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A NOVEL SEGREGATED ALGORITHM FOR INCOMPRESSIBLE FLUID FLOW AND HEAT TRANSFER PROBLEMS—CLEAR (COUPLED AND LINKED EQUATIONS ALGORITHM REVISED) PART II: APPLICATION EXAMPLES

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In Part I of this article a novel algorithm, CLEAR, was introduced. In this article the relative performance of the CLEAR algorithm and the SIMPLER algorithm is evaluated for six incompressible fluid flow and heat transfer problems with constant property. The six examples cover three two-dimensional orthogonal coordinates. Comprehensive comparisons are made between the two algorithms on the subject of iteration number for obtaining a converged solution, and the consumed CPU time. It is found that CLEAR can appreciably enhance the convergence rate. For the six problems tested, the ratio of iteration numbers of CLEAR over that of SIMPLER ranges from 0.15 to 0.84, and the ratio of the CPU time from 0.19 to 0.92.

INTRODUCTION

In Part I of this article [1] a novel algorithm was introduced. The new algorithm is called CLEAR (Coupled and Linked Equations Algorithm Revised). It differs from all SIMPLE-like algorithms in that it solves the improved pressure directly, rather than by adding a correction term, and no term is dropped in the derivation of the pressure equation. Thus the effects of the neighboring velocity values are fully taken into account. The coupling between velocity and pressure is therefore fully guaranteed, greatly enhancing the convergence rate of the iteration process.

In this article the CLEAR algorithm is applied to solve six fluid flow and heat transfer problems with available numerical solutions. Comparisons are made with the solutions from the SIMPLER algorithm. In the following, the comparison conditions and the convergence criterion are described first, followed by detailed

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NOMENCLATURE

a	fluid thermal diffusivity	u^*, v^*	temporary velocity
A	surface area	U, V	dimensionless velocity in two coordinates
D	diameter		
E	time step multiple	U_{lid}	moving velocity of lid
$flow_{ch}$	characteristic (reference) flow rate	x, y	coordinates
		X, Y	dimensionless coordinates
g	gravitational acceleration	α	underelaxation factor
H_1, H_2	height defined in Figure 18	β	relaxation factor
L	length of square cavity	δ	gap width
L_1, L_R, L_2	dimensions defined in Figure 18	ΔT	temperature difference
		ρ	fluid density
L_{in}, L_x	length defined in Figure 8	μ	fluid dynamic viscosity
r	radius	ν	fluid kinetic viscosity
R	radius of tube wall	ϖ	angular velocity
Ra	Rayleigh number		
Re	Reynolds number	**Subscripts**	
Rs_{cv}	relative mass flow rate unbalance of control volume	in	inlet; inner
		max	maximum
u, v	velocity component in x, y directions	mean	averaged
		out	outlet

presentations of the computational results of the six examples. Finally, some conclusions are drawn.

NUMERICAL COMPARISON CONDITIONS

In order to make a meaningful comparison between SIMPLER and CLEAR, the numerical treatments of all other aspects should be the same. These include:

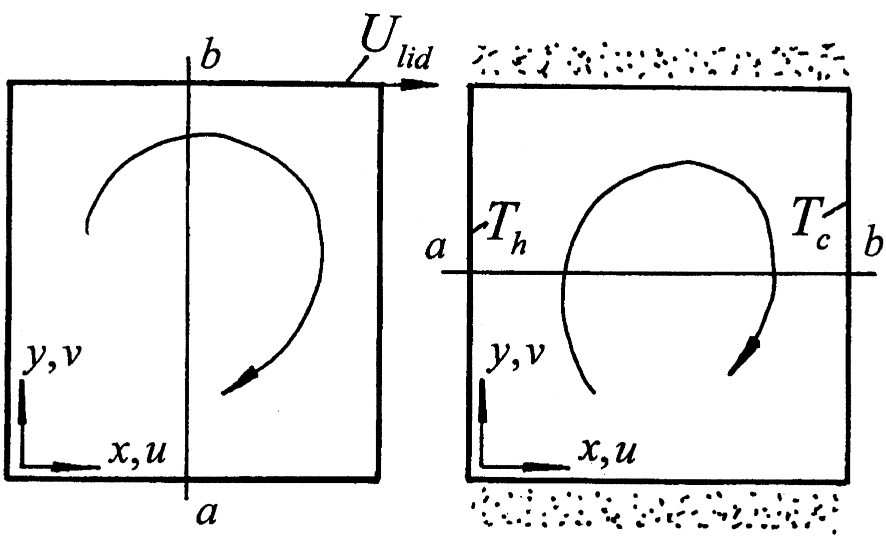


Figure 1. Definition of reference flow rate for fluid flow in an enclosure.

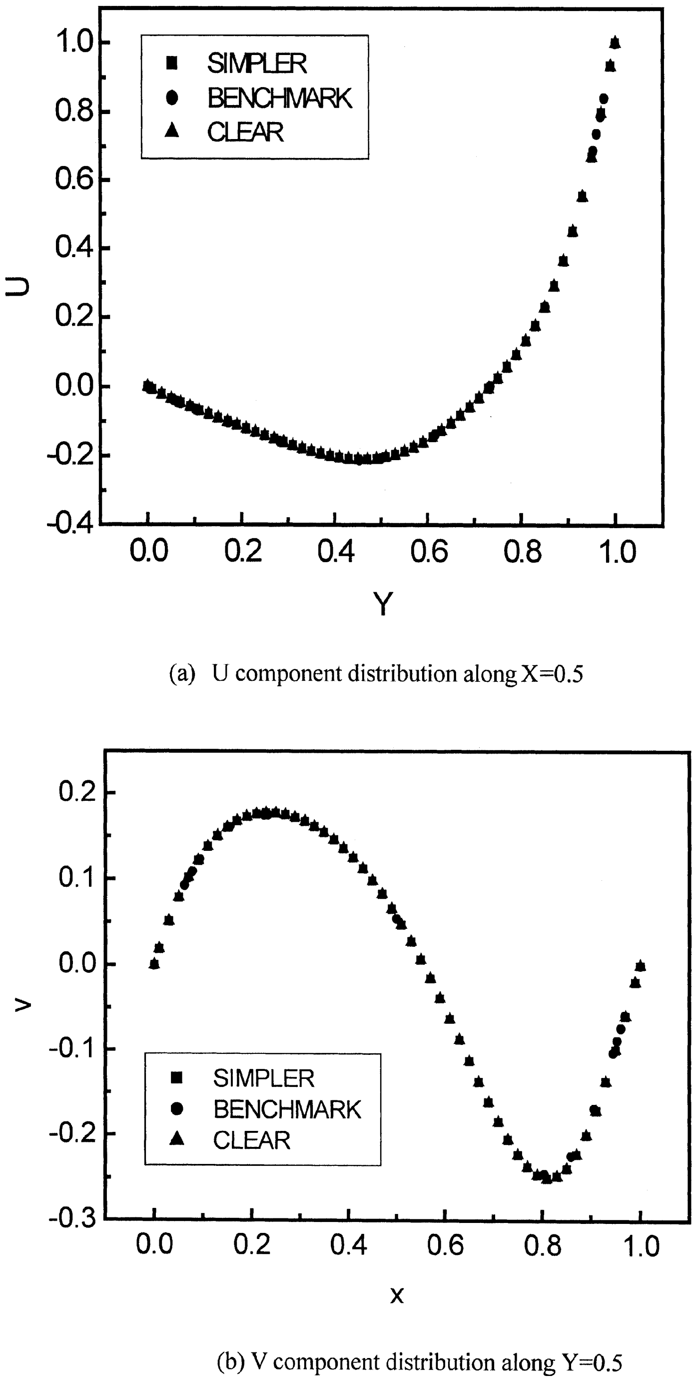
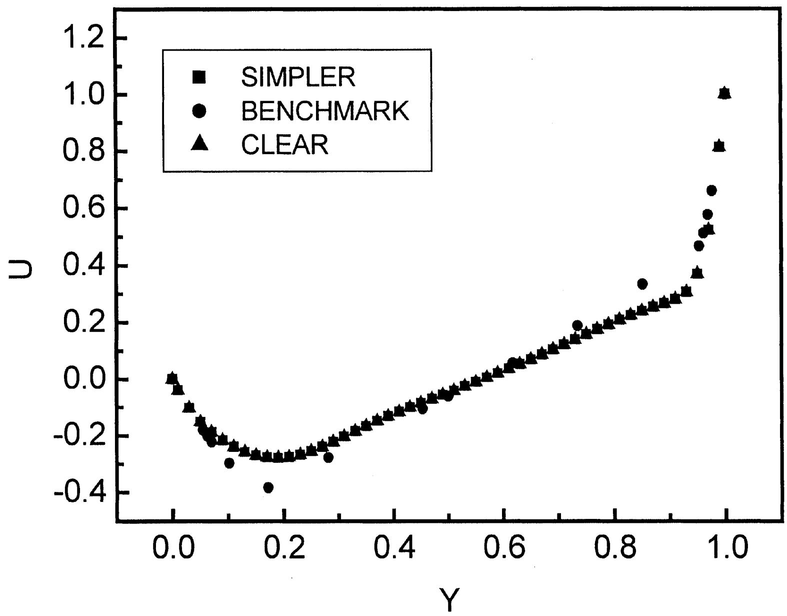
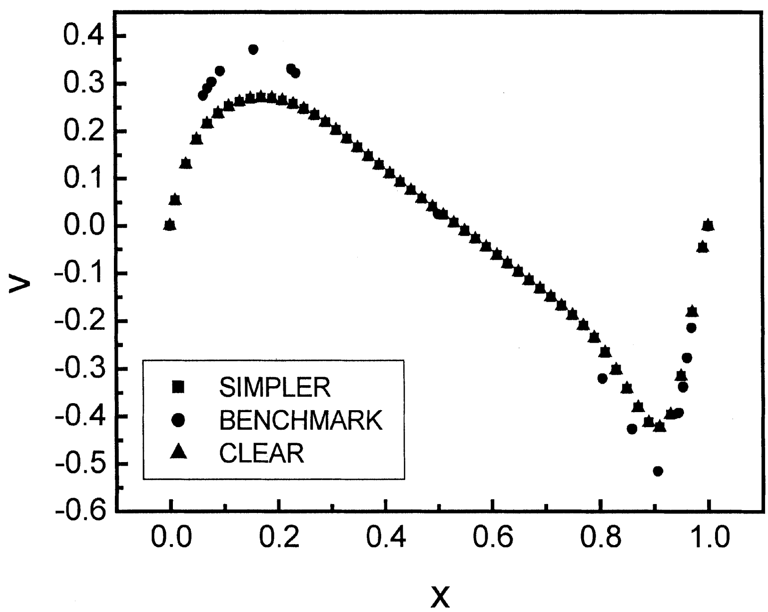


Figure 2. Predicted velocity distributions for Re = 100 in Problem 1.

