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A NEW STABILITY-GUARANTEED SECOND-ORDER DIFFERENCE SCHEME

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Based on the stability-controllable second-order difference (SCSD) scheme, a new stability-guaranteed second-order difference (SGSD) scheme is proposed whose merits are absolutely stable and adaptive. Its numerical accuracy is at least no less than that of the central difference (CD) and second-order upwind difference (SUD) schemes and sometimes higher than that of the QUICK scheme. The SGSD scheme can automatically choose a different difference scheme according to the available local field information in difference space or time. It automatically approaches the central difference scheme where or when diffusion is dominant, and approaches the second-order upwind difference scheme where or when convection is dominant. Computations for two benchmark problems using the SGSD and the other three schemes show its feasibility in engineering computations.

1. INTRODUCTION

When numerically solving convective-diffusive equations, discretization of the convective terms is one of the most challenging and interesting tasks, since discretization schemes for the convective terms in the Navier-Stokes equations and scalar transport equations are connected directly to the solution accuracy, efficiency, and convergenece. To make reference to all past works in connection with difference schemes for convection terms would be a task for the introduction of a book rather than of the present article. Here we focus our attention on the numerical simulation of conventional fluid flow and heat transfer problems. By "conventional" we mean those problems in which no sharp gradient (such as sharp gradients of a shock) exists in the computational domain. A close look at some previous comparison studies [1–6] indicates that the choice of a scheme which performs well in all situations is rather difficult, if not impossible. This situation is related directly to the following fact: for most existing schemes used in computing conventional fluid flow and heat transfer problems, the requirements for numerical stability and computational

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NOMENCLATURE

$a_p, a_{N,E,W,S}$	coefficients in discretization equation
D	diffusive conductance, for example, $D_e = \Gamma_e \Delta y/(\delta x)_e$
Er	channel expansion ratio
F	flow rate at the interface, for example, $F_e = (\rho u)_e \Delta y$
$P_{\Delta x}$, $P_{\Delta y}$	grid Peclet number in x and y directions, respectively
R	residual of discretization equation
Re	Reynolds number
S	nominal source term
u, v	velocity components in x and y directions, respectively
U, V	nondimensional velocity components in x and y directions, respectively
U_0	sliding velocity of the lid
x	coordinate
β	parameter indicating the percentage of central difference
Γ	nominal diffusion coefficient
δx, δy	distance between two neighboring grid points in x and y directions, respectively
Δx, Δy	distance between two neighboring interfaces in x and y directions, respectively
ν	fluid kinetic viscosity
ϕ	general dependent variable

Subscripts

e, n, s, w	interfaces
E, N, S, W	near neighboring grid points
EE, NN, SS, WW	far neighboring grid points

accuracy are often contradictory. Lower-order schemes such as first-order upwind are stable but often lead to severe false diffusion. Second-order schemes such as central difference (CD) and higher-order schemes such as QUICK eliminate false diffusion but may produce wiggles and often fail to converge [7]. Taking the central difference scheme as an example, it is well known that the CD is prone to oscillation when a grid Peclet (or Reynolds) number is beyond a specific value. Even though recent study has shown that the one-dimensional stability analysis methods (such as [8]) give only the most severe critical Peclet number—for practical multidimensional cases CD may work well even if the local grid Peclet number is as large as 180 [9]—the presence of the Peclet number limit beyond which oscillation will occur is still an undesirable feature of the scheme. A significant amount of research effort has been directed toward convective discretization schemes, and many remedies have been proposed [5, 10–13]. For example, in [5] it is shown that, compared to the CD, the second-order upwind difference scheme (SUD) performs better and its implementation method in discretization is recommended. From stability considerations, the SUD is perfect since it is absolutely stable; however, our numerical practice has shown that it is somewhat diffusive [14]. Thus, as concluded in [7], the matter is far from being solved, and the need for a definitive study of the formulation of convection and diffusion still remains.

Before going into the detail of the construction of a new scheme, we shall briefly discuss the essence of discretization of the convective term. Inspection of the discretization process of the convective term, written in a nonconservative form $(u\partial\phi/\partial x)_i$, we can see that actually we encounter two kinds of quantities, i.e., convective velocity u and the scalar derivative $\partial\phi/\partial x$. The physical mechanism of convective flow is totally directional in that only the information upstream can be transferred downstream, not vice versa. From this regard it is better to adopt directional discretization schemes such as first-order upwind or second-order upstream schemes. However, from the first-order derivative itself, approximations with equal points at the two sides of position i are better than bias approximations in that the information at the two sides may be equally taken into account, especially when diffusion is the dominant mechanism. Thus we may expect that an approximation combining symmetry and nonsymmetry structures of difference formulations with their percentage being adjusted automatically will give better performance. This is our basic consideration in constructing a new discretization scheme for the convection term.

Starting from this point, the following schemes, which are combinations of several existing schemes, caught our attention. These are second-order hybrid (SHYBRID) [13] and stability-controllable second-order difference (SCSD) schemes [15]. In [13], the CD, QUICK, and SUD are organized in a general form SHYBRID. In [15] the CD and SUD are combined to generate the SCSD scheme. Because of its simplicity of definition, in this article we take SCSD scheme as our starting point, and a further refinement will be made to make the scheme self-adaptive in that the percentage of central difference can be automatically adjusted according to the problem itself. In such a way we shall formulate a new scheme whose stability is guaranteed with at least second-order accuracy. It is named stability-guaranteed second-order difference (SGSD).

In the following the details of the construction of the scheme, and its performance comparison with CD, SUD, and QUICK for lid-driven cavity flow and flow over a back-facing step will be presented. In the numerical simulation the diffusion term is always discretized by the second-order CD schemes, hence the difference in numerical solutions is from the convection term.

