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A simple method for improving the SIMPLER algorithm for numerical simulations of incompressible fluid flow and heat transfer problems

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Abstract

Purpose – To provide an improved version of SIMPLER algorithm which can enhance the convergence rate of the iterative solution procedure in the field of computational fluid dynamics analysis.

Design/methodology/approach – The improved version of SIMPLER algorithm is developed by modifying the coefficients of the velocity correction equation and implementing the correction of pressure within an iteration cycle.

Findings – The CSIMPLER algorithm (the improved version) can enhance the convergence rate for almost all cases tested, especially for the low under-relaxation factor situations. The pressure correction term even can be overrelaxed to further enhance the convergence rate.

Research limitations/implications – The CSIMPLER algorithm can enhance the rate of convergence to different degree for different problems. It can only be adopted to solve the incompressible fluid flow and heat transfer.

Practical implications – CSIMPLER is a simple and effectual method to enhance the convergence rate of the iterative process for the computational fluid dynamics analysis. The existing code of SIMPLER can be easily changed to CSIMPLER.

Originality/value – The paper developed an improved version of SIMPLER algorithm with some minor changes in the existing SIMPLER code.

Keywords Programming and algorithm theory, Flow, Heat transfer, Fluid dynamics, Numerical analysis

Paper type Research paper

Nomenclature

a_P, a_E, a_W, a_N, a_S = coefficients in the discretized equation
A = surface area
b = constant term in the discretized equation
d_e, d_n = coefficients in the velocity correction equation
E = time step multiple
p = pressure

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p^*	= temporary pressure	u', v'	= velocity correction
p'	= pressure correction	$\hat{u}, \hat{v}$	= pseudovelocity
q_m	= reference mass flow rate	x, y	= coordinates
R_{max}	= maximum of absolute values of the mass flow rate residuals	α	= under-relaxation factor
		$\delta x, \delta y$	= distance between two adjacent grid points in x and y direction
R_{sum}	= total mass flow rate residuals of entire domain	$\Delta x, \Delta y$	= control volume width in x, and y direction
S_{max}	= relative maximum of absolute values of the mass flow rate residuals	ϕ	= general variable
		ρ	= fluid density
		Γ	= nominal diffusion coefficient
S_{sum}	= relative total mass flow rate residuals of entire domain	ν	= fluid kinetic viscosity
S_ϕ	= source term	*Subscripts*	
u, v	= velocity component in x, y direction	e, n	= east, north interface
		nb	= neighboring grid points

Introduction

In the numerical solution of incompressible fluid flow and heat transfer problems, the pressure-correction approach is the most popular method used in CFD/NHT community. The first pressure-correction algorithm was the SIMPLE proposed by Patankar and Spalding (1972). The acronym SIMPLE stands for semi-implicit method for the pressure-linked equation. The major approximations made in the SIMPLE algorithm are (Tao, 2001):

- The initial pressure field and the initial velocity fields are independently assumed, hence the inherent interconnection between pressure and velocities are neglected, leading to some inconsistency between them.
- The effects of the pressure corrections of the neighboring grids are arbitrary dropped in order to simplify the solution procedure, thus make the algorithm semi-implicit.

These assumptions will not affect the final solutions if the iterative process converges (Tao, 2001; Patankar, 1980). However, they do affect the convergence rate. As described in Shyy and Mittal (1998), the great simplicity of the SIMPLE algorithm comes from the neglecting the terms that couples neighboring velocity values in the equation for the velocity correction. However, this can also cause slow convergence of the SIMPLE algorithm and it has been found this neglect tends to overpredict the pressure correction and underrelaxation for the pressure correction has to be resorted to in order to stabilize the iterative procedure. Therefore, since the propose of the SIMPLE algorithm, a number of its variants were proposed in order to overcome one or both of the approximations (Patankar, 1981; van Doormaal and Raithby, 1984, 1985; Raithby and Schneider, 1988; Issa, 1985; Connell and Stow, 1986; Chatwani and Turan, 1991; Lee and Tzong, 1992; Yen and Liu, 1993; Sheng *et al.*, 1998; Yu *et al.*, 2001; Gjesdal and Lossius, 1997; Wen and Ingham, 1993).

The SIMPLER algorithm (Patankar, 1981) successfully overcome the first approximation, and is widely used in the current CFD/NHT community. Even though there are more than ten variants of the SIMPLE-like algorithm, the second approximation, i.e. the drop of the neighboring grid effects, have not been successfully resolved so far and many attempts have been made to resolve the problem. In 1984, van Doormaal and Raithby (1984) proposed the SIMPLEC algorithm, in which by changing the definition of the coefficients of the velocity correction equation the effects of this drop is partially compensated. In the algorithm SIMPLEX (van Doormaal and Raithby, 1985; Raithby and Schneider, 1988), by solving a set of algebraic equation for the coefficients in the velocity correction equations, the effects of dropping the neighboring grids are also taken into account in some degree. In 1985, the PISO method is proposed by Issa (1985) to implement two or more correction steps of pressure correction. In 1986 Connell and Stow (1986) proposed two variants of pressure correction process. Chatwani and Turan (1991) proposed a pressure-velocity coupling algorithm in 1991 to determine the under-relaxation factor in the pressure correction equation based on the minimization of the global mass residual norm. In 1992, Lee and Tzong (1992) introduced an artificial source term into the pressure-linked equation to improve the convergence performance. In 1993, Yen and Liu (1993) proposed the explicit correction step method to accelerate the convergence by making the velocity explicitly satisfy the momentum equation. For buoyancy driven fluid flows Sheng *et al.* (1998) introduced a temperature correction into the velocity correction equation. In 2001, Yu *et al.*(2001) modified the SIMPLER algorithm by artificially changing the under-relaxation term to match the variable to be solved. The revised method was called MSIMPLER. All the above-mentioned algorithms and some others not mentioned above (for example, SIMPLESSEC, SIMPLESSE of Gjesdal and Lossius (1997), and the method proposed in Wen and Ingham (1993) are usually called SIMPLE-like or SIMPLE-family algorithm. The character common to all these algorithms is that a pressure correction term is introduced to the segregated solution process to improve the velocity and the effects of the pressure corrections of the neighboring grid points are neglected. Recently, Moukalled and Darwish (2000) made a comprehensive review and reorganization of the express format for all the pressure correction algorithms. It can be seen that the SIMPLER algorithm successfully overcomes the first approximation, while almost all other variants of the SIMPLE algorithm concentrate on overcoming the second approximation. There seems no such attempt in the literature to combine the SIMPLER algorithm and one of the other variants so that the effects of both of the two approximations can be alleviated in a better degree in one algorithm.