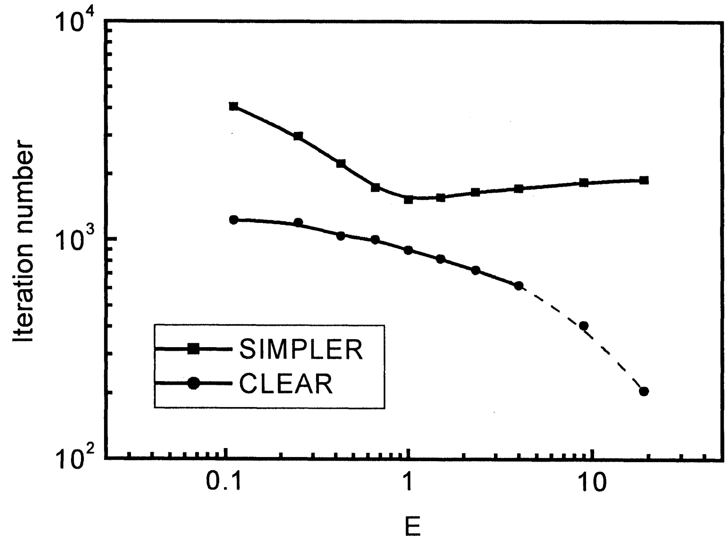


(a) U component distribution along X=0.5

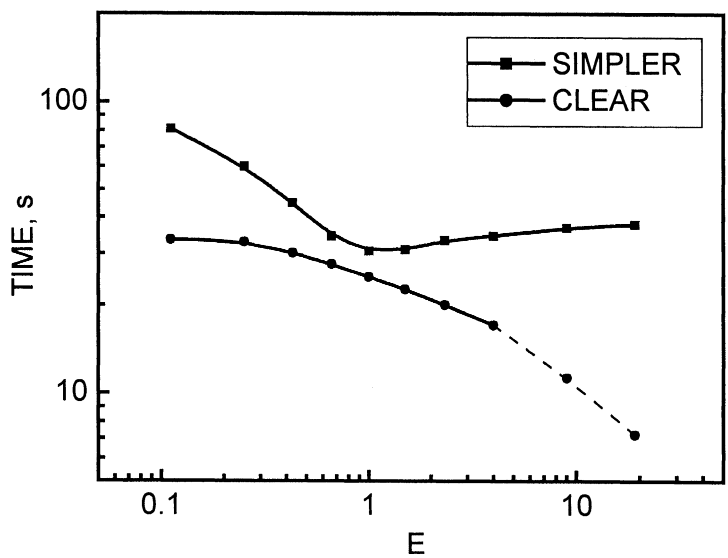


(b) V component distribution along Y=0.5

Figure 3. Predicted velocity distributions for Re = 1,000 in Problem 1.



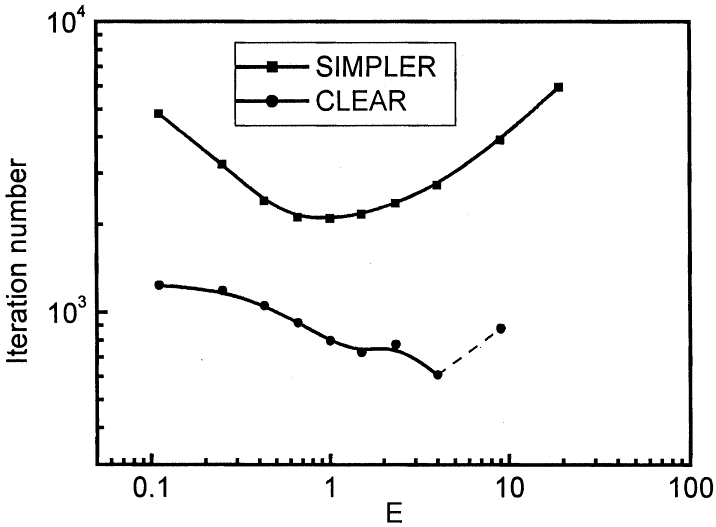
(a) Iteration number



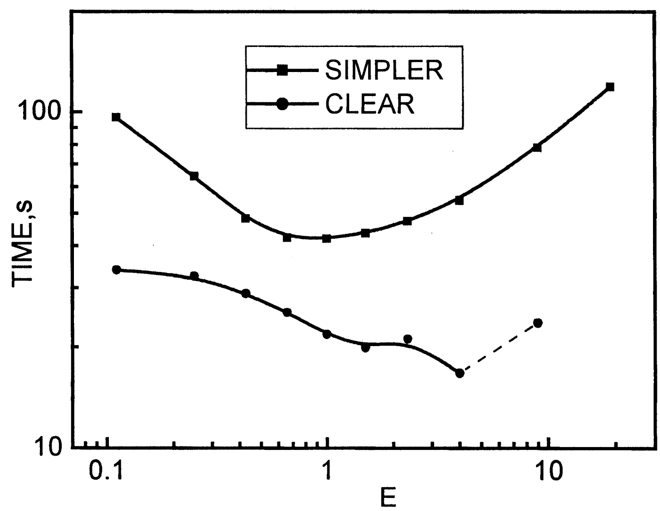
(b) CPU time

Figure 4. Comparison of iteration numbers and CPU time for Re = 100 in Problem 1.

1. Discretization scheme: For the stability of solution procedure and the simplicity of implementation, the absolutely stable scheme, power-law scheme [2], is adopted.



(a) Iteration number



(b) CPU time

Figure 5. Comparison of iteration numbers and CPU time for Re = 1,000 in Problem 1.

2. Solution method of the algebraic equations: The algebraic equations are solved by the alternative direction implicit method (ADI) incorporated by the block-correction technique [3].
3. Underrelaxation factor: For both the SIMPLER and CLEAR algorithm, the same value is adopted for the underrelaxation factor α. For the convenience of presentation, the time step multiple, E, is used in the following presentation, which relates to the underrelaxation factor α by

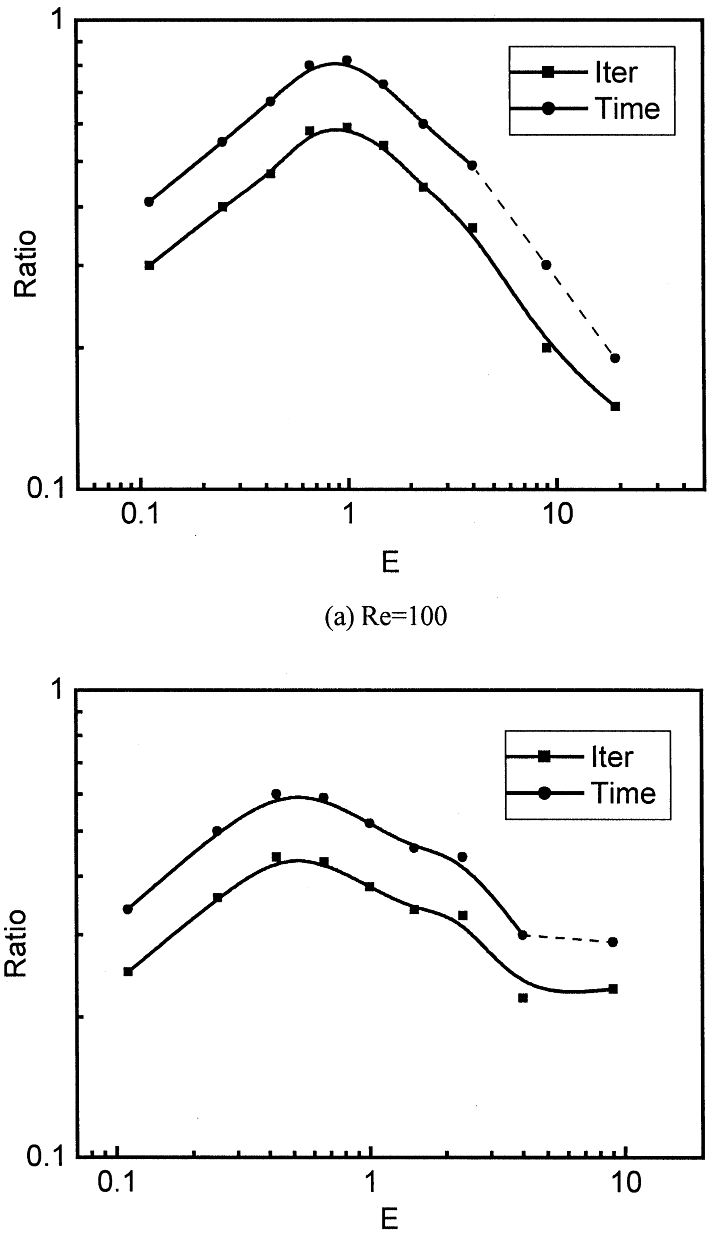


Figure 6. Ratios of iteration number and CPU time of CLEAR versus SIMPLER.

Eq. (1) [4]:

$$E = \frac{\alpha}{1-\alpha} \qquad (0 < \alpha < 1) \tag{1}$$

Table 1. Some correspondence between α and E

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
E	0.111	0.25	0.428	0.66	1	1.5	2.33	4	9	19

Some correspondence between α and E is presented in Table 1. It can be seen that with the time step multiple, we have a much wider range in which to show the performance of the algorithm in the high-value region of the underrelaxation factor.

As far as the second relaxation factor of the CLEAR algorithm is concerned, usually it takes values according to the following relation:

$$\beta = \begin{cases} 0.5 & 0 < \alpha \le 0.5 \\ 1 & 0.5 < \alpha \le 1 \end{cases} \tag{2}$$

For cases where a larger value of β is used, special description will be provided.

