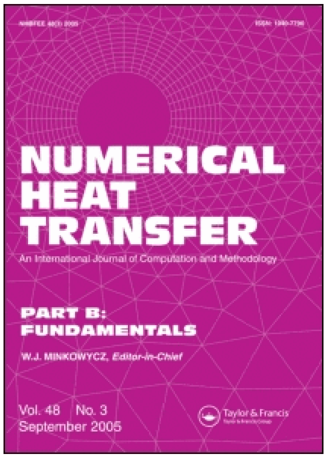

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DISCUSSION ON NUMERICAL TREATMENT OF PERIODIC BOUNDARY CONDITION FOR TEMPERATURE

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For periodic fully developed fluid flow and heat transfer problems, it is sufficient to confine attention to a single cycle. The temperature periodic conditions at the computational domain inlet/exit for fluid temperature can be implemented by two methods: either by extending the computational domain by several control volumes and replacing the field data at the domain inlet and outlet by each other, or by using linear interpolation while restricting the computation within one module. By carefully examining the numerical solutions of the fluid streamwise bulk temperature obtained from the two methods, it is found that at the fluid bulk temperature varies abruptly at the domain exit. A comprehensive analysis reveals that this abnormal phenomenon is caused by the fact that the domain exit fluid bulk temperature is actually not updated with iteration. A remedy is proposed: The domain exit bulk temperature is updated by an upwind-based interpolation method. Numerical examples show that the second-order interpolation method is feasible and reliable.

INTRODUCTION

Channels whose geometric shapes are periodic are widely used in compact heat exchangers to enhance heat transfer. Usually the streamwise cycle (module) number is large enough such that in the major part of such channels the fluid flow and heat transfer can be regarded as periodically fully developed. The concept of periodically fully developed flow and heat transfer was first developed by Patankar et al. [1]. Under the periodically fully developed condition, the velocity components vary periodically in the main flow direction and the pressure can be transformed to a periodic function after subtraction of a linear term which is related to the global mass flow rate. For the case of uniform wall temperature boundary condition, the dimensional fluid temperature approaches more and more closely the wall temperature, therefore the dimensional fluid temperature field is not periodic. However, an

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NOMENCLATURE

C_p	specific heat at constant pressure	T_w	wall temperature
$\mathbf{L}$	translation vector	u	velocity in the x direction
p	pressure	$\mathbf{U}$	velocity vector
Pr	Prandtl number	v	velocity in the y direction
Re	Reynolds number	Θ	dimensionless temperature
T	temperature	η	fluid dynamic viscosity
T_b	local bulk temperature	ρ	fluid density

appropriately defined nondimensional temperature varies streamwisely in a periodic manner. Thus, to reveal the flow and heat transfer characteristics in the periodic fully developed region, it is sufficient to confine attention to a single cycle (module). Because of the inherent characteristics, i.e., the streamwise periodicity in velocities, nondimensional temperature, etc., the solution of the resulting algebraic equations needs some special treatment. Broadly speaking, there are two types of solution methodologies. One methodology was proposed in [1], where the discretized algebraic equations were solved by a cyclic tridiagonal matrix algorithm (CTDMA) and the energy equation in nondimensional temperature form was solved in conjunction with the solution of an eigenvalue which represents a streamwise nondimensional temperature gradient. This solution procedure is somewhat complicated. Later, in [2, 3], a different solution methodology, an iterative method, was suggested, in which the dimensional governing equations were solved and the periodic boundary conditions were implemented by extending the computational domain and replacing the field data at the inlet by the computed solutions at the outlet of one module. For the outlet boundary, the so-called continuing outflow boundary condition (i.e., derivatives of the physical variables with respect to the streamwise direction were zero) was used in [2]. As indicated in [4], since all the variables change along the streamwise direction, this treatment for the outflow boundary might lead to some inaccuracy. In addition, it should be noted that in order to apply the periodic boundary conditions by the proposed method, extension of the solution domain was necessary even through our interest was in only one module. Subsequently, in [4, 5], the periodically fully developed fluid flow and heat transfer in wavy channels were simulated, and the iterative method was extended to mutual replacements of the inlet and outlet data for accelerating convergence, by which the periodic boundary condition was implemented not only by replacing the field data at the domain inlet by the computed solutions at the module outlet but also by replacing the field data at the domain outlet by those at the position within the domain corresponding to the domain outlet. As indicated above for the execution of the mutual replacement method, the computational domain was composed of one module and several additional control volumes in the streamwise direction. The energy equation was solved via the dimensional temperature. However, it is the dimensionless temperature that possesses periodic variation. The dimensional temperature at the domain inlet and outlet were then calculated using the dimensionless temperature at these locations. In [4, 5], the resulting algebraic equations were solved by the successive line underrelaxation method (SLUR) incorporated by the conventional tridiagonal matrix algorithm (TDMA). Being aware of the complexity for solving the eigenvalue problem for the nondimensional temperature, Kelkar and Patankar [6] solved the dimensional temperature equation and determined the domain inlet/outlet dimensional temperature through the nondimensional temperature in the solution domain by using the periodicity of the nondimensional temperature. The CTDMA was used and the solution domain was only half of a module, using the symmetric character pertinent to the problem solved. In order to restrict the computational domain exactly within one model and use the conventional TDMA, in [7–9] another mutual replacement method of linear interpolation was suggested to treat the temperature periodic boundary condition. This method implemented the periodic boundary condition for temperature at the inlet/outlet by linear interpolation between the field data on the first interior point near the inlet and those near the outlet.

Recently, in [10], the stream–vorticity function method with periodic boundary conditions was used to solve a mixed-convection heat transfer problem. In [11], the periodic boundary condition was used to solve the convective heat transfer in a pipe with periodically arranged surface vortex generators. In both reports, the details of implementing the periodic boundary conditions were not presented clearly. In [12], the commercial software CFX was used to solve for laminar flow and heat transfer in a periodic serpentine channel. The mutual replacement method was adopted to treat the periodic boundary condition at the inlet and outlet. In order to guarantee the periodic condition, the computational domain was pre-extended such that in the fluid flow and heat transfer reach the periodic state. For the geometry studied in [12] (serpentine tube), the authors found that three units (modules) were enough to reach the periodic condition. They picked up the data in the central unit as the boundary conditions. This practice for the implementation of periodic boundary condition does not restrict the computational domain mainly to one module. In addition, if three units are required to reach the periodic state, then the fluid flow and heat transfer in the central (i.e., second) unit have not reached the periodic state. Hence the data picked up from this unit are questionable for use as the periodic boundary condition.

From the authors' numerical experiences, the boundary condition treatment is one of the major concerns of numerical simulation, and in some cases a globally correct numerical solution may contain some undesirable features just because of some inappropriate treatment of the related boundary conditions. This article provides some examples.