In the present work, the idea of SIMPLEC is incorporated into the SIMPLER algorithm to overcome the second approximation in some extent. The revised algorithm is called consistent-SIMPLER (CSIMPLER) hereafter. Numerical experiments showed that CSIMPLER can generally accelerate the rate of convergence, especially for natural convection and the cases with low under-relaxation factor.

Mathematical formulation

In the following, the major steps in the algorithm SIMPLER and SIMPLEC are briefly reviewed, and then the idea of CSIMPLER is presented.

Brief review of the SIMPLER algorithm

In all cases considered, the flow was assumed to be Newtonian, laminar and two-dimensional. Viscous dissipation is omitted. All thermophysical properties except density are presumed constant. The Boussinesq approximation is used for natural convection problem. In Cartesian coordinates, the governing equations are as follows:

Mass:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{1}$$

Momentum:

$$\frac{\partial(\rho uu)}{\partial x} + \frac{\partial(\rho vu)}{\partial y} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho vv)}{\partial y} = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

Discretizing the governing equations by the finite volume method (Tao, 2001; Moukalled and Darwish, 2000) on a staggered grid system (Figure 1), we have

$$a_P \Phi_P = \sum a_{\text{nb}} \Phi_{\text{nb}} + b \tag{4}$$

where Φ is the general valuable standing for u and v, the subscripts P and nb refer to the gird point P and its neighboring grids, respectively, a_p is the coefficient for the main grid point, a_{nb}'s are the coefficients of neighboring grid points and b is the source term.

For the discretized momentum equations separating the pressure gradient term from the b-term and replace the general valuable by u or v, we have:

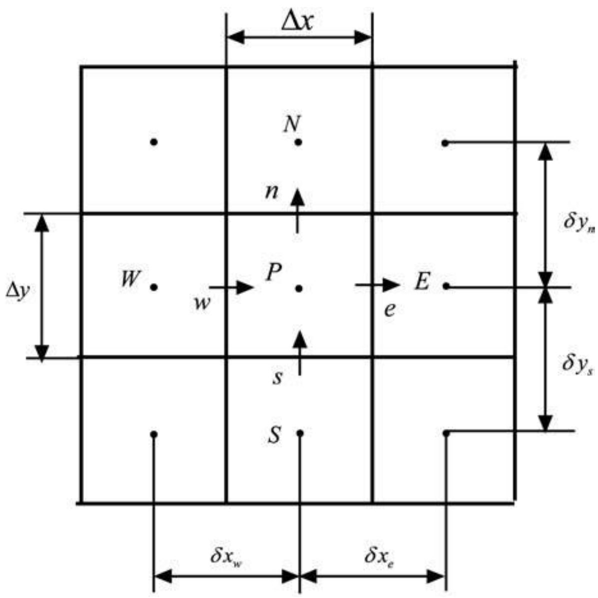


Figure 1.
Control volumes in 2D Cartesian coordinates

$$a_e u_e = \sum a_{\mathrm{nb}} u_{\mathrm{nb}} + (p_P - p_E) A_e + b \tag{5a}$$

$$a_n v_n = \sum a_{\mathrm{nb}} v_{\mathrm{nb}} + (p_P - p_N) A_n + b \tag{5b}$$

The discretized pressure equation is deduced from the momentum equations and the continuity equation and can be expressed as

$$a_P p_P = \sum a_{\mathrm{nb}} p_{\mathrm{nb}} + b_P \tag{6}$$

For details, references (Tao, 2001; Patankar, 1980) can be consulted.

By solving equations (5a) and (5b), we can obtained the intermediate solutions, symbolized by u^* and v^* which need to be improved such that the improved velocities can satisfy the mass conservation condition for each control volume.

By introducing a pressure correction term, p', and the corresponding velocity correction terms u' and v', the improved velocities can be expressed by

$$u = u^* + u' \tag{7a}$$

$$v = v^* + v' \tag{7b}$$

These improved velocities are required to satisfy the continuity condition.

The equations for the velocity correction terms, u', v', can be derived by some substitution and rearrangement (Tao, 2001; Patankar, 1980), and take the following form:

$$a_e u'_e = \sum a_{\mathrm{nb}} u'_{\mathrm{nb}} + (p'_P - p'_E) A_e \tag{8a}$$

$$a_n v'_n = \sum a_{\mathrm{nb}} v'_{\mathrm{nb}} + (p'_P - p'_N) A_n \tag{8b}$$

At this point an approximation, i.e. the second approximation in the SIMPLE algorithm mentioned above, is introduced: dropping the terms $\sum a_{\mathrm{nb}} u'_{\mathrm{nb}}$ and $\sum a_{\mathrm{nb}} v'_{\mathrm{nb}}$ in the above equations to simplify the expressions. Then we obtain

$$u'_e = d_e (p'_P - p'_E), \quad v'_e = d_n (p'_P - p'_N) \tag{9a}$$

where d_e, d_n are defined as

$$d_e = \frac{A_e}{a_e}; \quad d_n = \frac{A_n}{a_n} \tag{9b}$$

Then the improved velocities are rewritten as follows:

$$u_e = u_e^* + d_e \left(p'_P - p'_E \right) \tag{10a}$$

$$v_n = v_n^* + d_n \left(p'_P - p'_N \right) \tag{10b}$$

Substitution of the improved velocities of equations (10a) and (10b) into continuity equation, the equation for the pressure correction term is then derived

$$a_P p'_P = \sum a_{nb} p'_{nb} + b \quad (11a)$$

where

$$b = (\rho u^* A)_w - (\rho u^* A)_e + (\rho v^* A)_s - (\rho v^* A)_n \quad (11b)$$

In equation (11b) the coefficients are the same as those in the equation (6) except the b-term, where the velocities take the values of the previous iteration, rather than the intermediate solutions.