4. Convergence criterion: From the presentation of the SIMPLER and the CLEAR algorithms in Part I, it can be seen that when the solution approaches convergence, the temporary solution of velocity from the momentum equation, u^*, v^*, should satisfy the mass conservation condition. This is taken as the convergence criterion, which is expressed as

$$\mathrm{Rs_{cv}} = \underset{\mathrm{cv}}{\mathrm{MAX}} \left[\frac{(\rho u^* A)_w - (\rho u^* A)_e + (\rho v^* A)_s - (\rho v^* A)_n}{\mathrm{flow_{ch}}} \right] \le 5.0 \times 10^{-8} \tag{3}$$

where $\mathrm{Rs_{cv}}$ is the maximum relative mass flow rate unbalance of all the control volumes in the computational domain; $\mathrm{flow_{ch}}$ is the characteristic (or reference) flow rate of the problem studied. For problems with inflow and outflow boundaries, $\mathrm{flow_{ch}}$ takes the mass flow rate at the inflow boundary; for fluid flow in an enclosure, $\mathrm{flow_{ch}}$ is defined by Eq. 4 [5] (Figure 1):

$$\mathrm{flow_{ch}} = \int_a^b \rho |u|\, dy \qquad \mathrm{flow_{ch}} = \int_a^b \rho |v|\, dx \tag{4}$$

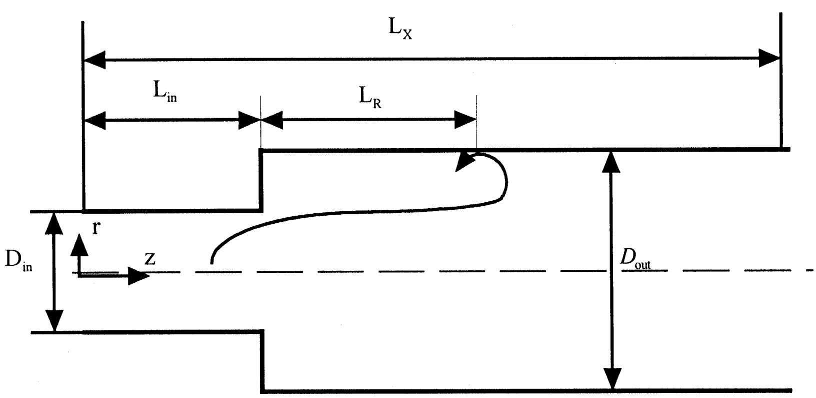
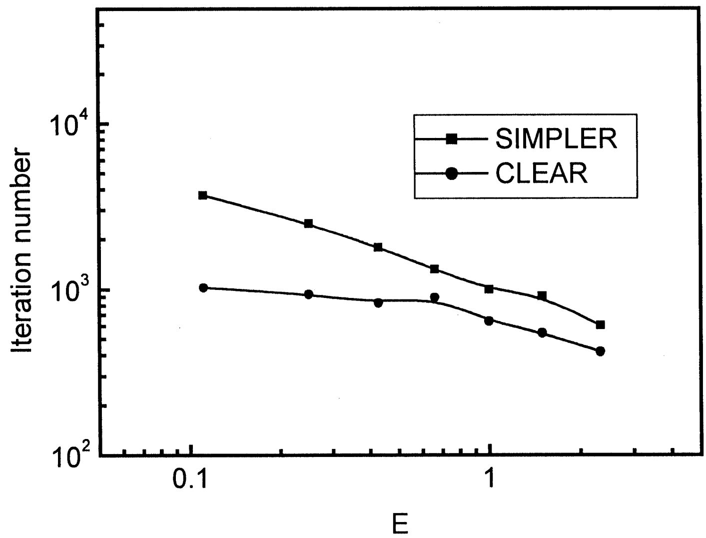
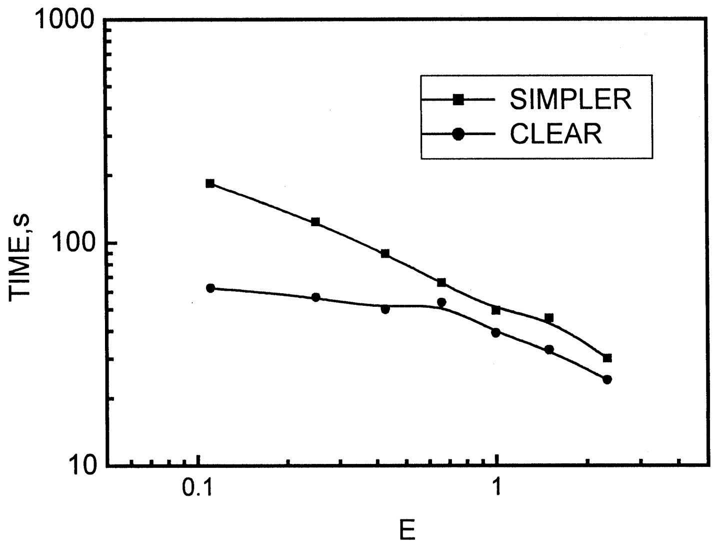


Figure 7. Laminar flow over an annular backward step.



(a) Iteration number



(b) CPU time

Figure 8. Comparison of iteration number and CPU time for Re = 150 in Problem 2.

5. Grid system: Grid system character is specified individually. For each problem the same grid system is used for execution of both the SIMPLER and CLEAR algorithms.

The SIMPLER and CLEAR algorithms are applied to six two-dimensional problems of fluid flow and heat transfer. They are (1) lid-driven cavity flow in a

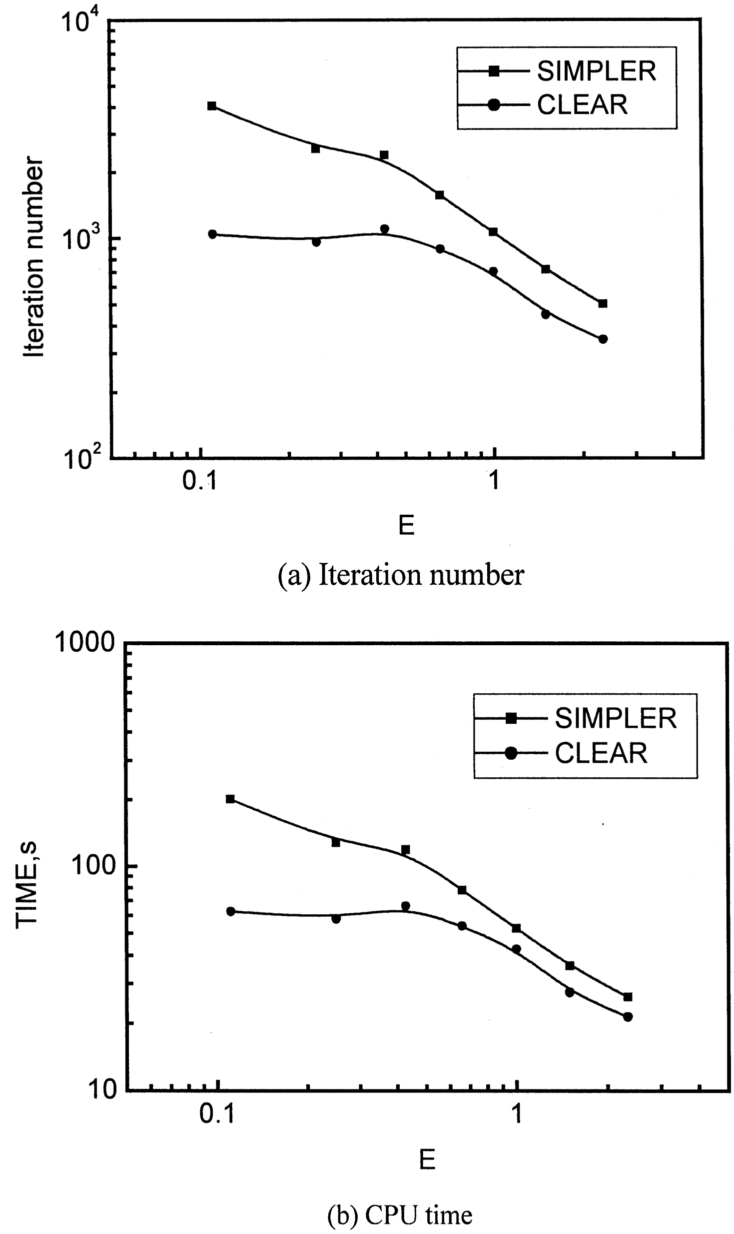
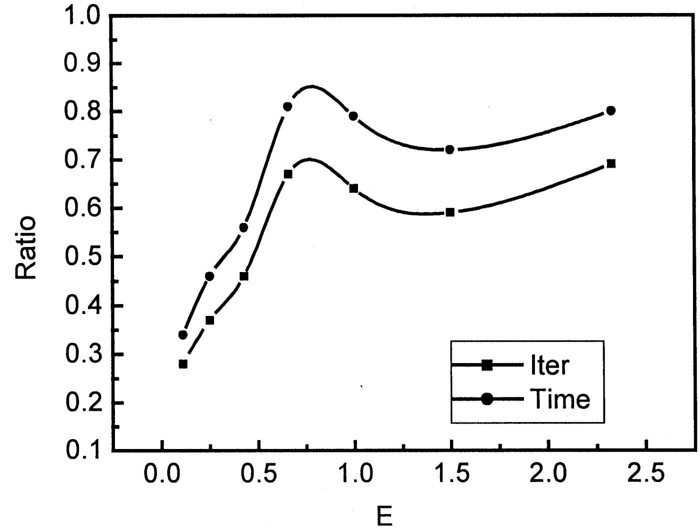
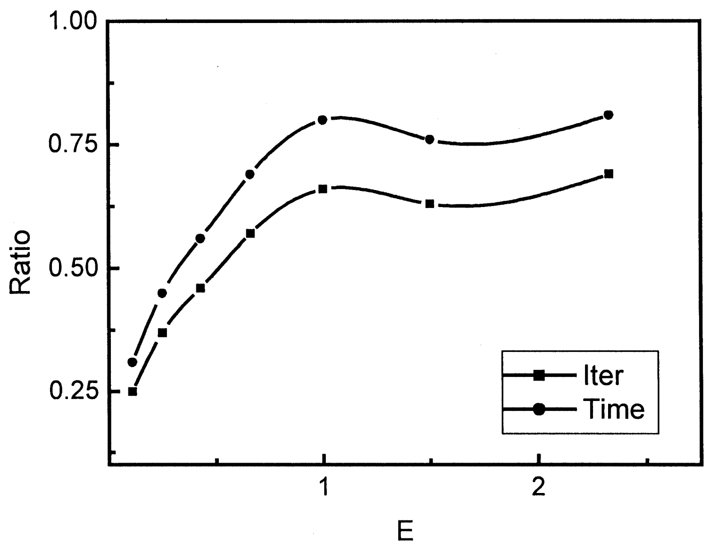


Figure 9. Comparison of iteration number and CPU time for Re = 200 in Problem 2.

square cavity; (2) laminar fluid flow over an annular backward step; (3) lid-driven cavity flow in a polar cavity; (4) laminar fluid flow over a rectangular backward-facing step; (5) natural convection in an annulus enclosure; and (6) natural convection in a square cavity. These six problems cover the three 2-D orthogonal coordinates. The number of iterations for obtaining a converged solution, the CPU



(a) Re=150



(b) Re=200

Figure 10. Ratios of iteration numbers and CPU time for Problem 2.

time, and the robustness of the algorithms are compared. To save space, the governing equations of each problem are omitted. All of the six problems are based on the following assumptions: laminar, incompressible, steady-state, and constant fluid property.

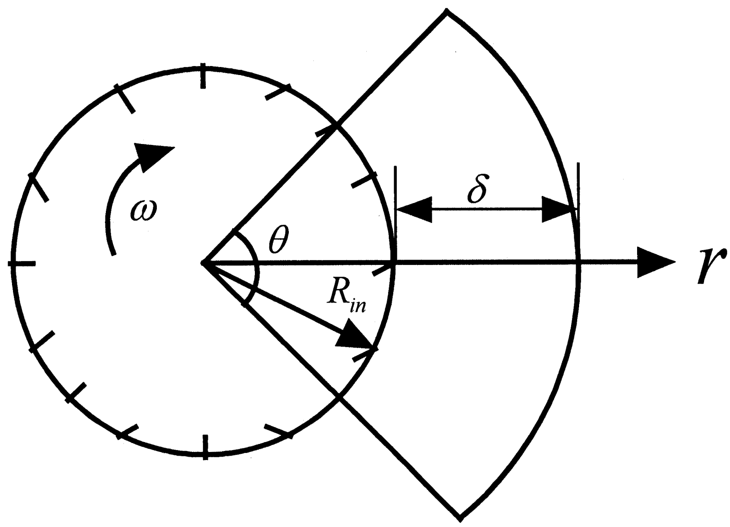


Figure 11. Lid-driven cavity flow in a polar cavity.