2. DERIVATION OF THE DIFFERENCE SCHEMES FOR CONVECTIVE TERMS

The general differential equations for a steady-steady convection-diffusion problem for the general variable ϕ can be written in a conservative form as

$$\frac{\partial(\rho u_j \phi)}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\Gamma_\phi \frac{\partial \phi}{\partial x_j}\right) + S_\phi \tag{1}$$

where ρ is the density of the fluid, u_j is the jth component of the velocity, Γ_ϕ is the diffusion coefficient, and S_ϕ is the source term for the variable ϕ.

For discretization of the above equation, the finite-volume method is adopted here. A typical control volume is shown in Figure 1. For two-dimensional problems, after integration over the control volume, the discretized equation can be obtained as follows:

$$\begin{aligned} F_e\phi_e - F_w\phi_w + F_n\phi_n - F_s\phi_s = S_\phi\,\Delta x\,\Delta y + D_e(\phi_E - \phi_P) - D_w(\phi_W - \phi_P) \\ + D_n(\phi_N - \phi_P) - D_s(\phi_S - \phi_P) \end{aligned} \tag{2}$$

The variables $\phi_{n,e,w,s}$ at control-volume interfaces can be determined using an interpolation or extrapolation involving the values of the neighboring grid points.

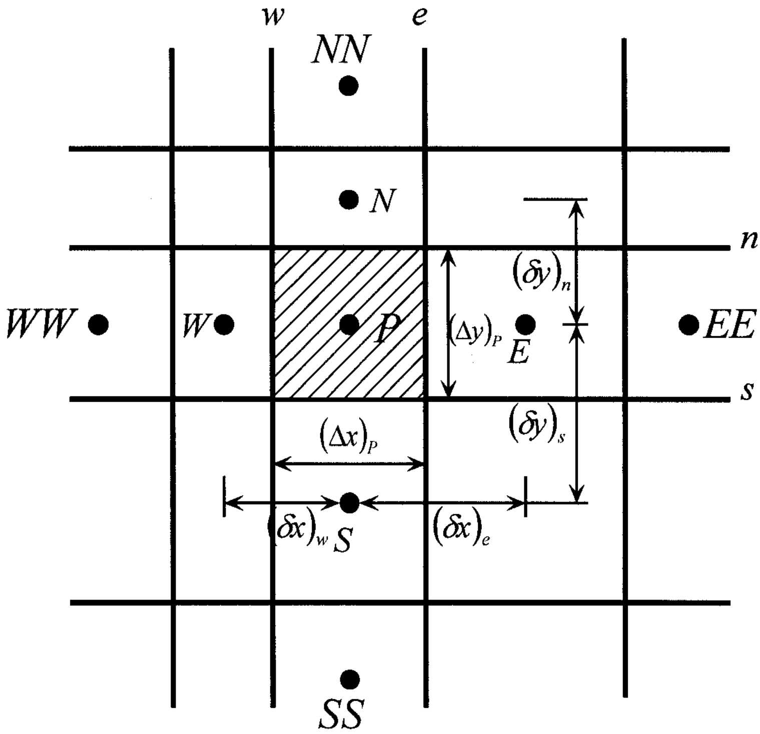


Figure 1. Schematic of the control volume.

In the finite-volume method, the choice of interpolation or extrapolation method is the scheme problem.

2.1. SGSD Scheme in Uniform Grid System

To complete the discretization, the interface values of ϕ are first interpolated by the SCSD scheme [15]. For example, the value of variable ϕ on the east surface is interpolated as

$$\phi_e = \beta\phi_e^{\mathrm{CD}} + (1-\beta)\phi_e^{\mathrm{SUD}} \qquad (0 \leq \beta \leq 1) \tag{3}$$

where the superscripts CD and SUD designate the central difference and the second-order upwind difference, and the quantity β is a prespecified parameter. Obviously, the scheme is of second-order accuracy. And when $\beta = 0$, it becomes SUD, and CD for $\beta = 1$. Further, it can be easily shown that when $\beta = 3/4$, it leads to QUICK.

By the one-dimensional stability analysis method [8], it can be shown that when

$$P_{\Delta x} = \frac{u\,\delta x}{\Gamma} \leq \frac{2}{\beta} \qquad P_{\Delta y} = \frac{v\,\delta y}{\Gamma} \leq \frac{2}{\beta} \tag{4}$$

the scheme is stable. Thus, by selecting the value of β we can control its stability.

Although the stability of the SCSD scheme is controllable, the value of the factor β is given before starting the iteration. For a given problem, all information of the physical field exists objectively. No matter what algorithms and difference scheme are employed, no change can be made to the existing physical field. Moreover, the interaction of convection and diffusion is different in a different direction or space in the physical field of a given convective-diffusive problem: somewhere the convection is dominant and elsewhere not. For example, for the flow in a straight duct the convection is dominant in the main flow direction, while the interaction of convection and diffusion may be of counterbalance on the cross section. We expect to employ the central difference scheme where the diffusion is dominant and to use a nonsymmetric structure formulation such as the second-order upwind difference scheme where the convection is very strong. In this way, the physical problems may be predicted more objectively.

According to the statement above, the factor β should not be given beforehand but should be obtained interactively from the iterative results. It is difficult to construct an accurate function for β. However, one-dimensional model analysis indicates that the critical grid Peclet number of the SCSD scheme is $2/\beta$. Thus, we may select a simple function as follows:

$$\beta = \frac{2}{2 + |P_\Delta|} \tag{5}$$

where P_Δ is the grid Peclet number.