The solution procedure of the SIMPLER algorithm is as follows:

- guess a initial velocity field u^0, v^0;
- calculate the coefficients of the discretized momentum equations and the pseudo-velocities $\hat{u}$ and $\hat{v}$ by following equations:

$$\hat{u} = \frac{\sum a_{nb} u^0_{nb} + b}{a_e}; \quad \hat{v} = \frac{\sum a_{nb} v^0_{nb} + b}{a_n}$$

- solve pressure equation to get p^*;
- solve the discretized momentum equations with p^* to get u^* and v^*;
- solve the pressure correction equation to get p';
- correct the velocities by equations (7a) and (7b);
- solve the discretized equations for other scalar variables if necessary; and
- return to step 2 until convergence condition is satisfied.

It is to be noted that in the SIMPLER algorithm, the pressure correction term is only used to correct the velocities, but not used to correct the pressure. The pressure correction values are overpredicted by solving equation (11a), because the effects of the velocity corrections at neighboring grid points are totally neglected. The obtained pressure correction values are appropriate to correct the velocities, but not to pressure values.

Since the discretized equations are all solved by iterative method, the solutions of velocities of the current iteration are based on the coefficients and source term determined by the solutions of the last iteration. In particular, the pressure field is solved according to the velocities of the previous iteration, and it is in this aspect that the solved velocity field and the pressure field are not consistent. It is the authors' consideration that the pressure filed may be further revised within the iteration that the consistency between the two fields can be refined.

Review of the SIMPLEC method

Subtracting the two sides of equations (8a) and (8b) by $\sum a_{nb} u'_e$, $\sum a_{nb} v'_n$, respectively, we have:

$$(a_e - \sum a_{nb}) u'_e = \sum a_{nb} (u'_{nb} - u'_e) + (p'_P - p'_E) A_e \quad (12a)$$

$$(a_n - \sum a_{\mathrm{nb}})v'_n = \sum a_{\mathrm{nb}}(v'_{\mathrm{nb}} - v'_n) + (p'_P - p'_N)A_n \tag{12b}$$

Now dropping the first term of the right hand side of the above two equations, we obtain the following velocity correction equations:

$$u'_e = d'_e(p'_P - p'_E), \quad v'_e = d'_n(p'_P - p'_N) \tag{13a}$$

where

$$d'_e = \frac{A_n}{(a_e - \sum a_{\mathrm{nb}})}, \quad d'_n = \frac{A_n}{(a_n - \sum a_{\mathrm{nb}})} \tag{13b}$$

Obviously the drop of the first term at the right hand side of equations (12a) and (12b) have less effect than that of dropping the corresponding term in equations (8a) and (8b). This means that the SIMPLEC algorithm alleviates in some degree the effect of the second approximation in the SIMPLE algorithm.

The pressure correction equation in the SIMPLEC algorithm is the same as that in the SIMPLE algorithm except that the *d*-terms are calculated from equation (13b). The solution procedure of the SIMPLEC algorithm is identical to that of SIMPLE (Tao, 2001; Patankar, 1980).

Presentation of the CSIMPLER algorithm

Now we incorporate the major idea of the SIMPLEC algorithm into the SIMPLER algorithms as follows:

- the pressure correction equation of the SIMPLEC is adopted in the SIMPLER algorithm, i.e. the *d*-terms are calculated from equation (13b);
- the pressure is also corrected after the pressure correction equation is solved:

$$p = p^* + \alpha_p p' \tag{14}$$

where α_p is the relaxation factor for the pressure correction. When $\alpha_p < 1$, it is underrelaxation of the pressure correction, while $\alpha_p > 1$ implies the overrelaxation. Our practices have shown that overrelaxation of the pressure correction term is often useful for the acceleration of the convergence procedure which will be discussed later.

By adopting above two treatments into SIMPLER algorithm and keeping the solution procedures the same, the resulting solution algorithm is called consistent SIMPLER, simplified by CSIMPLER.

We consider that the introduction of the pressure correction term into the present pressure will improve the coupling between velocity and pressure, hence, may accelerate the convergence of the iterative process. It is to be noted that for any existing code based on the SIMPLER algorithm the implementation of the CSIMPLER algorithm is very simple and easy.

Numerical comparisons of CSIMPLER and SIMPLER

Comparison conditions

The performance of CSIMPLER and SIMPLER algorithm is compared for seven different problems of fluid flow and heat transfer. In order to make a meaningful comparison between SIMPLER and CSIMPLER, the numerical treatments of all other aspects should be the same. These includes:

- *Discretization scheme.* For the stability of solution procedure and the simplicity of implementation, the absolutely stable scheme, power-law scheme (Tao, 2001), is adopted.
- *Solution method of the algebraic equations.* The algebraic equations are solved by the alternative direction implicit method (ADI) incorporated by the block-correction technique (Moukalled and Darwish, 2000).
- *Under-relaxation factor.* For both the SIMPLER and CSIMPLER algorithm, the same value is adopted for the under-relaxation factor α. For the convenience of presentation, the time step multiple, E, is used in the following presentation, which relates to the under-relaxation factor α by equation (15) (van Doormaal and Raithby, 1984):

$$E = \frac{\alpha}{1 - \alpha} \quad (0 < \alpha < 1) \tag{15}$$

  Some correspondence between α and E is presented in Table I. It can be seen that with the time step multiple, we have a much wider range to show the performance of the algorithm in the high value region of the under-relaxation factor.
- *Convergence criterion.* The convergence criterion is adopted as follows:

$$S_{\text{sum}} = \frac{R_{\text{sum}}}{q_m} \leq \varepsilon, \quad S_{\max} = \frac{R_{\max}}{q_m} \leq \varepsilon \tag{16}$$

  where R_{sum} is the sum of the mass flow rate residuals of all the internal control volumes, $R_{\max}$ is the maximum of absolute values of the mass flow rate residuals, q_m is a reference flow rate. For the seven examples tested the value of ε is taken as 5×10^{-8}.

