

Numerical Heat Transfer

Chapter 3 Numerical Methods for Solving Diffusion Equation (2)



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3.4 TDMA & ADI Methods for Solving ABEqs

3.4.1 TDMA algorithm (算法) for 1-D conduction problem

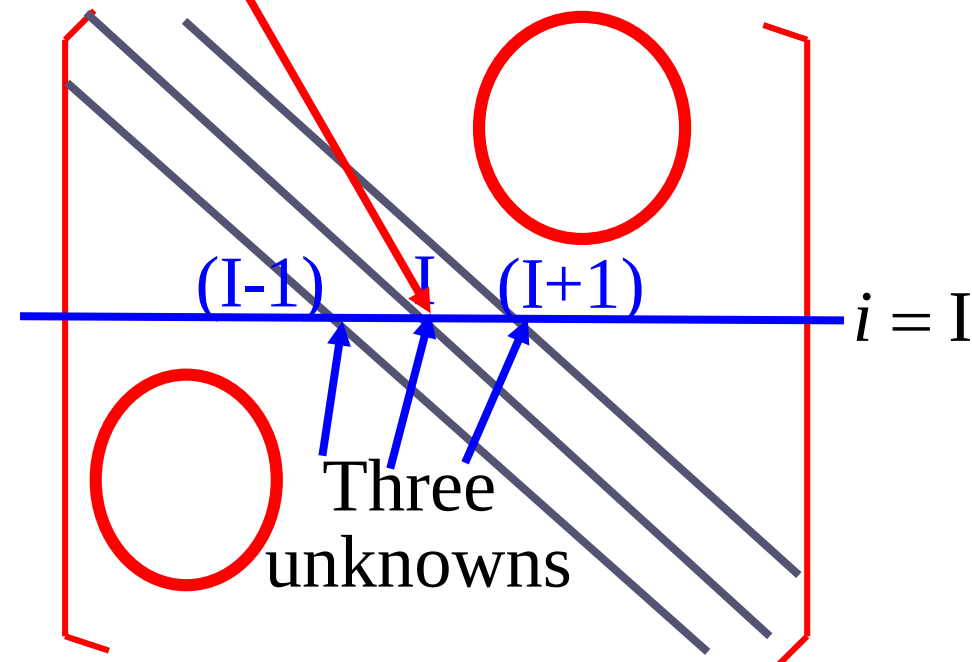
1. General form of algebraic equations of 1-D conduction problems

The ABEqs for steady and unsteady ($f > 0$) problems take the following form

$$a_P T_P = a_E T_E + a_W T_W + b$$

The matrix (矩阵) of the coefficients is a tri-diagonal (三对角) one.

$$a_1 T_1 + a_2 T_2 + \dots + a_i T_i + \dots + a_{M1} T_{M1} = b \quad (i = 1, M1)$$



2. Thomas algorithm(算法)

The numbering method of W-P-E is humanized (人性化), but it can not be accepted by a computer!

Rewrite above equation:

$$A_i T_i = B_i T_{i+1} + C_i T_{i-1} + D_i, i = 1, 2, \dots, M1 \quad (a)$$

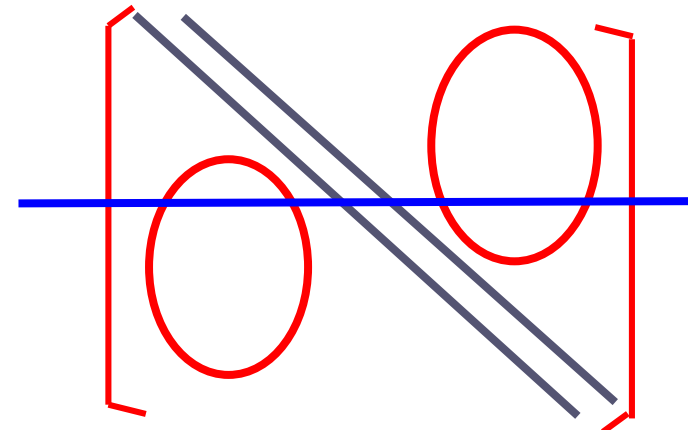
End conditions: $i=1, C_i=C_1=0; i=M1, B_i=B_{M1}=0$

(1) Elimination (消元) – Reducing the unknowns at each line from 3 to 2

Assuming the eq. after elimination as

$$T_{i-1} = P_{i-1} T_i + Q_{i-1} \quad (b)$$

Coefficient has been treated to 1.



The purpose of the elimination procedure is to find the relationships between P_i , Q_i with A_i , B_i , C_i , D_i :

Multiplying Eq.(b) by C_i , and adding to Eq.(a):

$$A_i T_i = B_i T_{i+1} + \cancel{C_i T_{i-1}} + D_i \quad (a)$$

$$\cancel{C_i T_{i-1}} = C_i P_{i-1} T_i + C_i Q_{i-1} \quad (b)$$

$$A_i T_i - C_i P_{i-1} T_i = B_i T_{i+1} + D_i + C_i Q_{i-1}$$

Yielding
$$T_i = \left(\frac{B_i}{A_i - C_i P_{i-1}} \right) T_{i+1} + \frac{D_i + C_i Q_{i-1}}{A_i - C_i P_{i-1}}$$

Comparing with $T_{i-1} = \underbrace{P_{i-1} T_i}_{\downarrow} + \underbrace{Q_{i-1}}_{\downarrow}$

$$P_i = \frac{B_i}{A_i - C_i P_{i-1}}; \quad Q_i = \frac{D_i + C_i Q_{i-1}}{A_i - C_i P_{i-1}};$$

The above equations are **recursive (递归的)**—i.e.,

In order to get P_i , Q_i , P_1 and Q_1 must be known.

In order to get P_1 , Q_1 , use Eq.(a)

$$A_i T_i = B_i T_{i+1} + C_i T_{i-1} + D_i, \quad i = 1, 2, \dots, M-1 \quad (a)$$

and the left end condition: $i=1, C_i=0$

Applying Eq.(a) to $i=1$, and comparing it with Eq.(b)

$$T_{i-1} = P_{i-1} T_i + Q_{i-1}$$

the expressions of P_1 , Q_1 can be obtained:

From $i = 1, C_1 = 0$, Eq.(a) becomes: $A_1 T_1 = B_1 T_2 + D_1$

$$T_1 = \frac{B_1}{A_1} T_2 + \frac{D_1}{A_1} \longrightarrow P_1 = \frac{B_1}{A_1}; \quad Q_1 = \frac{D_1}{A_1}$$

(2) Back substitution(回代) – Starting from M1 via Eq.(b) to get T_i sequentially (顺序地)

$$T_{M1} = P_{M1} T_{M1+1} + Q_{M1}, \quad P_i = \frac{B_i}{A_i - C_i P_{i-1}};$$

End condition:
 $i = M1, B_i = 0$

$$\longrightarrow P_{M1} = 0 \longrightarrow T_{M1} = Q_{M1}$$

$$T_{M1} = Q_{M1} \xrightarrow{\quad} T_{i-1} = P_{i-1} T_i + Q_{i-1} \text{ to get: } T_{M1-1}, \dots, T_2, T_1.$$

3. Implementation of Thomas algorithm for 1st kind B.C.

For 1st kind B.C., the solution region is from $i=2$...to $M_{1-1}=M_2$, because T_1 and T_{M1} are known.