As indicated above, one-dimensional stability analysis gives the most severe stability condition, and for many practical cases a conditionally stable difference scheme is inclined to oscillate beyond the critical grid Peclet number predicted by the one-dimensional model. However, since at present we have no way to obtain the stability condition for multidimensional practical problems, to be on the safe side we prefer to use the most severe condition to construct our new scheme. Thus the definition of the interface variable (taking ϕ_e as an example) is as follows:

$$\phi_e = \beta\phi_e^{\mathrm{CDS}} + (1-\beta)\phi_e^{\mathrm{SUD}} \qquad \beta = \frac{2}{(2 + |P_\Delta|)} \tag{6}$$

It can be easily shown that the scheme defined by Eq. (6) is absolutely stable. It is a stability-guaranteed second-order difference scheme (SGSD). The most attractive feature of the SGSD is that the percentage of CD and SUD is self-adaptive. In practical computations, information from previous solutions is used to determine the Peclet number for every grid point in such coordinate. When the local velocity is large, the related grid Peclet will be large too, and hence the percentage of the CD will be small. Thus not only for every grid point but also for every direction of coordinates, the percentage of CD and SUD in the discretization of the convection terms is automatically adjusted in each iteration, leading to better consistency between the discretization scheme and the physical problem itself, which we believe will enhance the accuracy and robustness of the numerical solution.

2.2 Implementation of SGSD Scheme on the Nonuniform Grid System

For stability of the iteration procedures, the deferred-correction method is adopted, which was proposed in [16] and latter enhanced in [17]. In the deferred-correction method, the interpolation for the interface variable is expressed as the sum of a low-order (such as first-order upwind) difference scheme and a correction term corresponding to the scheme employed. Thus Eq. (3) may be written as follows:

$$\phi_e = \phi_e^{\mathrm{FUD}} + \{[\beta\phi_e^{\mathrm{CD}} + (1-\beta)\phi_e^{\mathrm{SUD}}] - \phi_e^{\mathrm{FUD}}\}^{\mathrm{old}} \tag{7}$$

where the superscript FUD designates the first-order upwind difference and old designates the previous iterative results.

Using Patanakar's notation [18], Eq. (2) can be written in the form

$$\begin{gathered} a_P\phi_P = a_E\phi_E + a_W\phi_W + a_N\phi_N + a_S\phi_S + b + S_{ad} \\ a_P = a_E + a_W + a_N + a_S - S_P\,\Delta x\,\Delta y \qquad b = S_c\,\Delta x\,\Delta y \end{gathered} \tag{8}$$

$$\begin{aligned} a_E &= D_e + [|-F_e, 0|] \qquad & a_W &= D_w + [|F_w, 0|] \\ a_N &= D_n + [|-F_n, 0|] \qquad & a_S &= D_s + [|F_s, 0|] \end{aligned} \tag{9}$$

$$S_{\mathrm{ad}} = -S_{\mathrm{ad},e} + S_{\mathrm{ad},w} - S_{\mathrm{ad},n} + S_{\mathrm{ad},s} \tag{10}$$

where S_{ad} is the additional source term generated by employing the deferred-correction method.

The above discretization procedure applies for both uniform and nonuniform grid systems and for any kind of discretization scheme of the convection term. To get practical expressions for S_{ad}, the grid system and the scheme definition have to be specified. Since a nonuniform grid system is widely used in numerical heat transfer, in the following we extend the definition of SGSD to a nonuniform grid system.

Figure 2 shows a one-dimensional nonuniform grid system. Employing the Lagrange interpolation method, the variable ϕ at the east and west interfaces can be obtained:

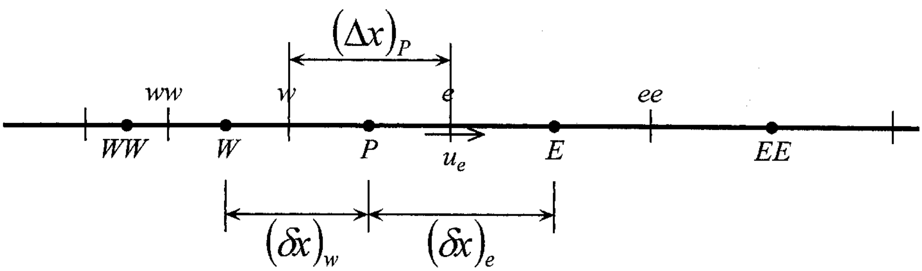


Figure 2. 1-D nonuniform grid system.

$$\phi_e = \phi_P + \left[\beta\left(\frac{x_e - x_P}{x_E - x_P}\phi_E + \frac{x_E - x_e}{x_E - x_P}\phi_P\right)\right.$$

$$\left. +(1-\beta)\left(\frac{x_e - x_W}{x_P - x_W}\phi_P - \frac{x_e - x_P}{x_P - x_W}\phi_W\right) - \phi_P\right] \qquad (u_e > 0) \tag{11a}$$

$$\phi_e = \phi_E + \left[\beta\left(\frac{x_e - x_P}{x_E - x_P}\phi_E + \frac{x_E - x_e}{x_E - x_P}\phi_P\right)\right.$$

$$\left. +(1-\beta)\left(\frac{x_{EE} - x_e}{x_{EE} - x_E}\phi_E - \frac{x_E - x_e}{x_{EE} - x_E}\phi_{EE}\right) - \phi_E\right] \qquad (u_e < 0) \tag{11b}$$

$$\phi_w = \phi_W + \left[\beta\left(\frac{x_P - x_w}{x_P - x_W}\phi_W + \frac{x_w - x_W}{x_P - x_W}\phi_P\right)\right.$$

$$\left. +(1-\beta)\left(\frac{x_w - x_{WW}}{x_W - x_{WW}}\phi_W - \frac{x_w - x_W}{x_W - x_{WW}}\phi_{WW}\right) - \phi_W\right] \qquad (u_w > 0) \tag{11c}$$

$$\phi_w = \phi_P + \left[\beta\left(\frac{x_P - x_w}{x_P - x_W}\phi_W + \frac{x_w - x_W}{x_P - x_W}\phi_P\right)\right.$$

$$\left. +(1-\beta)\left(\frac{x_E - x_w}{x_E - x_P}\phi_P - \frac{x_P - x_w}{x_E - x_P}\phi_E\right) - \phi_P\right] \qquad (u_w < 0) \tag{11d}$$