  For an open system with inlet and outlet boundaries, q_m is the inlet mass flow rate. For a close system, it is given by

$$q_m = \int_a^b \rho |u| \mathrm{d}y \tag{17}$$

  where a, b stand for the bottom and top of any section of the system (Figure 2(a)).

Table I. Some correspondence between α and E

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
E	0.111	0.25	0.428	0.66	1	1.5	2.33	4	9	19

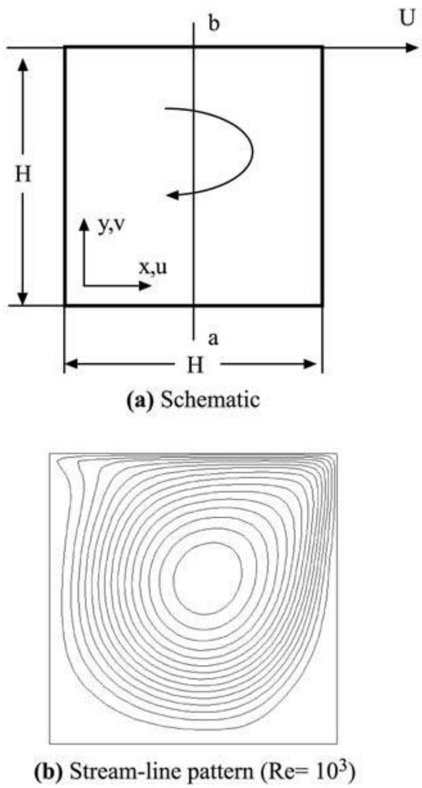


Figure 2.
Lid-driven flow in a square cavity

- *Grid system.* Grid system character is specified individually. For each problem the same grid system is used for the two algorithms.

The SIMPLER and CSIMPLER algorithms are applied to seven two dimensional problems of fluid flow and heat transfer. They are:

(1) lid-driven flow in a square cavity;
(2) flow in a tube with sudden expansion;
(3) natural convection in a square cavity;
(4) natural convection in a horizontal annulus;
(5) natural convection in a vertical annulus;
(6) nature convection in a square cavity with an internal isolated plate; and
(7) natural convection in a horizontal annulus with a slotted inner cylinder.

These seven problems cover the three 2D orthogonal coordinates. For saving space, the governing equations of each problem are omitted. For all the seven examples, numerical or experimental results are available in the literatures. Our computational results using the CSIMPLER and SIMPLER are almost identical and agree with the available results. For simplicity, in the following the comparisons with the available data are omitted, and only some results (stream-line pattern or isothermal contour pattern) and the iteration numbers for obtaining converged solutions are presented. Since in the CSIMPLER algorithm the extra computational effort for execution of one iteration is only spent for the explicit computation of the *d*-coefficients and the pressure correction, the saving in computational time is almost the same as the saving in the iteration numbers.

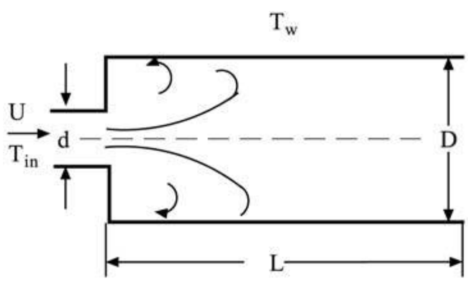
Comparison examples

The lid-driven cavity flow. The simulations are carried out for Reynolds number of 10^3 with two different uniform grid systems. The predicted streamlines are shown in Figure 2(b) and the iteration numbers required for convergence are listed in Table II. It can be seen that in a wide rage of the under-relaxation factor, the CSIMPLER algorithm has a faster convergence rate, especially for the cases with low values of the under-relaxation factor (Ghie *et al.*, 1982) (Figure 2(a)).

Flow in a tube with a sudden expansion. The simulations are conducted for $Re = 10^3$ with two different grid systems. The simulated flow pattern is shown in Figure 3(b) and the iteration numbers are compared and listed in Table III. It can be observed that for this case, CSIMPLER does not offer benefit, however, the convergence rate of the

Table II.
Required iteration numbers for case 1

E	0.25	0.5	1	3	5	7	9	11	15
α	0.200	0.333	0.500	0.750	0.833	0.875	0.900	0.917	0.938
$Re = 10^3$; 42×42 meshes; $S_{sum} \leq 5 \times 10^{-8}$ and $S_{max} \leq 5 \times 10^{-8}$									
SIMPLER	1,670	1,040	612	237	140	134	120	114	619
CSIMPLER	1,466	947	574	230	140	126	119	142	Div
$Re = 10^3$; 102×102 meshes; $S_{sum} \leq 5 \times 10^{-8}$ and $S_{max} \leq 5 \times 10^{-8}$									
SIMPLER	5,019	3,218	2,071	899	646	427	378	353	Div
CSIMPLER	4,530	2,856	1,890	867	561	401	358	Div	Div



(a) Schematic

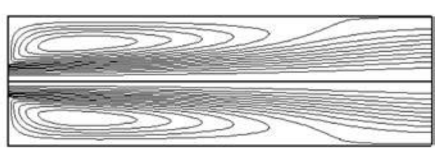

(b) Stream-line pattern (Re=10^3)

Figure 3.
Flow in a tube with a sudden expansion

CSIMPLER is not worse than that of the SIMPLER algorithm (Macagno and Hung, 1967) (Figure 3(a)).

