Control of convergence in a computational fluid dynamic simulation using fuzzy logic

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Abstract A fuzzy control method was used to accelerate iteration convergence in numerical fluid dynamic simulation using SIMPLER algorithm. The residual ratio of momentum or energy equation between two successive iterations was used as the input variable. A fuzzy logic algorithm was developed in order to obtain the relative increment of the under-relaxation factor and its new value was then used for the next iteration. The algorithm was tested by four benchmark problems. In all cases considered, when the fuzzy control logic was used, convergence was achieved with nearly the minimum number of iterations, showing the feasibility of the proposed method.

Keywords: fuzzy control, relaxation factor, convergence, SIMPLER algorithm.

SIMPLE-like algorithm is widely used in the computational fluid dynamic (CFD) simulations. Because of the high nonlinearity, under-relaxation is often used to promote convergence. The optimum value of the under-relaxation factor, which can ensure convergence with the minimum iterations, is highly problem-dependent, and it cannot be known in advance. Actually the judgement itself of the variation trend of the iterative process possesses some fuzzy characteristics, and the convergence can be regarded as a fuzzy phenomenon. Therefore, a fuzzy logic method may be useful to control the iterative process.

Ryoo et al. developed a fuzzy algorithm for control of convergence in a CFD simulation[1]. They regarded residuals of momentum equations and energy equation as the best indicators of convergence. If the current residuals increase in magnitude, the relaxation factor should be reduced, and *vice versa*. The details of how to carry out the fuzzy logic reasoning were not explained in ref. [1]. In the present work, a fuzzy algorithm is developed using the reasoning method introduced in refs. [2, 3]. Meanwhile the membership functions of input in ref. [1] are revised for a better control. The algorithm is tested by four benchmark problems. Comparison of the iteration number needed for convergence using fuzzy control and using various fixed relaxation factors is presented.

1 Basic idea of fuzzy control

Fuzzy mathematics has been developed as a new branch of mathematics in the last twenty

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years. It is extensively applied in many fields of science and technology. Fuzzy control is its application in the control process. For complicated process, satisfactory results can be obtained by using fuzzy control technique.

The process of fuzzy logic control consists of three steps. As shown in fig. 1, the first step is to fuzzify the input using membership functions selected. The second is to implement fuzzy logic reasoning by fuzzy relation obtained from control rules and calculate the fuzzy set of the output. The last is to defuzzify it to get the definite value of the output.

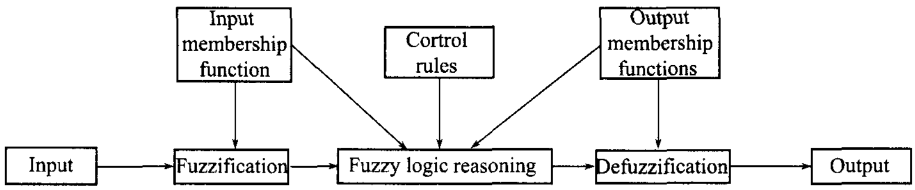

Fig. 1. Schematic of the fuzzy logic control.

2 Execution of a fuzzy control method

2.1 Selecting input variable and output variable.

In all cases, the flow is assumed to be Newtonian, laminar and two-dimensional (2D). Viscous dissipation is neglected. All thermophysical properties except density are assumed to be constant. The Boussinesq approximation is used for the natural convection. The conservation equations are discretized by the finite volume approach as

$$a_P\phi_P = \Sigma a_{nb}\phi_{nb} + b, \tag{1}$$

where ϕ_P is the value of the main grid point P for the variable under consideration, ϕ_{nb} are the values of the neighboring grid points, a_P is the coefficient for the main grid point, a_{nb} are the coefficients of neighboring grid points, and b is the source term.

After under-relaxation, the equation is transformed into

$$\phi_P^n = \phi_P^{n-1} + \alpha\left(\frac{\sum a_{nb}\phi_{nb}^{n-1} + b}{a_P} - \phi_P^{n-1}\right), \tag{2}$$

$$\frac{a_P}{\alpha}\phi_P^n = \sum a_{nb}\phi_{nb}^{n-1} + b + (1-\alpha)\frac{a_P}{\alpha}\phi_P^{n-1}, \tag{3}$$

where α is the relaxation factor and the superscript n, $n-1$ is the iterative time number.