NUMERICAL EXPERIMENTS

Problem 1: Lid-Driven Cavity Flow in a Square Cavity

Calculations are conducted for Re number $= 100$ and 1,000. A uniform grid of 52×52 is employed. The Reynolds number is defined by

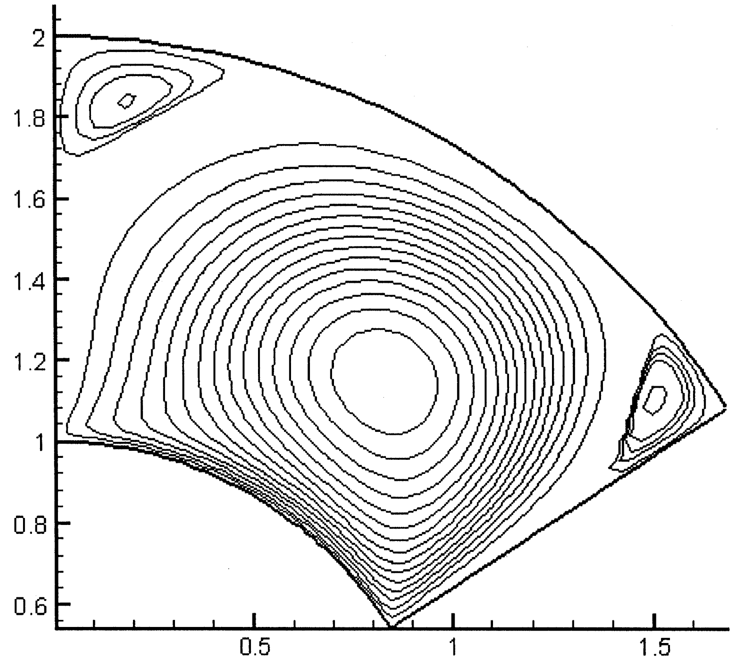
$$\mathrm{Re} = \frac{U_{\mathrm{lid}} L}{\nu} \tag{5}$$

In Figures 2 and 3 the velocity distribution along the two centerlines are shown, and the benchmark solutions from [6] are also presented, where X and Y are nondimensional coordinates, normalized by the cavity height. It can be seen that the solutions from the SIMPLER and the CLEAR are almost identical.

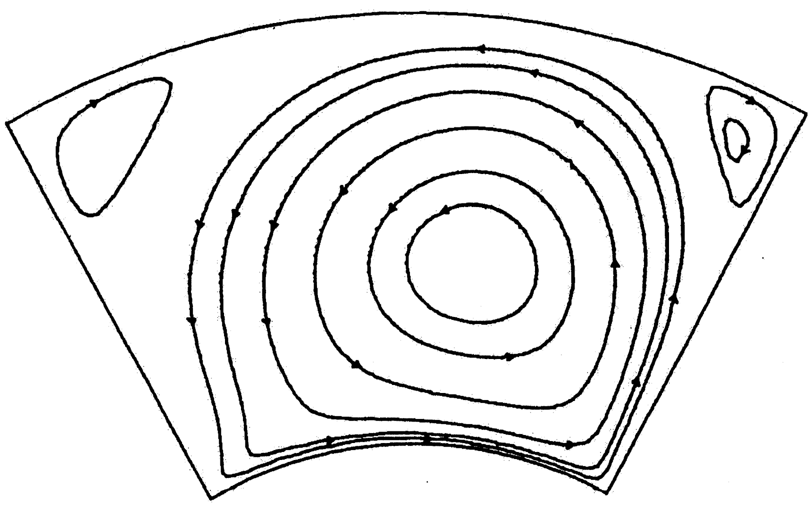
The number of iterations and the consumed CPU time are plotted in Figure 4 ($\mathrm{Re} = 100$) and Figure 5 ($\mathrm{Re} = 1{,}000$). In Figure 6, the ratios of iteration number and CPU time of CLEAR over that of SIMPLER are presented. In the figures, the part shown by the dashed lines is obtained by taking $\beta = 1.2$, i.e., here underrelaxation of the improved pressure must be taken in order to get a converged solution because the intermediate velocity is predicted by a large value of the underrelaxation factor (around 0.9). It can be seen that for $\mathrm{Re} = 100$ the ratio of iteration number ranges from 0.15 to 0.59 and that of CPU time from 0.19 to 0.82; and for $\mathrm{Re} = 1{,}000$, the two ranges are 0.22–0.44 and 0.29–0.59, respectively. The saving of iteration number and CPU time is appreciable.

Problem 2: Laminar Fluid Flow over an Annular Backward Step

The computational configuration is shown in Figure 7, where $L_x/D_{\mathrm{in}} = 30$, $L_{\mathrm{in}}/D_{\mathrm{in}} = 5$, and $D_{\mathrm{out}}/D_{\mathrm{in}} = 2$. Macagno and Hung [7] carried out experimental and



(a) Our prediction



(b) Results of [7]

Figure 12. Predicted stream function in Problem 4 (Re = 350).

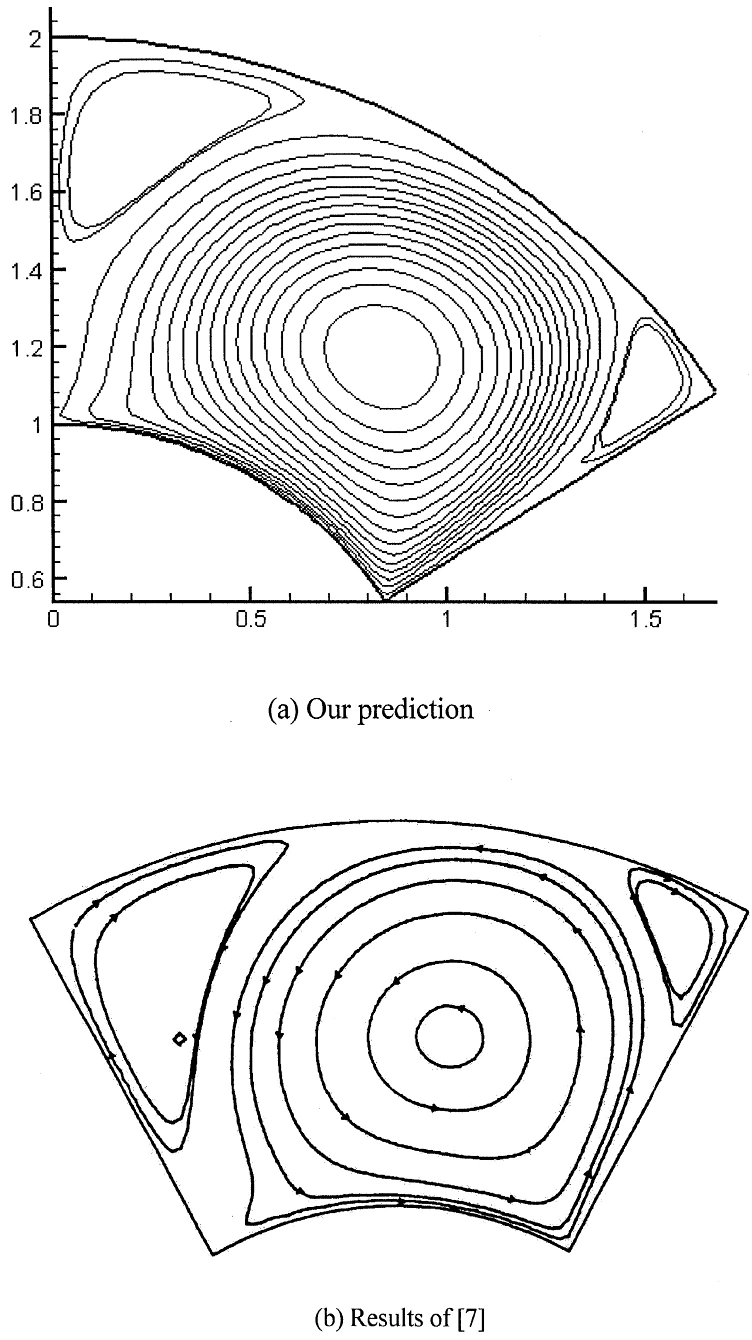


Figure 13. Predicted stream function in Problem 4 (Re = 1,000).