Natural convection in a square cavity. The computations are conducted for $Pr = 0.7$ and $Ra = 10^4$ with two different systems. Ra number is defined as $Ra = \rho^2 g\beta\Delta TL^3 Pr/\eta^2$. Figure 4(b) and (c) show the streamlines and the isothermals. Table IV lists the iteration numbers for CSIMPLER and SIMPLER algorithms. Obviously, the CSIMPLER can enhance the convergence rate (Barakos and Mitsoulis, 1994) (Figure 4(a)).

Natural convection in a horizontal annulus. The computations are carried out for $Ra = 10^4$ with two different grid systems. In Figure 5(b) and (c) the predicted streamlines and isothermal are provided. The required iteration numbers are listed in Table V (Kuehn and Goldstein, 1969) (Figure 5(a)).

Natural convection in a vertical cylinder annulus. The computations are carried out for $Ra = 10^4$ and $Pr = 0.7$ with two different grid systems. Figure 6(b) and (c) show the streamline and isothermal distribution patterns for $Ra = 10^4$. Table VI presents the required iteration numbers for the CSIMPLER and SIMPLER methods. For the cases with low values of under-relaxation factor, the CSIMPLER has some benefit (Eisherbiny, 1983) (Figure 6(a)).

Natural convection in a square enclosure with an internal isolated vertical plate. The inner plate and the bounding wall of the enclosure are maintained at uniform but different temperatures. The computations are carried out for $Ra = 10^4$ and 10^6 and $Pr = 0.7$ with two respective grid systems. The predicted fields for $Ra = 10^4$ are shown in Figure 7(b) and (c), which agree with (Wang *et al.*, 1994) very well. The required iteration numbers for CSIMPLER and SIMPLER are compared in Table VII (Wang *et al.*, 1994) (Figure 7(a)).

Natural convection in horizontal annulus with a slotted inner cylinder. As shown in Figure 8(a), the inside and outside walls have constant but different temperatures. The computations are carried out for $Ra = 10^5$ and 10^6 ($Pr = 0.7$) with two different grid systems. The predicted results for $Ra = 10^5$ are shown in Figure 8(b) and (c). Table VIII compared the required iteration numbers for the CSIMPLER and SIMPLER methods (Yang and Tao, 1992) (Figure 8(a)).

Discussion on the comparison results

The above comparison results display the following features.

- For almost all cases considered under different conditions and with different coordinates systems, the iteration numbers using CSIMPLER algorithm are less

E	0.25	0.5	1	3	5	7	9	11	15
α	0.200	0.333	0.500	0.750	0.833	0.875	0.900	0.917	0.938
$Re = 10^3$; 32×22 meshes; $S_{sum} \leq 5 \times 10^{-8}$ and $S_{max} \leq 5 \times 10^{-8}$									
SIMPLER	450	257	162	99	99	96	97	94	101
CSIMPLER	449	257	162	99	99	96	94	93	98
$Re = 10^3$; 62×42 meshes; $S_{sum} \leq 5 \times 10^{-8}$ and $S_{max} \leq 5 \times 10^{-8}$									
SIMPLER	1,460	834	510	185	150	127	Div	Div	Div
CSIMPLER	1,442	826	506	185	152	Div	Div	Div	Div

Table III. Required iteration numbers for case 2

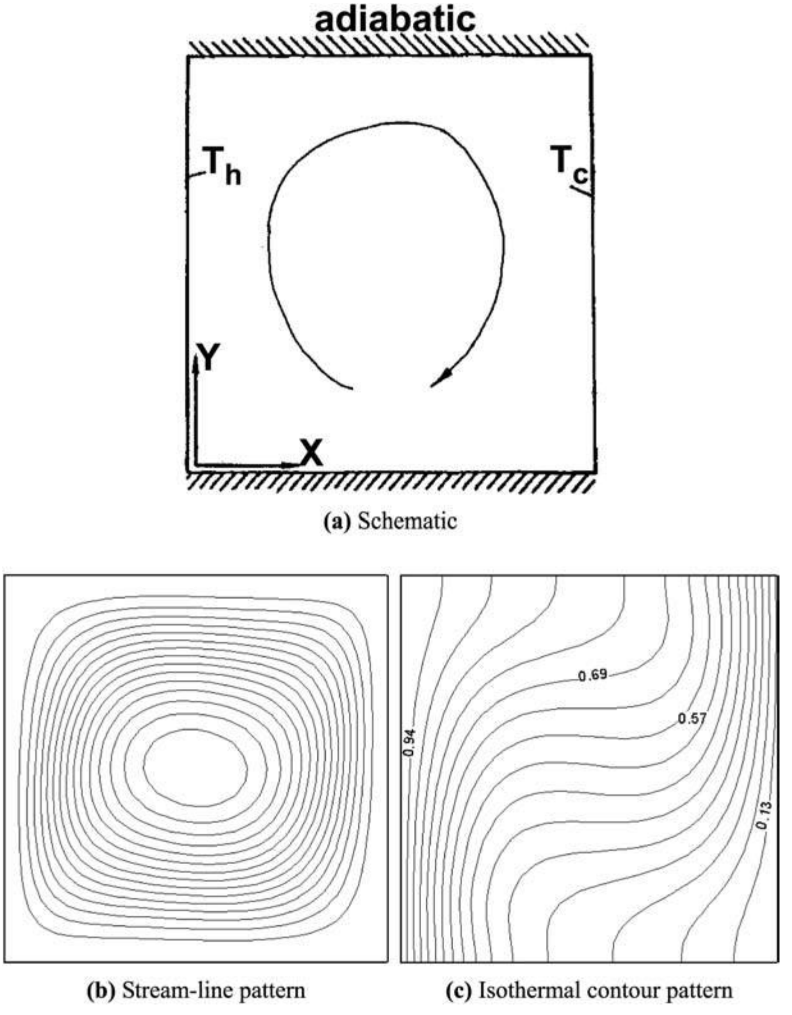
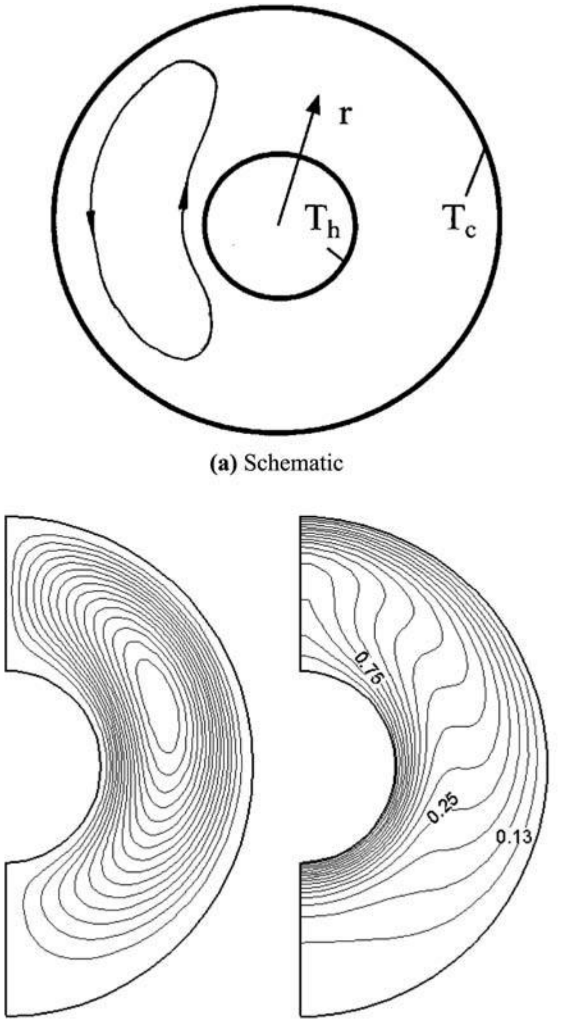