The rest of the article is organized as follows. First the basic concepts of the periodic fully developed fluid flow/heat transfer and the numerical method, including the numerical techniques of mutual replacements, are briefly reviewed. Two numerical examples in which the periodic temperature boundary condition is treated by linear interpolation are presented, and an undesirable feature of the temperature distribution is presented. A detailed analysis is presented to reveal the reason and a corrected treatment technique is proposed. The proposed method is then adopted in the computations of three examples, including flow and heat transfer in periodically converging-diverging channels, followed by presentation of numerical results showing the feasibility of the proposed method. Finally, some conclusions are drawn.

2. BRIEF REVIEW OF PERIODICALLY FULLY DEVELOPED FLUID FLOW/ HEAT TRANSFER AND ITS BOUNDARY CONDITIONS

2.1. Basic Concepts of Periodically Fully Developed Fluid Flow/Heat Transfer

Consider a 2-D duct of geometric periodic shape with uniform wall temperature T_w separated by a translation vector **L** within which a fluid with constant properties is moving. One such module is presented schematically in Figure 1. If the fluid flow and heat transfer are periodically fully developed, the fluid velocity $\mathbf{U}(\mathbf{r})$, the pressure $p(\mathbf{r})$, and the dimensionless temperature $\Theta(\mathbf{r})$ in successive modules have following relationships.
Velocity:

$$\mathbf{U}(\mathbf{r}) = \mathbf{U}(\mathbf{r}+\mathbf{L}) = \mathbf{U}(\mathbf{r}+2\mathbf{L})\ldots \tag{1}$$

Pressure:

$$p(\mathbf{r}) - p(\mathbf{r}+\mathbf{L}) = p(\mathbf{r}+\mathbf{L}) - p(\mathbf{r}+2\mathbf{L})\ldots \tag{2}$$

Dimensionless temperature:

$$\Theta(\mathbf{r}) = \Theta(\mathbf{r}+\mathbf{L}) = \Theta(\mathbf{r}+2\mathbf{L})\ldots \tag{3}$$

which is defined as

$$\Theta(\mathbf{r}) = \frac{T(\mathbf{r}) - T_W}{T_b(\mathbf{r}) - T_W} \tag{4}$$

Here T_b is the local bulk temperature and is defined as

$$T_b = \frac{\int_A uT\,dA}{\int_A u\,dA} \tag{5}$$

The task of numerical simulations is to find the fluid velocity and temperature distributions and the related friction factor and Nusselt number of the module. The major issue in the numerical solution is the numerical treatment of the periodic

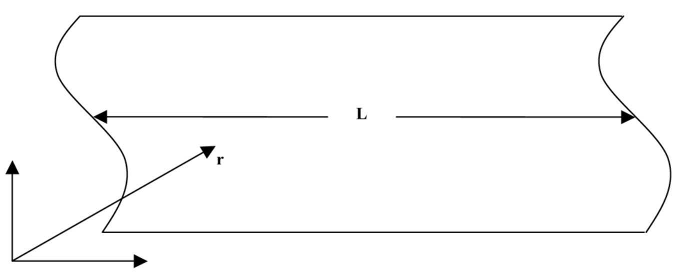


Figure 1. Schematic of a geometric periodic domain.

boundary conditions, especially for the fluid temperature. In this article, attention is focused on the methods proposed and developed in [4–9], where the energy equation is solved via a dimensional temperature and the inlet/outlet periodic boundary conditions are treated via replacement or interpolation techniques, so that the computational domain can be restricted in one unit.

2.2. Numerical Implementation for Temperature Periodic Boundary Condition

In this section, the replacement method and the linear interpolation method for the inlet/outlet temperatures are described. For execution of the replacement method, an extended computational domain is required; while for the linear interpolation, computation is restricted within exactly one module. For simplicity of presentation the methods will be called simply the domain extension method (DEM) and the linear interpolation method (LIM), respectively. In the following, the two methods will be introduced briefly, taking the periodically fully developed fluid flow/heat transfer in a converging-diverging channel (Figure 2) as an example.

2.2.1. Domain extension method (replacement method). One module of the converging-diverging channel is represented by the geometry A–B–C–D in Figure 2. To implement the replacement method, the computational domain is extended in the streamwise direction by at least one control volume. This is shown schematically by the region A–B–G–H, where the dashed line *EF* within the module is the counterpart of GH.

When the flow and heat transfer are fully developed along the flow direction, the velocity and the dimensionless temperature on lines AB and EF should be equal to ones on lines DC and HG, respectively. So the periodicity of the velocity and nondimensional temperature can be expressed as follows:

$$u(\mathrm{AB}, y) = u(\mathrm{DC}, y) \qquad v(\mathrm{AB}, y) = v(\mathrm{DC}, y) \tag{6}$$

$$u(\mathrm{GH}, y) = u(\mathrm{EF}, y) \qquad v(\mathrm{GH}, y) = v(\mathrm{EF}, y) \tag{7}$$

$$\Theta(\mathrm{AB}, y) = \Theta(\mathrm{DC}, y) \qquad \Theta(\mathrm{HG}, y) = \Theta(\mathrm{EF}, y) \tag{8}$$

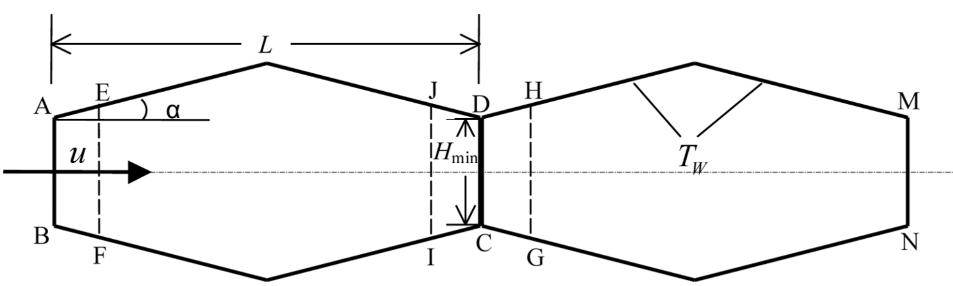


Figure 2. Geometric model of a 2-D periodically converging-diverging channel.

Equations (6) and (7) can be used directly as the periodic boundary conditions for velocity components at the computational domain inlet and outlet, while Eq. (8) should be transformed into dimensional form via definition (4):

$$T(\mathrm{AB}, y) = T_W + \Theta(\mathrm{DC}, y)[T_b(\mathrm{AB}) - T_W] \tag{9}$$

$$T(\mathrm{HG}, y) = T_W + \Theta(\mathrm{EF}, y)[T_b(\mathrm{HG}) - T_W] \tag{10}$$

where the local bulk temperature $T_b(\mathrm{AB})$ is a prespecified value (different from T_w) and the local bulk temperature $T_b(\mathrm{HG})$ is updated from iteration to iteration during the solution procedure. It should be noted that in Eq. (9) the replacement method is implemented in that $\Theta(\mathrm{AB}, y)$ is replaced by its counterpart $\Theta(\mathrm{DC}, y)$. Similar understanding can be applied to Eq. (10). From Figure 2 it can be clearly observed that only when the domain is extended at least by one control volume can we update both the inlet (line AB) and outlet (line HG) boundary conditions using the replacement method. Cartesian coordinates are used in this article. However, if the inlet and the outlet boundary grids are everywhere one-to-one corresponding, then the replacement method can also be used for unstructured grids.