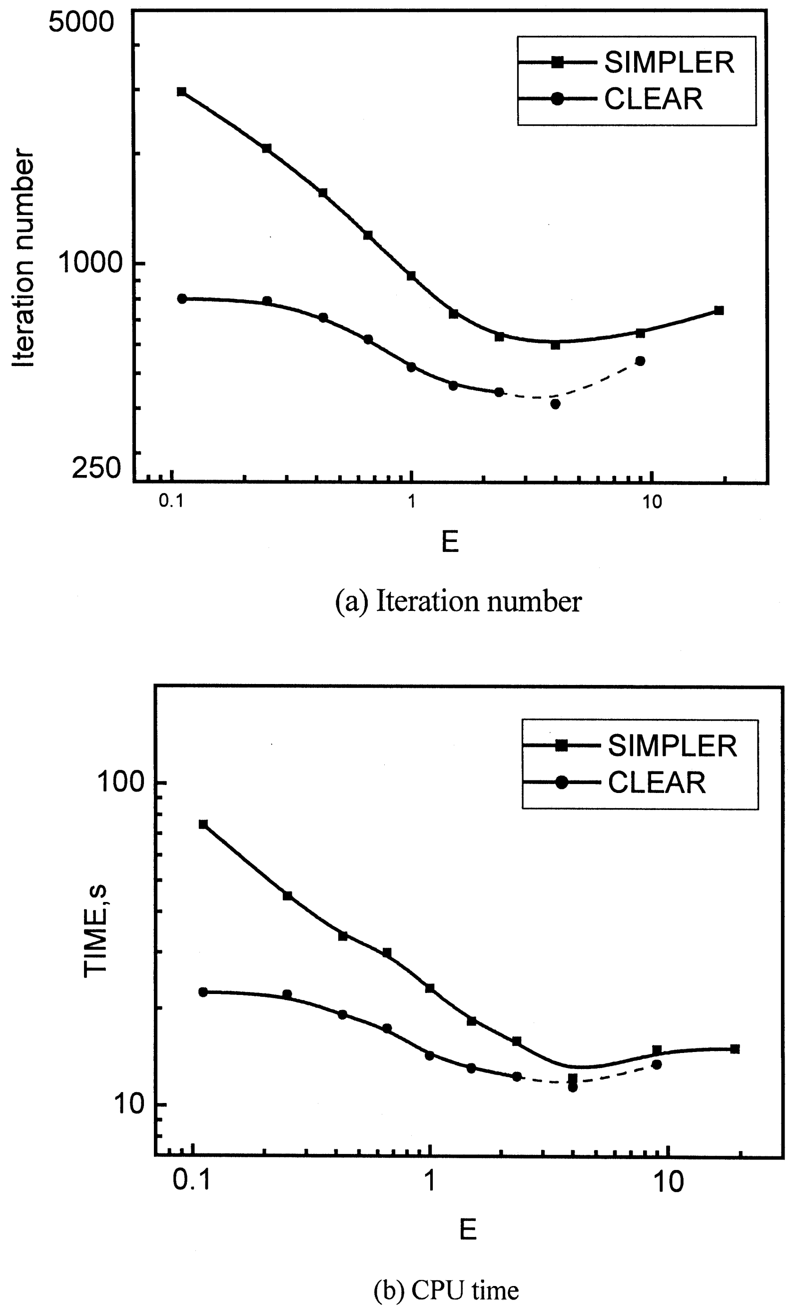
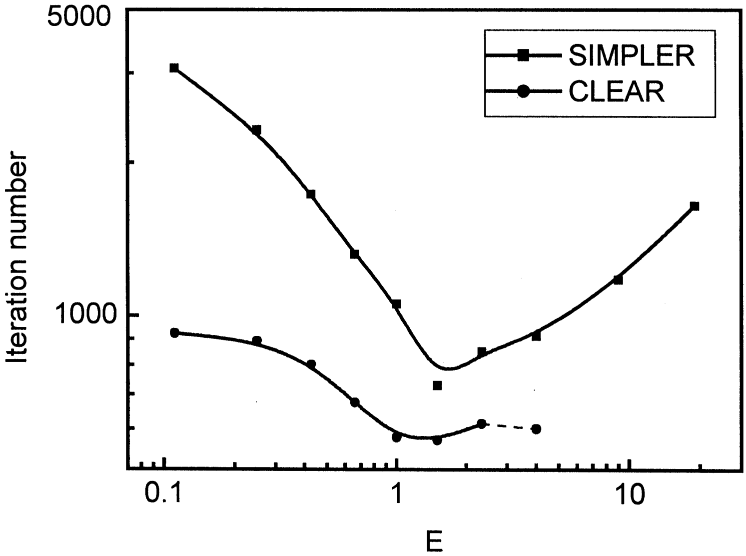
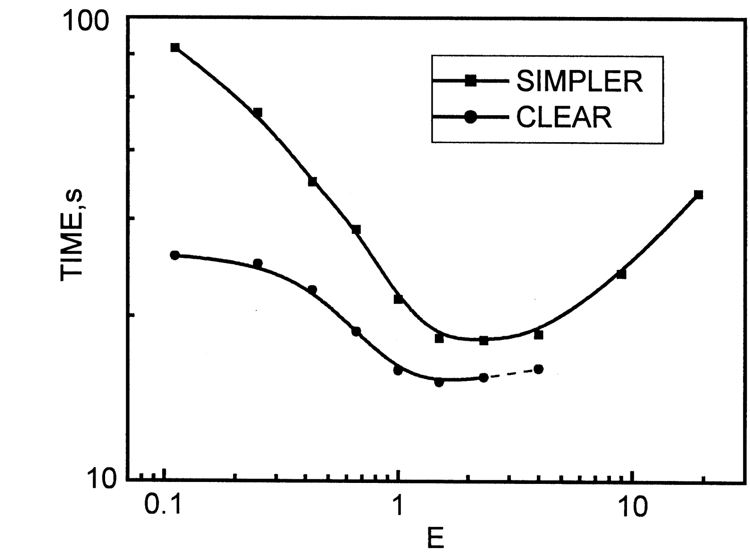


Figure 14. Comparison of iteration number and CPU time for Re = 350 in Problem 3.

numerical study of this problem and provided the following results: the ratios of the reattachment length over inlet diameter, L_R/D_{in}, as 6.5 and 8.8 for Re numbers 150 and 200, respectively. In the present study a grid system of 202×42 is adopted. The



(a) Iteration number



(b) CPU time

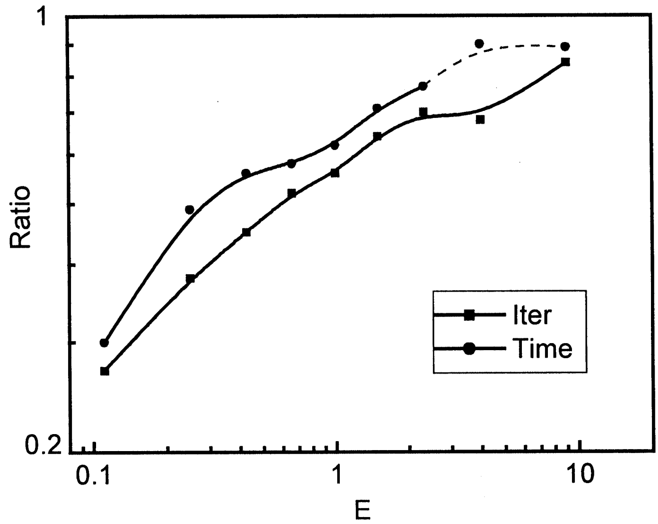
Figure 15. Comparison of iteration number and CPU time for Re = 1,000 in Problem 3.

domain extension method is used: the inlet step region of the solid is treated as a special fluid with very large viscosity [5]. The inlet velocity distribution is supposed to be fully developed:

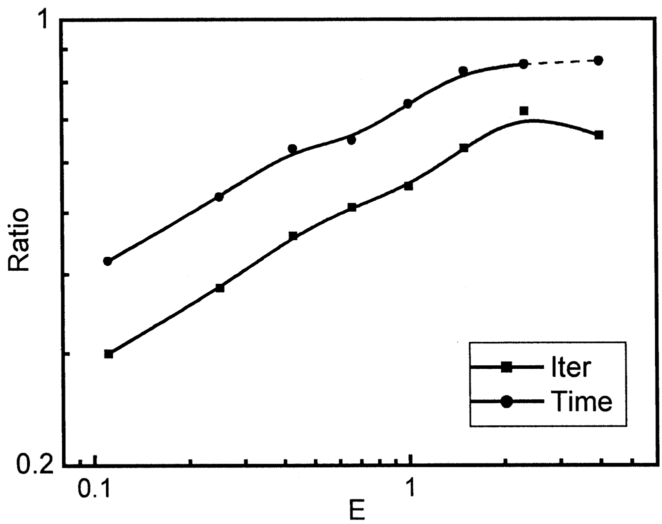
$$u = u_{\max}\left(1 - \frac{r^2}{R_{in}^2}\right) \qquad R_{\mathrm{in}} = \frac{D_{\mathrm{in}}}{2} \qquad u_{\max} = 2u_{\mathrm{mean}} \tag{6}$$

The fully developed boundary condition is assigned to the outflow boundary.

The predicted L_R/D_{in} from the two algorithms is the same: 6.62 for Re number = 150, and 8.85 for Re number = 200.



(a) Re=350



(b) Re=1000

Figure 16. Ratios of iteration numbers and CPU time for Problem 3.

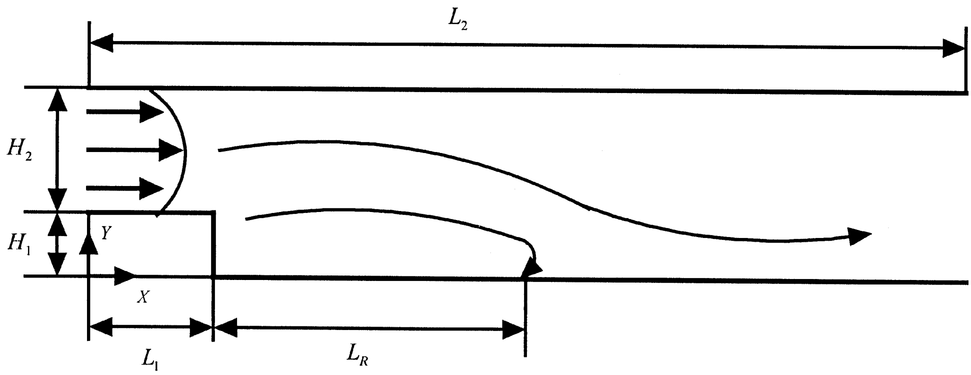


Figure 17. Flow over a rectangular backward step.

The iteration number and CPU time of the two algorithms are displayed in Figures 8 and 9 for the two Re numbers. The ratios of iteration number and CPU time are presented in Figure 10. For Re = 150, the ratio of the iteration number ranges from 0.28 to 0.69, that of the CPU time ranges from 0.34 to 0.81. For Re number = 200, the two ranges are from 0.25 to 0.69 and from 0.31 to 0.81, respectively. The maximum saving in CPU time is up to 69%.

Problem 3: Lid-Driven Cavity Flow in a Polar Cavity

The configuration is presented in Figure 11 ($\theta = 1$ radian, $\delta/R_{\text{in}} = 1$). This example was studied by Fuchs and Tillmark using both experimental and numerical methods [8]. The Reynolds number is defined as

$$\text{Re} = \frac{U_{\text{lid}}\delta}{\nu} \tag{7}$$

where U_{lid} is the circumferential velocity of the moving lid, $U_{\text{lid}} = R_{in} \times \omega$.

Our computations are conducted on a grid system of 52×52. The predicted stream functions for the two Re numbers (350 and 1,000) from the two algorithms are almost identical and are shown in Figures 12 and 13, respectively, where the results of [8] are also presented for reference.

In Figures 14 and 15, the comparisons are presented. Again the dashed lines are obtained with a β value greater than 1 (here it is 1.5). It can be observed from Figure 16 that the new algorithm performance is highly superior to that of the SIMPLER algorithm. The iteration number and CPU time of the CLEAR are only 0.27–0.84 and 0.30–0.92, respectively, of that of the SIMPLER for Re = 350, and 0.30–0.72 and 0.42–0.86, respectively, for Re = 1,000.

