Numerical Heat Transfer
(数值传热学)
Chapter 2 Discretization of Simple Domain and Two Ways for Establishing Discretized Equation

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2.1 Grid Generation

2.1.1 Task, method and classification

1. Task （任务） of domain discretization

Discretizing the computational domain into a number of sub-domains which are not overlapped(重叠) and can completely cover the entire computational domain.

Four kinds of information can be obtained:

(1) Node（节点） :the position at which the values of dependent variables are solved;

(2) Control volume （CV, 控制容积） ：the minimum volume to which the conservation law is applied;

(3) Interface（界面） :boundary of two neighboring （相邻的） CVs.

(4) Grid lines（网格线）：Curves formed by connecting two neighboring nodes.

The spatial (空间的) relationship between two neighboring nodes, the **influencing coefficients (影响系数)**, will be decided in the procedure of the equation discretization.

2. Classification of domain discretization method

(1) According to node relationship: structured (结构化) vs. unstructured（非结构化）

(2) According to node position: inner node vs. outer node

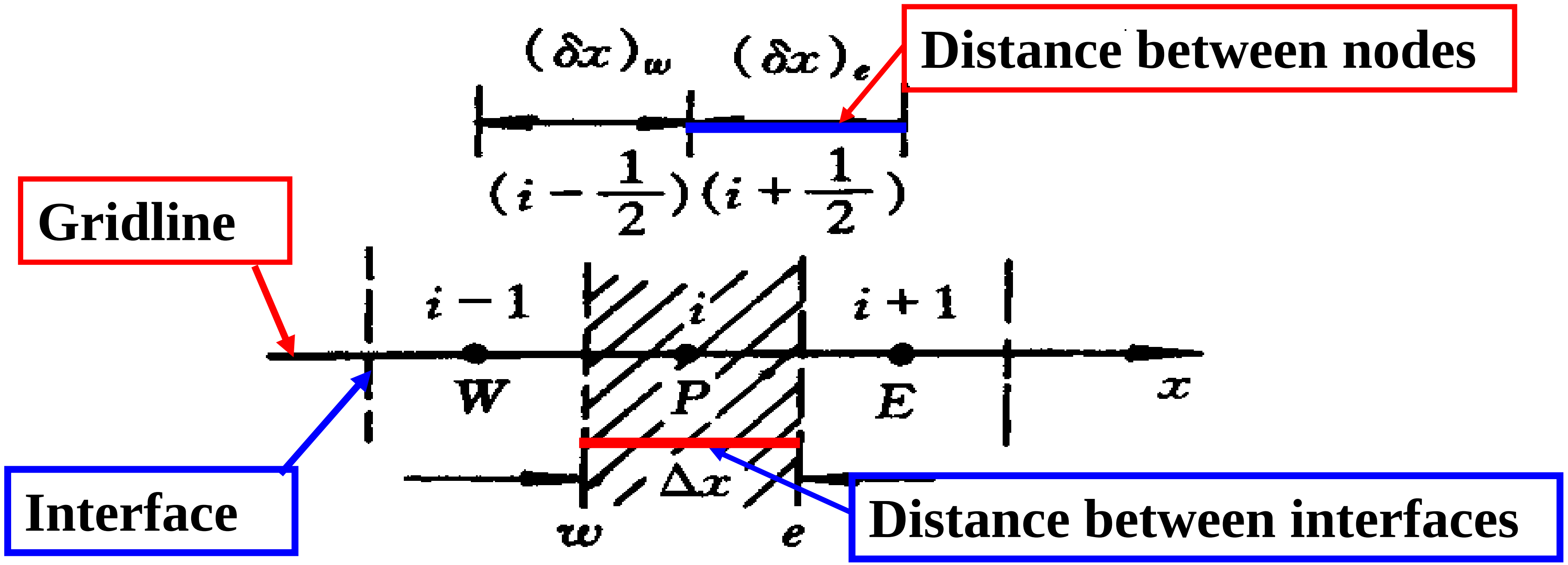
2.1.2 Expression of grid system (网格系统表示)

Grid line－solid line; Interface-dashed line（虚线）;

Distance between two nodes－ δx

Distance between two interfaces－ Δx

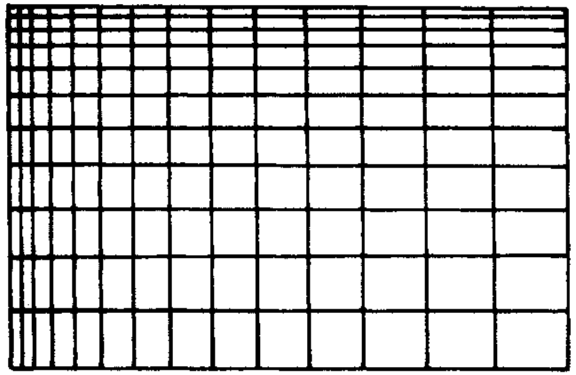
Interfaces by lower cases(小写字母) ***w*** and ***e*** .