Applying Eq.(b) to $i=1$ with given $T_{1,given}$:

$$T_1 = P_1 T_2 + Q_1 \longrightarrow P_1 = 0; Q_1 = T_{1,given}$$

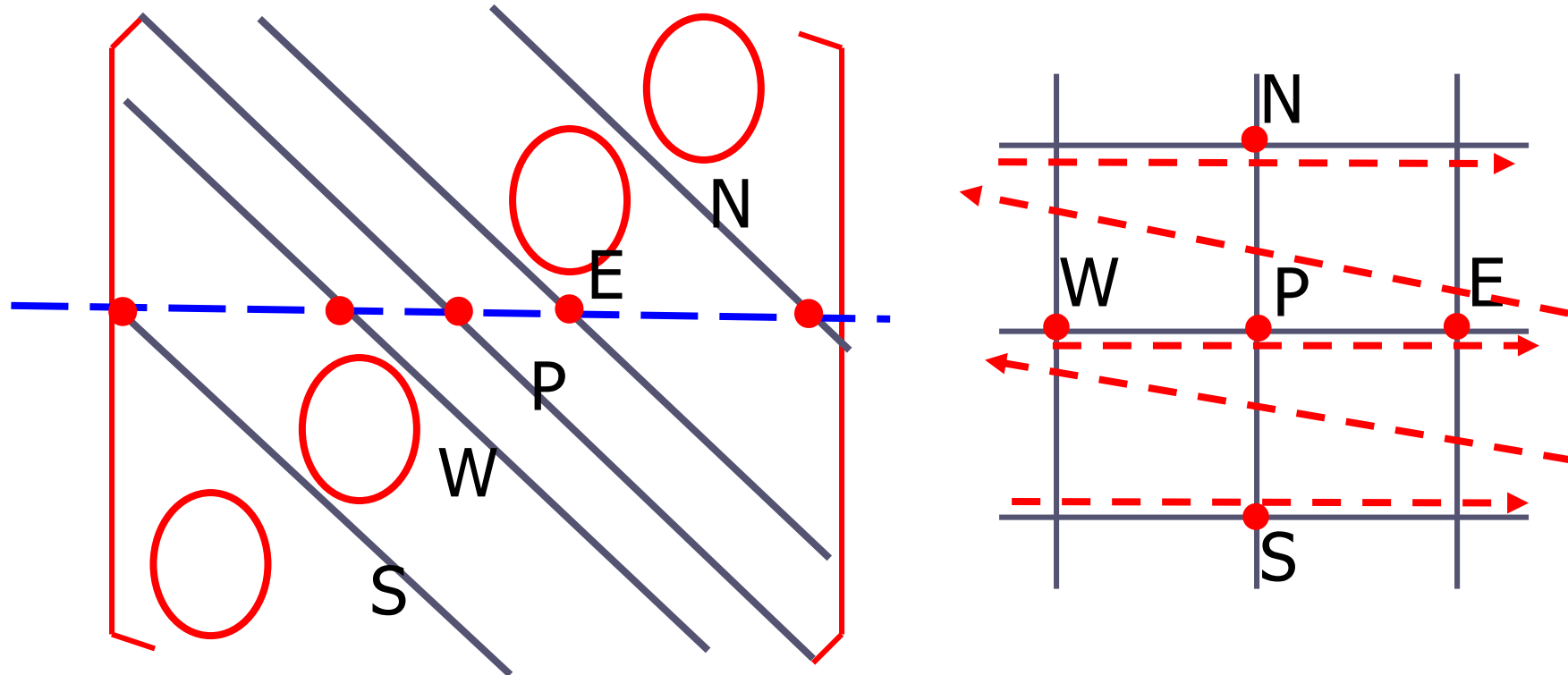
Because T_{M1} is known, back substitution should be started from M_2 :

$$T_{M2} = P_{M2} T_{M1} + Q_2$$

When the ASTM is adopted to deal with B.C. of the 2nd and 3rd kind, **the numerical B.C. for all cases is regarded as 1st kind**, and the above treatment should be adopted.

3.4.2 ADI method for solving multi-dimensional problem

1. Introduction to the matrix of 2-D problem



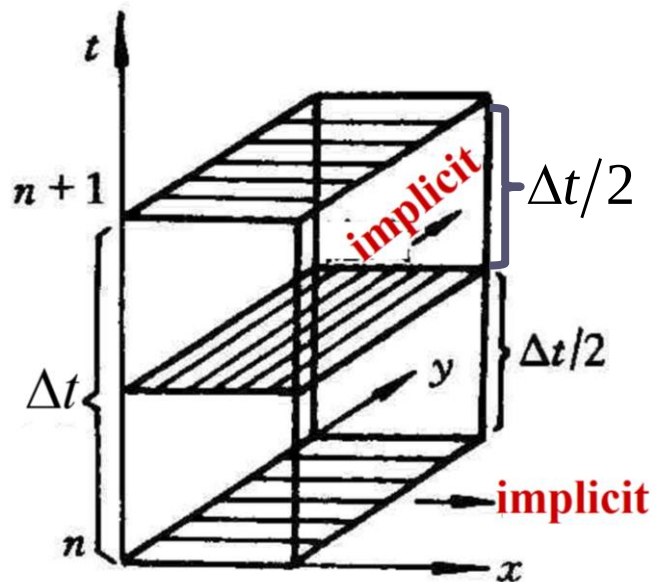
1-D storage (一维存储) of variables and its relation to matrix coefficients

Numerical methods for solving ABEqs. of 2-D problems.

- (1) Penta-diagonal algorithm(PDMA,五对角阵算法)
- (2) Alternative (交替的)-direction implicit (ADI, 交替方向隐式方法)

Iteration process is very similar to solving an unsteady problem: finishing one iteration is corresponding to go one time step forward!

2. 2-D Peaceman-Rachford ADI method



Dividing Δt into two uniform parts;

In the 1st $\Delta t / 2$ implicit in x direction,
and explicit in y direction;

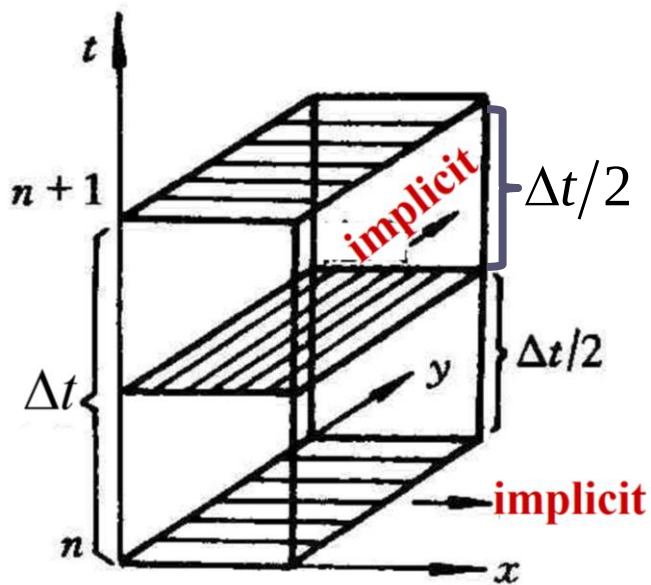
In the 2nd $\Delta t / 2$ implicit in y direction,
and explicit in x direction.

Set $u_{i,j}$ the temporary(临时的) solutions at the first sub-time levels

$\delta_x^2 T_{i,j}^n$ ---CD scheme for 2nd derivative at n time level in x direction

$$\delta_x^2 T_{i,j}^n = \frac{T_{i+1,j}^n - 2T_{i,j}^n + T_{i-1,j}^n}{\Delta x^2}$$

2-D ADI



1st sub-time level: $\frac{u_{i,j} - T_{i,j}^n}{\Delta t / 2} = a(\delta_x^2 u_{i,j} + \delta_y^2 T_{i,j}^n)$

The solution of $u_{i,j}$ can be obtained by TDMA by taking $\delta_y^2 T_{i,j}^n$ as b-term with known values at n time level

2nd sub-time level: $\frac{T_{i,j}^{n+1} - u_{i,j}^n}{\Delta t / 2} = a(\delta_x^2 u_{i,j} + \delta_y^2 T_{i,j}^{n+1})$

$T_{i,j}^{n+1}$ is solved by TDMA and is the solution at time level of (n+1).