Problem 4: Laminar Fluid Flow over a Rectangular Backward Step

The problem is shown schematically in Figure 17. Computations are conducted for Re = 100 and 300. The geometric parameters are taken from Kondoh et al. [9]: $H_2/H_1 = 2$, $L_1/H_1 = 5$, $L_2/H_1 = 30$. The inlet velocity distribution is fully developed:

$$X = 0 \qquad 1 < Y < \frac{H_1 + H_2}{H_1} \qquad U = 1.5\left\{1 - \left[\frac{Y - 0.5(H_2/H) - 1}{0.5(H_2/H_1)}\right]\right\} \tag{8}$$

$$V = 0$$

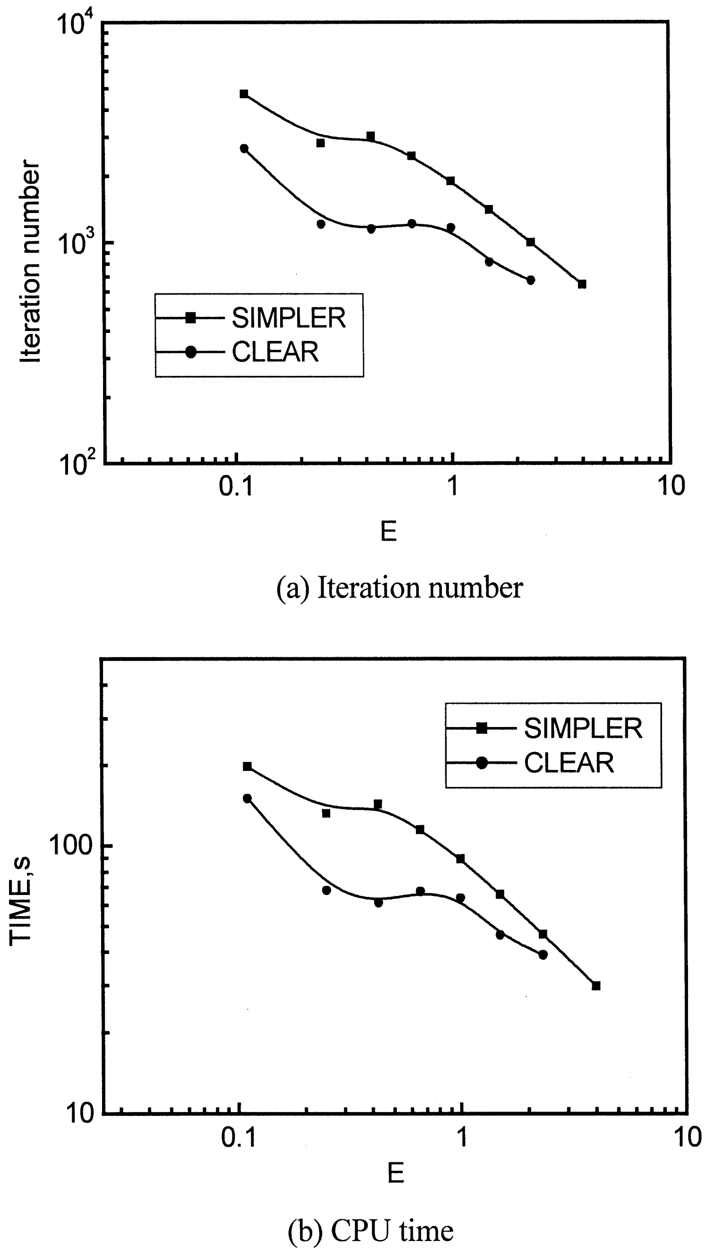


Figure 18. Comparison of iteration number and CPU time for Re = 100 in Problem 4.

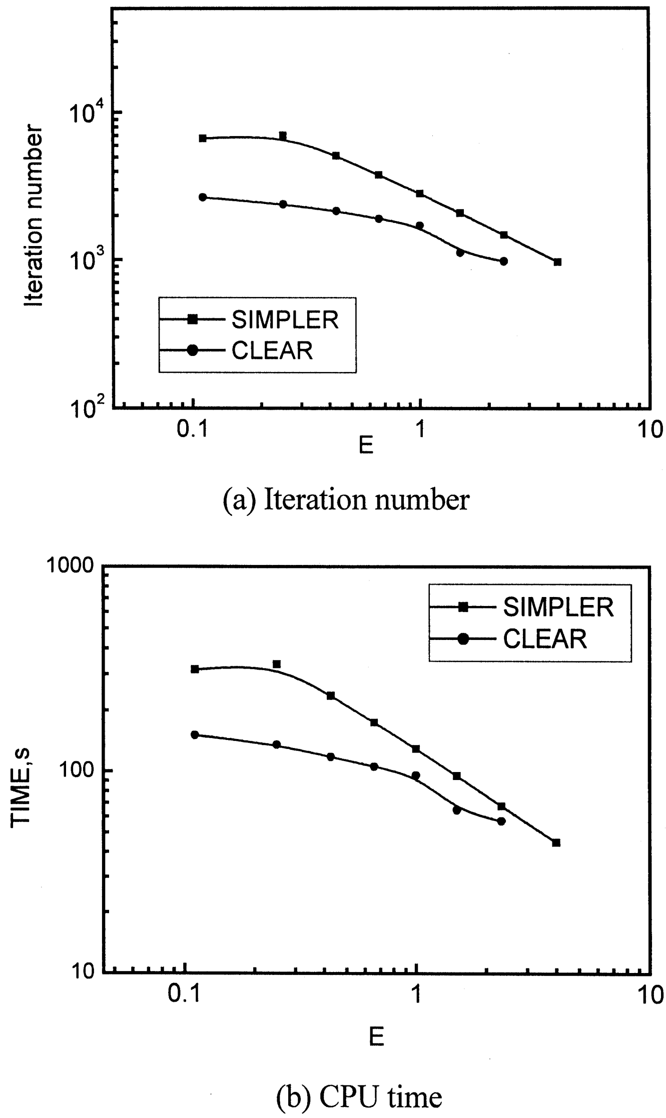
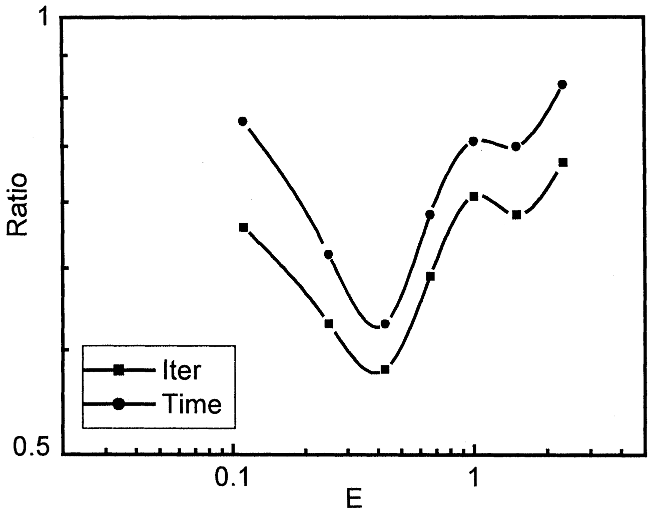


Figure 19. Comparison of iteration number and CPU time for Re = 300 in Problem 4.

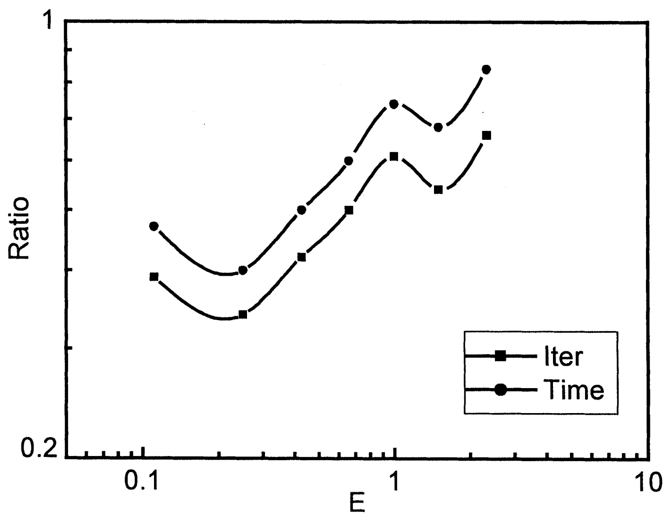
At the outflow boundary, fully developed condition is assumed. The Reynolds number is defined as

$$\mathrm{Re} = \frac{u_{\mathrm{mean}} H_1}{\nu} \tag{9}$$

where u_{mean} is the mean velocity at the inlet section. A grid system of 122×62 is used. In the domain $0 < X < L_1/H_1$, $0 < Y < 1$, the domain extension method [5] is used to deal with the solid region.



(a) Re=100



(b) Re=300

Figure 20. Ratios of iteration numbers and CPU time for Problem 4.

The compared results shown in Figures 18, 19, and 20 once again show the superior performance of the CLEAR to that of the SIMPLER. The two ratios are as follows:

For Re = 100, from 38% to 67% (ITER), from 43% to 83% (CPU time)
For Re = 300, from 34% to 66%(ITER), from 40% to 84%(CPU time)

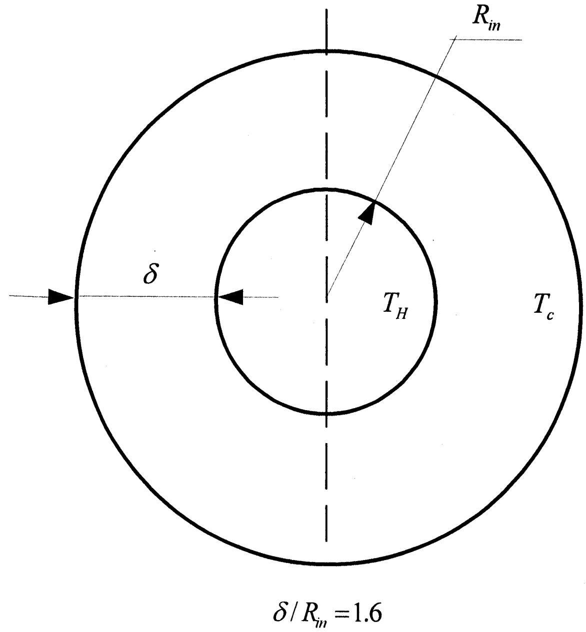


Figure 21. Natural convection in an annular space.

Problem 5: Natural Convection in an Annulus Enclosure

The fifth problem tested is laminar natural convection between two horizontal concentric cylinders, depicted in Figure 21. Two cases are tested, $\mathrm{Ra} = 10^3$ and 10^4, where Rayleigh number is defined as

$$\mathrm{Ra} = \frac{\rho g \beta \delta^3 \Delta T}{a\mu} \tag{10}$$

The Boussinesq assumption is adopted. Computations are conducted on a uniform grid system with 42×32 mesh.

Figure 22 shows the numerical results including flow field and temperature field, where the results of [10] are also provided for comparison. Figures 23 and 24 show the comparison results of iteration number and CPU time. Obviously, the performance of the CLEAR is much better than that of the SIMPLER. The augmentation of convergence rate is shown in Figure 25. For $\mathrm{Ra} = 10^3$, the ratio of the iteration number varies from 0.26 to 0.48, and the CPU time ratio varies from 0.36 to 0.57, which means about half the time is saved. And for $\mathrm{Ra} = 10^4$ the two ratios range from 0.29 to 0.42 and from 0.38 to 0.56, respectively.