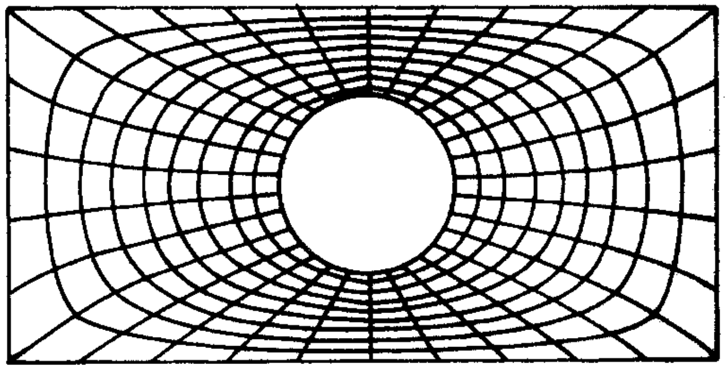
2.1.3 Introduction to different types of grid system and generation method

(1) **Structured grid (结构化网格)**: Node position layout（布置） is in order (有序的）, and fixed for the entire domain.

（2） **Unstructured grid (非结构化网格)**： Node position layout(布置) is in disorder, and may change from node to node. The generation and storage **(存储)** of the relationship of neighboring nodes are the major work of grid generation.

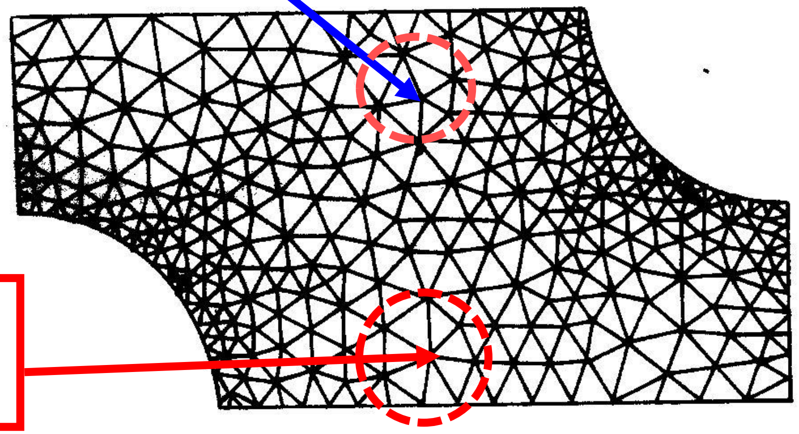


Structured （a)



Structured （b)

5 elements



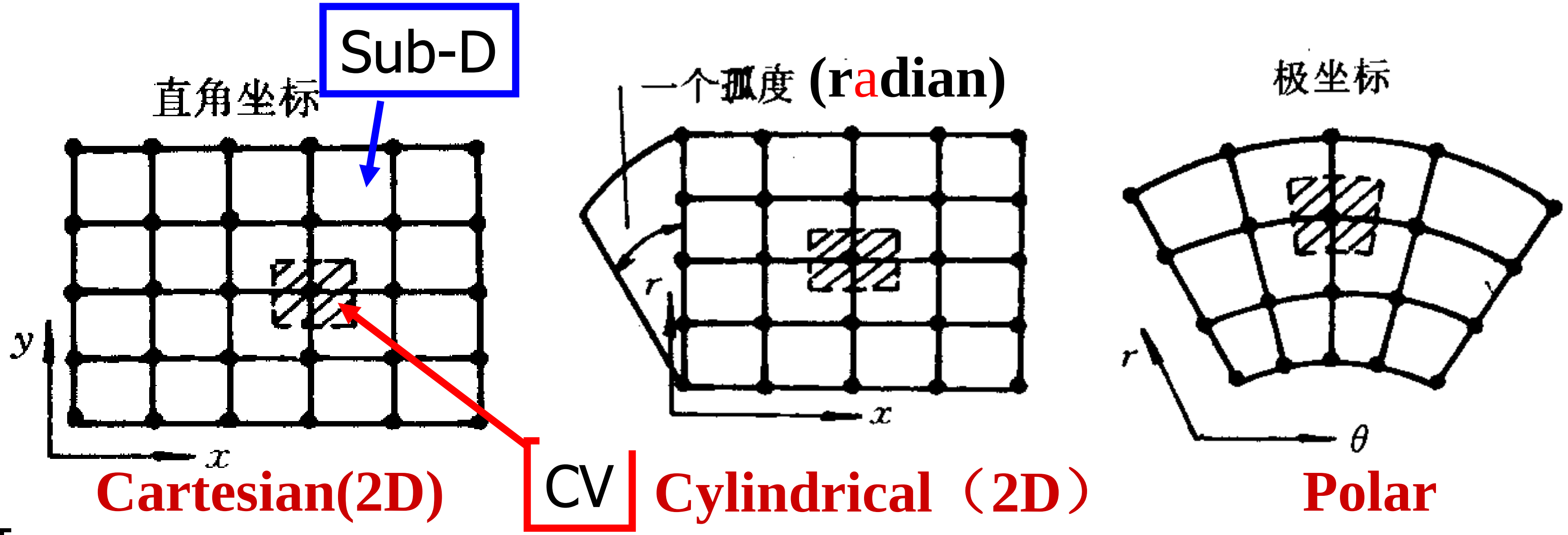
Un-structured

6 neighboring elements

Both structured and unstructured grid layout (节点布置) have two practices（实施）： outer node and inner node.