3. 3-D Peaceman-Rachford ADI method

Dividing Δt into three uniform parts; In the 1st $\Delta t / 3$ implicit in x , and explicit in y, z directions; In the 2nd and 3rd $\Delta t / 3$ implicit in y ,z direction, and explicit in x, z directions and x,y , respectively; Set $u_{i,j,k}$, $v_{i,j,k}$ the temporary(临时的) solutions at two sub-time levels

$$\text{1st sub-time level: } \frac{u_{i,j,k} - T_{i,j,k}^n}{\Delta t / 3} = a(\delta_x^2 u_{i,j,k} + \delta_y^2 T_{i,j,k}^n + \delta_z^2 T_{i,j,k}^n)$$

$$\text{2nd sub-time level: } \frac{v_{i,j,k} - u_{i,j,k}^n}{\Delta t / 3} = a(\delta_x^2 u_{i,j,k} + \delta_y^2 v_{i,j,k} + \delta_z^2 u_{i,j,k})$$

$$\text{3rd sub-time level: } \frac{T_{i,j,k}^{n+1} - v_{i,j,k}^n}{\Delta t / 3} = a(\delta_x^2 v_{i,j,k} + \delta_y^2 v_{i,j,k}^n + \delta_z^2 T_{i,j,k}^{n+1})$$

The algebraic equations of 3D transient HC problem

is updated for one time step by such ADI method:
adopting TDMA three times in x,y,z direction respectively.

It's obvious that this solution procedure is not fully implicit, and for 3D case the time step is limited by following stability condition:

$$a\Delta t\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}\right) \leq 1.5$$

If the time step is larger than the value specified by the above eq., the resulted numerical solutions will be oscillating (振荡) . We call that the solution procedure is not stable .

More discussion on the numerical stability will be presented in Chapters 4 and 7.

Major numerical methods (concepts) introduced in this chapter

1. Fully implicit scheme (全隐格式) of transient problem, which can guarantee (保证) stable and physically meaningful numerical solution;
2. Harmonic mean (调和平均) for determination of interface conductivity;
3. Unified coefficient expression by introducing a scaling factor and a nominal radius;
$$\frac{(\delta x)_e}{\lambda_e} = \frac{(\delta x)_{e^+}}{\lambda_E} + \frac{(\delta x)_{e^-}}{\lambda_P}$$
4. Linearization of source term (源项线性化) by $S = S_C + S_P \phi_P$, $S_P \leq 0$;
5. Additional source term method (ASTM, 附加源项法) for treating 2nd and 3rd kinds of boundary conditions;
6. TDMA (三对角矩阵算法) for solving algebraic equation;
7. General expression of discretized heat conduction eq.

$$a_P T_P = a_E T_E + a_W T_W + b = \sum a_{nb} T_{nb} + b \quad \text{Physical meanings of } a_E, a_W:$$

Reciprocal of thermal resistance between two points, thermal conductance.

3.5 Fully Developed Heat Transfer(FDHT) in Circular Tubes

3.5.1 Introduction to FDHT in tubes and ducts

3.5.2 Physical and mathematical models

3.5.3 Governing equations and their non dimensional forms

3.5.4 Conditions for unique solution

3.5.5 Numerical solution method

3.5.6 Treatment of numerical results

3.5.7 Discussion on numerical results

3.5 Fully Developed HT in Circular Tubes

3.5.1 Introduction to FDHT in tubes and ducts

1. Simple fully developed heat transfer

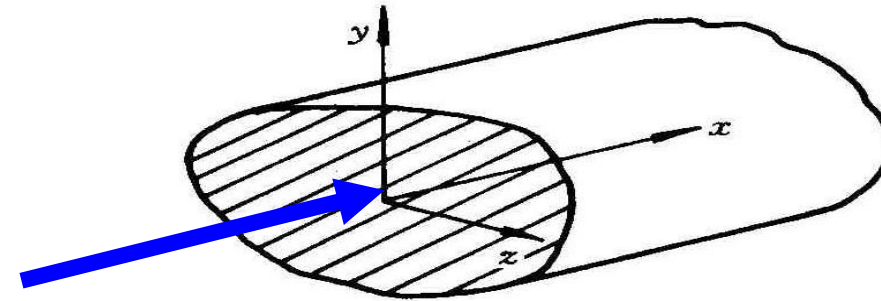
Physically: Velocity components normal to flow direction equal zero; Fluid **dimensionless** temperature distribution is independent on (无关) the position in the flow direction

Mathematically: Both dimensionless momentum and energy equations are of **diffusion type**.

Present chapter is limited to the simple cases.

FDHT in straight duct is an example of simple cases.

$$\frac{\partial}{\partial x} \left(\frac{T_{w,m} - T}{T_{w,m} - T_b} \right) = 0$$



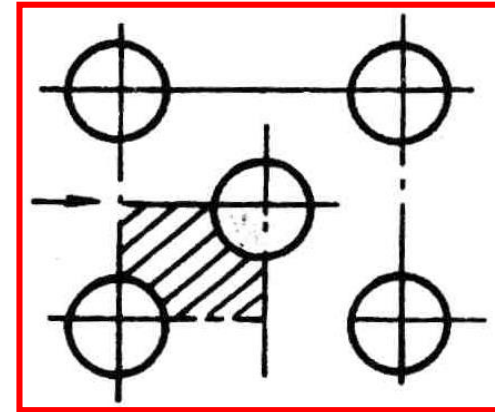
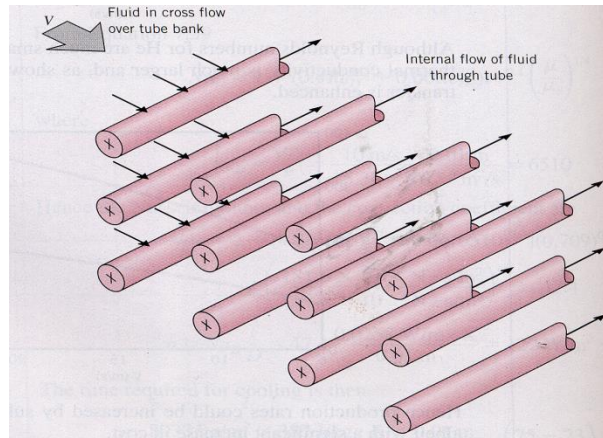
where $T_{w,m}$ is the circumferential (圆周的) average wall temperature, and T_b is the fluid cross-sectional (截面的) average temperature.

2. Complicated FDHT

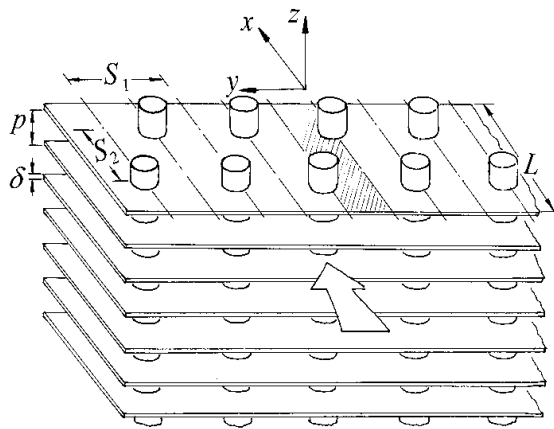
In the cross section normal to flow direction there exist velocity components, and the dimensionless temperature depends on the axial position, often exhibits (呈现) periodic (周期的) character. The full Navier-Stokes equations must be solved.

This subject is discussed in Chapter 11 of the textbook.