2.2.2. Linear interpolation method. The linear interpolation method is used when the computational domain is strictly limited within one module, such as A–B–C–D in Figure 2. When the flow and heat transfer are periodically fully developed along the flow direction, the velocity and the dimensionless temperature on line EF should be equal to ones on line HG. To update the velocity components at the outlet of the module, linear interpolation between the values at lines JI and GH may be used. By adopting periodicity, the variables at HG may be replaced by those at EF. Thus we have

$$u(\mathrm{AB}, y) = u(\mathrm{DC}, y) = \frac{u(\mathrm{EF}, y) + u(\mathrm{JI}, y)}{2} \tag{11}$$

$$v(\mathrm{AB}, y) = v(\mathrm{DC}, y) = \frac{v(\mathrm{EF}, y) + v(\mathrm{JI}, y)}{2} \tag{12}$$

$$\Theta(\mathrm{AB}, y) = \Theta(\mathrm{DC}, y) = \frac{\Theta(\mathrm{EF}, y) + \Theta(\mathrm{JI}, y)}{2} \tag{13}$$

Then the module inlet and outlet dimensional temperatures can be described as

$$T(\mathrm{AB}, y) = T_w + \frac{1}{2}[\Theta(\mathrm{EF}, y) + \Theta(\mathrm{JI}, y)][T_b(\mathrm{AB}) - T_w] \tag{14}$$

$$T(\mathrm{DC}, y) = T_w + \frac{1}{2}[\Theta(\mathrm{EF}, y) + \Theta(\mathrm{JI}, y)][T_b(\mathrm{DC}) - T_w] \tag{15}$$

where the local bulk temperatures $T_b(\mathrm{AB})$ and $T_b(\mathrm{DC})$ are updated from iteration to iteration. This LIM was adopted in [7–9]. To the authors' knowledge, in [6] a similar method was used, although those authors did not explicitly use the term "linear interpolation." In the finite-volume method, the interface interpolation method implies a scheme of discretization, and the linear interpolation corresponds to the central difference [13]. Hence, mathematically, this interpolation has second-order accuracy. When a rather complicated variation pattern of the dependent variable exists near the domain inlet and outlet, a finer grid distribution should be located around the two boundaries in order to maintain the accuracy.

3. AN UNDESIRABLE FEATURE CONCEALED IN THE DEM AND LIM

Using the DEM and LIM, we can solve the energy equation with dimensional temperature in quite a simple and straightforward manner. However, in a series study of periodically fully developed fluid flow and heat transfer, we gradually became aware of an undesirable feature caused by using these techniques. In this section, we illustrate this undesirable feature by solving the laminar flow and heat transfer in the converging-diverging channel shown in Figure 2. Since the channel is symmetrical about the centerline, the actual solution domain is only the upper part of one module.

The governing conservation equations in the Cartesian coordinates are the following.

Mass conservation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{16}$$

x-Direction momentum:

$$\rho\,\mathrm{div}(u\mathbf{U}) = \eta\,\mathrm{div}(\mathrm{grad}\,u) - \frac{\partial p}{\partial x} \tag{17}$$

y-Direction momentum:

$$\rho\,\mathrm{div}(v\mathbf{U}) = \eta\,\mathrm{div}(\mathrm{grad}\,v) - \frac{\partial p}{\partial y} \tag{18}$$

Energy conservation:

$$\mathrm{div}(T\mathbf{U}) = \frac{\lambda}{\rho C_p}\mathrm{div}(\mathrm{grad}\,T) \tag{19}$$

The boundary conditions are as follows.

At the top wall:

$$u = v = 0 \qquad T = T_w \tag{20}$$

At the centerline:

$$\frac{\partial u}{\partial y} = 0 \qquad v = 0 \qquad \frac{\partial T}{\partial y} = 0 \tag{21}$$

The inlet and outlet boundary conditions for velocity and temperature are represented by Eqs. (6), (7) and (9), (10) for the domain extension method, and by Eqs. (11), (12) and Eqs. (14), (15) for the linear interpolation method.

Computations were conducted for the following conditions. The channel dimensions are $L = 0.02\,\mathrm{m}$, $H_{\min} = 0.005\,\mathrm{m}$, and $\alpha = 20^\circ$ (see Figure 2). The fluids, with $\mathrm{Re} = 100$, $\mathrm{Pr} = 0.7$, and $T_{b,\,\mathrm{in}} = 400\,\mathrm{K}$ are cooled with $T_w = 300\,\mathrm{K}$. The finite-volume method is adopted to discretize the governing equations [13, 14], and the CLEAR algorithm [15, 16] is adopted to deal with the coupling between velocity and pressure. The SGSD scheme [17] is used to discretize the convective term, and the stepwise approximation technique is used for the irregular boundaries. The grid system is 170 (in x) $\times$ 52 (in y) in each module. The convergence conditions are expressed as

$$Rs_{\mathrm{cv}} = \underset{\mathrm{CV}}{\mathrm{MAX}}\left[\frac{(\rho u^* A)_w - (\rho u^* A)_e + (\rho v^* A)_s - (\rho v^* A)_n}{\mathrm{flow_{ch}}}\right] \leq 5.0 \times 10^{-8} \tag{22}$$