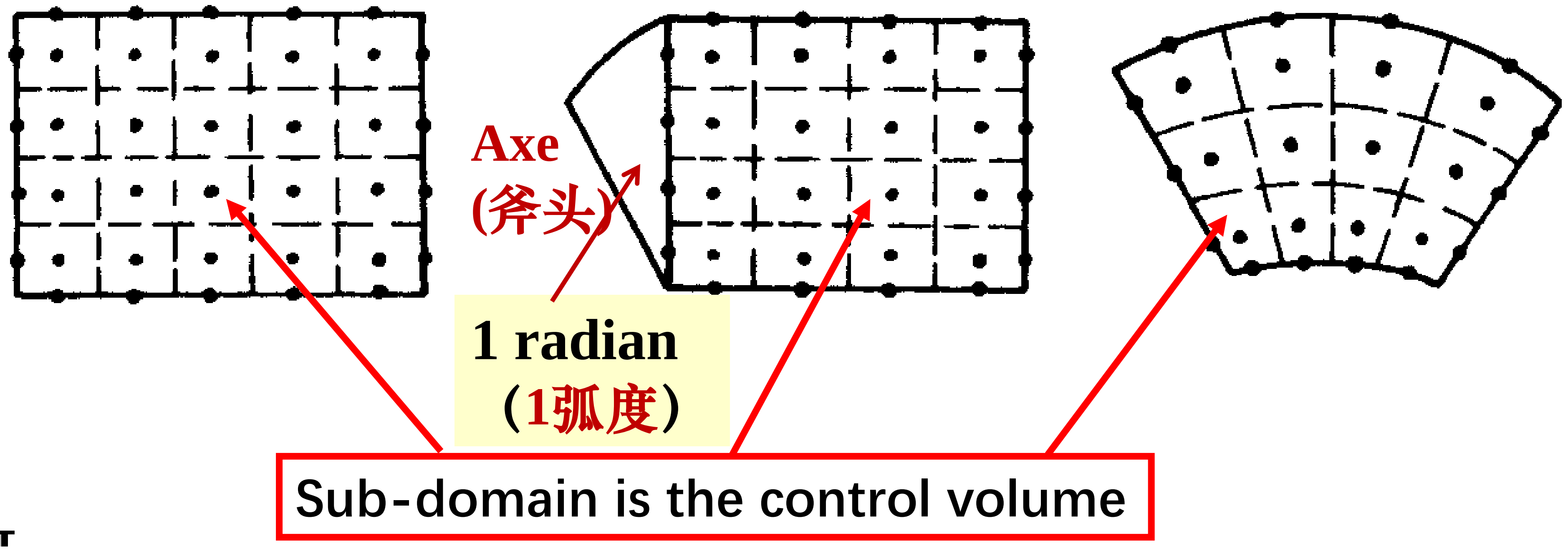
（3）Outer node and inner node for structured grid

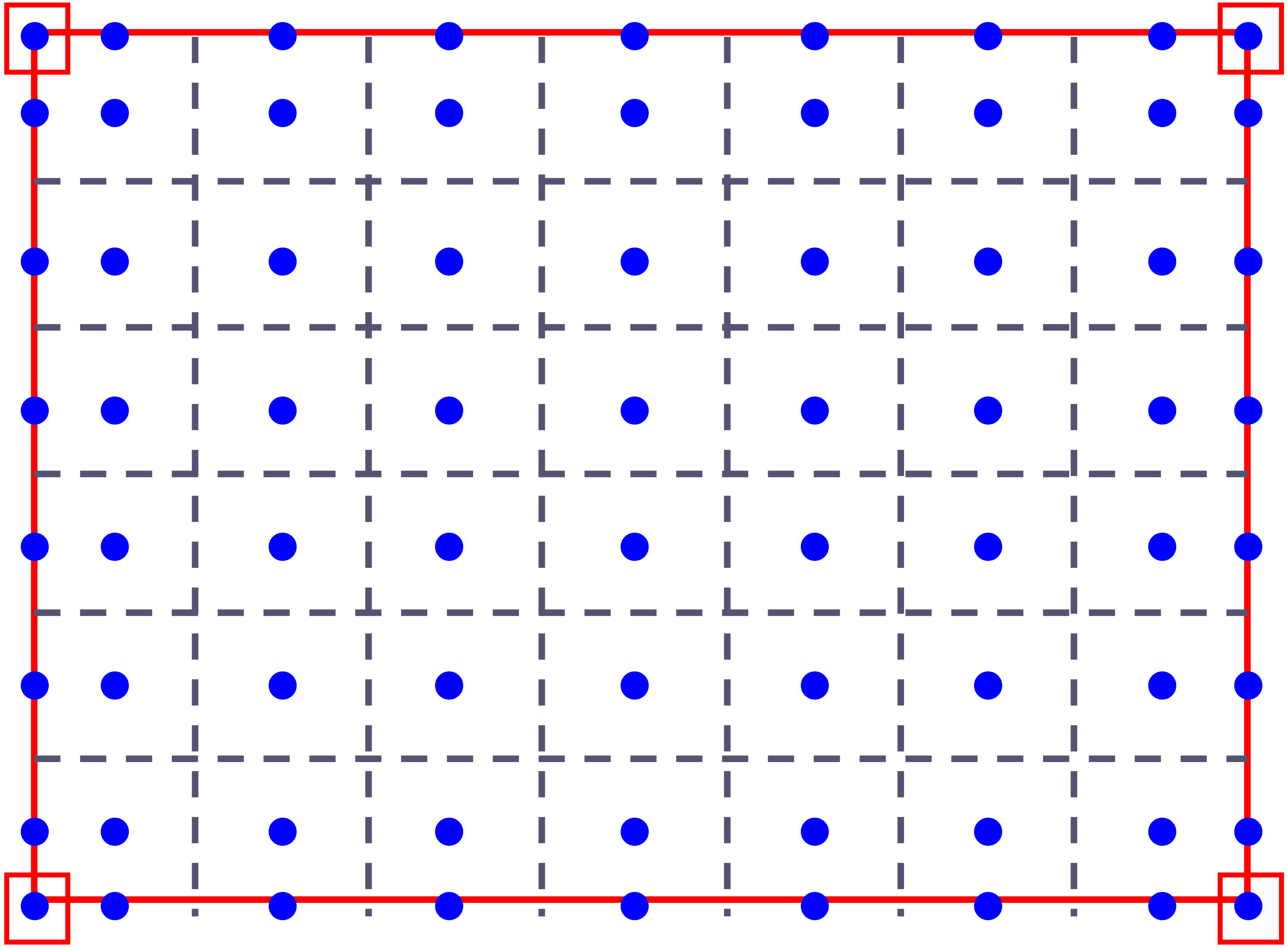
(a) Outer node method： Node is positioned at the vertex of a sub-domain(子区域的角顶)； The interface is between two nodes； Generating procedure： Node first and interface second---called **Practice A** (by Patankar)， or cell-vertex method (单元顶点法).