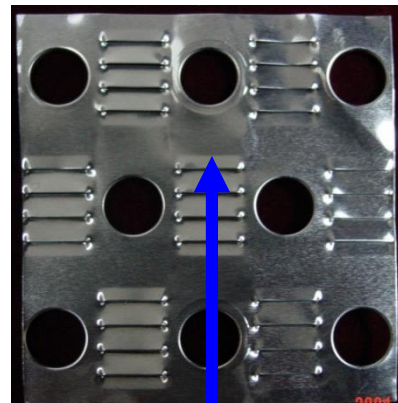
Examples of complicated FDHT



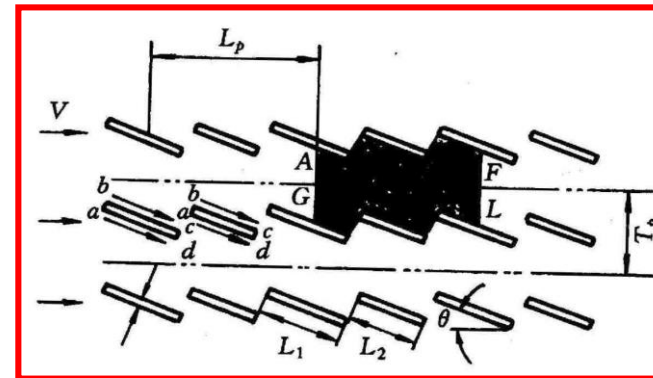
Tube bundle (bank) (管束)



Fin-and-tube
heat exchanger

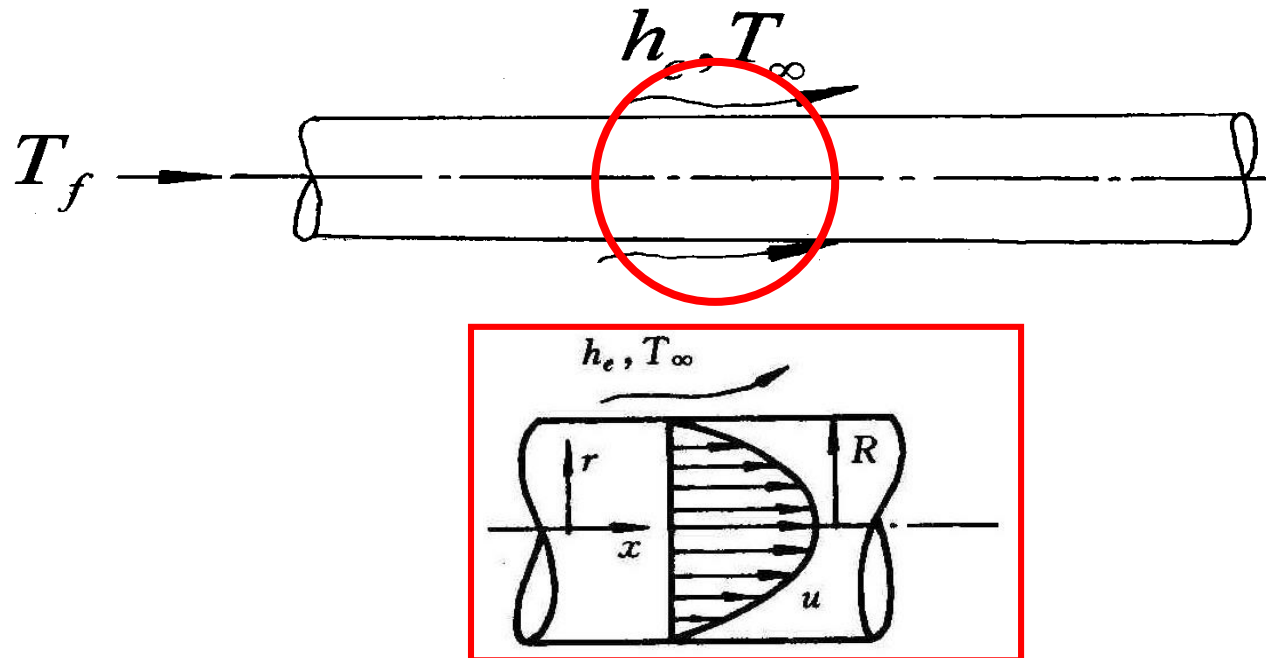


Louver fin (百叶窗翅片)



3.5.2 Physical and mathematical models of FDHT in circular tube

A laminar flow in a long tube is cooled (heated) by an external fluid with temperature T_∞ and heat transfer coefficient h_e . Determine the in-tube heat transfer coefficient and Nusselt number in the FDHT region.



1. Simplification (简化) assumptions

- (1) Thermo-physical properties (热物性) are constant ;
- (2) Axial (轴向的) heat conduction in the fluid is neglected;
- (3) Viscous dissipation (耗散) is neglected;
- (4) Natural convection (自然对流) is neglected;
- (5) Tube wall thermal resistance is neglected;
- (6) The flow in tube is steady , laminar and fully developed:

$$\frac{u}{u_m} = 2\left[1 - \left(\frac{r}{R}\right)^2\right]; \quad v = 0, u_m - \text{Mean velocity}$$

2. Mathematical formulation (描述)

(1) Energy equation

Cylindrical coordinate, symmetric temperature distribution, no natural convection (A4) and steady (A6):

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left(\lambda r \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) + s_T$$

FD flow
(A6)

No axial
cond. (A2)

No dissipation
(A3)

$$\rho c_p u \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left(\lambda r \frac{\partial T}{\partial r} \right)$$

Mathematically, what type is this eq.?

2-D parabolic eq.!

(2) Boundary condition

$$r = 0, \frac{\partial T}{\partial r} = 0 \quad (\text{Symmetric condition}) ;$$

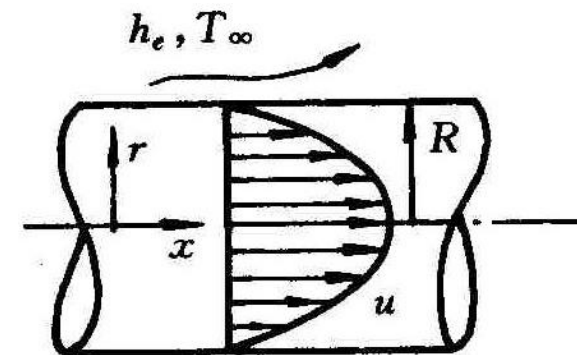
$$r = R, -\lambda \frac{\partial T}{\partial r} = h_e (T - T_\infty)$$

(External convective condition!)

Internal fluid thermal conductivity

External (外部) convective heat transfer coefficient (given)

No wall thermal resistance (A5), equivalent to wall thickness equals zero, tube outer radius = tube inner radius = R



3.5.3 Governing eqs. and dimensionless forms

From fully developed condition a dimensionless temperature can be introduced, transforming the PDE to ordinary differential eq..

Defining $\Theta = \frac{T - T_\infty}{T_b - T_\infty}$

Annotations:
 - "Given temp." points to T_∞ in the numerator.
 - "Cross section average temp." points to T_b in the denominator.
 - Red arrows indicate the transformation from the general form $\frac{T - T}{T_b - T}$ to the specific form $\frac{T - T_\infty}{T_b - T_\infty}$.

Then: $T = \Theta(T_b - T_\infty) + T_\infty$; $\frac{\partial T}{\partial x} = \Theta \frac{\partial T_b}{\partial x} = \Theta \frac{dT_b}{dx}$

Defining two dimensionless spatial coordinates:

$\eta = \frac{r}{R}$; $X = \frac{x}{R \bullet Pe}$ $Pe = \frac{2R \rho c_p u_m}{\lambda} = \frac{2Ru_m}{a}$

Annotations:
 - A red arrow points to Pe in the denominator of X .
 - A blue dashed box encloses $\rho c_p u_m$ in the numerator of Pe .
 - A red arrow points to a in the denominator of the second form of Pe .

