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A comparison study of the convergence characteristics and robustness for four variants of SIMPLE-family at fine grids

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Abstract *A comparative study is performed to reveal the convergence characteristics and the robustness of four variants in the semi-implicit method for pressure-linked equations (SIMPLE)-family: SIMPLE, SIMPLE revised (SIMPLER), SIMPLE consistent (SIMPLEC), and SIMPLE extrapolation (SIMPLEX). The focus is concentrated in the solution at fine grid system. Four typical fluid flow and heat transfer problems are taken as the numerical examples (lid-driven cavity flow, flow in an axisymmetric sudden expansion, flow in an annulus with inner surface rotating and the natural convection in a square enclosure). It is found that an appropriate convergence condition should include both mass conservation and momentum conservation requirements. For the four problems computed, the SIMPLEX always requires the largest computational time, the SIMPLER comes the next, and the computational time of SIMPLE and SIMPLEC are the least. As far as the robustness is concerned, the SIMPLE algorithm is the worst, the SIMPLER comes the next and the robustness of SIMPLEX and SIMPLEC are superior to the others. The SIMPLEC algorithm is then recommended, especially for the computation at a fine grid system. Brief discussion is provided to further reveal the reasons which may account for the difference of the four algorithms.*

Nomenclature

A = area of a cell faces
b = source term in a discretized equation
d = ratio of area to momentum coefficient
E = time step multiple [$= \alpha/(1-\alpha)$]
g = acceleration due to gravity
D = characteristic length of cavity
L = characteristic length of sudden expansion
p = pressure
p^* = intermediate pressure
p' = pressure correction
q_m = mass flow rate
r = radial coordinate
Ra = Rayleigh number [$= g\beta(T_H - T_C)D^3/(a\nu)$]
Re = Reynolds number
RSMAX = the maximum residual of control-volume mass flow rate
T = temperature

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u, v	=	velocity components in the x and y direction
u^*, v^*	=	intermediate velocities
u', v'	=	velocity correction
x, y	=	spatial coordinates
α	=	underrelaxation factor
β	=	volumetric coefficient of expansion
ρ	=	density
$\varepsilon_1, \varepsilon_2$	=	pre-specified small values to control convergence

Superscripts

*	=	intermediate value

Subscripts

e, w, n, s	=	cell faces
C	=	lowest temperature
H	=	highest temperature
IN	=	inlet diameter
m	=	mass
nb	=	neighbor points
OUT	=	out diameter

1. Introduction

Since the semi-implicit method for pressure-linked equations (SIMPLE) algorithm was proposed by Patankar and Spalding (1972), it has been widely applied to the fields of computational fluid dynamics (CFD) and numerical heat transfer(NHT). Over the last three decades about ten variants (Acharya and Moukalled, 1989; Date, 1986; Gjesdal and Lossius, 1997; Patankar, 1980, 1981; Sheng *et al.*, 1998; Van Doormaal and Raithby, 1984, 1985; Yen and Liu, 1993; Yu *et al.*, 2001) were proposed to improve the convergence performance, and these algorithms consist the so-called SIMPLE-series or SIMPLE-family solution algorithm. Today the SIMPLE-family is probably the most popular algorithm for solving incompressible Navier-Stokes equations with primitive variables. In the past decade, the SIMPLE-series algorithms were also successfully extended to solving compressible fluid flow (Demirdzic *et al.*, 1993; Karki and Patankar, 1989; Shyy *et al.*, 1992). Among the different variants, the most often used algorithms are SIMPLE, SIMPLE revised (SIMPLER). SIMPLE consistent (SIMPLEC) and SIMPLE extrapolation (SIMPLEX). The SIMPLEST algorithm (Spalding, 1980) is essentially the same as SIMPLE, with a difference only in the discretization scheme for the convection term. By algorithm we mean the way to deal with the coupling between the velocity and pressure, thus we do not take it as a new variant of the SIMPLE-family.

A number of comparisons between the different variants of the SIMPLE-family have been conducted (Barton, 1998; Jang *et al.*, 1986; Latimer and Pollard, 1985; McCuirk and Palma, 1993). The emphasis of these comparison works is often concentrated on the convergence rate for solving some typical problems, and the grid number used is usually in the range of 10×10 to 30×30. With the rapid advances in computer technologies and the constantly-reduced prices of computers, nowadays even using a PC, people can easily deal with a network with 100×100 grids. Thus the convergence characteristics of these algorithms remain an interesting subject for further study. To the authors knowledge, only Moukalled and Darwish (2000) and Van Doormaal and Raithby (1985) have clearly indicated the convergence characteristics of the above algorithms at fine grids.

When Van Doormaal and Raithby (1985) proposed the SIMPLEX algorithm, they regarded that the SIMPLEX experienced an optimization in convergence with grid refinement when compared to the SIMPLEC method. Recently, Moukalled and Darwish (2000) have made a unified study about ten algorithms belonging to the SIMPLE-series. In that article, the authors indicated that in all SIMPLE-based methods, no care is taken to ensure that the rate of convergence will not degrade with grid refinement. This concern is addressed in SIMPLEX. It is emphasized that the SIMPLEX algorithm has a lower degradation in the rate of convergence with grid refinement as compared to other SIMPLE-like algorithms. And the following conclusion made in Van Doormaal and Raithby (1985) was recited in Moukalled and Darwish (2000): for sufficiently fine grids SIMPLEX is more efficient than SIMPLE, SIMPLER and SIMPLEC. Since the meaning of words "fine" and "coarse" are only qualitative and relative, it is necessary to reveal specifically what is the fine grid used in Van Doormaal and Raithby (1985). It turns out that the so-called fine grid in Van Doormaal and Raithby (1985) is just 25×25 for 2D case. Definitely, the grid system of (25×25) could not be regarded as a fine one today. So a question comes into being as whether the statement made in Van Doormaal and Raithby (1985) and Moukalled and Darwish (2000) is still applicable? In the present study, by fine grid we mean a grid with grid number in the level of 100×100 or so for 2D case. We compare the convergence characteristics of the SIMPLE, SIMPLEC, SIMPLER and SIMPLEX for four cases and get a conclusion different from that of Van Doormaal and Raithby (1985).