Cartesian(2D) Cylindrical（2D） Polar

(b) **Inner node method**： Node is positioned at the center of sub-domain; Sub-domain is identical to control volume; Generating procedure: Interface first and node second, called **Practice B** (by Patankar) , or cell-centered method（单元中心法）.



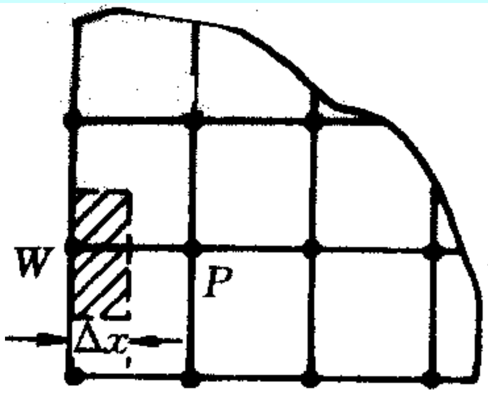


Generating procedure of Practice B

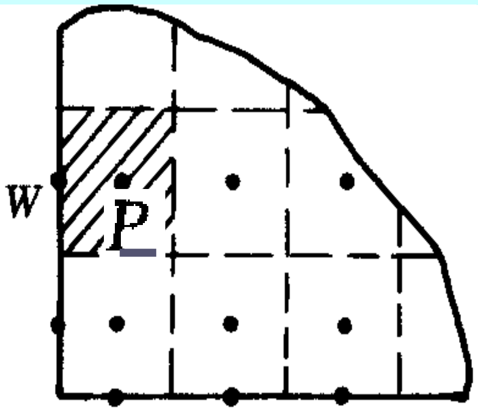
2.1.4 Comparison between Practices A and B

(a) Boundary nodes have different CV.

Practice A



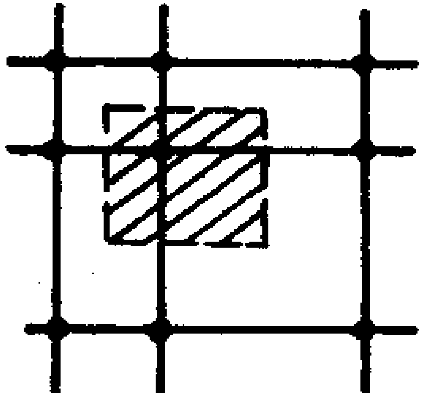
Practice B



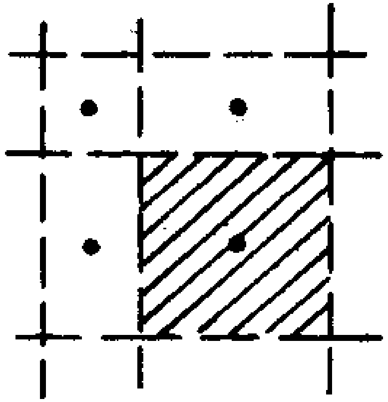
Boundary point has half CV. Boundary point has zero CV.

(b) Practice B is more feasible（适用） for non-uniform grid layout.

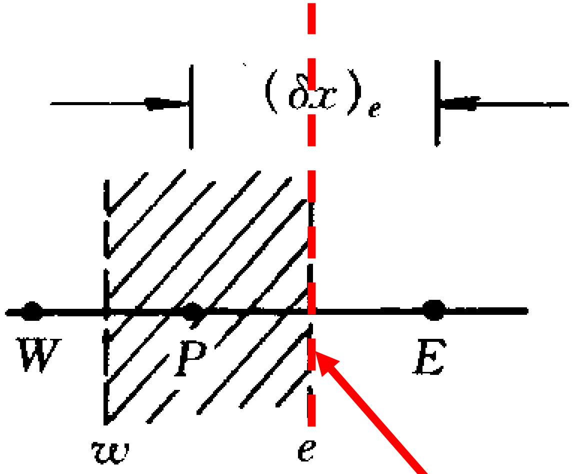
Practice A



Practice B



(c) For non-uniform grid layout, Practice A can guarantee (保证) the discretization accuracy of interface derivatives（界面导数）.