The residual normalized by the coefficient a_P after n times of iterations is given by

$$d = \frac{\sum a_{nb}\phi_{nb}^n + b - a_P\phi_P^n}{a_P}. \tag{4}$$

The square norm of the residual is defined as

$$\| d \| = \sqrt{\sum d^2}. \tag{5}$$

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The input variable is taken as

$$e = \frac{\| d \|_n}{\| d \|_{n-1}}. \tag{6}$$

The output variable is the relative increment in relaxation factor $\Delta\alpha$. After defuzzification, the relaxation factor is updated by

$$\alpha_{n+1} = (1 + \Delta\alpha)\alpha_n. \tag{7}$$

2.2 Membership functions and control rules

A fuzzy set is featured by its membership function. The computation of fuzzy sets is realized through the computation of their membership functions. A general set is described by its characteristic function, whose value is either 0 or 1. When an element belongs to the set, the function value is 1. Otherwise, the value is 0. The membership function of a fuzzy set is regarded as the extension of the characteristic function of a general set, in that the value of a membership function can be anyone between 0 and 1. The specific form of the membership function is highly problem-dependent, and cannot be determined in advance. It can only be determined in the numerical practice. The criterion for judging a good membership function is to see whether it can result in a good control.

The input variable e and the output variable $\Delta\alpha$ are fuzzy variables. The variable e possesses three values: positive big (PB), positive medium (PM) and positive small (PS). The variable $\Delta\alpha$ also has three values: negative big (NB), negative small (NS) and positive small (PS). (PS). Triangle and trapezoidal shape membership functions are adopted for the fuzzy sets in this work. They can be generalized by a trapezoidal model as shown in fig. 2. The ordinate denoted by μ is the membership value, while the abscissa represents the elements of the set that possess some characteristics in common. The values of parameters a, b, c and d for each function are given in table 1. They are determined by the numerical computation practices. With table 1, we can determine to what degree an element belongs to the fuzzy set (membership-degree). For example, for input variable e, the membership of the element 2 to PS is zero, to PM is 1 and to PB is 0.5.

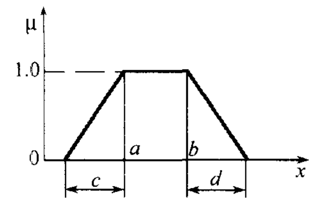

Fig. 2. Trapezoidal membership function model

Table 1 Membership functions for input and output

Membership functions for input				Membership functions for output			
e	PS	PM	PB	$\Delta\alpha$	NB	NS	PS
a	0	2.0	2.5	a	−0.4	−0.25	0.2
b	0	2.0	∞	b	−0.4	−0.25	0.2
c	0	1.0	1.0	c	0	0	0.4
d	1.0	0.5	0	d	0.5	0.5	0

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Table 2 Fuzzy control rules

e	PB	PM	PS
$\Delta\alpha$	NB	NS	PS

Three control rules are given in table 2. They can also be presented by if-statement. For example, the first rule means that if the input variable is PB, the output is NB.

2.3 Implementation of fuzzy logic algorithm

The control mode adopted is the one of the single input and the single output. The method of fuzzy reasoning introduced in ref. [3] is used. As indicated above, the fuzzy control algorithm consists of three steps. The first is to fuzzify the input into a fuzzy set. The second is to calculate fuzzy logic relation determined by control rules and get the fuzzy set of the output. The last is to defuzzify it to get the definite value of the output.

A given residual ratio can be considered as a special fuzzy set which is named as fuzzy single point. Fuzzy single point is such a fuzzy set that in which there is only one element with positive membership function value. Represented in a membership function figure, it is a one-unit-long vertical line through the point. This is the procedure for the fuzzification of the input variable. The resultant value of e is the fuzzy single point A.

The three control rules are rewritten as follows:

$$R_1 \text{ :if } e = A_1, \text{ then } \Delta\alpha = B_1;$$

$$R_2 \text{ :if } e = A_2, \text{ then } \Delta\alpha = B_2;$$

$$R_1 \text{ :if } e = A_3, \text{ then } \Delta\alpha = B_3;$$

where the fuzzy sets $A_1 \sim A_3$ represent PB, PM, and PS and $B_1 - B_3$ represent NB,NS, and PS.