In the following presentation, the major features of the four algorithms will be very briefly reviewed at the staggered grid system, and then convergence comparison by using the four algorithms will be conducted for the four selected problems, including the forced convection and natural convection in three 2D coordinates. Two criterias will be selected to judge the convergence: the criterion of mass conservation and the criterion of both mass conservation and momentum conservation. The robustness of the four algorithms will also be compared. Finally, some conclusions will be drawn.

2. Mathematical formulation of the four algorithms compared

The problems we solve here are assumed to be at steady state with constant properties. Thus for the simplicity of presentation, only the discretized mass and momentum equations are dealt with. The governing equation of temperature does not effect the algorithm we compared here, hence, will be omitted. Furthermore, to show the major features of the different algorithms, the pressure-correction equation is derived for a 2D incompressible fluid flow problem in Cartesian coordinates. For the details of the derivation, Patankar (1981) and Tao (2001) may be consulted. The symbols used in Patankar (1981) are adopted here.

The discretized flow governing equations are as follows:
Mass conservation:

$$(\rho u\Delta y)_e - (\rho u\Delta y)_w + (\rho v\Delta x)_n - (\rho v\Delta x)_s = 0 \tag{1}$$

Momentum conservation:

$$\left.\begin{aligned} a_e u_e &= \sum a^u_{nb} u_{nb} + b^u + (p_P - p_E)A_e \\ a_n v_n &= \sum a^v_{nb} v_{nb} + b^v + (p_P - p_N)A_n \end{aligned}\right\} \tag{2}$$

The solution consequence is: guess velocity fields to evaluate the coefficients of the momentum equations; guess a pressure field p^* and solve the discretized momentum equations to obtain temporary solutions of velocity denoted by u^*, v^*. To improve u^*, v^* such that the improved velocity satisfy the mass conservation condition, a pressure correction term p' and the corresponding velocity correction terms u', v' should be added to their current values. Then by subtraction of the momentum equations for u^*, v^*

$$\left.\begin{aligned} a_e u^*_e &= \sum a^u_{nb} u^*_{nb} + b^u + (p^*_P - p^*_E)A_e \\ a_n v^*_n &= \sum a^v_{nb} v^*_{nb} + b^v + (p^*_P - p^*_N)A_n \end{aligned}\right\} \tag{3}$$

from the momentum equations for $u = u^* + u'$, $v = v^* + v'$

$$\left.\begin{aligned} a_e(u^*_e + u'_e) &= \sum a^u_{nb}(u^*_{nb} + u'_{nb}) + b^u + [(p^*_P + p'_P) - (p^*_E + p'_E)]A_e \\ a_n(v^*_n + v'_n) &= \sum a^v_{nb}(v^*_{nb} + v'_{nb}) + b^v + [(p^*_P + p'_P) - (p^*_N + p'_N)]A_n \end{aligned}\right\} \tag{4}$$

we can yield the following expressions:

$$\left.\begin{aligned} a_e u'_e &= \sum a^u_{nb} u'_{nb} + (p'_P - p'_E)A_e \\ a_n v'_n &= \sum a^v_{nb} v'_{nb} + (p'_P - p'_N)A_n \end{aligned}\right\} \tag{5}$$

Equation (5) tells us that the velocity correction consists of two parts. One is the pressure correction difference between two adjacent points which are in the same direction as the velocity, and this part is the direct motive force bringing the velocity correction. The other part is caused by the neighborhood velocity correction which can be regarded as the indirect influence of the pressure corrections at nearby locations. The main approximation made in the SIMPLE algorithm is to neglect the influence of these nearby velocity corrections. This hypothesis is equivalent to set coefficients $a_{nb} = 0$ in equation $\sum a^u_{nb} u'_{nb}$, then we can get the velocity correction equation

$$\left.\begin{aligned} u'_e &= d_e(p'_P - p'_E), \quad d_e = \tfrac{A_e}{a_e} \\ v'_n &= d_n(p'_P - p'_N), \quad d_n = \tfrac{A_n}{a_n} \end{aligned}\right\} \tag{6}$$

Equation (6) is used to compute the velocity correction value in the SIMPLE algorithm. And the resulting velocity $u = u^* + u'$, $v = v^* + v'$ are taken as the solution of this iteration level. Substituting $u_e = u_e^* + d_e(p'_P - p'_E)$ and $v_n = v_n^* + d_n(p'_P - p'_N)$ into the continuity equation (1), we obtain the pressure correction equation

$$a_P p'_P = \sum a_{nb} p'_{nb} + b_P \tag{7}$$

where $a_E = \rho_e d_e \Delta y$, $a_N = \rho_n d_n \Delta x$, $a_P = \sum a_{nb}$ and $b_P = (\rho u^* \Delta y)_e^w + (\rho v^* \Delta x)_n^s$. In the SIMPLE algorithm, under-relaxation is needed for p', since it is considered that the value of p' is exaggerated because of the neglect of the nearby velocity corrections in equation (5).

The primary distinction between the SIMPLEX and SIMPLE and the SIMPLEC and SIMPLE is the determination of the coefficient d in equation (6). Van Doormaal and Raithby (1984) take the following form for d:

$$d_e = \frac{A_e}{\sum a_{nb} - a_e}; \qquad d_n = \frac{A_n}{\sum a_{nb} - a_n} \tag{8}$$

The subtraction of the diagonal coefficient from the summation of coefficients of the neighboring velocities in the denominator of equation (8) greatly alleviate the influence of neglecting the nearby velocity corrections in equation (5). This improvement leads to the SIMPLEC algorithm, and no underrelaxation is needed for the pressure correction in the SIMPLEC algorithm.