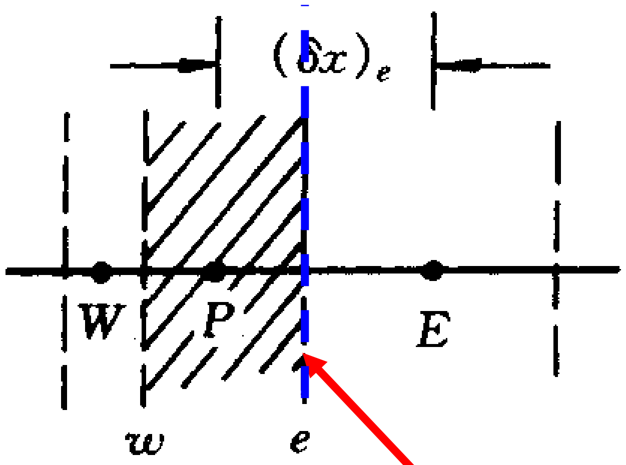


Interface in middle

$$\left(\frac{\partial \phi}{\partial x}\right)_e \cong \frac{\phi_E - \phi_P}{(\delta x)_e}$$

2nd-order accuracy

Practice A



Interface is biased（偏置）

$$\left(\frac{\partial \phi}{\partial x}\right)_e \cong \frac{\phi_E - \phi_P}{(\delta x)_e}$$

Lower than 2nd order accuracy

Practice B

Example 2-1 Discretization of 1-D domain by inner point method

For a 1-D domain,

1) descretizing it by adopting the inner node method , and determining:

(1) the node coordinates,

(2) the position of the CV interfaces;

(3) the width of the CV .

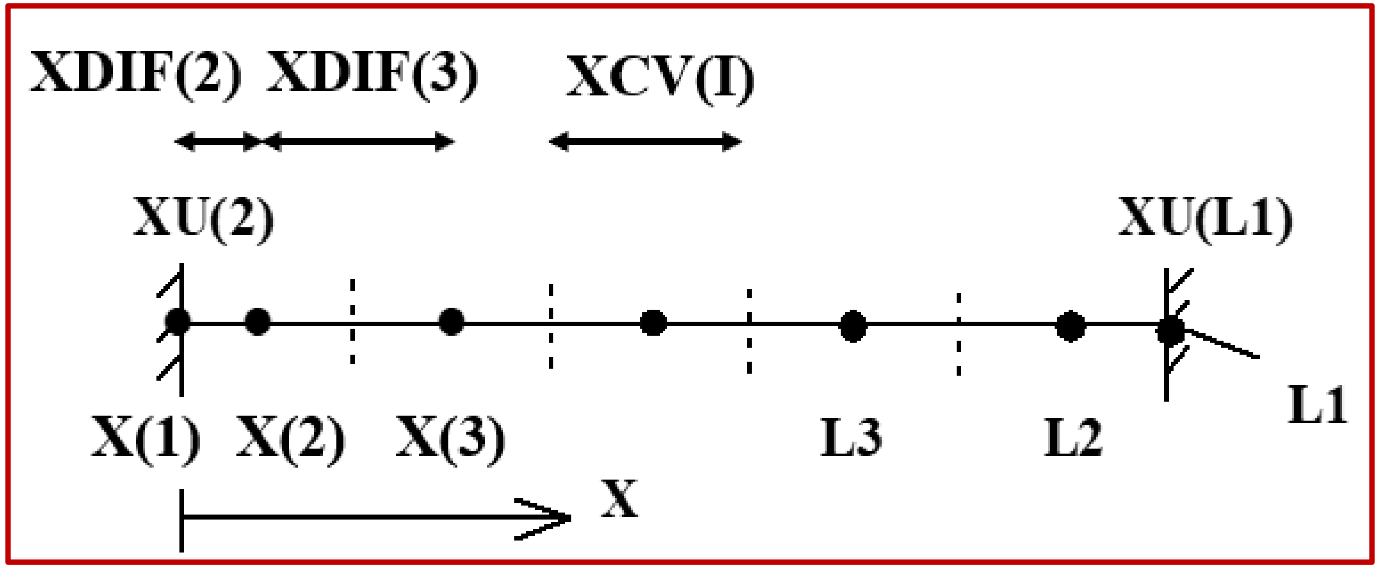
2) developing the code for implementing this discretization.

Analysis：

For the inner node method, N uniform divisions will have (N + 2) node (including the boundary nodes).

Solution:

Adopting five uniform divisions, obtaining L1=7 nodes. The grid system of the inner node is as follows.



Following geometric data are obtained:

(1) X(i): coordinate of the node(grid); From 1 to L1

(2) XCV(i): width of the control volume; From 2 to L2 (= L1 - 1)

(3) XDIF(i): difference between two naighboring nodes; From 2 to L1

defined by : XDIF(i) = X(I) - X(I - 1)

(4) XU(i): position of u - velocity for flow simulation; From 2 to L1.

Detailed discussion of the C++ code will be conducted in the after-class t**u**toring given by Prof. Fang thrpugh Vedio. Teaching assistant will announce the exact date and time.