$$DT = \mathrm{MAX}\frac{T^n - T^{n-1}}{T^n} \leq 1.0 \times 10^{-8} \tag{23}$$

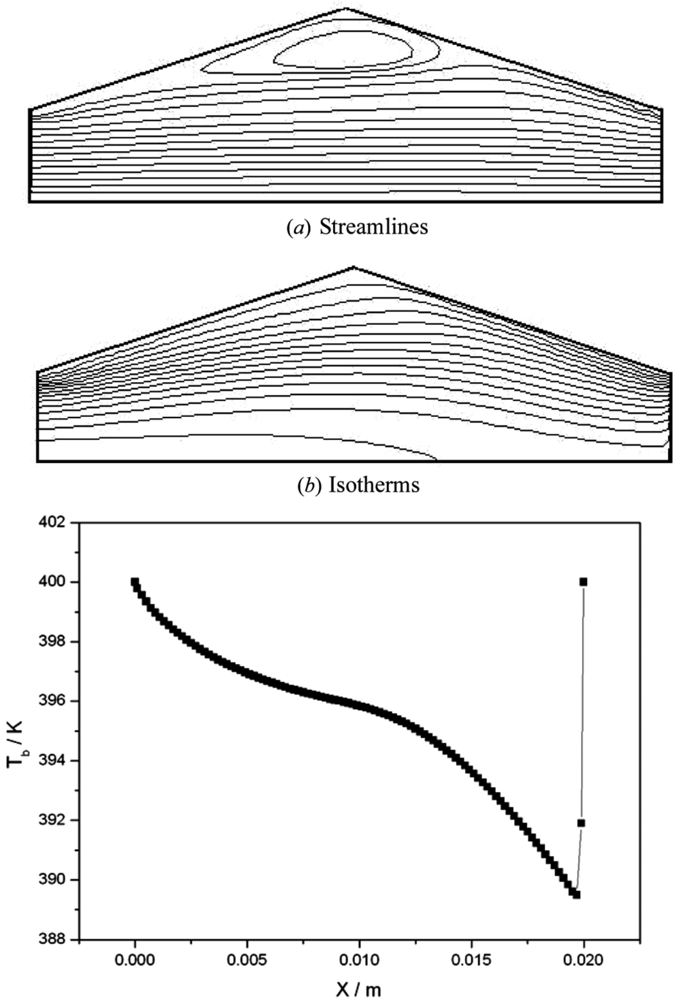
where Rs_{cv} is the maximum relative mass flow rate unbalance of all the control volumes in the computational domain [15] and DT is the maximum relative temperature variation in successive two iterations of all the nodes in the computation domain.

The predicted results of the stream functions, isotherms, and bulk temperature distributions using the LIM are shown in Figure 3. It can be seen that while the streamlines and the major part of the isotherms and the bulk temperature distributions show reasonable variation patterns, the isotherms and the bulk temperature distributions have an unreasonable feature at the domain exit: The local values increase very quickly to the prespecified bulk temperature. The solutions using the domain extension method are presented in Figure 4, where two modules constitute a computational domain. It can be seen from Figure 4 that the same feature exists.

Why did this phenomenon appear? Did the stepwise approximation technique cause the fallacious phenomenon? The periodic boundary condition was then applied to solve the periodic fully developed fluid flow/heat transfer problem in a parallel-plate channel, which is the simplest periodic channel where the Cartesian coordinates can fit the geometric boundary perfectly.

Theoretically, for fully developed fluid flow and heat transfer in a parallel-plate channel, the results should be periodic alone the flow direction with an arbitrary distance as a computational module. The linear interpolation method is applied to solve this problem. The fluid with $T_{b,\,\mathrm{in}} = 400\,\mathrm{K}$ and $\mathrm{Re} = 100$ is cooled with $T_{\mathrm{w}} = 300\,\mathrm{K}$. The results are presented in Figure 5.

From Figure 5*a* it can be seen that the velocity distribution is consistent with the theoretical result, which reflects the validity of the periodic velocity boundary treatment. But the results of temperature distributions exhibit the same abnormal phenomenon at the channel outlet, as shown in Figures 5*b* and 5*c*. This implies that it is not the stepwise approximation technique that causes the inconsistent



(c) The local bulk temperature

Figure 3. Results of applying linear interpolation method in converging-diverging channel.

temperature distribution at the domain exit. At the same time, this gives us a hint that probably the numerical techniques adopted to deal with the periodic temperature boundary condition cause the error. After a detailed analysis an inadvertent assumption in the DEM and LIM has been revealed, and this analysis is presented below.

4. ANALYSIS AND REMEDIES

4.1. Analysis of the Abnormal Phenomenon

The analysis is conducted for the DEM, and it can be shown that the conclusion is also valid for the LIM. The following analysis shows that in the

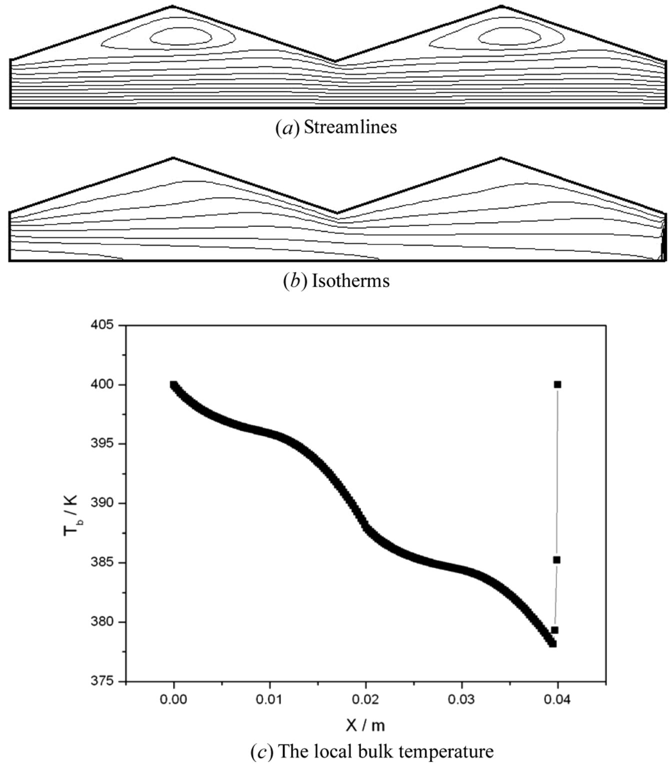


Figure 4. Results of applying extending one module method in converging-diverging channel.

implementation of the LIM or DEM, the domain exit fluid bulk temperature actually remains unchanged during the solution procedure, leading to the above-mentioned abnormal phenomenon.

Referring to the module A–B–G–H in Figure 2, from the periodicity we have

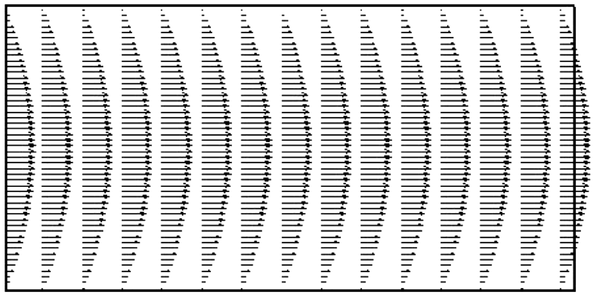
$$\Theta(\mathrm{HG}) = \Theta(\mathrm{EF}) \tag{24}$$

At the $(n+1)$th iteration, the dimensionless temperature can be expressed by:

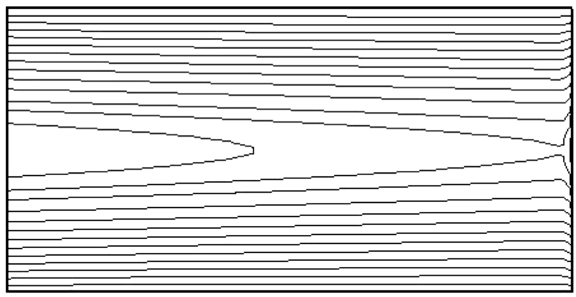
$$\Theta^{(n+1)}(\mathrm{HG}, y) = \frac{T^{(n+1)}(\mathrm{HG}, y) - T_W}{T_b^{(n+1)}(\mathrm{HG}) - T_W} = \Theta^{(n+1)}(\mathrm{EF}, y) \tag{25}$$