Similar expressions can be obtained for the variable ϕ at the north and south interfaces. For simplicity of presentation, we define

$$F_i^+ = \frac{F_i + |F_i|}{2} \qquad F_i^- = \frac{F_i - |F_i|}{2} \qquad (i = e, w, n, s) \tag{12}$$

where F designates the flow rate at the control-volume surfaces. The additional source terms at the control-volume surfaces are

$$S_{\mathrm{ad},e} = F_e^+\left[\beta\left(\frac{x_e - x_P}{x_E - x_P}\phi_E^{\mathrm{old}} + \frac{x_E - x_e}{x_E - x_P}\phi_P^{\mathrm{old}}\right)\right.$$

$$\left. +(1-\beta)\left(\frac{x_e - x_W}{x_P - x_W}\phi_P^{\mathrm{old}} - \frac{x_e - x_P}{x_P - x_W}\phi_P^{\mathrm{old}}\right) - \phi_P^{\mathrm{old}}\right]$$

$$+ F_e^-\left[\beta\left(\frac{x_e - x_P}{x_E - x_P}\phi_E^{\mathrm{old}} + \frac{x_E - x_e}{x_E - x_P}\phi_P^{\mathrm{old}}\right)\right.$$

$$\left. +(1-\beta)\left(\frac{x_{EE} - x_e}{x_{EE} - x_E}\phi_E^{\mathrm{old}} - \frac{x_E - x_e}{x_{EE} - x_E}\phi_{EE}^{\mathrm{old}}\right) - \phi_E^{\mathrm{old}}\right] \tag{13a}$$

$$S_{\mathrm{ad},w} = F_w^+ \left[\beta \left(\frac{x_P - x_w}{x_P - x_W} \phi_W^{\mathrm{old}} + \frac{x_w - x_W}{x_P - x_W} \phi_P^{\mathrm{old}} \right) \right.$$
$$\left. + (1-\beta) \left(\frac{x_w - x_{WW}}{x_W - x_{WW}} \phi_W^{\mathrm{old}} - \frac{x_w - x_W}{x_W - x_{WW}} \phi_{WW}^{\mathrm{old}} \right) - \phi_W^{\mathrm{old}} \right]$$
$$+ F_w^- \left[\beta \left(\frac{x_P - x_w}{x_P - x_W} \phi_W^{\mathrm{old}} + \frac{x_w - x_W}{x_P - x_W} \phi_P^{\mathrm{old}} \right) \right.$$
$$\left. + (1-\beta) \left(\frac{x_E - x_w}{x_E - x_P} \phi_P^{\mathrm{old}} - \frac{x_P - x_w}{x_E - x_P} \phi_E^{\mathrm{old}} \right) - \phi_P^{\mathrm{old}} \right] \tag{13b}$$

$$S_{\mathrm{ad},n} = F_n^+ \left[\beta \left(\frac{y_n - y_P}{y_N - y_P} \phi_N^{\mathrm{old}} + \frac{y_N - y_n}{y_N - y_P} \phi_P^{\mathrm{old}} \right) \right.$$
$$\left. + (1-\beta) \left(\frac{y_n - y_S}{y_P - y_S} \phi_P^{\mathrm{old}} - \frac{y_n - y_P}{y_P - y_S} \phi_P^{\mathrm{old}} \right) - \phi_P^{\mathrm{old}} \right]$$
$$+ F_n^- \left[\left[\beta \left(\frac{y_n - y_P}{y_N - y_P} \phi_N^{\mathrm{old}} + \frac{y_N - y_n}{y_N - y_P} \phi_P^{\mathrm{old}} \right) \right. \right.$$
$$\left. + (1-\beta) \left(\frac{y_{NN} - y_n}{y_{NN} - y_N} \phi_N^{\mathrm{old}} - \frac{y_N - y_n}{y_{NN} - y_N} \phi_{NN}^{\mathrm{old}} \right) - \phi_N^{\mathrm{old}} \right] \tag{13c}$$

$$S_{\mathrm{ad},s} = F_s^+ \left[\beta \left(\frac{y_P - y_s}{y_P - y_S} \phi_S^{\mathrm{old}} + \frac{y_s - y_S}{y_P - y_S} \phi_P^{\mathrm{old}} \right) \right.$$
$$\left. + (1-\beta) \left(\frac{y_s - y_{SS}}{y_S - x_{SS}} \phi_W^{\mathrm{old}} - \frac{y_s - y_S}{y_S - y_{SS}} \phi_{SS}^{\mathrm{old}} \right) - \phi_S^{\mathrm{old}} \right]$$
$$+ F_s^- \left[\left[\beta \left(\frac{y_P - y_s}{y_P - y_S} \phi_W^{\mathrm{old}} + \frac{y_s - y_S}{y_P - y_S} \phi_P^{\mathrm{old}} \right) \right. \right.$$
$$\left. + (1-\beta) \left(\frac{y_N - y_s}{y_N - y_P} \phi_P^{\mathrm{old}} + \frac{y_P - y_s}{y_N - y_P} \phi_N^{\mathrm{old}} \right) - \phi_P^{\mathrm{old}} \right] \tag{13d}$$

3. TEST CALCULATIONS FOR TWO BENCHMARK PROBLEMS

In this section, numerical solutions from the SGSD scheme are compared with those of the SUD, QUICK, and CD schemes. The resulting discretization equations are solved by the block implicit method (BIM). The BIM is a direct method in that the discretization equations for one control volume are solved simultaneously, including velocities and pressure. The solution procedure is then advanced from control volume to control volume. Once all the control volumes are visited, this completes one outer iteration. Details about the BIM may be found in [14, 19, 20].