In another development Van Doormaal and Raithby (1985) extend the equation $u'_e = d_e(p'_P - p'_E) = d_e \Delta p'_e$ to the computation of the neighborhood velocity correction by supposing $u'_{nb} = d_{nb} \Delta p'_{nb}$. This practice is equivalent to extrapolate the expression of the velocity correction at location studied to all the nearby locations. The substitution of the above equation into the discretized momentum equation (3), results in the equation

$$a_e d_e \Delta p'_e = \sum a_{nb} d_{nb} \Delta p'_{nb} + A_e \Delta p'_e \tag{9}$$

Further approximation is made in Van Doormaal and Raithby (1985) that $\Delta p'_e = \Delta p'_{nb}$. Substitution of this approximation into equation (9) leads to the following equation for d_e

$$a_e d_e = \sum a_{nb} d_{nb} + A_e \tag{10}$$

The equation for d_n, d_s and d_w can be obtained similarly. Because the coefficients of the discretized momentum equation for u and v are already calculated and available, the values of d_e and d_n can be determined by solving the algebraic equations on each iteration of the segregated solution procedure. It is considered that the d_e and d_n such determined have considered the influence of the nearby velocity corrections. To identify the use of the extrapolation in determining d, the character X is appended to SIMPLE, leading to a new variant SIMPLEX (Moukalled and Darwish, 2000). Obviously, the pressure correction p' in the SIMPLEX algorithm need not be underrelaxed either. The SIMPLEX algorithm is implanted by executing the following sequence of steps:

(1) guess a velocity field, u^0 and v^0 to evaluate the coefficients and the constant of momentum equations;

(2) guess a pressure field, p^*;

(3) compute the equations u, d_e, v and d_n to get u_e^*, d_e, v_n^* and d_n equations;

(4) solve the pressure correction equation to acquire p' according to the value of u_e^*, d_e, v_n^* and d_n;

(5) improve the velocity field according to p', d_e and d_n;

(6) using the improved velocity field $(u^* + u', v^* + v')$ and the pressure field $(p^* + p')$, return to step 3. Repeat this cycle until convergence is achieved. The velocity need to be underrelax ed except for the pressure.

As far as the SIMPLER algorithm is concerned, its major difference from the SIMPLE algorithm is that the pressure field is solved from the previous velocity field, rather than assumed. And the pressure correction is only used to correct the velocity fields, not the pressure.

The four algorithms discussed above have been implemented in this article. The code developed is verified through three benchmark problems (lid-driven cavity flow in rectangular coordinates, flow in a 2D axisymmetric sudden expansion and natural convection in a square cavity). The results agree well with benchmark solutions available in the literature. Then the code is used to perform the comparison study for the convergence characteristics. The results are presented in the following section.

3. Results of comparison study

Four flow and heat transfer problems (lid-driven cavity flow, flow in a 2D axisymmetric sudden expansion, flow in annulus with the inner wall rotating about the axis and natural convection in a square cavity) are used to compare the convergence rate and robustness. The four test cases are depicted schematically in Figure 1. The governing equations for the four problems are those for 2D incompressible fluid. For the natural convection, we adopt the Boussinesq assumption. All of these are well documented in Patankar (1980) and Tao (2001), and will not be restated here for simplicity.

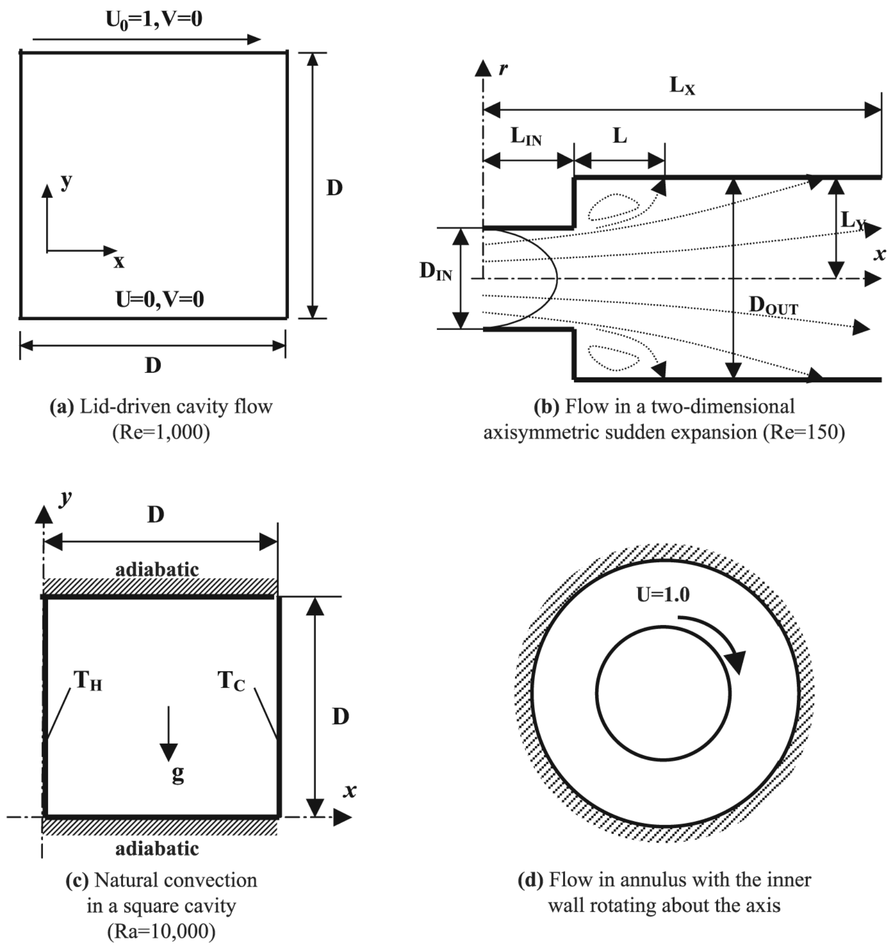


Figure 1.
Four problems for performance study

The comparisons for the convergence characteristics of the four algorithms are conducted under two criteria for the iteration convergence. The first one is the maximum relative residual of control-volume (SMAX) in the continuity equation which is less then a pre-specified value.