Constant properties (A1)

Thermal diffusivity

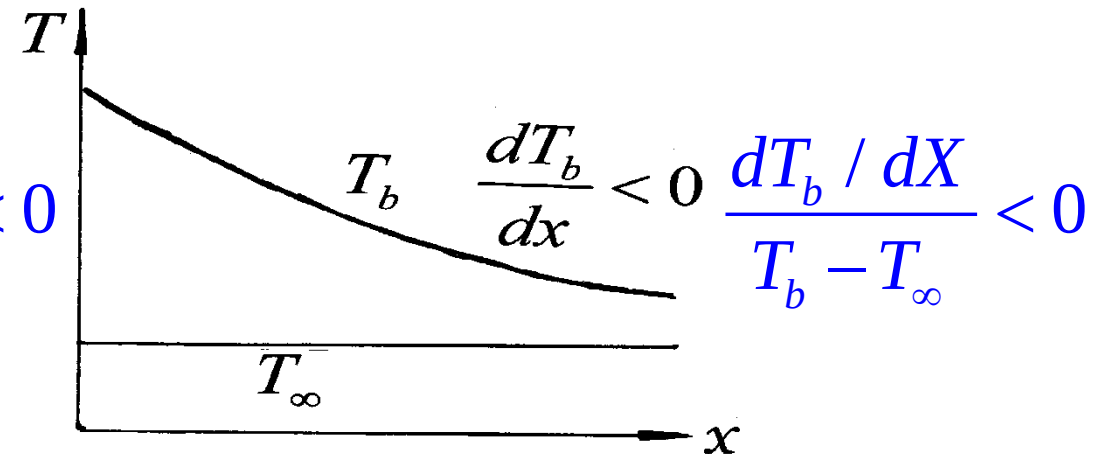
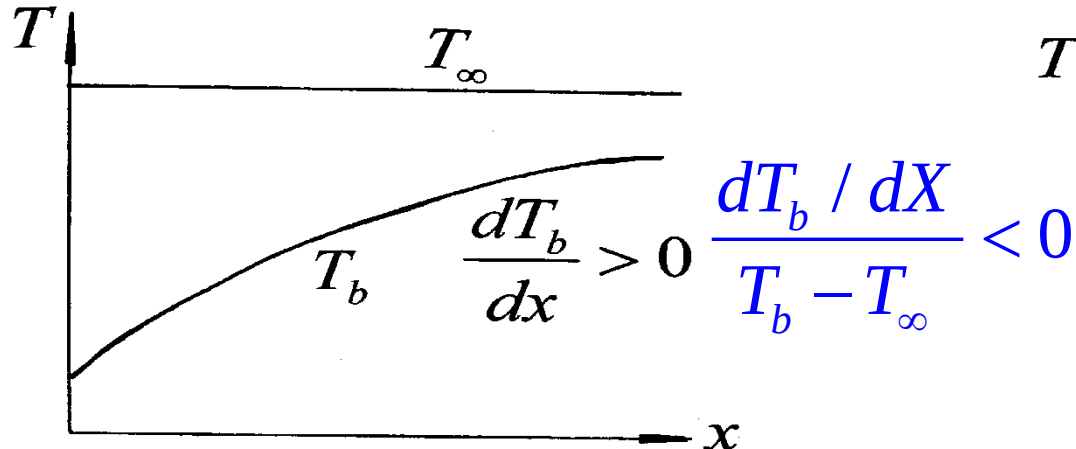
热扩散率

Energy eq. can be rewritten as:

$$\frac{dT_b / dX}{T_b - T_\infty} = \frac{1}{\eta} \frac{d}{d\eta} \left(\eta \frac{d\Theta}{d\eta} \right) / \left(\frac{1}{2} \Theta \frac{u}{u_m} \right) = -\Lambda \quad \boxed{\Lambda > 0}$$

Dependent on X only

Dependent on η only



Λ is called **eigenvalue** (特征值)

Following ordinary differential equation for the dimensionless temperature can be obtained

$$\frac{1}{\eta} \frac{d}{d\eta} \left(\eta \frac{d\Theta}{d\eta} \right) / \left(\frac{1}{2} \Theta \frac{u}{u_m} \right) = -\Lambda \quad (a)$$

The inner B.C is transformed (转换成) into: $\eta = 0, \frac{d\Theta}{d\eta} = 0$ (b)

The outer B.C $r = R, -\lambda \frac{\partial T}{\partial r} = h_e (T - T_\infty)$ is transformed into:

$$\eta = 1, -\frac{d\left(\frac{T - T_\infty}{T_b - T_\infty}\right)}{d\left(\frac{r}{R}\right)} = \left(\frac{h_e R}{\lambda}\right) \frac{T - T_\infty}{T_b - T_\infty} \longrightarrow \frac{d\Theta}{d\eta} \Big|_{\eta=1} = -Bi \Theta_w \quad (c)$$

Question: whether from Eqs. (a)-(c) a unique (唯一的) solution can be obtained?

3.5.4 Analysis of condition for unique solution

Because of the **homogeneous** (齐次性) character :

Every term in the differential equation contains **a linear part** of dependent variable or its 1st/2nd derivative.

$$\frac{1}{\eta} \frac{d}{d\eta} \left(\eta \frac{d\Theta}{d\eta} \right) / \left(\frac{1}{2} \Theta \frac{u}{u_m} \right) = -\Lambda \longrightarrow \frac{1}{\eta} \frac{d}{d\eta} \left(\eta \frac{d\Theta}{d\eta} \right) = -\Lambda \left(\frac{1}{2} \Theta \frac{u}{u_m} \right)$$

In addition, the given B.Cs. are also **homogeneous**:

$$\eta = 0, \frac{d\Theta}{d\eta} = 0; \quad \left. \frac{d\Theta}{d\eta} \right|_{\eta=1} = -Bi\Theta_w$$

For the above mathematical formulation there exists an **uncertainty** (不确定性) of being able to be multiplied by a constant for its solution.

While in order to solve the problem, the value of Λ in the formulation has to be determined.

In order to get a unique solution and to specify the eigenvalue, we need **to supply one more condition!**

We examine the definition of dimensionless temperature:

$$\Theta_b = \left(\frac{T - T_\infty}{T_b - T_\infty} \right)_b = \frac{T_b - T_\infty}{T_b - T_\infty} \equiv \mathbf{1.0}$$

Physically, the averaged temperature is defined by

$$\Theta_b = \frac{\int_0^R 2\pi r u \Theta dr}{\pi R^2 u_m} = \underline{2 \int_0^1 \frac{r}{R} \frac{u}{u_m} \Theta d\left(\frac{r}{R}\right)} = \mathbf{1}$$

Thus the complete formulation is:

$$\frac{1}{\eta} \frac{d}{d\eta} \left(\eta \frac{d\Theta}{d\eta} \right) + \Lambda \left(\frac{1}{2} \Theta \frac{u}{u_m} \right) = 0 \quad (a)$$

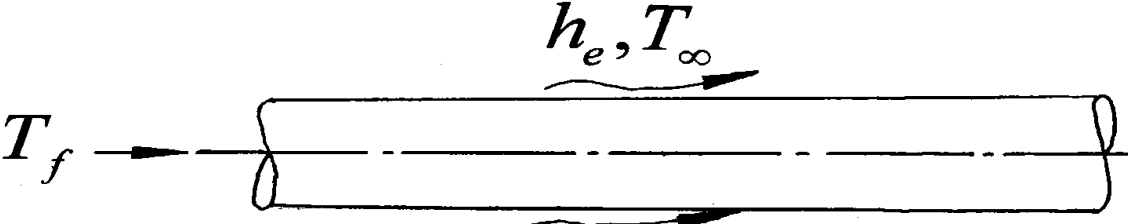
$$\eta = 0, \quad \frac{d\Theta}{d\eta} = 0; \quad (b)$$

$$\left. \frac{d\Theta}{d\eta} \right|_{\eta=1} = -Bi\Theta_w \quad (c)$$

$$\int_0^1 \eta \frac{u}{u_m} \Theta d\eta = 1/2 \quad (d)$$

Non-homogeneous term!