Two benchmark problems will be used to examine the proposed scheme. First, numerical calculations are performed for the lid-driven cavity flow investigated by

Ghia et al. [21] using a multigrid method. Constant-property laminar flows are assumed for the Reynolds number 10^3. The Reynolds number is defined as $\mathrm{Re} = U_0 L/\nu$, where L is the length of the square enclosure side wall, U_0 is the speed of the sliding lid, and ν is the fluid kinetic viscosity. Second, numerical computations are conducted for the flow over a backward-facing step for Reynolds numbers 100 and 300.

3.1. Lid-Driven Cavity Flow

Both uniform and nonuniform grids consisting of 42×42 nodes are used. The nonuniform grid distribution is of sine-type profile, in which the grid becomes finer and finer close to the cavity wall. Each calculation is terminated when the control-volume maximum residual of the discretized continuity and momentum equations becomes smaller than 10^{-6}, i.e.,

$$\text{Residual} = \max(R^c_{\max}, R^u_{\max}, R^v_{\max}) < 1. \times 10^{-6} \tag{14}$$

where the superscripts $c, u,$ and v stand for continuity, u-momentum, and v-momentum equations.

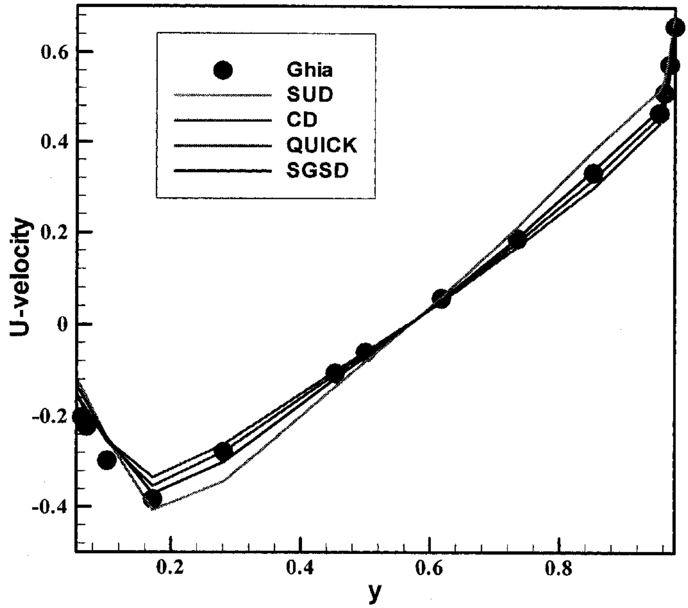
Centerline velocity profiles for the u (x-direction) and v (y-direction) velocity components are shown in Figures 3 and 4 for $\mathrm{Re} = 10^3$ with a uniform grid system. Each figure provides a comparison among the four schemes. Also shown there are the results calculated by Ghia et al. on finer (129×129) grids as the standards for comparison. In Figure 5, centerline velocity distributions obtained by using the nonuniform grid system (42×42) are presented. From theses three figures, it can be observed that for the uniform 42×42 grid system, solutions from QUICK and SGSD are better than those from CD and SUD, and the solutions in the nonuniform 42×42 grid system have appreciably better accuracy than those in the uniform grid system of the same grid number.

Tables 1–4 list the relative errors of the centerline velocities between the present numerical solutions using uniform and nonuniform grids and the results provided by Ghia et al. These tables further enhance our understanding of the numerical accuracy of the four schemes: the numerical accuracy of the SGSD scheme is, generally speaking, at least no less than that of the CD and SUD schemes, and sometimes it is even better than that of the QUICK scheme (Table 2).

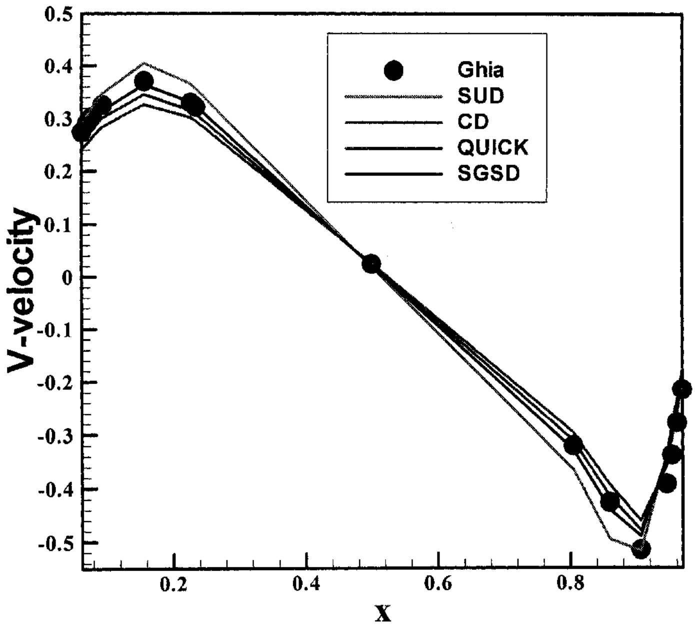
Since in the execution of SGSD extra computations have to be conducted to determine the local directional Peclet number, a question may arise about its convergence rate compared with other schemes. Our numerical practices show that in the nonuniform grids, the iteration will converge more rapidly when using the SGSD scheme. Table 5 lists the CPU time consumed using the four schemes, where the CPU time consumed using the SGSD scheme in a uniform 42×42 grid is taken as 1. Table 5 shows that for the uniform grid, the CPU time of SGSD may be 20–30% higher than that of QUICK and CD, while for the nonuniform grids, the CPU time of SGSD is about 40–50% off compared with that of QUICK and CD.