$$\mathrm{RSMAX} = \frac{\mathrm{SMAX}}{q_{\mathrm{m}}} \leq \varepsilon_{\mathrm{mass}} \tag{11}$$

where q_{m} is the reference mass flow rate. For the open system, such as flow in a 2D axisymmetric sudden expansion (Figure 1(b)), we take the inlet mass flow rate as the referenced q_{m}. For the closed system, for example lid-driven cavity flow (Figure 1(a)) and natural convection in a square cavity (Figure 1(c)), we make a numerical integration for the flow rate along any section in the field to obtain the reference q_{m} (Tao, 2001) by adopting the absolute value of the velocity. The second criterion requires both the relative maximum mass residual and the relative residual module in the momentum equations are also less than pre-specified small values. Thus apart from equation (11), following condition is added

$$\left(\sum_{\text{node}}\left\{a_e u_e - \left[\sum_{nb} a_{nb} u_{nb} + b + A_e(p_P - p_E)\right]\right\}^2\right)^{1/2} \Big/ \rho u_{\mathrm{m}}^2 \leq \varepsilon_{\mathrm{mom}} \quad (12)$$

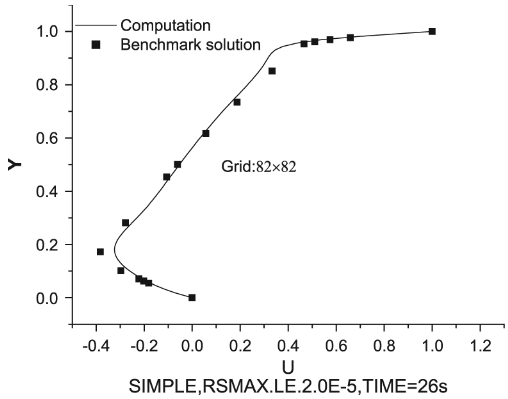
We take the inlet momentum as the referenced momentum in the above equation for the open system. For the closed system, first we get the numerical integral of the momentum along any section, then we adopt their average value as the reference momentum.

3.1 Convergence comparison under the first criteria

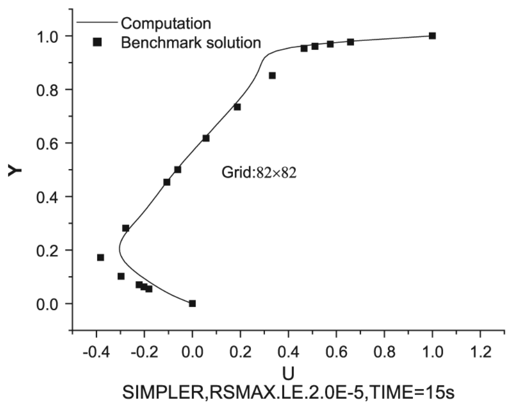
Under this condition, we compare the four algorithms for the two problems: lid-driven cavity flow and flow in annulus with the inner wall rotating about the axis. We adopted the underrelaxation factor of $\alpha_{u,v} = 0.8$ for all the algorithm compared and $\alpha_p = 0.3$ for the SIMPLE only. It was found that the SIMPLER algorithm needs the least computation time but the results are inferior to all others. The behavior of SIMPLEX is the opposite. The phenomenon becomes severe with grid refinement. The details are presented below.

Problem 1: Lid-driven cavity flow. Figure 2 shows the velocity distributions at the horizontal centerline together with the computational benchmark solutions for Re = 1,000 (Ghia *et al.*, 1982). In the abscissa of each figure the required CPU time of a PC with 128 M memory and 400 MHz frequency is indicated. It can be observed that the solution differences among the four schemes are insignificant when the grids are not fine (82×82), but the differences increase with grid refinement. Because of the space limitation only the results of the finest grid system (202×202) are provided. From the figure, it can be seen that the solution accuracy of SIMPLER is the worst, and the results of SIMPLEX is the best. As far as the CPU time is concerned, the SIMPLER algorithm needs the least, while the SIMPLEX the most. The phenomenon becomes more severe with grid refinement. The solutions of SIMPLEC is superior to that of SIMPLE but inferior to that of SIMPLEX at finest grid.

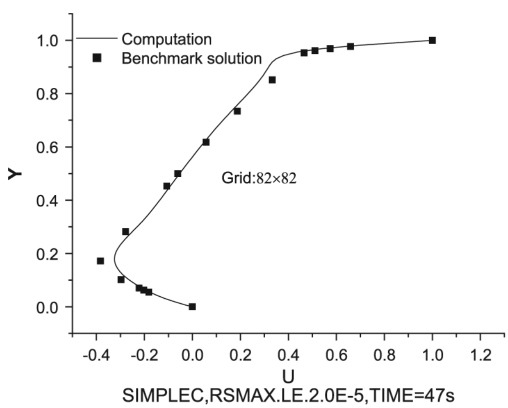
Problem 2: Flow in annulus with the inner wall rotating about the axis. For this case, the analytical solution of the tangential velocity distribution along the radius is adopted as the benchmark solution (Bird *et al.*, 2002). The relative results are shown in Figure 3. The behavior of the four algorithms is the same as that in problem 1.



(a) SIMPLE, 82 × 82



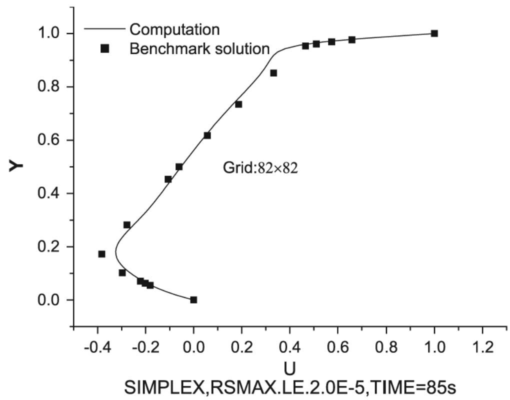
(b) SIMPLER, 82 × 82



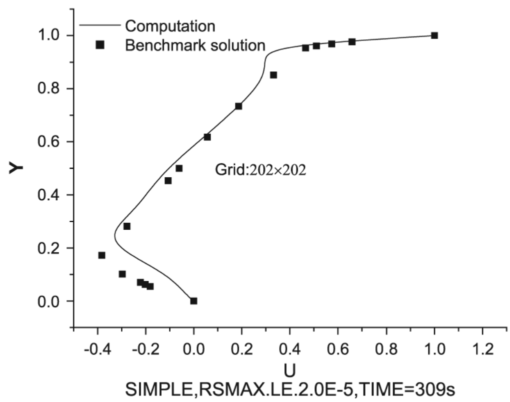
(c) SIMPLEC, 82 × 82

(continued)