From Eq. (25) it can be obtained

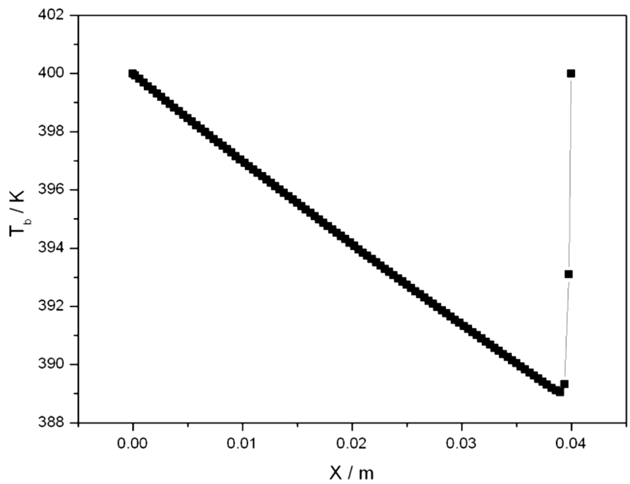
$$T^{(n+1)}(\mathrm{HG}, y) = T_w + \Theta^{(n+1)}(\mathrm{HG}, y)\left[T_b^{(n+1)}(\mathrm{HG}) - T_w\right] \tag{26}$$



(*a*) Velocity vector



(*b*) Isotherms



(*c*) The local bulk temperature

Figure 5. Results of applying periodic boundary condition in flat channel.

In the iterative computational process, $T_b^{(n+1)}(\mathrm{HG})$ is actually replaced by $T_b^{(n)}(\mathrm{HG})$, and $\Theta^{(n+1)}(\mathrm{HG}, y)$ is replaced by $\Theta^{(n+1)}(\mathrm{EF}, y)$. Thus we have

$$T^{(n+1)}(\mathrm{HG}, y) = T_w + \Theta^{(n+1)}(\mathrm{EF}, y)\left[T_b^{(n)}(\mathrm{HG}) - T_w\right] \tag{27}$$

By substituting $T^{(n+1)}$ (HG, y) into the expression of $T_b = \int_{\hat{A}} uT\,dA / \int_{\hat{A}} u\,dA$, we get

$$
\begin{aligned}
T_b^{(n+1)}(\mathrm{HG}) &= \frac{\int_{\hat{A}} T_w u(\mathrm{EF}, y)\,dA + \int_{\hat{A}} \Theta^{(n+1)}(\mathrm{EF}, y) u(\mathrm{HG}, y) \left[T_b^{(n)}(\mathrm{HG}) - T_w\right] dA}{\int_A u(\mathrm{HG}, y)\,dA} \\
&= T_w + \frac{\left[T_b^{(n)}(\mathrm{HG}) - T_w\right] \int_A \frac{T^{(n+1)}(\mathrm{EF}, y) - T_w}{T_b^{(n+1)}(\mathrm{EF}) - T_w} u(\mathrm{HG}, y)\,dA}{\int_A u(\mathrm{HG}, y)\,dA} \\
&= T_w + \frac{\left[T_b^{(n)}(\mathrm{HG}) - T_w\right] \int_A \left[T^{(n+1)}(\mathrm{EF}, y) - T_w\right] u(\mathrm{EF}, y)\,dA}{\left[T_b^{(n+1)}(\mathrm{EF}) - T_w\right] \int_A u(\mathrm{EF}, y)\,dA} \\
&= T_w + \frac{\left[T_b^{(n)}(\mathrm{HG}) - T_w\right]\left[T_b^{(n+1)}(\mathrm{EF}) - T_w\right]}{\left[T_b^{(n+1)}(\mathrm{EF}) - T_w\right]} = T_b^{(n)}(\mathrm{HG})
\end{aligned} \tag{28}
$$

The above analysis reveals the fundamental reason for the above-mentioned fallacious temperature variation at the module outlet: The local bulk temperature at the module outlet is not updated with the iteration. It is interesting to note that even though in the right-hand side of Eq. (27) the present value of $T_b^{n+1}(\mathrm{HG})$ is replaced by its previous value in the iteration calculation, without the above-mentioned derivation, we cannot get directly the conclusion that $T_b^{n+1}(\mathrm{HG}) = T_b^{n}(\mathrm{HG})$. This is because in Eq. (27), the value of Θ^{n+1} (EF) is updated. If this practice is correct, Eq. (27) should lead finally to the updated value of $T_b^{n+1}(\mathrm{HG})$.

The higher exit fluid temperature will cause fluid heat conduction from the exit to the inner region of the domain. Even though the fluid conduction is can often be neglected when the fluid Peclet number is greater than 100 [18], and hence the major results of heat transfer presented in [4, 5, 7–9] are right when either the DEM or the LIM is used to treat the temperature periodic boundary condition, the abrupt change of the exit bulk temperature is surely unreasonable and undesirable. The present article proposes following two techniques to overcome this undesirable feature.

4.2. Remedies for Correction

It is proposed that during the iteration the local bulk temperature at the module (domain) outlet boundary be updated by upwind interpolation from the local fluid bulk temperatures within the domain.

To be specific, we propose the following two upwind interpolation schemes:

$$T_b(\mathrm{L1}) = T_b(\mathrm{L2}) \tag{29}$$

$$T_b(\mathrm{L1}) = 1.5T_b(\mathrm{L2}) - 0.5T_b(\mathrm{L3}) \tag{30}$$

where L1, L2, and L3 are the last, last but one, and last but two grids in the steamwise direction, respectively, Equations (29) and (30) are similar to the first-order upwind scheme (FUD) and second-order upwind scheme (SUS) of the discretization of convection term. For convenience, Eqs. (29) and (30) are denoted by FUSET (first-order upwind scheme for exit temperature) and SUSET (second-order upwind scheme for exit temperature). Numerically, the above upwind interpolation method is incorporated into the iterative solution procedure as follows. Within each iteration, the periodic condition for the nondimensional temperature is implemented by either the DEM or LIM, and at the end of each outer iteration the module exit bulk temperature is updated by Eq. (29) or (30). For simplicity of later presentation, the original DEM or LIM will be called simply DEM or LIM, and that incorporating the upwind interpolation of the exit bulk temperature will be called corrected DEM (LIM).