Figure 4.
Natural convection in a square cavity ($Ra = 10^4$, $Pr = 0.7$)

Table IV.
Required iteration numbers for case 3

E	0.25	0.5	1	3	5	7	9	11	15
α	0.200	0.333	0.500	0.750	0.833	0.875	0.900	0.917	0.938
$Ra = 10^4$; 32×32 meshes; $S_{sum} \leq 5 \times 10^{-8}$ and $S_{max} \leq 5 \times 10^{-8}$									
SIMPLER	3,752	2,460	1,498	625	407	305	246	207	158
CSIMPLER	3,296	2,260	1,420	611	401	302	244	205	157
$Ra = 10^4$; 62×62 meshes; $S_{sum} \leq 5 \times 10^{-8}$ and $S_{max} \leq 5 \times 10^{-8}$									
SIMPLER	8,924	6,674	4,394	1,908	1,286	971	660	505	376
CSIMPLER	5,791	5,725	4,025	1,879	1,257	950	653	501	374



(a) Schematic

(b) Stream-line pattern **(c)** Isothermal contour pattern

Figure 5.
Natural convection in a horizontal annulus ($Ra = 10^4$, $Pr = 0.7$)

E	0.25	0.5	1	3	5	7	9	11	15
α	0.200	0.333	0.500	0.750	0.833	0.875	0.900	0.917	0.938
$Ra = 10^4$; 32×32 meshes; $S_{sum} \leq 5 \times 10^{-8}$ and $S_{max} \leq 5 \times 10^{-8}$									
SIMPLER	2,514	1,721	1,076	463	306	232	228	Div	Div
CSIMPLER	2,155	1,572	1,020	454	302	230	Div	Div	Div
$Ra = 10^4$; 62×62 meshes; $S_{sum} \leq 5 \times 10^{-8}$ and $S_{max} \leq 5 \times 10^{-8}$									
SIMPLER	2,471	2,986	2,346	1,177	805	621	Div	Div	Div
CSIMPLER	2,181	1,790	2,054	1,130	787	611	Div	Div	Div

Table V.
Required iteration numbers for case 4

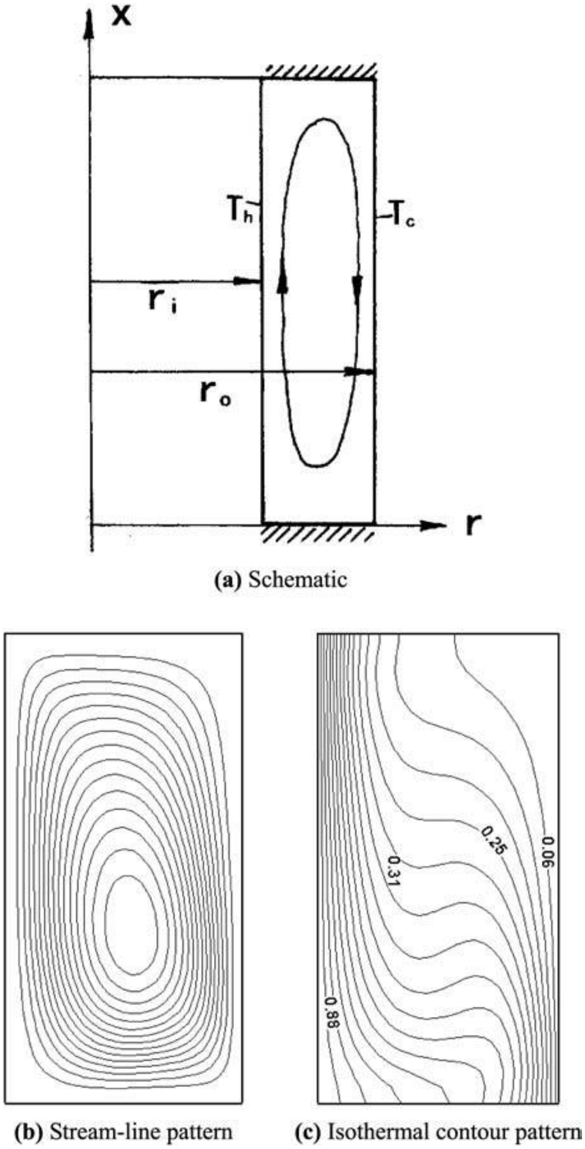


Figure 6.
Natural convection in a vertical annulus ($Ra = 10^4$, $Pr = 0.7$)

than or at worst equal to those using SIMPLER. To different problems, CSIMPLER can enhance the rate of convergence to different degree.