2.2 Taylor Expansion Method for Equation Discretization in FD

2.2.1 1-D model equation

2.2.2 Taylor expansion method

2.2.3 FD form of discretized 1-D model equation

2.2 Taylor Expansion Method for Equation discretization

2.2.1 1-D model equation（一维模型方程）

1-D model equation has four typical terms ：transient term, convection term, diffusion term and source term. It is specially designed for the study of discretization methods.

Non-conservative. $$\frac{\partial(\rho\phi)}{\partial t}+\rho u\frac{\partial\phi}{\partial x}=\frac{\partial}{\partial x}(\Gamma\frac{\partial\phi}{\partial x}) + S_\phi$$ For FDM

Conservative $$\frac{\partial(\rho\phi)}{\partial t}+\frac{\partial(\rho u\phi)}{\partial x}=\frac{\partial}{\partial x}(\Gamma\frac{\partial\phi}{\partial x})+S_\phi$$ For FVM

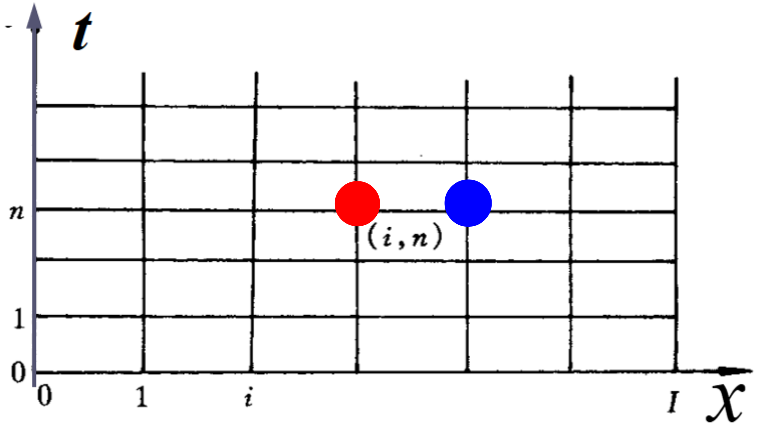
Trans **Conv.** **Diffus.** **Source**

Small but complete---"麻雀虽小，五脏俱全！"

2.2.2 Taylor expansion for FD form of derivatives

1. FD form of 1st order derivative

Expanding $\phi(x,t)$ at $(i+1,n)$ with respect to (对于) point (i,n):



$$\phi(i+1,n)=\phi(i,n)+\frac{\partial\phi}{\partial x})_{i,n}\Delta x+\frac{\partial^2\phi}{\partial x^2})_{i,n}\frac{\Delta x^2}{2!}+.....$$

$$\frac{\partial\phi}{\partial x})_{i,n}=\frac{\phi(i+1,n)-\phi(i,n)}{\Delta x}-\frac{\Delta x}{2}(\frac{\partial^2\phi}{\partial x^2})_{i,n}+...$$

$$\frac{\partial \phi}{\partial x})_{i,n} = \frac{\phi(i+1,n)-\phi(i,n)}{\Delta x} + O(\Delta x)$$

$O(\Delta x)$ is called **truncation error** （**TE, 截断误差**）：

With $\Delta x \to 0$ replacing $\frac{\partial \phi}{\partial x})_{i,n}$ by $\frac{\phi(i+1,n)-\phi(i,n)}{\Delta x}$ will lead to an error $\le$ $K\Delta x$ where K is independent of Δx. ----Mathematical meaning of $O(\Delta x)$

The exponent (**指数**) of Δx is called order of TE(**截差的阶数**) .

Replacing analytical solution $\phi(i,n)$ by approximate value ϕ_i^n, yields:

Forward difference:

（向前差分）

$$\boxed{\frac{\partial \phi}{\partial x})_{i,n} \cong \frac{\phi_{i+1}^n - \phi_i^n}{\Delta x} = \frac{\delta \phi}{\delta x})_i^n , O(\Delta x)}$$

Backward difference:
（向后差分）

$$\left.\frac{\partial \phi}{\partial x}\right)_{i,n} \cong \frac{\phi_i^n - \phi_{i-1}^n}{\Delta x}, O(\Delta x)$$

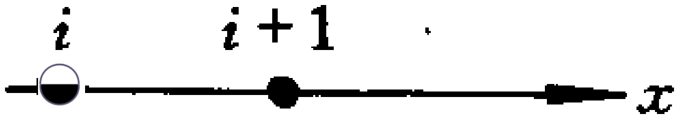
Central difference:
（中心差分）

$$\left.\frac{\partial \phi}{\partial x}\right)_{i,n} \cong \frac{\phi_{i+1}^n - \phi_{i-1}^n}{2\Delta x}, O(\Delta x^2)$$

2. Different FD forms of 1st ad 2nd order derivatives

Stencil (格式图案) of FD expression

$$\frac{\phi_{i+1}^n - \phi_i^n}{\Delta x}$$



○ For the node where FD form is constructed

● For node which is used in the construction of FD form

◒ For the node for which FD form is constructed and which is also used in the construction.

