 0.5$, the CPU time can be saved from 67.5 to 85.4 percent and from 43.9 to 79.9 percent for an open system and for a closed system, respectively.

Finally, it should be noted that the proposed under-relaxation factor control method can be used for the elliptic problems, but not for the parabolic problems. And more research work is needed in order to apply the method for high *Re* and *Ra* cases and 3D situations.

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