- When the value of the under-relaxation factor is lower, the enhancement of the CSIMPLER is more appreciable. With the increase in the under-relaxation factor, however, its enhancement function gradually disappears. It should be noted that the larger under-relaxation factor (larger than 0.9) is seldom used in the computation for actual engineering problem and the factor less than 0.5 is often used in the cases of strongly coupled problems and it is for such cases that acceleration of the convergence rate is especially desired. Therefore those numerical methods which show benefits at the low under-relaxation factor do make sense for engineering computations.

- The robustness of CSIMPLER algorithm is seemingly a bit worse than that of the SIMPLER algorithm in that for some examples, at the very high value region of the under-relaxation factor, usually larger than 0.9 or more, CSIMPLER may lead to diverge while SIMPLER can still get converged solution.

But this would not affect the application of the CSIMPLER algorithm. Apart from the reason mentioned above, this is because of following two factors. First, as it can be seen from Tables V and VIII, this situation usually occurs at the relatively coarse grid; second, the difference between the variation ranges of the under-relaxation factor within which the converged solution can be obtained is quite small.

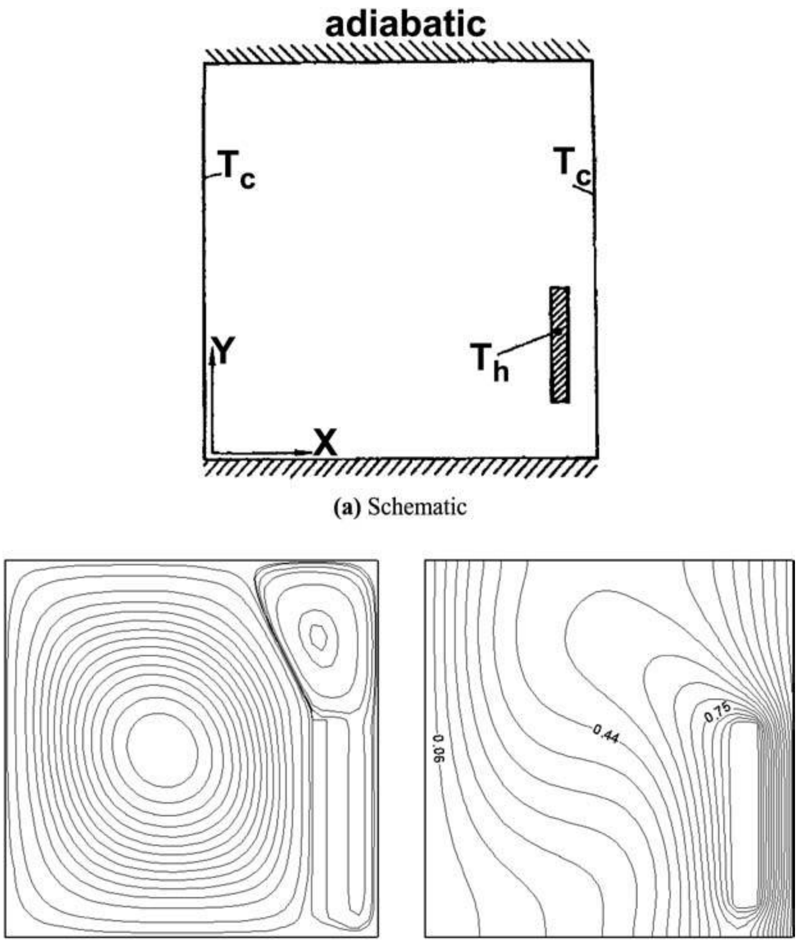
Finally attention is turned to the relaxation of the pressure correction term. From our practices the pressure correction term can even be overrelaxed, i.e. the value of $\alpha_p$in equation (14) can be larger than 1. Numerical tests have been performed and the results are presented in Table IX for comparison. It can be seen that for all the cases tested the overrelaxation of pressure correction is of advantage to enhance the rate of solution convergence, and the larger relaxation factor the more advantage. To give a precise description of the overrelaxation of the pressure correction term, following two facts should be added. First, the robustness of the algorithm will be a bit deteriorated while overrelaxation of the pressure correction is adopted; second, too large value of the over-relaxation factor may lead to divergence of the solution procedure. Our practices show that the relaxation factor larger than 5 may lead to such an outcome, although the accurate value is problem dependent.

Conclusion

The CSIMPLER algorithm is proposed based on the SIMPLER algorithm by modifying the coefficients of the velocity correction and pressure correction equations and correcting pressure by the pressure correction term. Seven numerical examples show that the CSIMPLER algorithm can enhance the convergence rate for almost all cases tested and especially appreciable when the under-relaxation factor is low. The robustness of the CSIMPLER is nearly as good as that of SIMPLER algorithm. For the seven examples tested, the pressure correction term even can be overrelaxed to further enhance the convergence. The adoption of the CSIMPLER algorithm for any existing code based on the SIMPER is very simple and easy.