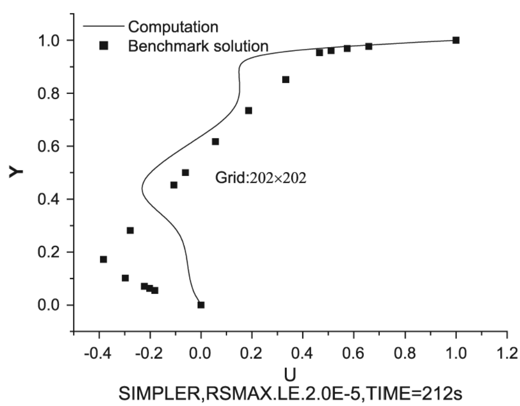
Figure 2.
Comparison between four algorithms under first convergence criterion for lid-driven cavity flow



(d) SIMPLEX, 82 × 82



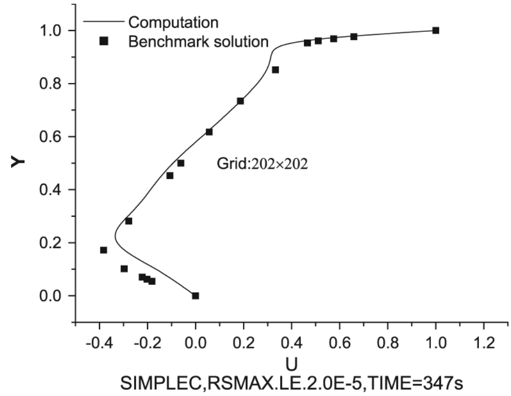
(e) SIMPLE, 202 × 202



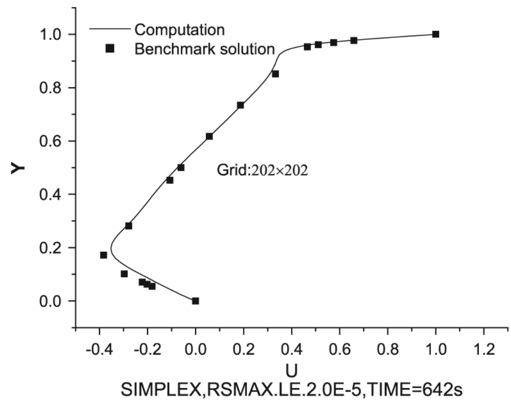
(f) SIMPLER, 202 × 202

(continued)

Figure 2.



(g) SIMPLEc, 202 × 202



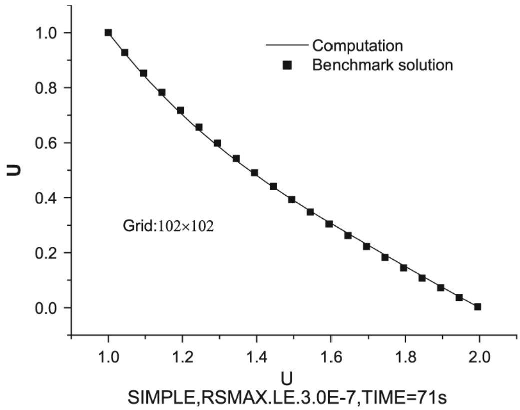
(h) SIMPLEX, 202 × 202

Figure 2.

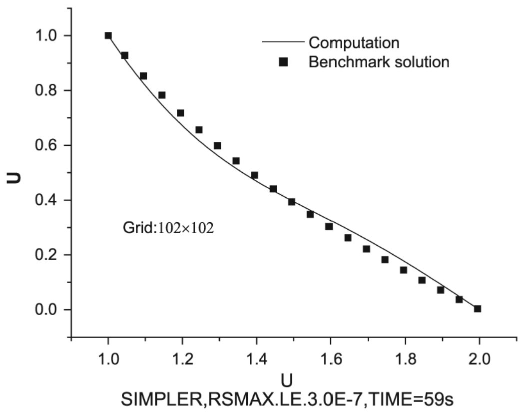
3.2 Convergence comparison under the second criterion

As indicated above, the second criterion includes the requirement for both mass conservation and the momentum conservation. The underrelaxation factors used are $\alpha_{u,v} = 0.8$, $\alpha_T = 0.8$ and $\alpha_p = 0.3$. Computations are performed for four problems mentioned above. Since the qualitative results are more or less the same, to save the space, only the results for flow in a 2D axisymmetric sudden expansion and natural convection in a square cavity are provided here. These two problems cover the forced convection and natural convection, and also represent the Cartesian coordinates and cylindrical coordinates.

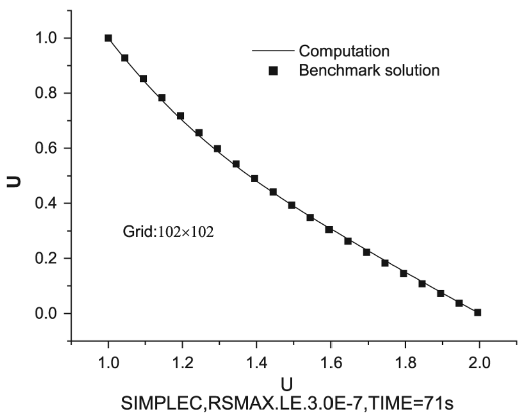
Problem 3: Flow in a 2D axisymmetric sudden expansion. The computational results are presented in Table I. We can see there that the predicted value of the representative parameter L_r/D_{in} are almost identical under the same grid density for the four algorithms compared. Such a uniformity in numerical results are reasonable and expected. Since it is generally considered that