All the fuzzy rules comprise the fuzzy relation R:

$$R = \bigcup_{i=1}^{3} R_i, \tag{8}$$

$$R_i = A_i \times B_i, \tag{9}$$

where R_i represents the fuzzy relation implied in the ith rule, symbol $\cup$ stands for the dyadic production, and $A_i \times B_i$ is the direct production of two fuzzy sets.

The output is determined by

$$B = A \circ R,$$

where $\circ$ is the synthesizing operator, which is one rule of the fuzzy reasoning.

The output of the fuzzy reasoning process, B, is a fuzzy set. Its membership function is denoted by $\mu_B(x)$. To get an exact value, it is necessary to defuzzify the output.

A center of gravity method is used for defuzzification,

$$x^* = \frac{\int x\mu_B(x)\mathrm{d}x}{\int \mu_B(x)\mathrm{d}x}, \tag{10}$$

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where $\mu_B(x)$ is the membership function of the fuzzy set of output, x is the abscissa of the membership function figure, and x^* is the relative increment of relaxation-factor $\Delta\alpha$.

The above fuzzy logic algorithm is employed to adjust the relaxation factor during iterations in implementing SIMPLER algorithm[4]. On every iteration, residuals for the momentum equations and energy equation are calculated. Then the module for the fuzzy control algorithm is run and the change ratio for the relaxation factors is obtained. The relaxation factors for velocities and temperature are then updated for the next iteration.

3 Examples and results

3.1 Application examples

The above fuzzy logic algorithm is tested by four benchmark problems using SIMPLER algorithm. Different fixed relaxation factors are also tested for comparison .The convergence criterion adopted is as follows:

$$\frac{R_{\text{sum}}}{q_m} \leqslant 1\times10^{-7}, \qquad \frac{R_{\max}}{q_m} \leqslant 1\times10^{-7}, \tag{11}$$

where R_{sum} is the sum of the residuals of all the internal grid points, $R_{\max}$ is the maximum of absolute values of the residuals, and q_m is the referenced flux. In the open system, q_m is the inlet flow rate; in the close system, such as the lid driven flow as shown in fig. 4, it is given by

$$q_m = \int_a^b \rho|u|dy . \tag{12}$$

It should be noted that in eq. (12) the absolute value of the velocity is used to guarantee the non-zero result of q_m for whatever case studied.

Case 1. The natural convection in a square cavity (fig. 3). The cavity is closed and filled with air. The left and right walls keep constant but different temperatures, and the top and bottom walls are adiabatic. The computations are conducted for Pr = 0.71 and Ra=10^3—10^7 with 20×20 control volumes. Rayleigh number, Ra, is defined as $Ra = \rho^2 g\beta\Delta TL^3 Pr/\eta^2$.

Case 2. The flow in lid-driven cavity flow, which is a cavity with a lid moving horizontally with constant velocity U (fig. 4). The simulations are carried out for Re=100 and 1000 with 20×20 control volumes.

Case 3. The flow in a tube with a sudden expansion (fig.5). The inlet flow temperature is lower than the wall's which is constant. The simulations are conducted for Re=100 and 1000 with 40×10 control volumes .

Case 4. The natural convection in a horizontal annulus (fig. 6). The walls of inner and outer cylinders are isothermal with inner surface being hot and outer one cold. The computations are carried out for Ra=10^4 and 10^5 with 20×30 control volumes.

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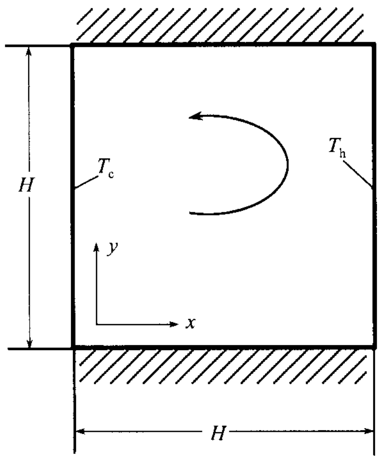

Fig. 3. The natural convection in a square cavity.