E	0.25	0.5	1	3	5	7	9	11	15
α	0.200	0.333	0.500	0.750	0.833	0.875	0.900	0.917	0.938
$Ra = 10^4$; 32 × 32 meshes; $S_{sum} \leq 5\times10^{-8}$ and $S_{max} \leq 5\times10^{-8}$									
SIMPLER	2,515	1,705	1,063	455	299	226	183	154	119
CSIMPLER	2,261	1,588	1,016	446	295	224	181	153	468
$Ra = 10^4$; 62 × 62 meshes; $S_{sum} \leq 5\times10^{-8}$ and $S_{max} \leq 5\times10^{-8}$									
SIMPLER	3,631	4,121	2,863	1,321	887	676	550	465	Div
CSIMPLER	3,239	3,373	2,608	1,272	865	664	542	460	Div

Table VI. Required iteration numbers for case 5



(a) Schematic

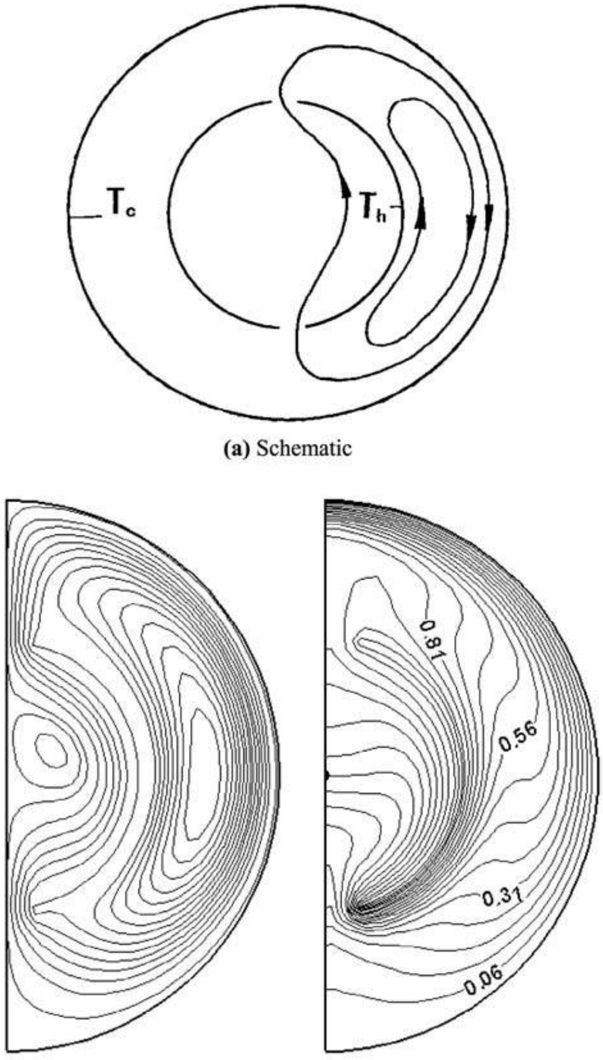
(b) Stream-line pattern

(c) Isothermal contour pattern

Figure 7.
Natural convection in a square cavity with an internal isolated plate ($Ra = 10^4$, $Pr = 0.7$)

Table VII.
Required iteration numbers for case 6

E	0.25	0.5	1	3	5	7	9	11	15
α	0.200	0.333	0.500	0.750	0.833	0.875	0.900	0.917	0.938
$Ra = 10^4$; 42×42 meshes; $S_{sum} \leq 5 \times 10^{-8}$ and $S_{max} \leq 5 \times 10^{-8}$									
SIMPLER	845	758	728	722	725	726	726	725	724
CSIMPLER	724	756	725	719	722	724	724	724	723
$Ra = 10^6$; 82×82 meshes; $S_{sum} \leq 5 \times 10^{-8}$ and $S_{max} \leq 5 \times 10^{-8}$									
SIMPLER	1,826	1,655	1,125	1,166	1,185	1,196	1,203	1,208	1,211
CSIMPLER	1,779	1,346	1,022	1,138	1,165	1,181	1,191	1,197	Div



(a) Schematic

(b) Stream-line pattern

(c) Isothermal contour pattern

Figure 8.
Natural convection in a horizontal annulus with a slotted inner cylinder ($Ra = 10^5$, $Pr = 0.7$)

E	0.25	0.5	1	3	5	7	9	11	15
α	0.200	0.333	0.500	0.750	0.833	0.875	0.900	0.917	0.938
$Ra = 10^5$; 32×32 meshes; $S_{sum} \leq 5 \times 10^{-8}$ and $S_{max} \leq 5 \times 10^{-8}$									
SIMPLER	1,102	1,142	1,041	1,090	1,122	1,136	1,145	1,158	1,186
CSIMPLER	1,100	1,048	1,040	1,089	1,120	1,135	1,143	1,149	Div
$Ra = 10^6$; 62×62 meshes; $S_{sum} \leq 5 \times 10^{-8}$ and $S_{max} \leq 5 \times 10^{-8}$									
SIMPLER	5,726	5,947	6,557	7,572	7,882	8,033	Div	Div	Div
CSIMPLER	5,080	5,791	6,424	7,497	7,833	7,997	Div	Div	Div

Table VIII.
Required iteration numbers for case 7

Table IX.
Required iteration numbers for the relaxation test for the pressure correction

α_p	0.5	1.0	1.5	2.0	2.5	3.0	SIMPLER
Case 1 ($Re = 10^3$, 42×42, $E = 0.5$)	988	947	912	881	864	855	1,040
Case 2 ($Re = 10^3$, 62×42, $E = 0.5$)	829	826	823	820	817	815	834
Case 3 ($Ra = 10^4$, 32×32, $E = 0.5$)	1,340	1,300	1,267	1,240	1,217	1,197	1,400
Case 4 ($Ra = 10^4$, 32×32, $E = 0.5$)	1,635	1,572	1,523	1,483	1,451	1,423	1,721
Case 5 ($Ra = 10^4$, 32×32, $E = 0.5$)	1,640	1,588	1,545	1,508	1,476	1,466	1,705
Case 6 ($Ra = 10^4$, 42×42, $E = 0.5$)	757	756	755	754	753	752	758
Case 7 ($Ra = 10^6$, 62×62, $E = 0.5$)	5,860	5,791	5,734	5,684	5,641	5,602	5,947

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