(a) SIMPLE, 102 × 102



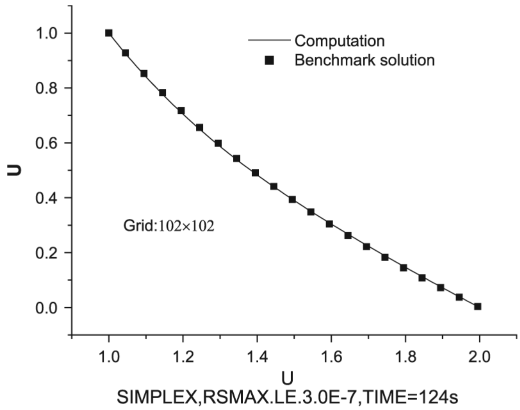
(b) SIMPLER, 102 × 102



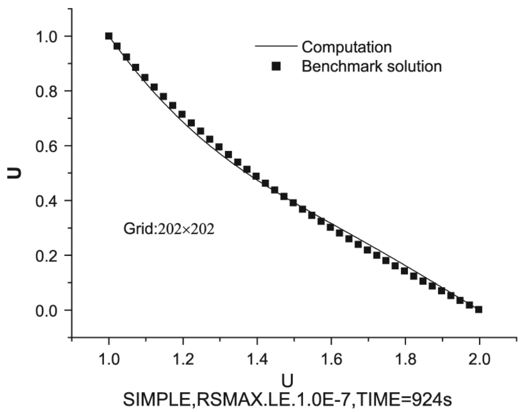
(c) SIMPLEC, 102 × 102

(continued)

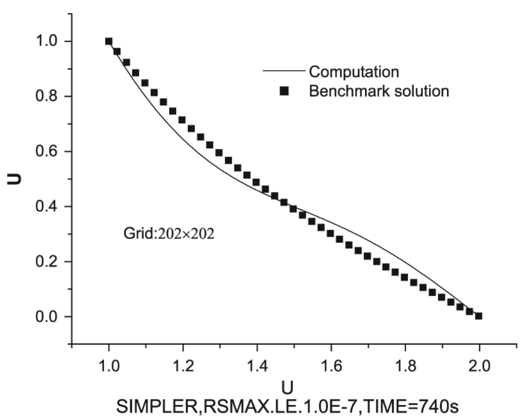
Figure 3.
Comparison between four algorithms under first convergence criterion for flow in an annulus with inner surface rotating



(d) SIMPLEX, 102 × 102



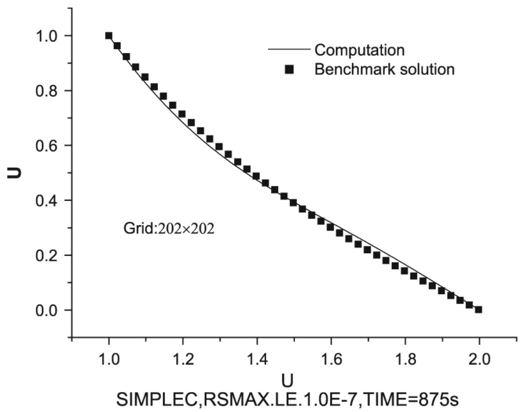
(e) SIMPLER, 102 × 102



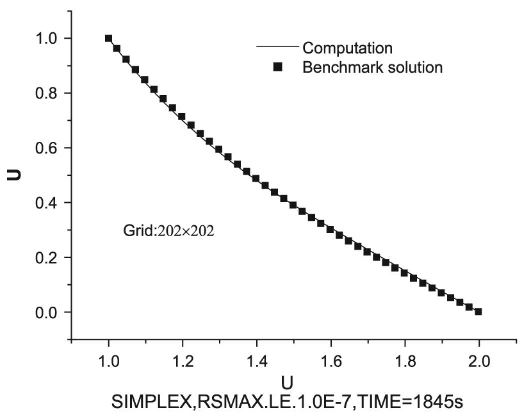
(f) SIMPLER, 202 × 202

Figure 3.

(continued)



(g) SIMPLEC, 202 × 202



(h) SIMPLEX, 202 × 202

Figure 3.

Table I. The predicted values of L_r/D_{IN} and the computation time with different grid density

	L_r/D_{IN} (Re = 150)				Time(s)			
Grid	SIMPLE	SIMPLER	SIMPLEC	SIMPLEX	SIMPLE	SIMPLER	SIMPLEC	SIMPLEX
32 × 12	6.510505	6.510505	6.510505	6.510505	0.200288	0.210302	0.280403	0.520748
52 × 12	6.288261	6.288261	6.288261	6.288261	0.350504	0.420604	0.450648	1.321901
102 × 18	6.709247	6.709247	6.709247	6.709247	1.682419	2.453528	1.542218	7.801218
152 × 22	6.633205	6.633205	6.633205	6.633205	4.816926	7.240411	4.696754	26.11756
202 × 42	6.628800	6.628800	6.628800	6.628800	51.88461	77.19099	52.36530	142.1244
Source: Macagno and Hung (1967)			6.5					

the accuracy of a numerical solution mainly depends on the discretization scheme and the grid fineness, while the iteration convergence rate depends on the algorithm dealing with the coupling between velocity and pressure (Tao, 2001). The solution uniformity of the four algorithms also implies that the second criterion is more appropriate for the judging of convergence. As far as the CPU time is concerned, the time spent by SIMPLEX is the most, and secondly is the SIMPLER.

Problem 4: Natural convection in a square cavity. The results are listed in Table II. Again we can see the uniformity of the four solutions. The expense of CPU time of SIMPLEX is the most, and that of the SIMPLE is the least (except for the 42×42 case) while that of SIMPLEC is somewhere in between.