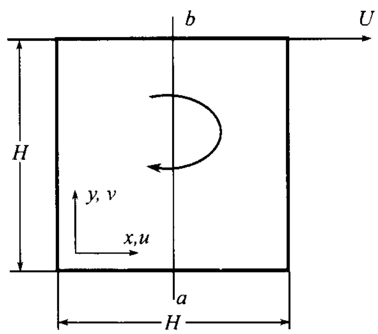

Fig. 4. A lid-driven flow in a square cavity.

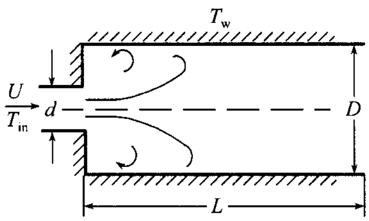

Fig. 5. The flow in a tube with a sudden expansion.

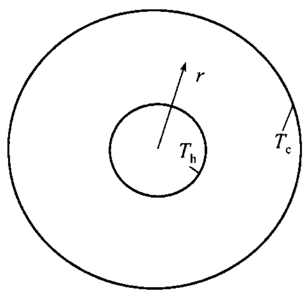

Fig. 6. The natural convection in a horizontal annulus.

3.2 Comparison of iterations number

For all the cases studied, the fuzzy control algorithm achieves convergence, while for the fixed value of relaxation factor, divergence occurs for some cases. A summary of the number of iterations required for convergence using fuzzy control is compared to that using various fixed relaxation factors (table 3). The comparison results are also presented in figs. 7—10 for intuitive inspection.

In all the cases studied, the fuzzy control algorithm can reach convergence with iteration number very close to the minimum number, showing great efficiency of the fuzzy control method.

In fig. 11, the convergence history of the case ($Ra=10^6$) is presented. As can be seen, the value of relaxation factor is reduced quickly at the beginning of the iterative process and then increased gradually followed by an up-and-down variation pattern during the whole iterative process. Such a kind of convergence history is typical in for all the cases studied.

4 Conclusion

For laminar, 2D fluid flow and heat transfer problems in three orthogonal coordinate

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Table 3 Comparison of the iterations number required for convergence

Natural convection in a square cavity (Cartesian coordinate system)								
Ra	Values of relaxation factor α							fuzzy control
	0.1	0.3	0.5	0.7	0.9	0.95	0.98	
10^3	2366	1433	831	440	147	80	308	162
10^4	2707	1330	721	368	118	63	238	169
10^5	2948	1163	597	295	93	Osc	Osc	221
10^6	2623	944	472	230	Osc	Osc	Div	238
10^7	3207	1133	565	Osc	Div	Div	Div	506
The lid-driven flow in a square cavity (Cartesian coordinate system)								
Re	values of relaxation factor α							fuzzy control
	0.1	0.3	0.5	0.7	0.9	0.95	0.98	
10^2	799	389	215	115	46	32	73	77
10^3	997	403	209	106	56	55	Div	77
The flow in a tube with a sudden expansion (Cylindrical coordinate system)								
Re	values of relaxation factor α							fuzzy control
	0.1	0.3	0.5	0.7	0.9	0.94	0.98	
10^2	801	244	118	60	27	58	Osc	81
10^3	1450	572	322	174	81	68	838	97
The natural convection in a horizontal annulus (polar coordinate system)								
Ra	values of relaxation factor α							fuzzy control
	0.1	0.3	0.5	0.7	0.9	0.95	0.98	
10^4	3756	2018	1130	590	195	Div	Div	646
10^5	5469	2226	1149	566	186	Div	Div	647

Osc: oscillation; Div: divergence.

systems, fast and stable convergence could be achieved by using fuzzy control algorithm during the iterative process of implementing SIMPLER algorithm. The fuzzy logic control

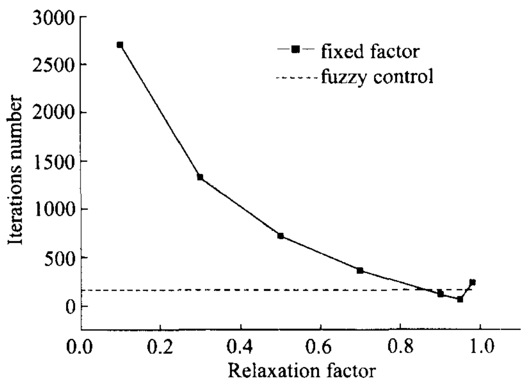

Fig. 7. Number of iterations needed for convergence of the natural convection in a square cavity at $Ra=10^4$.