Table 2-1 in the textbook

导数	差分表示式	格式图案	截差
$\left.\dfrac{\partial\phi}{\partial x}\right)_{i,n}$	$\dfrac{\phi_{i+1}^n-\phi_i^n}{\Delta x}$	$i+1$ x	$O(\Delta x)$
	$\dfrac{\phi_i^n-\phi_{i-1}^n}{\Delta x}$	$i-1$ x	$O(\Delta x)$
	$\dfrac{\phi_{i+1}^n-\phi_{i-1}^n}{2\Delta x}$	$i-1$ i $i+1$ x	$O(\Delta x^2)$
	$\dfrac{-3\phi_i^n+4\phi_{i+1}^n-\phi_{i+2}^n}{2\Delta x}$	$i+1$ $i+2$ x	$O(\Delta x^2)$
	$\dfrac{3\phi_i^n-4\phi_{i-1}^n+\phi_{i-2}^n}{2\Delta x}$	$i-2$ $i-1$ x	$O(\Delta x^2)$
	$\dfrac{4\phi_{i+1}^n+6\phi_i^n-12\phi_{i-1}^n+2\phi_{i-2}^n}{12\Delta x}$	$i-2$ $i-1$ $i+1$ x	$O(\Delta x^3)$
	$\dfrac{-2\phi_{i+2}^n+12\phi_{i+1}^n-6\phi_i^n-4\phi_{i-1}^n}{12\Delta x}$	$i-1$ $i+1$ $i+2$ x	$O(\Delta x^3)$
	$\dfrac{\phi_{i-2}^n-8\phi_{i-1}^n+8\phi_{i+1}^n-\phi_{i+2}^n}{12\Delta x}$	$i-2$ $i-1$ i $i+1$ $i+2$ x	$O(\Delta x^4)$

The stencil structure is biased.(偏置)

The stencil structure is symmetric, CD

The stencil structure is biased.(偏置)

The stencil structure is symmetric, CD

导数	差分表示式	格式图案	截差
$\left.\frac{\partial^2\phi}{\partial x^2}\right)_{i,n}$	$\frac{\phi_i^n - 2\phi_{i+1}^n + \phi_{i+2}^n}{\Delta x^2}$	i, $i+1$, $i+2$, x	$O(\Delta x)$
	$\frac{\phi_i^n - 2\phi_{i-1}^n + \phi_{i-2}^n}{\Delta x^2}$	$i-2$, $i-1$, i, x	$O(\Delta x)$
	$\frac{\phi_{i+1}^n - 2\phi_i^n + \phi_{i-1}^n}{\Delta x^2}$	$i-1$, i, $i+1$, x	$O(\Delta x^2)$
	$(-\phi_{i-2}^n + 16\phi_{i-1}^n - 30\phi_i^n + 16\phi_{i+1}^n - \phi_{i+2}^n)/12\Delta x^2$	$i-2$, $i-1$, i, $i+1$, $i+2$, x	$O(\Delta x^4)$

The stencil structure is biased.(偏置)

The stencil structure is symmetric, CD

Rule of thumb（大拇指原则） for judging (**判断**) correction of a FD form ：

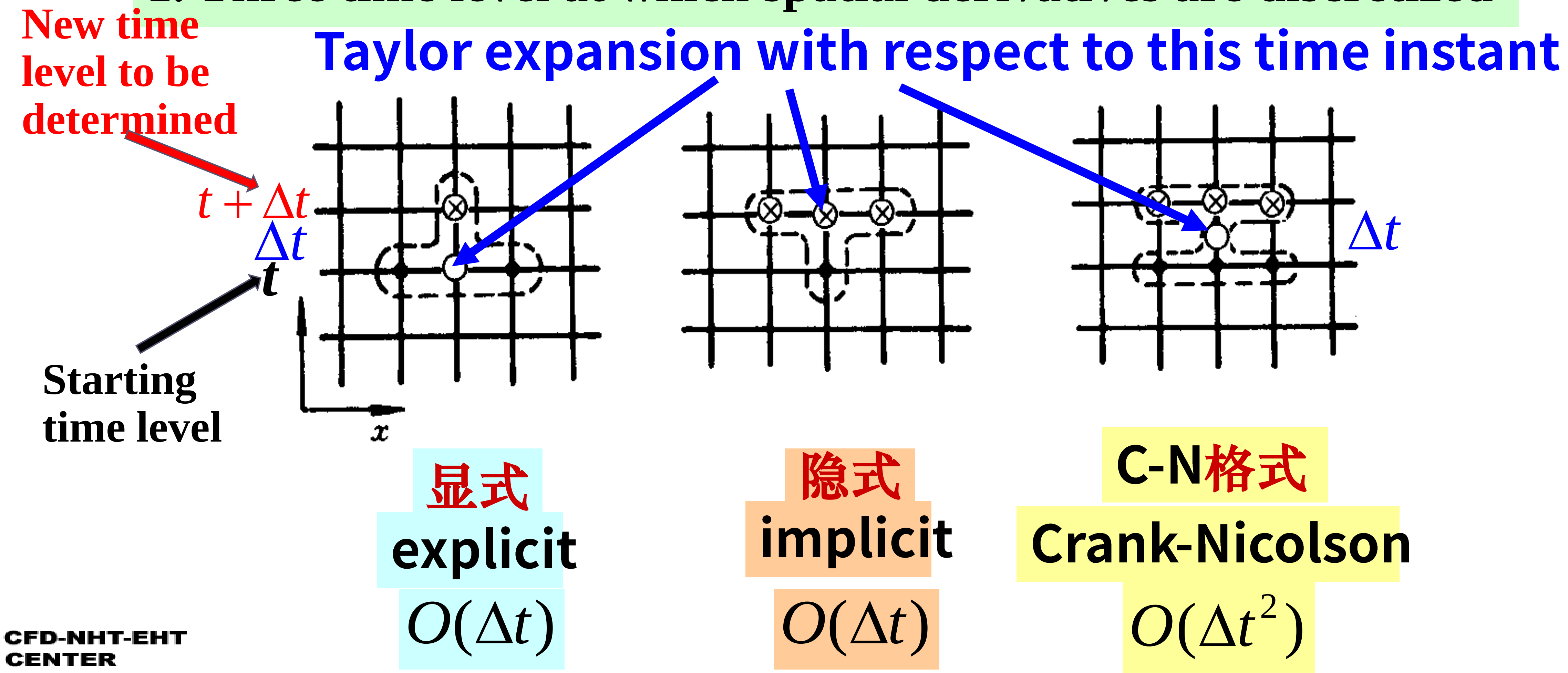
(1) Dimension (量纲) should be consistent(一致);