3.3 Comparison of algorithm robustness

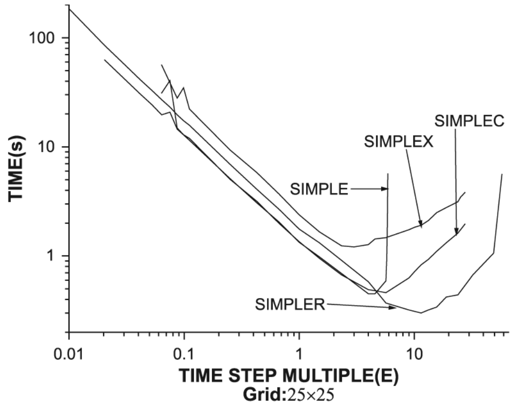
It is, generally, considered that if an algorithm can lead to a convergence solution within a wide rage of the relaxation factor, the algorithm possesses good robustness. It is based on this understanding that we performed the robustness comparison. In order to have a wide variation range, the so-called time step multiple (Van Doormaal and Raithby, 1984) is used instead of the underrelaxation factor. According to the above discussion, the second convergence criterion is adopted here. The results are presented in Figures 4-6. In the figures, the X-coordinate stands for the time step multiple E ($E = \alpha/(1 - \alpha)$), and the Y-coordinate is the computation time (TIME(s)). The variation range of the time step multiple within which a convergence solution can be acquired is regarded as the symbol of the robustness. The wider the range, the better the robustness. From these figures, the same conclusion can be made: the robustness of SIMPLE is always the worst whether the grid are coarse or not. The SIMPLER behaves as the SIMPLEC and SIMPLEX in coarse grid and experiences a degradation in convergence with grid refinement. The robustness of SIMPLEC and SIMPLEX are almost the same. These figures also tells us that the SIMPLER and SIMPLEX need more computation time than that of the SIMPLE and SIMPLEC.

4. Further discussion on the difference between the four algorithms

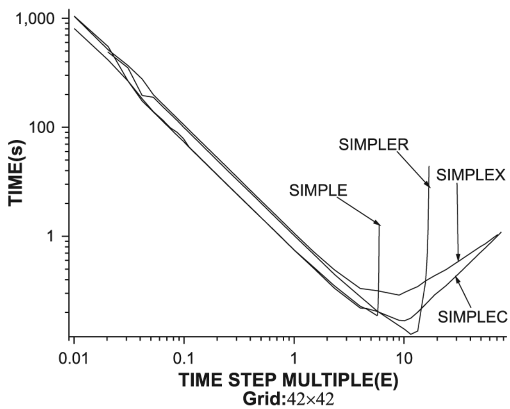
We have found that there are some differences among the four algorithms. Following discussion tries to further reveal the reasons that account for the difference. The main distinction among them is the determination of the coefficient d_e, d_n except that the SIMPLER needs to solve the pressure equation. The difference of d expression is listed in Table III. From our numerical

Table II. The results of the average Nusselt number and the computation time under different grid density

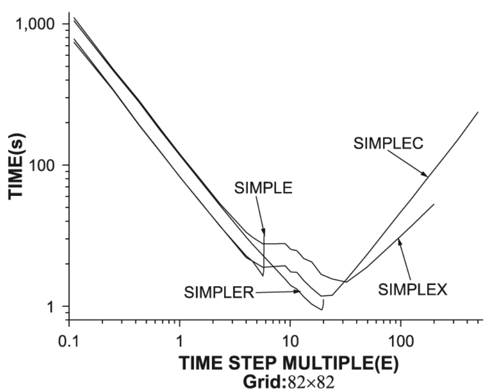
	Nu (Ra = 10,000)				Time(s)			
Grid	SIMPLE	SIMPLER	SIMPLEC	SIMPLEX	SIMPLE	SIMPLER	SIMPLEC	SIMPLEX
22×22	2.299	2.299	2.299	2.299	1.452088	1.81261	1.572261	2.443514
42×42	2.258	2.258	2.258	2.258	4.296177	3.34481	4.556552	6.269014
82×82	2.248	2.248	2.248	2.248	43.31228	50.3721	47.61847	61.38827
102×102	2.247	2.247	2.247	2.247	112.672	132.009	121.8552	165.7884
Source: Yu *et al.* (2001)	2.245							



(a) 25 × 25 grid system

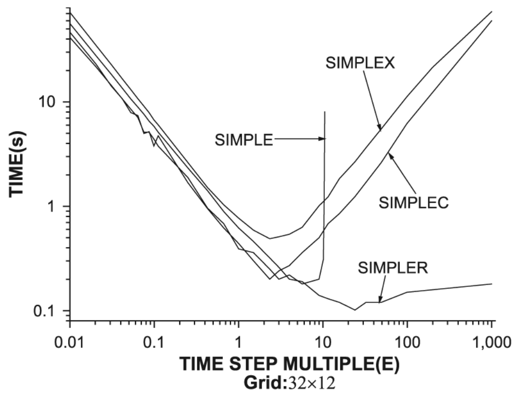


(b) 42 × 42 grid system

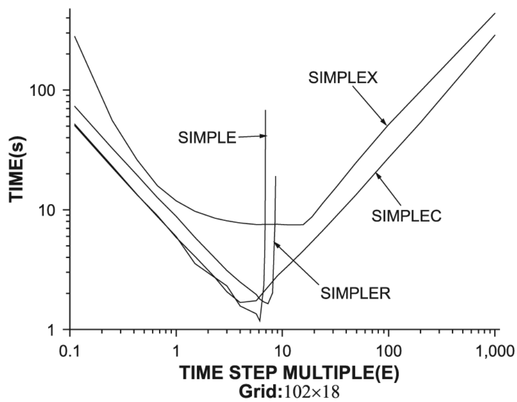


(c) 82 × 82 grid system

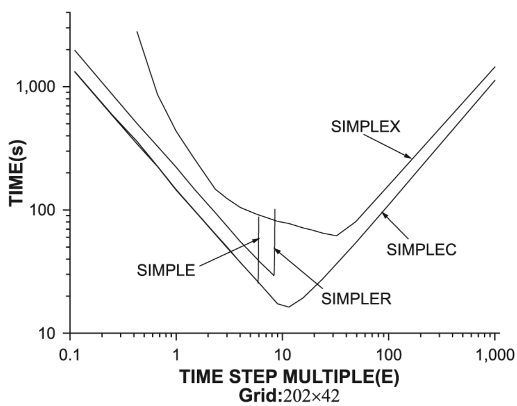
Figure 4.
Robustness comparison for lid-driven cavity flow