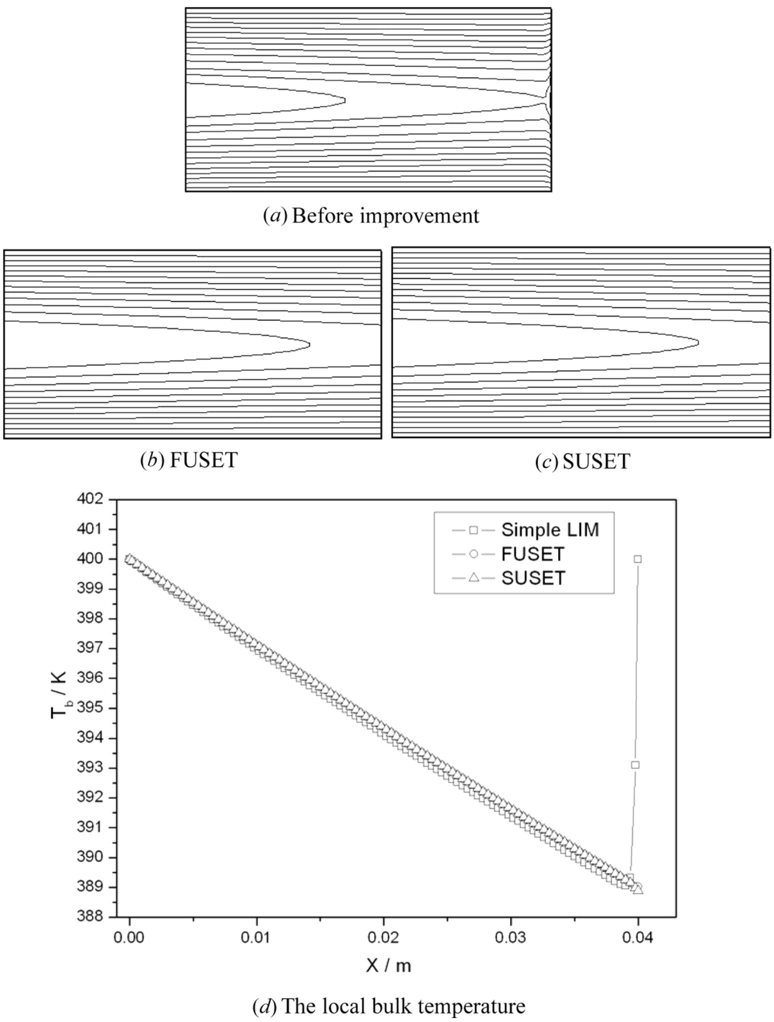


Figure 6. Comparison of predicted temperatures with different treatments of periodic temperature condition for heat transfer in a parallel-plate channel.

Table 1. Comparison of mean Nusselt numbers

Numerical method		$\overline{Nu}$	Analytic solution	Relative error (%)
Without correction	LIM	7.767		2.997
Corrected	FUSET	7.543	7.541	0.02652
	SUSET	7.540		0.01326

4.3. Results after Correction

The two corrections of the local bulk temperature T_b (L1) were applied to solve flow and heat transfer in flat channels, and the results are shown in Figure 6.

The results of heat transfer by two different corrections are more reasonable than the one without corrections. To compare the FUSET and SUSET, the mean Nusselt number and the local Nusselt number are calculated; the results are shown in Table 1 and Figure 7.

From Table 1 and Figure 7, it can be seen that the results by applying FUSET and SUSET are both better than the results without correction. And the results applying SUSET are better than the ones applying FUSET. The relative error of mean Nusselt number for FUSET is 0.02652%, and that for SUSET is only 0.01326%, which reflects the validity of the interpolative prediction method. The present authors thus recommend the adoption of the interpolative method of SUSET in conjunction with the LIM or DEM for implementation of the periodically fully developed fluid flow and heat transfer.

At this point a question may be raised as follows: In the domain extension method (simple DEM), if the extended control volumes are large enough, can the abnormal phenomenon be eliminated in the first module?

Numerical results with different numbers of extended control volumes for the heat transfer in a parallel channel are presented in Table 2 and Figure 8. It can be

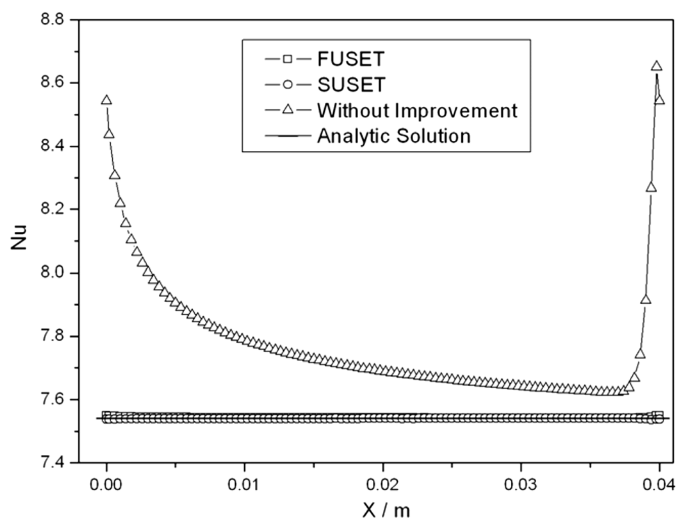


Figure 7. Comparison of local Nusselt number from corrected and uncorrected methods.

Table 2. Mean Nusselt numbers for different extensions

Numerical method	$\overline{Nu}$	Analytic solution	Relative error (%)
Not extended	7.767		2.997
Extending 1 C. V.	7.658		1.552
Extending 5 C. V.	7.559	7.541	0.2387
Extending 10 C. V.	7.543		0.02652
Extending 1 module	7.540		0.01326

observed that with an increase in the extended control volumes, the mean Nusselt number gradually approaches the exact solution. And with 10 extended control volumes, the error involved in the mean Nusselt number is of the same order as that by applying the FUSET (see Table 1). This comparison gives strong support to the application of the two proposed corrected methods.

5. APPLICATION EXAMPLES

In this section the SUSET method is applied to solve three periodic fully developed fluid flow and heat transfer problems. For comparison purposes, the results from FUSET are also provided.