Problem 6: Natural Convection in a Square Cavity

The square cavity has two adiabatic walls (top and bottom), with its two vertical walls being maintained at constant but different temperatures. Computations

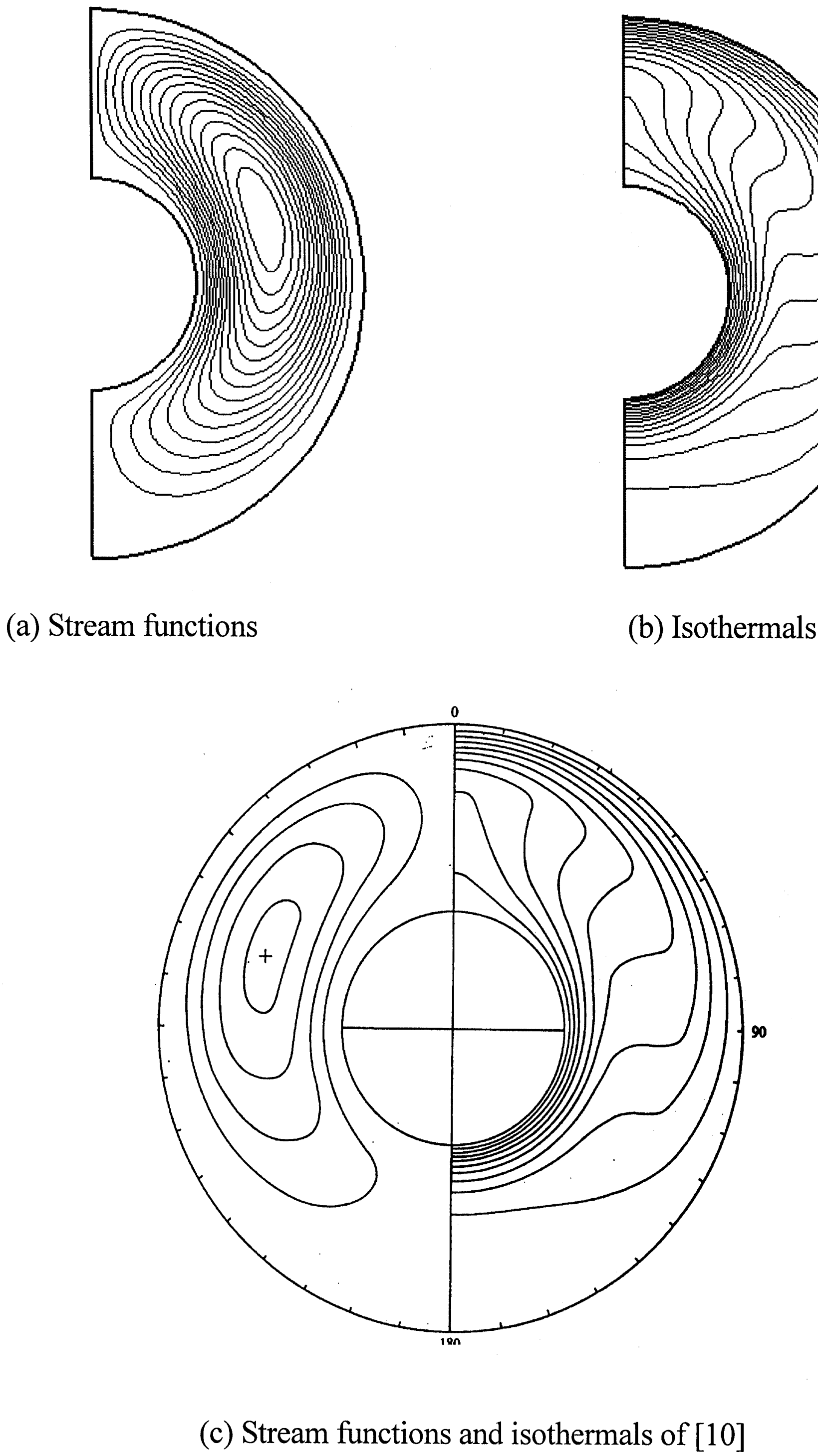
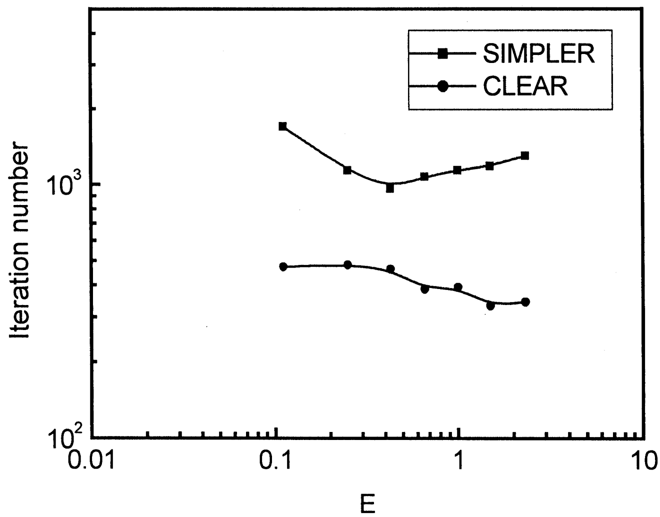
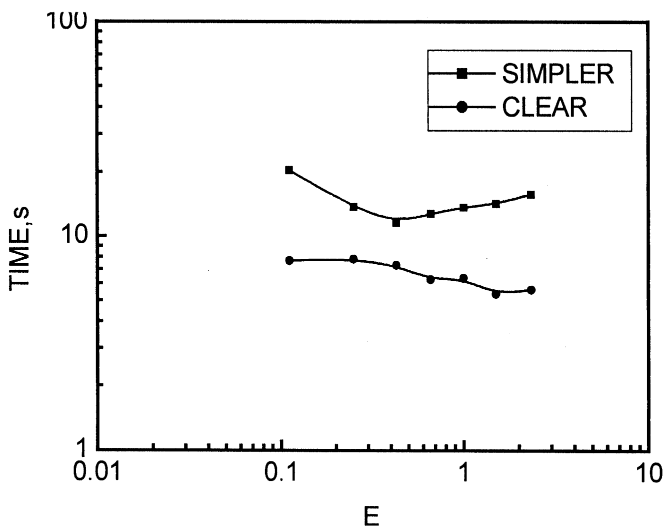


Figure 22. Predicted isothermals and stream functions for $Ra = 10^4$.



(a) Iteration number



(b) CPU time

Figure 23. Comparison of iteration number and CPU time for $\mathrm{Ra} = 10^3$ in Problem 5.

are performed for $\mathrm{Ra} = 10^4$ and 10^6 based on the Boussineq assumption. The Rayleigh number is defined by

$$\mathrm{Ra} = \frac{\rho g \beta L^3 \Delta T}{a\mu} \tag{11}$$

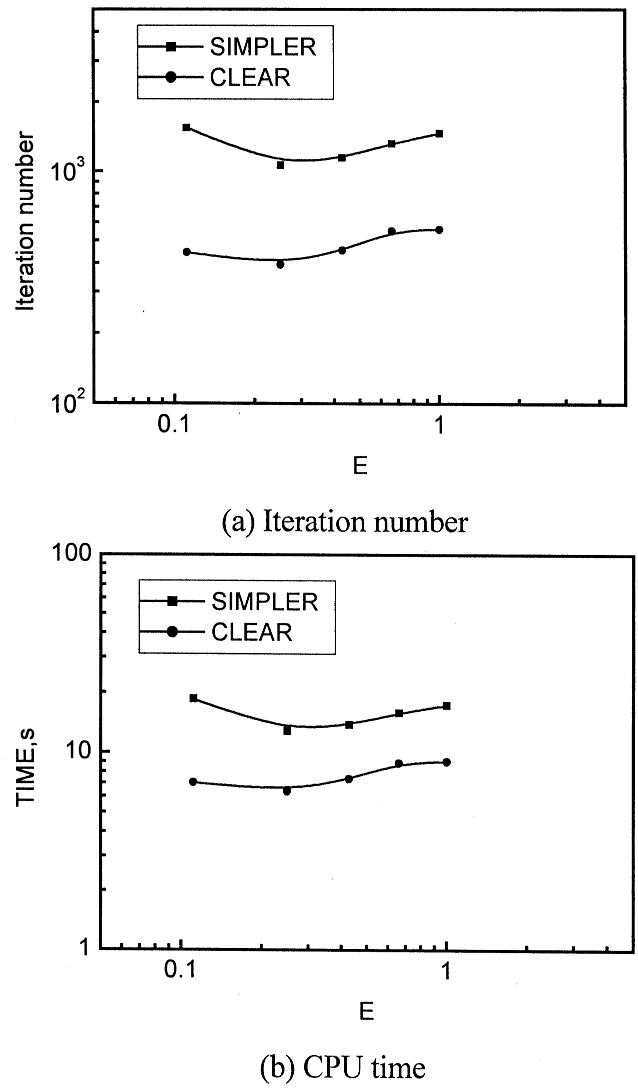


Figure 24. Comparison of iteration number and CPU time for $Ra = 10^4$ in Problem 5.

A uniform grid of 82×82 is applied.

The benchmark solution [11] of the cavity average Nusselt numbers for $Ra = 10^4$ and 10^6 are 2.238 and 8.903, respectively. In the present study the corresponding values are 2.24 and 9.08, showing good agreement. The variations of the iteration number and CPU time with the time-step multiple of the two algorithms are shown in Figures 26 and 27 for Ra number $= 10^4$ and 10^6 respectively. The ratios of the iteration number and the CPU time of the two algorithms are: for $Ra = 10^4$, from 0.19 to 0.33 (ITER), from 0.22 to 0.39 (CPU time); for $Ra = 10^6$, from 0.23 to 0.34 (ITER), and from 0.28 to 0.41 (CPU time) (Figure 28). Significant saving can be obtained by using the CLEAR algorithm.