3.5.5 Numerical solution method



$$\rho c_p u \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left(\lambda r \frac{\partial T}{\partial r} \right)$$

Defining $\Theta = \frac{T - T_\infty}{T_b - T_\infty}$; $\frac{dT_b / dX}{T_b - T_\infty} = \frac{1}{\eta} \frac{d}{d\eta} \left(\eta \frac{d\Theta}{d\eta} \right) / \left(\frac{1}{2} \Theta \frac{u}{u_m} \right) = -\Lambda$

$$\frac{1}{\eta} \frac{d}{d\eta} \left(\eta \frac{d\Theta}{d\eta} \right) = -\Lambda \left(\frac{1}{2} \Theta \frac{u}{u_m} \right) \quad \eta = 0, \frac{d\Theta}{d\eta} = 0; \quad \frac{d\Theta}{d\eta} \Big|_{\eta=1} = -Bi\Theta_w$$

$$\Theta_b = \left(\frac{T_b - T_\infty}{T_b - T_\infty} \right) \equiv 1.0 \longrightarrow \int_0^1 \eta \frac{u}{u_m} \Theta d\eta = 1/2$$

$$\frac{1}{\eta} \frac{d}{d\eta} \left(\eta \frac{d\Theta}{d\eta} \right) + \Lambda \left(\frac{1}{2} \Theta \frac{u}{u_m} \right) = 0$$

This is a 1-D conduction equation with a source term!

$\frac{\Lambda}{2} \Theta \frac{u}{u_m}$, whose value should be determined during the solution process **iteratively (迭代地)**.

Patankar—Sparrow proposed following numerical solution method:

1) Variable transformation **(变量变换)**

$$\text{Let } \Theta = \Lambda \phi$$

Because of the homogeneous character, the form of the equation is not changed only replacing Θ by ϕ .

**Complete
mathematical
formulation of
the problem**

$$\frac{1}{\eta} \frac{d}{d\eta} \left(\eta \frac{d\phi}{d\eta} \right) + \Lambda \left(\frac{1}{2} \phi \frac{u}{u_m} \right) = 0 \quad (a)$$

$$\eta = 0, \frac{d\phi}{d\eta} = 0; \quad (b)$$

$$\left(\frac{d\phi}{d\eta} \right)_{\eta=1} = -Bi\phi_w \quad (c)$$

$$\int_0^1 \eta \frac{u}{u_m} \Lambda \phi d\eta = 1/2 \quad (d) \longrightarrow$$

Non-homogeneous equ.

$\Lambda = 1 / \left(2 \int_0^1 \eta \frac{u}{u_m} \phi d\eta \right)$ It can be used to iteratively determine the **eigenvalue**.

2) Solution procedure

- (1) Assuming an initial field ϕ^* , to get Λ^*
- (2) Solving the algebraic equations of an ordinary differential eq. with a source term to get an improved ϕ
- (3) Repeating the above procedure until $|(\phi^* - \phi) / \phi| \leq \varepsilon$,
 $\varepsilon = 10^{-3} \sim 10^{-6}$

This iterative procedure is easy to approach(达到) convergence:

$$S = \Lambda \frac{1}{2} \frac{u}{u_m} \phi = \frac{(u/u_m)\phi}{4 \int_0^1 \eta (u/u_m) \phi d\eta} = \frac{(1-\eta^2)\phi}{4 \int_0^1 \eta (1-\eta^2) \phi d\eta}$$

ϕ exists in both numerator (分子) and denominator (分母), thus only the distribution, rather than absolute value will affect the source term.

From converged ϕ

$$\Lambda = 1 / (2 \int_0^1 \eta \frac{u}{u_m} \phi d\eta)$$

3.5.6 Treatment of numerical results

Two ways for obtaining heat transfer coefficient:

- 1. From solved temp. distribution using Fourier's law of heat conduction and Newton's law of cooling:**

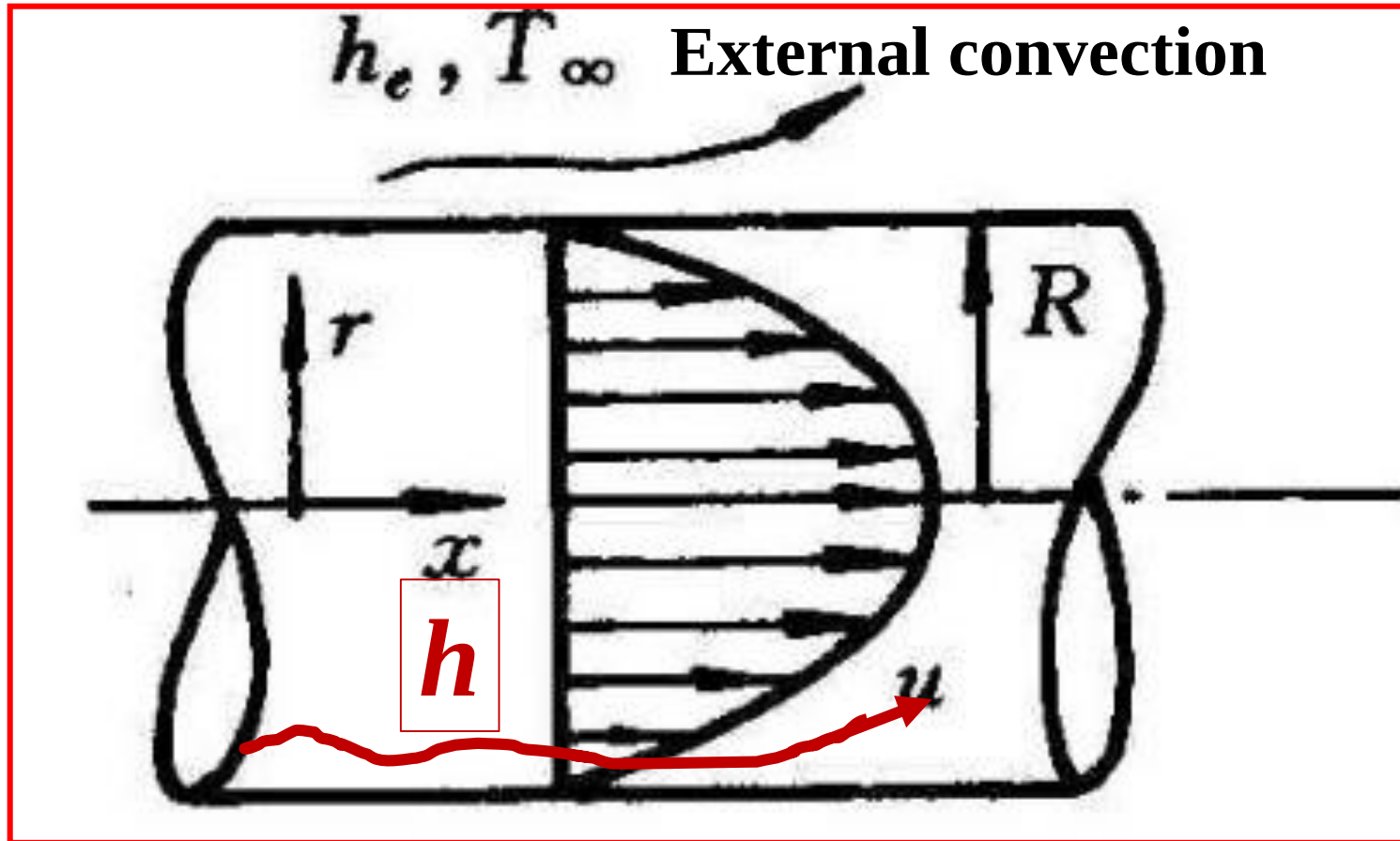
$$r = R, -\lambda \frac{\partial T}{\partial r} = h(T_w - T_b) \rightarrow h = -\lambda \left(\frac{\partial T}{\partial r} \right)_{r=R} \frac{1}{T_w - T_b}$$