Problem 1: Flow and Heat Transfer in a Periodic Converging-Diverging Channel

The channel dimensions are $L = 0.02\,\text{m}$, $H_{\text{min}} = 0.005\,\text{m}$, and $\alpha = 20°$. A uniform grid of 170×52 is used with stepwise approximation. Fluid with $\text{Re} = 100$

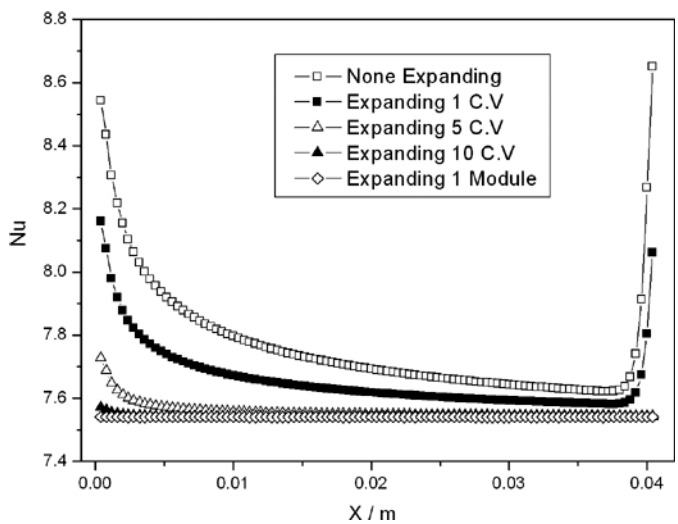


Figure 8. Comparison of local Nusselt number by applying expanding different numbers of control volumes.

and Pr $= 0.7$ is cooled with $T_{b,\,\mathrm{in}} = 400\,\mathrm{K}$ and $T_w = 300\,\mathrm{K}$. All the numerical aspects are the same as presented in Section 3. Computations were conducted for two kinds of domain: one composed of one module and another of two modules. For the one-module domain, both the simple LIM and the corrected LIM were used to treat the periodic boundary condition, while for the two-module domain, only the simple DEM was used and the converged solution of the first module was presented. The simulated results are presented in Figure 9. It can be seen from the figure that the local distribution of isotherms from SUSET are quite reasonable and the abrupt change of fluid temperature at the exit is totally discarded. The fluid bulk temperature distribution from the corrected LIM agrees well with that in the first module of the two-module results (shown by simple DEM in Fig. 9).

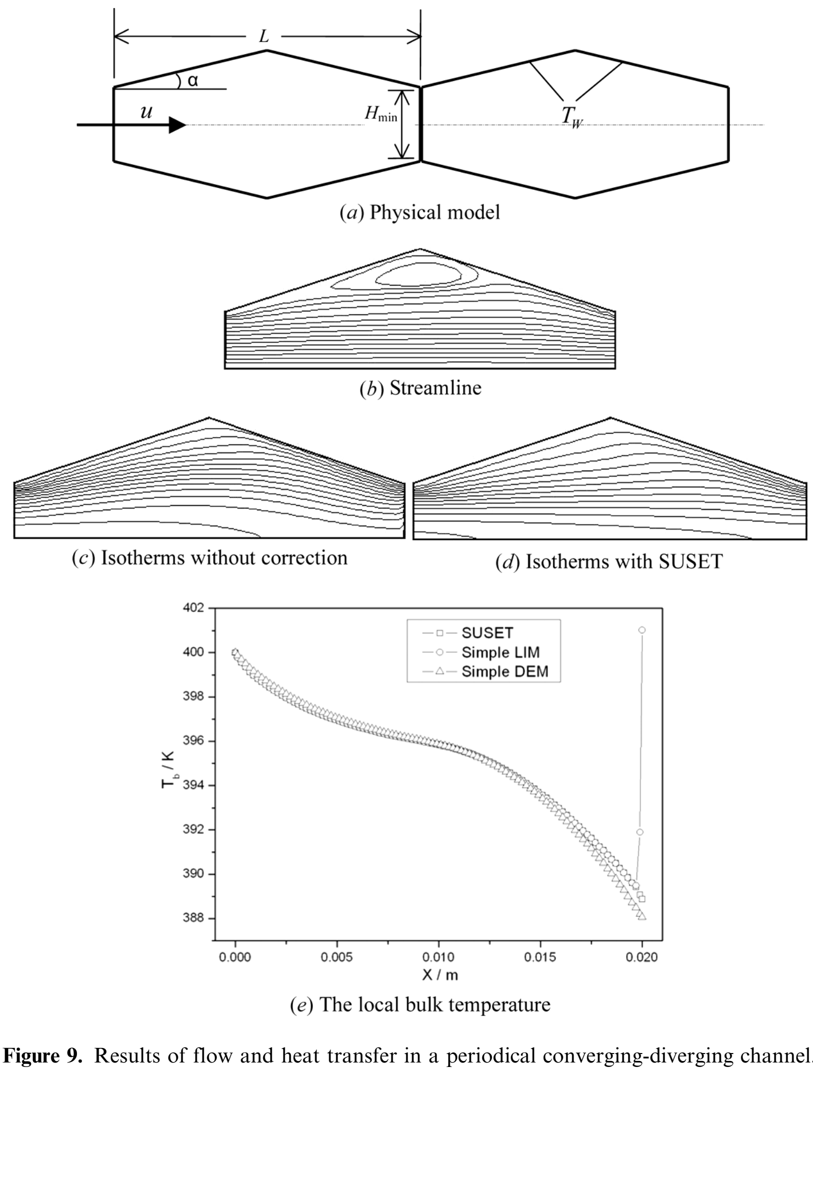
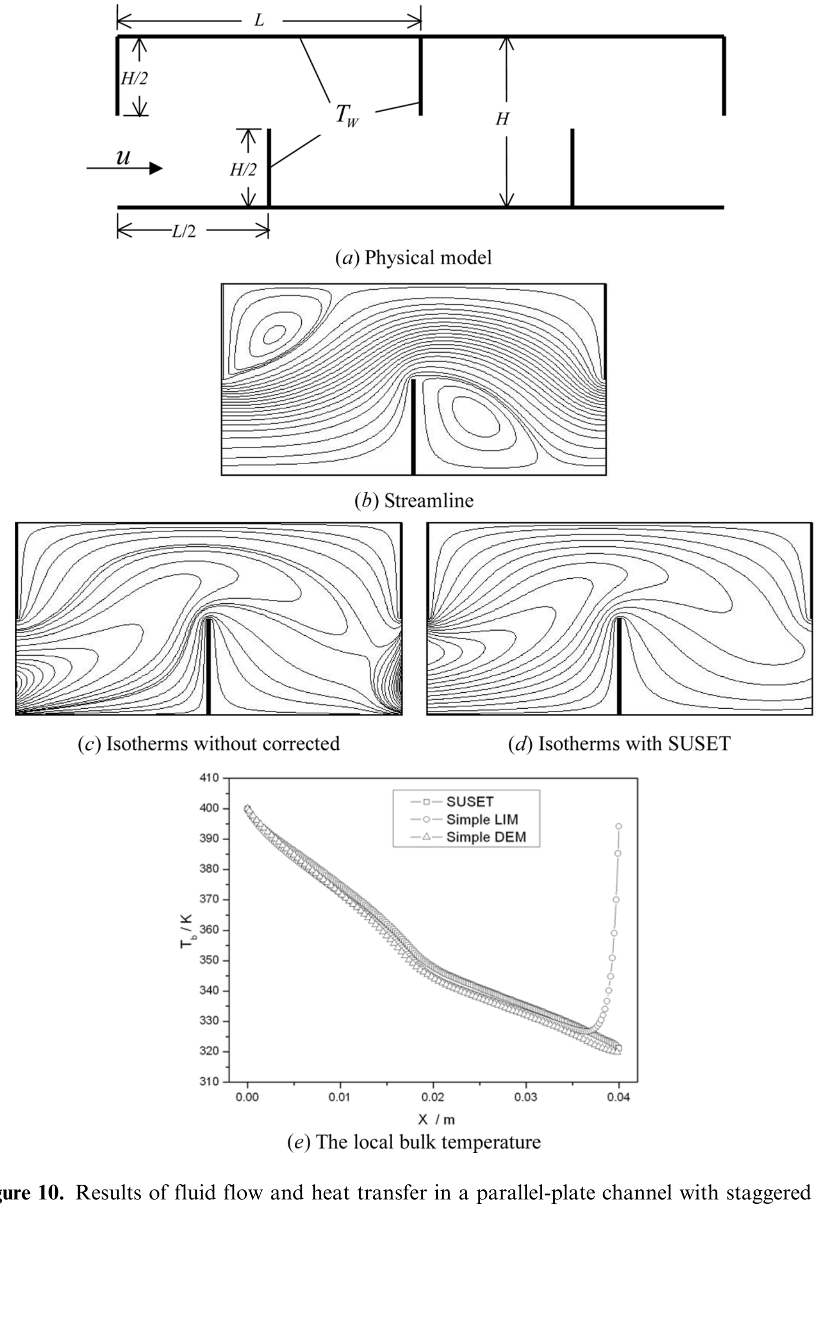