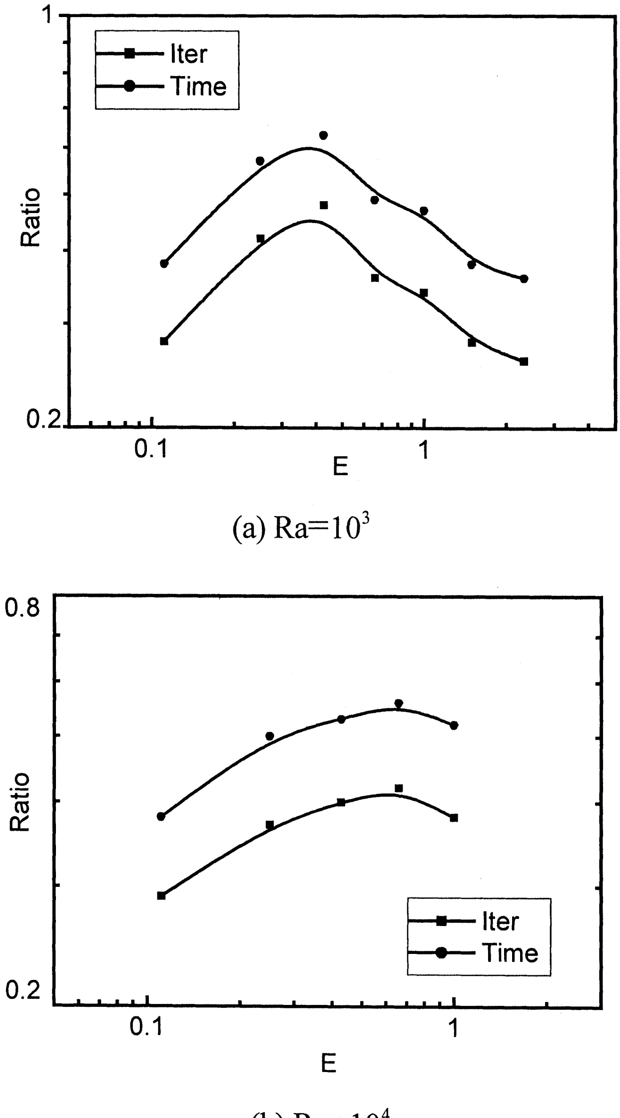
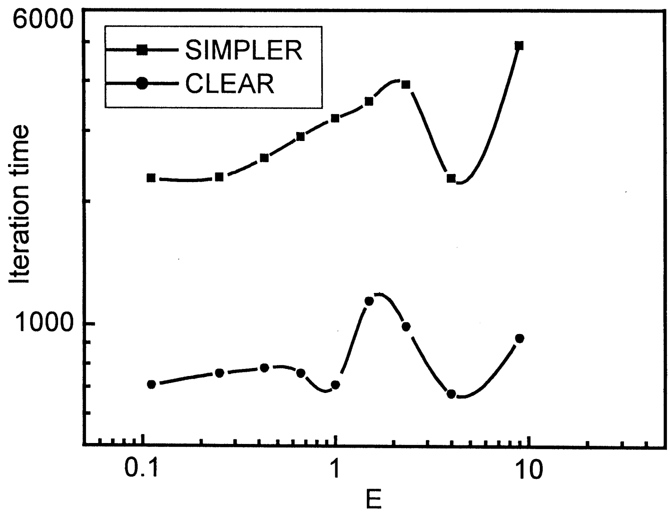


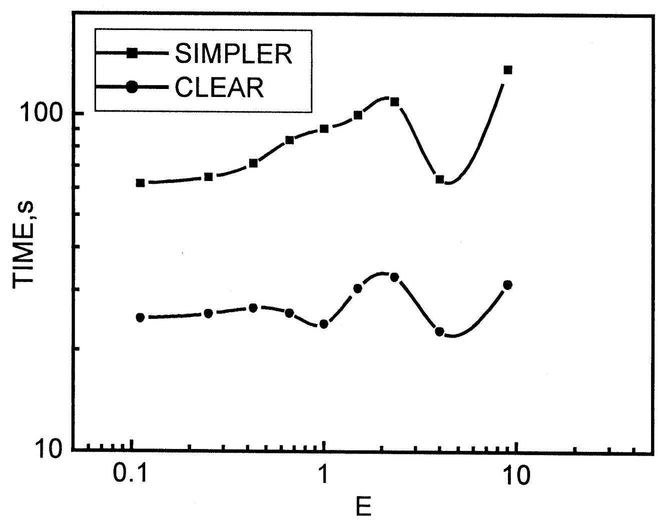
Figure 25. Ratios of iteration numbers and CPU time for Example 5.

Discussion

Through the above six examples, it is demonstrated that the CLEAR algorithm can greatly improve the convergence rate of the iterative process compared with the SIMPLER algorithm. We notice that the ratio of iteration numbers is smaller than that of the CPU time. This is because in one iteration of the CLEAR algorithm, extra computational effort is needed to compute the coefficients of the discretized momentum equation with the temporary velocity without solving the equation



(a) Iteration number

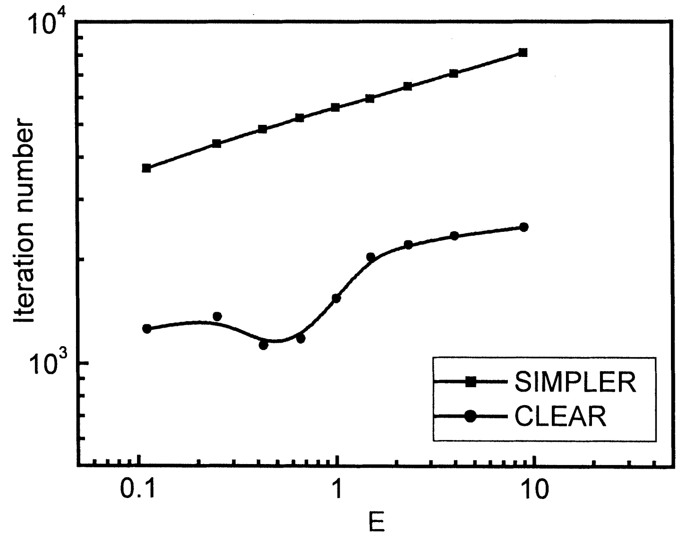


(b) CPU time

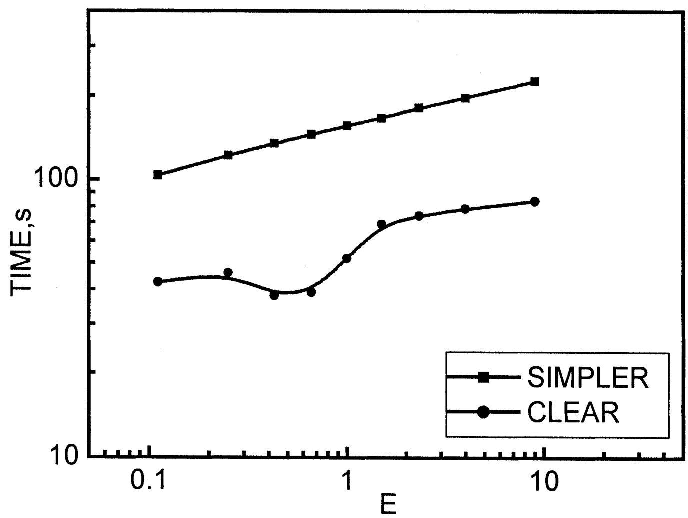
Figure 26. Comparison of iteration number and CPU time for $Ra = 10^4$ for Problem 6.

thereafter. Hence for each iteration the CPU time required in CLEAR is a bit larger than that in SIMPLER.

For the six tested problems, generally speaking, the total iteration numbers for CLEAR are about 15% to 84% of the SIMPER algorithm, and the proportion of CPU time is about 19–92%. We also noticed that for problems 1 and 3, in the region



(a) Iteration number



(b) CPU time

Figure 27. Comparison of iteration number and CPU time for $Ra = 10^6$ for Problem 6.

of high value of α, the robustness of the CLEAR algorithm is a bit weaker than that of the SIMPLER algorithm. For example, for the case of $Re = 350$ of problem 3, the SIMPLER algorithm can get a converged solution within the range of E from 10 to 20 $(\alpha = 0.952)$, while the CLEAR algorithm with $\beta = 1.5$ works within

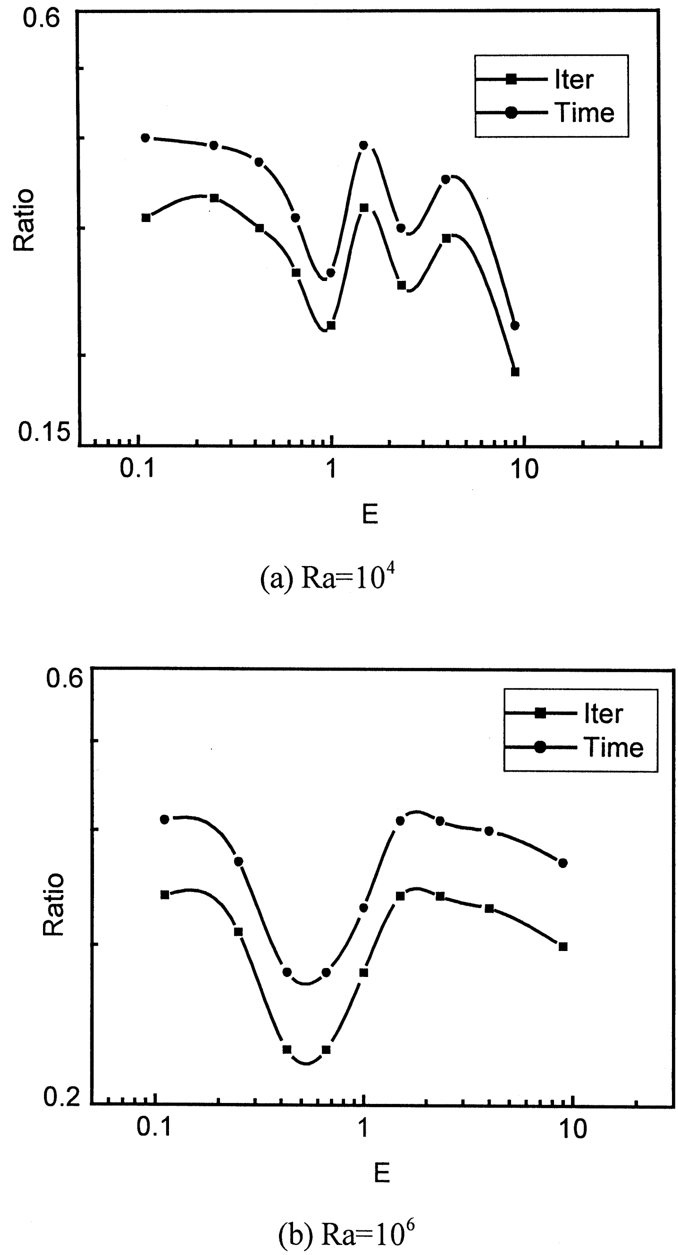


Figure 28. Ratios of iteration numbers and CPU time for Problem 6.

$E \leq 10\,(\alpha = 0.909)$. Seemingly this presents a weakness of the CLEAR algorithm. However, this will not affect the application of the CLEAR algorithm, simply because in the region of $E \leq 10$ the convergence rate of the CLEAR algorithm is much faster than that of the SIMPLER in the region of $E = 10$–20.

CONCLUSION

In this article, comprehensive numerical experiments have been conducted for the CLEAR algorithm proposed in [1] and the SIMPLER algorithm. The six tested incompressible laminar fluid flow and heat transfer problems cover three 2-D orthogonal coordinates. Numerical experiments definitely demonstrate that the CLEAR algorithm can significantly enhance the convergence rate of the iteration process compared with the SIMPLER algorithm. For the six problems tested, the CLEAR algorithm can reduce the iteration number by 16–85%, and the CPU time by 8–81%. Because of the good coupling of the CLEAR algorithm, the maximum value of the velocity underrelaxation factor for some situations may be a bit smaller than that of the SIMPLER algorithm, but this will not affect the application of the CLEAR algorithm, because of its very fast convergence rate in the normal region of the underrelaxation factor.

Extension of the CLEAR algorithm to problems of turbulent flow, the collocated grid system, and compressible fluid flow are now underway in the authors' group.

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