For inner fluid

Note: different from boundary condition

$$r = R, -\lambda \frac{\partial T}{\partial r} = h_e (T - T_\infty)$$

$$r = R, -\lambda \frac{\partial T}{\partial r} = h(T_w - T_b) \quad (\text{管内流体传热的热平衡条件})$$



In-tube fluid exchanges heat with tube wall (管内流体与管壁的热交换)

2. From the eigenvalue (特征值) :

From heat balance between inner and external heat transfer

$$h(T_b - T_w) = h_e(T_w - T_\infty)$$

Inner

Outer

Get:

$$\begin{aligned}
 h &= h_e \frac{T_w - T_\infty}{T_b - T_w} \rightarrow h = h_e \frac{1}{\frac{T_b - T_w}{T_w - T_\infty}} \rightarrow \frac{h_e}{\frac{T_b - T_\infty + T_\infty - T_w}{T_w - T_\infty}} \\
 &\rightarrow \frac{h_e}{\frac{T_b - T_\infty}{T_w - T_\infty} - 1} \rightarrow h = \frac{h_e}{\frac{1}{\frac{T_w - T_\infty}{T_b - T_\infty}} - 1} = \frac{h_e}{\frac{1}{\Theta_w} - 1} \rightarrow
 \end{aligned}$$

$$h = \frac{h_e}{\frac{1}{\Theta_w} - 1} = \frac{h_e \Theta_w}{1 - \Theta_w} = \frac{h_e \Lambda \phi_w}{1 - \Lambda \phi_w}$$

$$Nu = \frac{2Rh}{\lambda} = \frac{2R}{\lambda} \frac{h_e \Lambda \phi_w}{1 - \Lambda \phi_w} = \frac{2Bi \Lambda \phi_w}{1 - \Lambda \phi_w}$$

From the specified values Bi , the corresponding eigenvalues, Λ , can be obtained. Thus it is not necessary to find the 1st-order derivative at the wall of function ϕ for determining Nusselt number.

3.5.7 Discussion on numerical results

Table : Numerical results of FDHT in tubes

Bi	Λ	Nu
0	0	4.364
0.1	0.381 8	4.330
0.25	0.894 3	4.284
0.5	1.615	4.221
1	2.690	4.122
2	3.995	3.997
5	5.547	3.840
10	6.326	3.758
100	7.195	3.663
∞	7.314	3.657

$(Nu)_q$

$(Nu)_T$

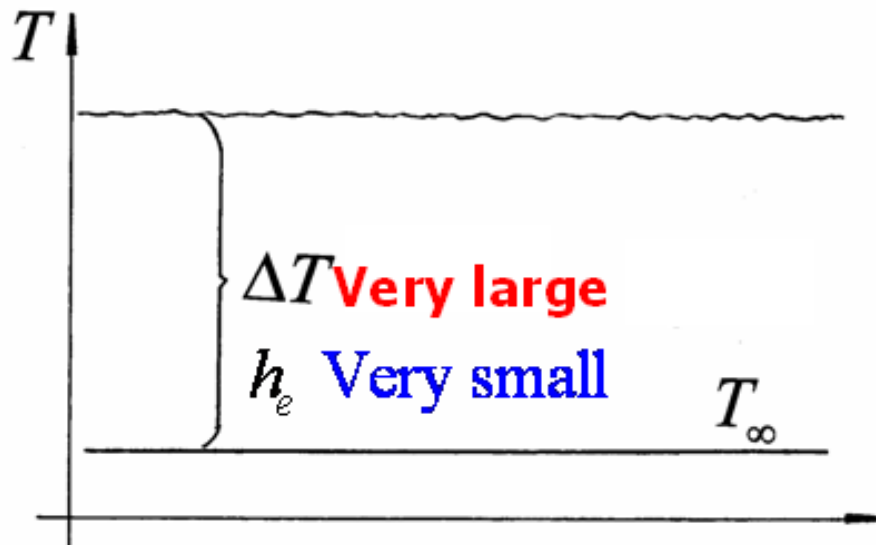
1. *Bi* effect:

From definition $Bi = \frac{Rh_e}{\lambda}$

$Bi \rightarrow \infty, h_e \rightarrow \infty$ External heat transfer is very strong, the wall temp. approaches fluid temp. This is corresponding to constant wall temp condition, thus

$$Nu = 3.66$$

$Bi \rightarrow 0, h_e \rightarrow 0$ Is this adiabatic? No!



Product of very small HT coefficient and very large temp. difference makes heat flux almost constant.

$$q = h_e \Delta T \approx \text{const}$$

2. Computer implementation of $Bi \rightarrow \infty$ and $Bi = 0$

$Bi \longrightarrow \infty$ by progressively (逐渐地) increasing Bi :

$$Bi = 10^5, 10^6, 10^7, \dots$$

$Bi = 0$ by progressively decreasing Bi :

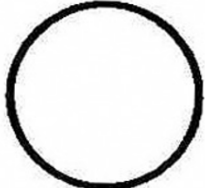
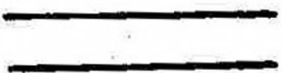
$$Bi = 0.1, 0.01, 0.001, 0.0001, 0.00001, \dots$$

Double decision (双精度) data must be used for the computation, because when Bi approaches zero, both numerator and denominator approach zero:

$$Nu = \frac{2Bi\Lambda\phi_w}{1 - \Lambda\phi_w}, \quad Bi \rightarrow 0, \Lambda \rightarrow 0, \Lambda\phi_w \rightarrow 1 \quad \longrightarrow \quad \frac{0}{0} \quad \text{An infinitive!} \\ \text{不定式!}$$

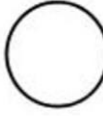
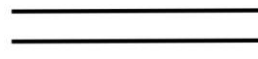
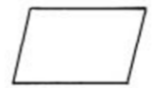
3. Collection of partial examples

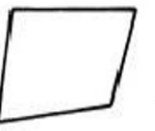
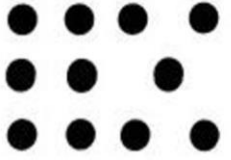
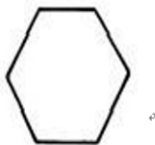
Numerical examples of simple FDHT

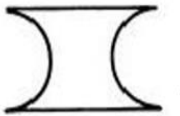
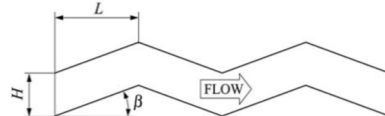
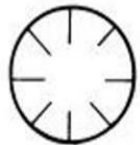
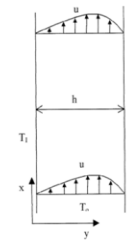
No	Cross section	B. Condition	Refs.
1		Uniform wall temp.; Uniform periphery wall heat flux; External convective heat transfer, etc.	[23,24, 25,26,27]
2		Uniform wall temp.; Uniform wall heat flux	[23]
3		Uniform wall temp.; Uniform axial wall heat flux Two opposite walls adiabatic and the other two opposite wall uniform temp.	[28,29,30]