3.2. Flow over a Backward-Facing Step

There are two important parameters that exert a great influence on the fluid mechanics in a two-dimensional backward-facin step geometry, i.e., the Reynolds

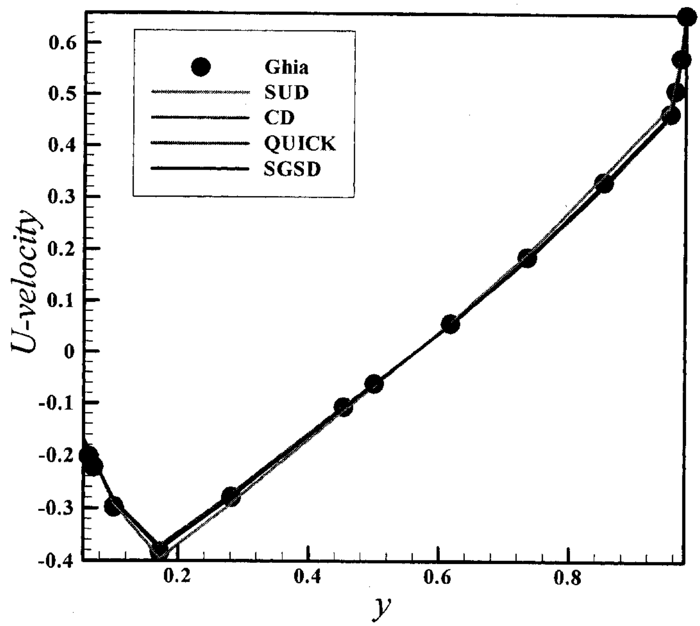


(a) u-velocity profile along the vertical centerline

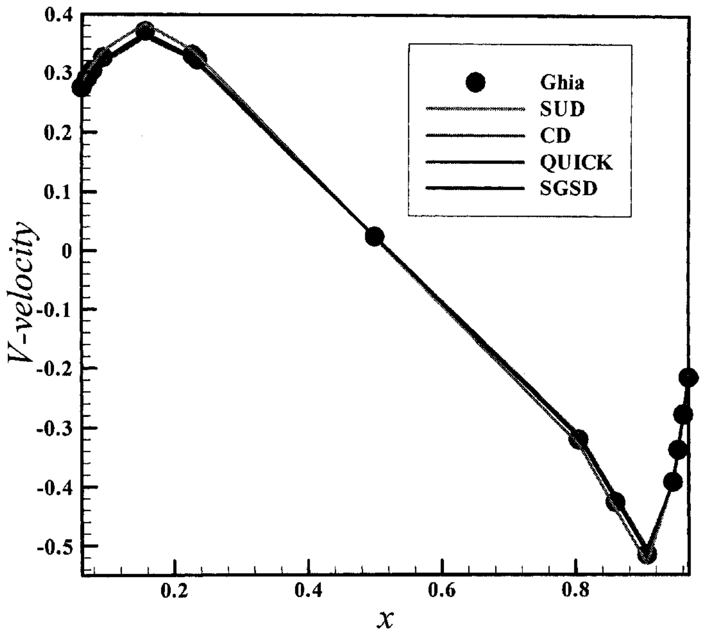


(b) v-velocity profile along the horizontal centerline

Figure 3. Velocity distributions for lid-driven cavity flow (Re = 1,000, 42 × 42 uniform grid system): (*a*) *u*-velocity profile along the vertical centerline; (*b*) *v*-velocity profile along the horizontal centerline.

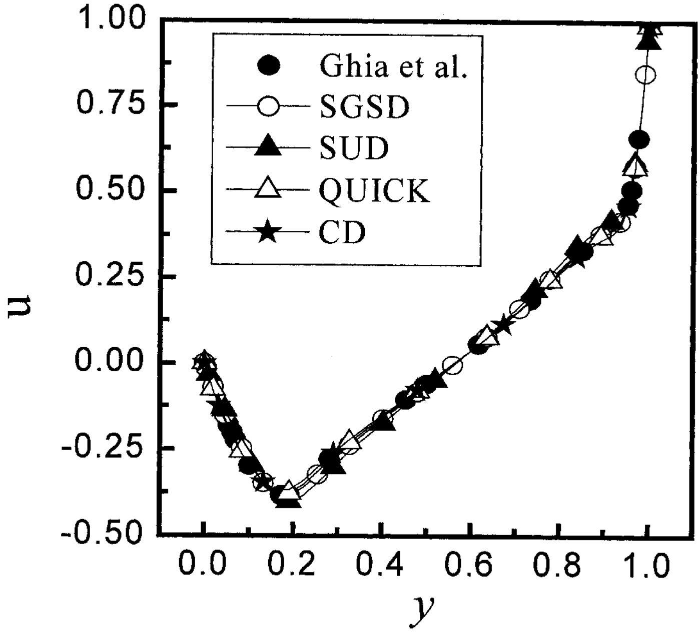


(a) u-velocity profile along the vertical centerline



(b) v-velocity profile along the horizontal centerline

Figure 4. Velocity distributions of lid-driven cavity flow (Re = 1,000, 82 × 82 uniform grid system): (*a*) *u*-velocity profile along the vertical centerline; (*b*) *v*-velocity profile along the horizontal centerline.



(a) u-velocity profile along the vertical centerline

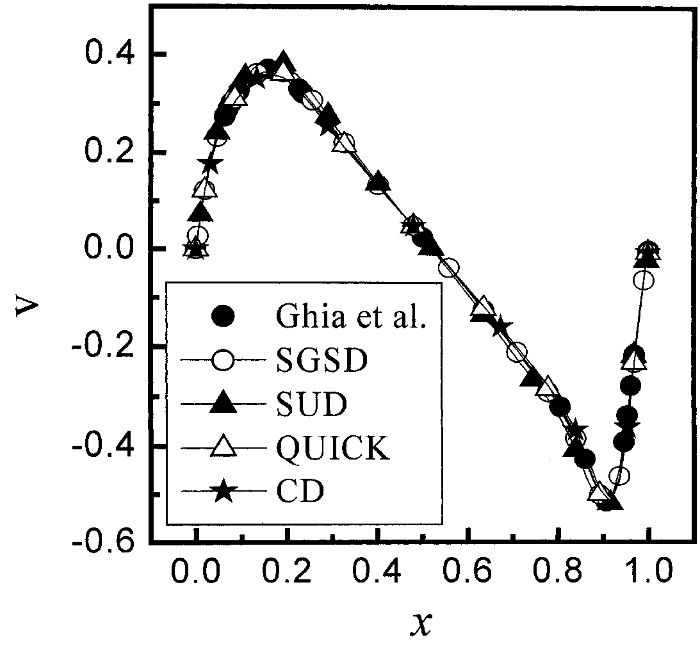


Figure 5. Velocity distributions of lid-driven cavity flow (Re = 1,000, 42×42 uniform grid system): (*a*) *u*-velocity profile along the vertical centerline; (*b*) *v*-velocity profile along the horizontal centerline.