Figure 9. Results of flow and heat transfer in a periodical converging-diverging channel.

Problem 2: Flow and Heat Transfer in a Parallel-Plate Channel with Staggered Fins [6]

The channel dimensions are $L = 0.04\,\text{m}$, $H = 0.02\,\text{m}$. A uniform grid of 102×52 is adopted. Fluid with $\text{Re} = 30$, $\text{Pr} = 0.7$, and $T_{b,\,\text{in}} = 400\,\text{K}$ is cooled with $T_w = 300\,\text{K}$. The computational results are shown in Figure 10. It can be observed



(*a*) Physical model

(*b*) Streamline

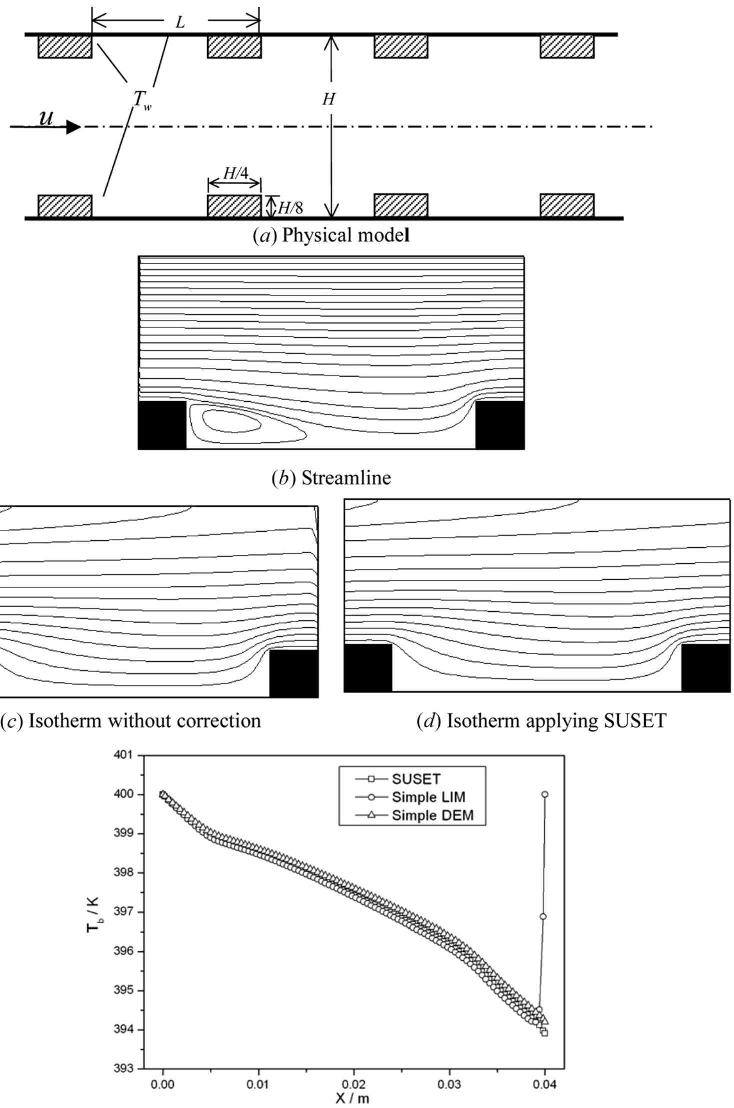
(*c*) Isotherms without corrected

(*d*) Isotherms with SUSET

(*e*) The local bulk temperature

Figure 10. Results of fluid flow and heat transfer in a parallel-plate channel with staggered fins.

that in the first half-compartment the isotherms from the simple and corrected LIM methods are quite similar, while in the second half-compartment the difference between the two solutions is appreciable. Figure 10*e* shows the big difference between the streamwise bulk temperature distributions, among which the curves from the corrected LIM and two-module domain agreewell. In addition, the present flow field distributions agree well with those in [6].



(*a*) Physical model
(*b*) Streamline
(*c*) Isotherm without correction
(*d*) Isotherm applying SUSET
(*e*) The local bulk temperature

Figure 11. Results of fluid flow and heat transfer in a channel with periodic square roughness elements.

Problem 3: Flow and Heat Transfer in a Channel with Periodic Square Roughness Elements

The channel dimensions are $L = 0.04\,\mathrm{m}$, $H = 0.04\,\mathrm{m}$. A uniform grid of 102×52 is used. Fluid with $\mathrm{Re} = 100$ and $\mathrm{Pr} = 0.7$ is cooled with $T_{b,\ \mathrm{in}} = 400\,\mathrm{K}$ and $T_w = 300\,\mathrm{K}$. The predicted results are presented in Figure 11. The same discussion can be conducted as for Problem 2.

6. CONCLUSIONS

In this article a qualitatively important feature concealed in the implementation of temperature periodic fully developed condition by the domain extension method or the linear interpolation is discussed. A remedy is proposed, and three application examples are provided. The following conclusions may be drawn.

1. When the simple DEM or LIM is used to deal with the temperature periodic fully developed condition, the module (domain) exit fluid bulk temperature remains unchanged during iteration, leading to an abrupt change of fluid temperature at the module (domain) exit.
2. Two upwind-based interpolations, FUSET and SUSET, are proposed to update the module (domain) exit fluid bulk temperature within the iteration. Numerical examples show that the unreasonable abrupt temperature change at the module (domain) exit has been discarded in the predicted fluid isotherms from the two corrected methods, with the results from SUSET being better.

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