(a) 32 × 12 grid system

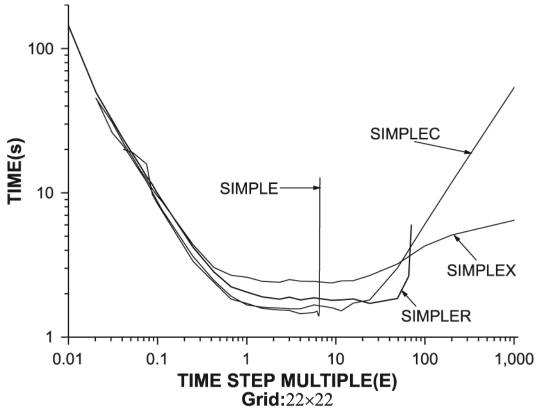


(b) 102 × 18 grid system

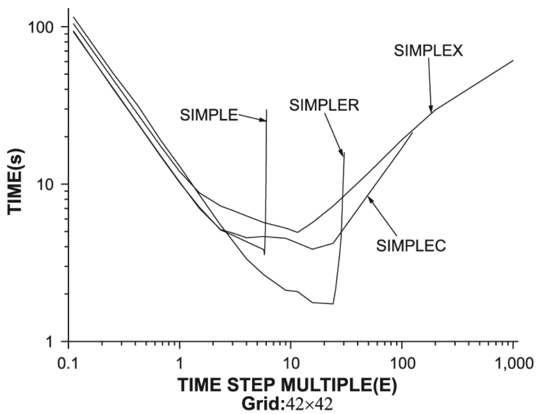


(c) 202 × 42 grid system

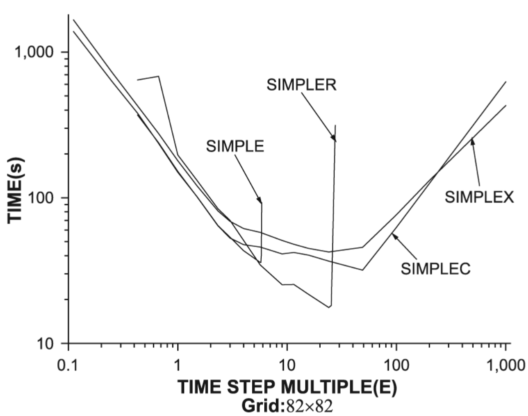
Figure 5.
Robustness comparison for flow in a 2D axisymmetric sudden expansion



(a) 22 × 22 grid system



(b) 42 × 42 grid system



(c) 82 × 82 grid system

Figure 6.
Robustness comparison for natural convection in a square cavity

practices, we found that if we solve a problem under the same condition (the same grid density, the same underrelaxation factors and the same convergence criterion, etc.), the values of d_e, d_n solved by SIMPLE and SIMPLER are close to each other when the convergence solution is approached, and the same applies to SIMPLEC and SIMPLEX. Tables IV-V show the value of d_e of two arbitrary points in two test cases. We can see from the tables that the value of d_e solved by the SIMPLEC and SIMPLEX is about five times larger than that of the SIMPLE and SIMPLER. Whether the d_e is the factor to affect the robustness or not is open to discussion. Now that the value of d_e solved by the SIMPLEC and SIMPLEX is nearly the same, while the SIMPLEC need not solve

Table III.
The contrast of four methods in approximation

Method	Approximation	d_e
SIMPLE	$\sum a^u_{nb} u'_{nb} = 0$	A_e / a_e
SIMPLER	$\sum a^u_{nb} u'_{nb} = 0$	A_e / a_e
SIMPLEC	$\sum a^u_{nb}(u'_{nb} - u'_e) = 0$	$A_e / (a_e - \sum a_{nb})$
SIMPLEX	$\Delta p'_e = \Delta p'_{nb}$	$a_e d_e = \sum a_{nb} d_{nb} + A_e$

Table IV.
The value of d_e of two arbitrary points (lid-driven cavity flow)

	25 × 25				42 × 42			
	SIMPLE	SIMPLER	SIMPLEC	SIMPLEX	SIMPLE	SIMPLER	SIMPLEC	SIMPLEX
d_u (12,7)	2.500	2.500	12.50	12.52	1.929	1.930	9.643	9.525
d_u (22,22)	2.064	2.064	10.32	8.593	1.874	1.873	9.368	9.265

	82 × 82			
	SIMPLE	SIMPLER	SIMPLEC	SIMPLEX
d_u (12,7)	0.9999	0.9999	4.999	4.976
d_u (22,22)	0.9612	0.9612	4.803	4.798

Table V.
The value of d_e of two arbitrary points (natural convection in a square cavity)

	22 × 22				42 × 42			
	SIMPLE	SIMPLER	SIMPLEC	SIMPLEX	SIMPLE	SIMPLER	SIMPLEC	SIMPLEX
d_u (10,6)	1.135	1.135	5.676	5.634	0.5927	0.5927	2.964	2.928
d_u (20,20)	1.183	1.183	5.916	6.060	0.5960	0.5960	2.980	2.979

	82 × 82			
	SIMPLE	SIMPLER	SIMPLEC	SIMPLEX
d_u (10,6)	0.2981	0.2981	1.490	1.474
d_u (20,20)	0.2975	0.2975	1.488	1.488

the d equations, the computation time by the SIMPLEC being less than that of the SIMPLEX is the natural outcome. From the four problems computed, it may be concluded that the SIMPLEC is superior to all others when fine grid system is used.

5. Conclusions

In this paper, the convergence character and the robustness of four variants in the SIMPLER-family are compared through four typical 2D fluid flow and heat transfer problems at fine grids (grid number being in the order of 100×100). The following conclusions can be made.

(1) The criterion for judging the iteration convergence of fluid flow and heat transfer problems is recommended to include both mass conservation and momentum conservation requirements. The mass conservation condition alone is not an adequate criterion, in that different algorithms may lead to different numerical solutions with other conditions being the same.

(2) For the four problems computed with the appropriate convergence criterion, the SIMPLEX needs the largest CPU time, the SIMPLER comes next, and the SIMPLE and SIMPLEC need the least computational time.

(3) Under the same conditions, the SIMPLE have the worst robustness and the next is the SIMPLER. The robustness of the SIMPLEC and SIMPLEX are almost the same and superior to that of the other two algorithms.

(4) To sum up, the SIMPLEC algorithm is recommended for the solution of incompressible fluid flow and heat transfer problems, especially when the grid density is fine, being in the order of 100×100.

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Further reading

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