Table 1. Relative error of centerline u-velocity obtained using uniform grid (42×42), %

y	SGSD	SUD	QUICK	CD
0.9766	0.5961	3.4477	−0.7099	−1.8278
0.9688	0.4940	5.5434	−1.3758	−3.3100
0.9609	−1.6179	5.8356	−4.1650	−7.0329
0.9531	1.7917	10.8210	−1.5664	−5.2377
0.8516	1.6725	14.6770	−4.4679	−10.0559
0.7344	1.0150	13.4836	−4.3165	−9.5678
0.6172	−7.2431	2.9113	−9.8562	−14.5212
0.5000	16.1020	33.9967	5.4276	−1.6118
0.4531	11.0819	27.5451	2.0098	−4.5642
0.2813	8.0669	23.6360	−0.2050	−5.9558
0.1719	−3.1419	6.3830	−7.6654	−12.6041
0.1016	−16.8012	−16.2563	−14.0868	−15.0858
0.0703	−23.0018	−27.9703	−16.1296	−14.2394
0.0625	−24.6782	−31.4765	−16.4884	−13.6463
0.0547	−25.9926	−33.9499	−17.0744	−13.7059
Mean error	9.5531	17.1956	7.0363	8.8644

Table 2. Relative error of centerline v-velocity obtained using uniform grid (42×42), %

x	SGSD	SUD	QUICK	CD
0.9688	−16.4017	−18.2859	−13.1989	−8.1213
0.9609	−14.9011	−17.2648	−11.9592	−7.0584
0.9531	−14.2403	−15.8331	−11.7725	−8.6462
0.9453	−12.5216	−13.5934	−10.3577	−8.4337
0.9063	−4.9311	0.57807	−7.0805	−10.5936
0.8594	3.1407	16.0975	−2.6930	−7.7979
0.8047	2.5182	14.1713	−2.8592	−7.6018
0.5000	−12.6286	−20.8630	−4.7901	4.1171
0.2344	0.5708	10.8422	−4.0980	−8.2425
0.2266	0.1746	10.6848	−4.4293	−8.6984
0.1563	−1.7118	9.0362	−6.7151	−11.8210
0.0938	−3.3040	6.5743	−7.6163	−12.7747
0.0781	−3.8282	5.7028	−7.9530	−13.0695
0.0703	−3.9121	5.4735	−7.9691	−13.0842
0.0625	−3.3509	5.9341	−7.3640	−12.5122
Mean error	6.5519	11.3957	7.3904	9.5048

number Re and the channel expansion ratio ER (the ratio of the channel widths downstream and upstream) [22]. Here the case of $\mathrm{Re}=100$, $\mathrm{ER}=1.5$ is taken as the test example. Under these conditions, the predicted reattachment length calculated by Kondoh et al. is 6.3 [22], which is slightly longer than the experimental value of 6.0. In the present study, the grids used are uniform and consist of 62×32 and 122×62 nodes. The predicted reattachment lengths obtained by using various schemes and grids are listed in Table 6. For the 62×32 uniform grid, the iteration diverges when the CD scheme is adopted. On these two grid systems the QUlCK scheme works well for $\mathrm{Re}=100$. However, when the Reynolds number increases to

Table 3. Relative error of centerline u-velocity obtained using nonuniform grid (42×42), %

y	SGSD	SUD	QUICK	CD
0.9766	1.1269	2.0628	0.8540	0.4216
0.9688	0.2870	2.0246	−0.4871	−0.7740
0.9609	1.9602	4.8359	1.5063	0.3286
0.9531	0.6974	4.8021	0.1073	−1.5663
0.8516	2.1528	7.7318	0.2913	−2.3510
0.7344	4.8506	8.1307	0.9723	−1.3675
0.6172	1.1750	3.3847	0.5963	−0.5086
0.5000	5.6743	15.9539	0.9868	−3.5197
0.4531	4.7614	13.3921	0.8922	−2.9676
0.2813	4.9379	12.7099	1.3954	−2.2621
0.1719	−1.1099	2.5490	−2.0266	−4.0194
0.1016	−3.6629	−5.3178	−1.4799	−0.6492
0.0703	−4.1269	−9.6804	0.3690	3.4833
0.0625	−4.0503	−10.8041	1.0843	4.8722
0.0547	−4.1084	−11.8449	1.4799	5.7706
Mean error	2.9788	7.6816	0.9393	2.3241

Table 4. Relative error of centerline v-velocity obtained using nonuniform grid (42×42), %

x	SGSD	SUD	QUICK	CD
0.9688	6.9945	0.3319	6.7140	9.1406
0.9609	6.2741	0.9397	6.2705	8.6305
0.9531	5.9233	0.1394	6.1191	8.4030
0.9453	3.3122	−1.0717	3.9119	5.6828
0.9063	−1.0766	1.0242	−0.7197	−0.38849
0.8594	0.7524	7.0385	−0.7828	−3.3798
0.8047	1.8425	7.3922	−0.2221	−2.5996
0.5000	9.1844	−0.7918	8.7886	12.3119
0.2344	2.6648	7.0451	1.0392	−1.0733
0.2266	2.5517	6.9387	1.0037	−1.1095
0.1563	−0.1105	4.1892	−1.0243	−3.0704
0.0938	−1.1953	3.0036	−1.3822	−3.1936
0.0781	−1.3705	2.7443	−1.3408	−3.0474
0.0703	−1.4338	2.6437	−1.3098	−2.9746
0.0625	−1.2370	2.8269	−1.0260	−2.6596
Mean error	3.0616	3.1517	2.7770	4.4854