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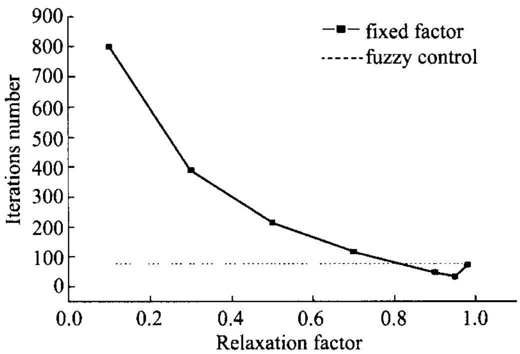

Fig. 8. Number of iterations needed for convergence of the lid-driven flow in a square cavity at $Re=10^2$.

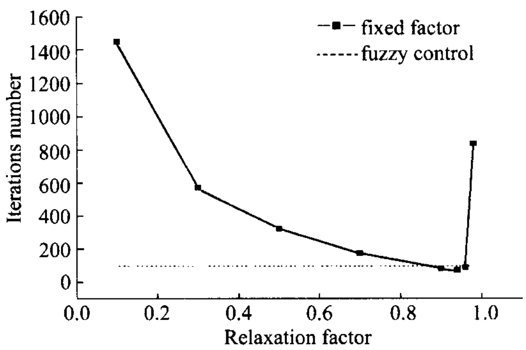

Fig. 9. Number of iterations needed for convergence of the flow in a tube with a sudden expansion at $Re=10^3$.

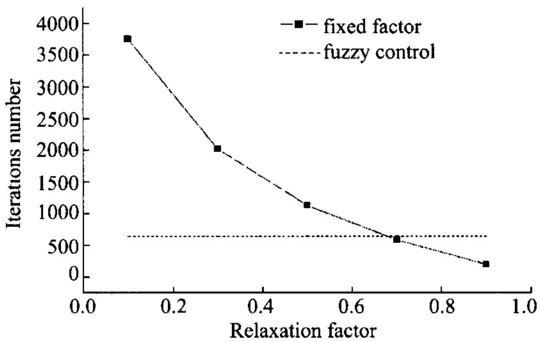

Fig. 10. Number of iterations needed for convergence of the natural convection in horizontal annulus at $Ra=10^3$.

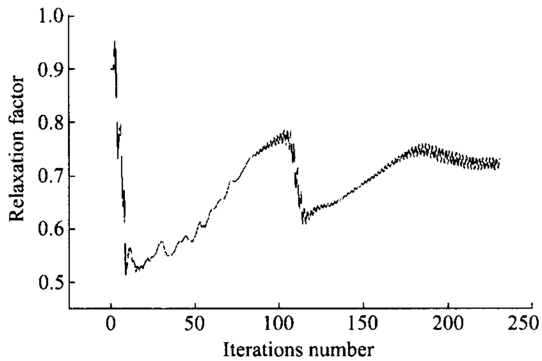

Fig. 11. Variation of the value of relaxation factor versus iterations number of case 1 at $Ra=10^6$.

not only accelerates the convergence process, but also enhances the robustness of the iteration procedure, such as the iteration of the natural convection in a square cavity for $Ra=10^7$.

It is worth noting that the fuzzy logic algorithm adopted in this paper has been tested only by 2D laminar problems. Extension to 3D and turbulent flows is necessary and is underway in the authors' group.

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References

1. Ryoo, J., Kaminski, D., Dragojlovic, Z., A residual-based fuzzy logic algorithm for control of convergence in a computational fluid dynamic simulation, ASME Journal of Heat Transfer, 1999, 121: 1076—1078.
2. Zhang, N. Y., Yan, P. F., Neural Net and Fuzzy Control (in Chinese), Beijing: Tsinghua University Press, 1998, 189—193.
3. Li, Y. S., Li J., Fuzzy Control Theory and the Application for the Process (in Chinese), Beijing: Press of National Defense Industry, 1993, 56—74.
4. Tao, W. Q., Numerical Heat Transfer (in Chinese), 2nd ed., Xi'an: Xi'an Jiaotong University Press, 2000, 1—5.

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