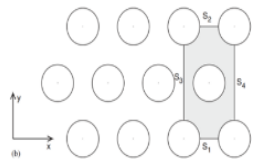
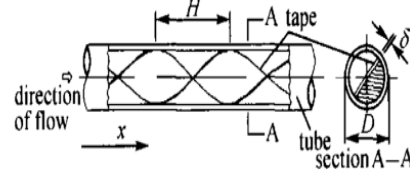
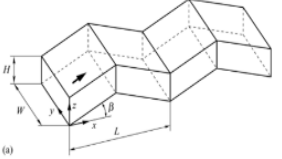
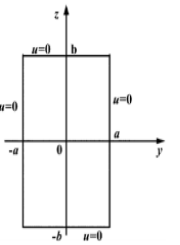
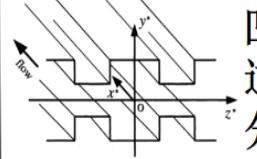
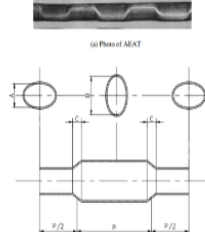
See QR code of this chapter

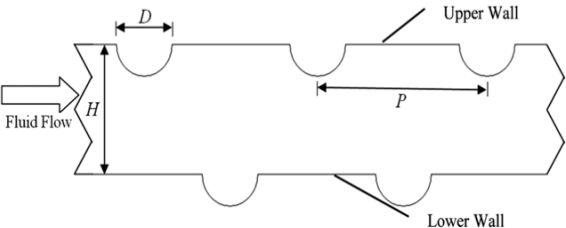
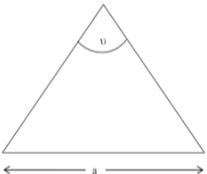
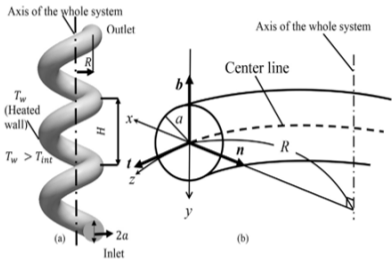
二维码资料 3: 现有部分管道内充分发展流动与传热解的汇总

No.	管道形状	热边界条件	文献
1 (1963)		均匀壁温; 给定周向热流分布; 轴向热流呈指数变化; 外部对流换热	[1-4]
2 (1978)		均匀壁温; 均匀热流及其组合	[1]
3 (1953)		均匀壁温; 周向任意分布热流; 轴向均匀热流; 一组对边均匀壁温, 另一线绝热	[5,6,7]
4 (1995)		外部对流换热条件	[8]
5 (1964)		周向均匀壁温、轴向均匀热流; 一个边均匀加热而其余两边绝热	[7-10]
6 (1972)		均匀壁温	[11]
7 (1972)		外部对流换热条件	[11]
8 (1969)		周向、轴向都是均匀热流	[12,13]
9 (1970)	 正弦曲线	均匀壁温; 周向均匀壁温、轴向均匀热流	[14]
10 (2000)	 环扇形	均匀壁温; 轴向均匀热流, 周向均匀壁温	[15]

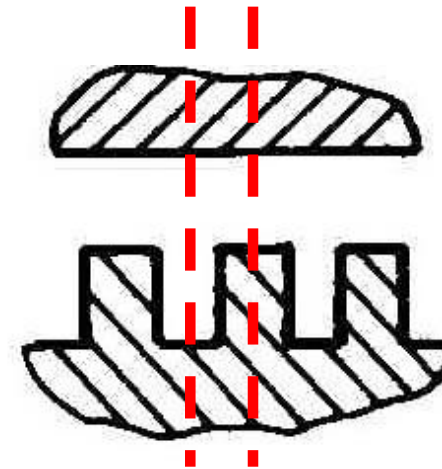
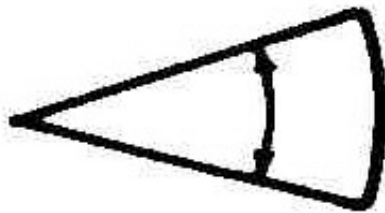
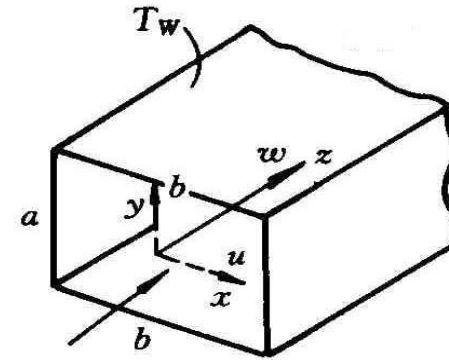
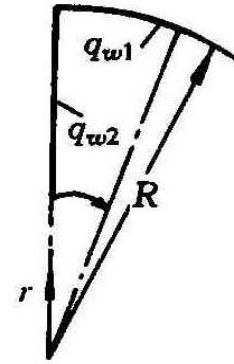
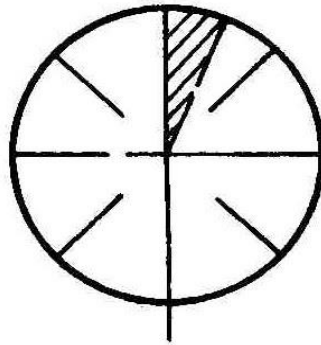
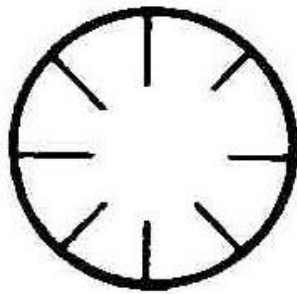
11 (1998)		均匀壁温。	[16]	14 (1966)		均匀热流：各组对边具有相同的均匀热流，不同组之间其值不同；均匀壁温；周向均匀壁温，轴向均匀热流。	[18]
12 (1973)		均匀壁温；周向均匀壁温、轴向均匀热流。	[17]	15 (1968)	 纵掠管束。	周向均匀壁温、轴向均匀热流；轴向、周向都是均匀热流。	[19-23]
13 (1995)		外部对流换热条件。	[8]	16 (1963)	 等圆相切。	均匀壁温。 (流场及阻力分析)。	[1,24,25]

17 (1963)		(流场及阻力分析)	[26]	21 (1984)		均匀壁温	[31]
18 (1991)		均匀壁温	[27]	22 (2002)		均匀壁温	[32]
19 (1978)		均匀壁温；均匀热流；轴向均匀热流、周向均匀壁温	[28,29]	23 (2002)	 微极性流体在垂直平板通道中充分发展自然对流	均匀热流对称加热	[33]
20 (1981)	 (垂直向上流动)	轴向均匀热流、周向均匀壁温，并考虑自然对流	[30]	24 (2003)	 含纽带的方形截面通道的层流充分发展换热	周向及轴向均匀热流	[34]

25 (2003)	 叉排管束中湿空气的充分发展对流换热	均匀壁温，壁面水蒸气分压力等于 t_w 下的饱和压力	[35]	28 (2004)	 含扭带圆管内充分发展对流换热	均匀壁温	[38]
26 (2003)	 三维矩形截面波纹形通道内的充分发展对流换热	均匀壁温	[36]	29 (2006)	 充满多孔介质的矩形通道内的充分发展对流换热	轴向均匀壁温、周向均匀热流；均匀壁温	[39]
27 (2003)	 展向有周期性凹槽的平直通道内的层流充分发展流动与发展中的换热	均匀壁温	[37]	30 (2006)	 交叉扭转椭圆管内充分发展对流换热	均匀壁温	[40]

31 (2013)	 带凹坑与凸起的流动通道的 周期性充分发展流动	仅模拟流场	[41]
32 (2016)	 对三角形截面通 道内多种热边界条 件下的的发展段及 充分发展段的对流 换热进行了详细的 综述	发现轴向均匀 热流周向均匀壁 温的条件最有利 于传热	[42]
33 (2017)	 三维螺旋 管内层流 充分发展 对流传热	均匀壁温，普朗 特数为 4.02, 7.5, 8.5	[43]

3.5.8 Other cases you can solve 20260929



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bank, where....