Table 5. CPU time using the four schemes

	SGSD	SUD	QUICK	CD
Uniform grid	1	1.1121	0.7023	0.6018
Nonuniform grid	0.5025	0.5309	0.7139	0.7436

Table 6. Predicted reattachment lengths

Grid	SGSD	SUD	QUICK	CD
62×32	6.3233	6.3323	6.1947	Divergent
122×122	6.2042	6.2098	6.1838	6.1772

300, oscillation of velocity occurs, while the SGSD scheme exhibits robustness and provides reasonable solutions. In Figure 6, the solution from SGSD is presented where the velocity profile is developing from upstream to downstream in a reasonable manner. In Figure 7 the results from QUICK are shown. Some irregularity of the velocity profiles in the downstream part can be observed. Careful inspection of the numerical solution found that velocity oscillation in the u component occurs from the first control volume near the bottom wall. This oscillation is shown Figure 8, where the solution from SGSD is a smooth continuous curve, while that of QUICK exhibits severe bumpiness around $u = 0$.

4. CONCLUSIONS

A new stability-guaranteed second-order difference (SGSD) scheme is proposed in this article. The comparison among the SGSD, SUD, QUICK, and CD schemes for two benchmark problems reveals that the SGSD scheme is more robust than the QUICK scheme and its numerical accuracy is at least no less than that of the CD scheme and sometimes higher than that of the QUICK scheme. For the two cases compared, the CPU time for SGSD on uniform grids is less than SUD, but about 20–30% more than the QUICK and CD. However, in the nonuniform grid

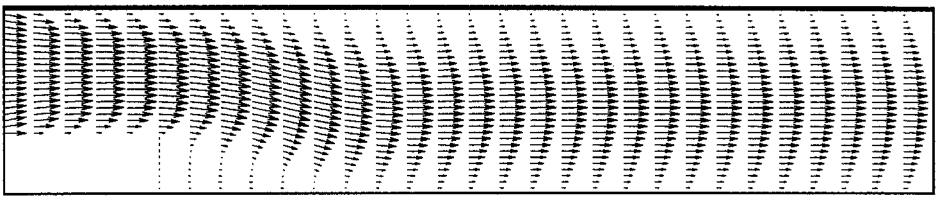


Figure 6. Velocity vectors obtained using SGSD scheme (Re = 300, Er = 1.5, 62×32 uniform grid).

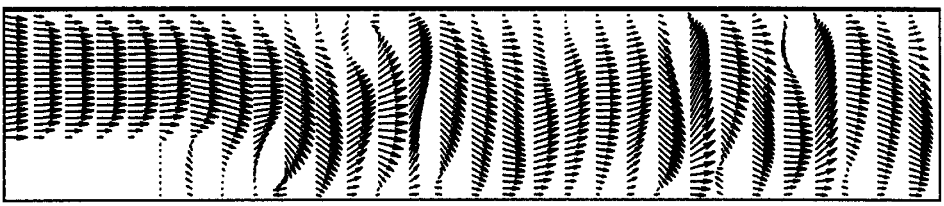


Figure 7. Velocity vectors obtained using QUICK scheme (Re = 300, Er = 1.5, 62×32 uniform grid).

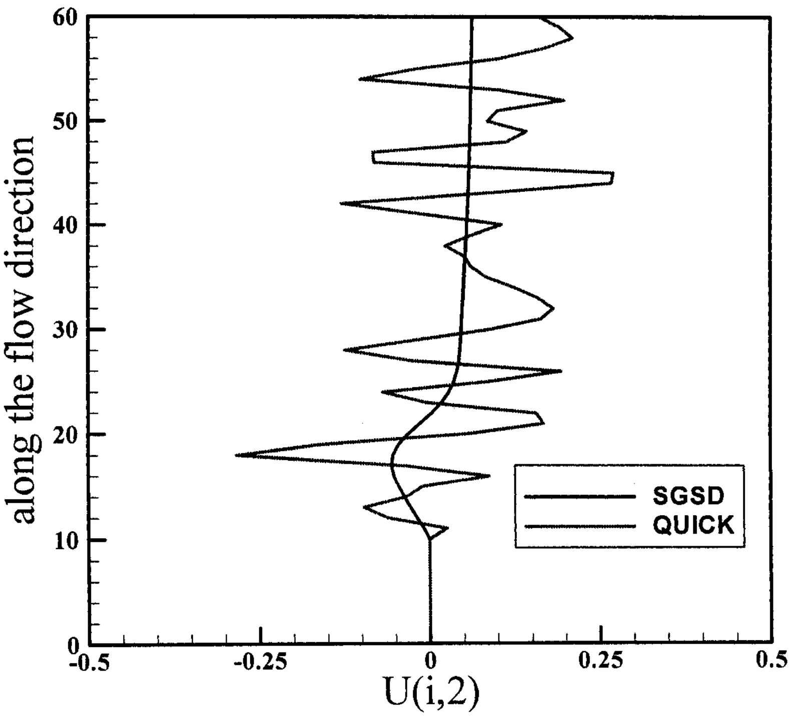


Figure 8. u-velocity profile at the first control-volume surface near the bottom wall (Re = 300, QUICK scheme, 62×32 uniform grid).

system the CPU time for SGSD is the shortest, being about 40–50% less than that of QUICK and CD. The perfect robustness and good efficiency make the scheme attractive in engineering computations for incompressible fluid flow and heat transfer problems.

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