(2) For a uniform field any order of derivatives should be zero .

2.2.3 Discretized form of 1-D model equation by FD

For an unsteady problem, it is to be determined at which time level to calculate the spatial (空间的) derivatives .

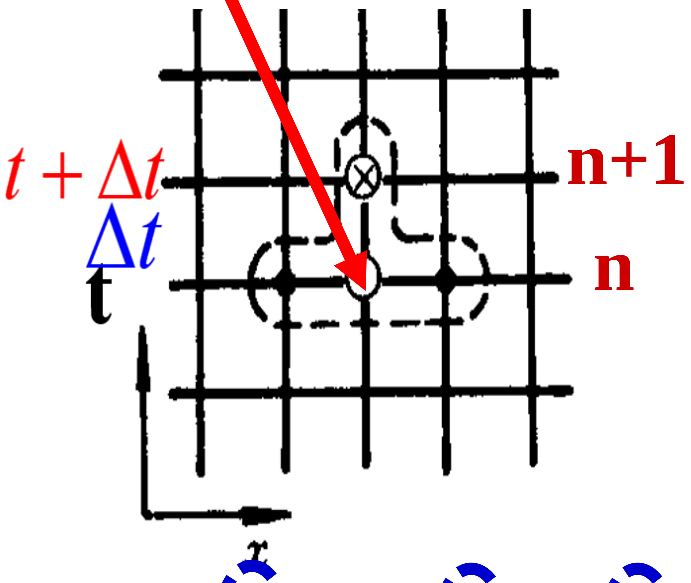
1. Three time level at which spatial derivatives are discretized

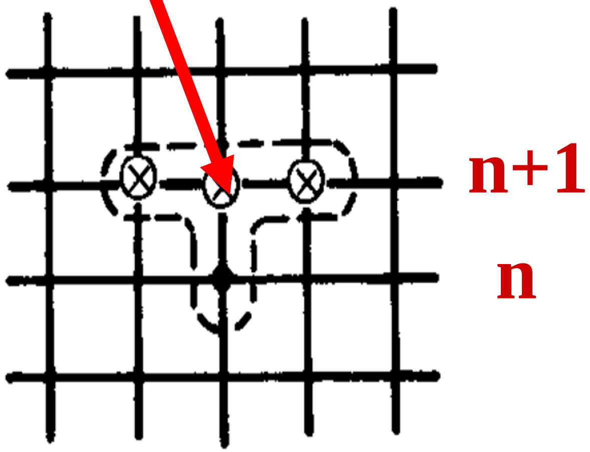


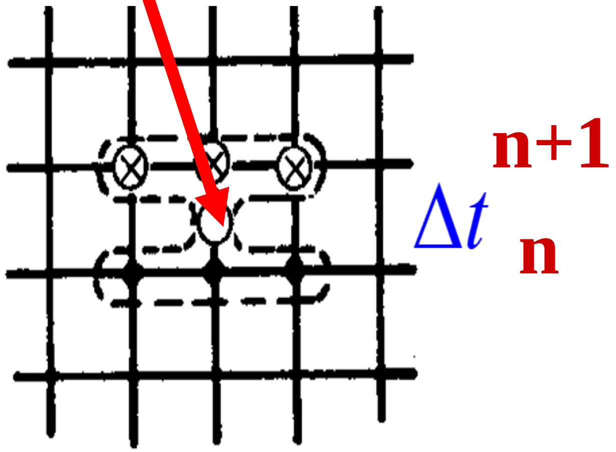
$$\frac{\partial T}{\partial t}=a\frac{\partial^2 T}{\partial x^2}:\quad \frac{\partial T}{\partial t}\approx\frac{T_i^{n+1}-T_i^n}{\Delta t};$$

Three choices of time level for $\frac{\partial^2 T}{\partial x^2}$

显式 explicit | **隐式 implicit** | **C-N格式**







$$\frac{\partial^2 T}{\partial x^2}\cong\frac{T_{i+1}^n-2T_i^n+T_{i-1}^n}{\Delta x^2}\qquad \frac{T_{i+1}^{n+1}-2T_i^{n+1}+T_{i-1}^{n+1}}{\Delta x^2}\qquad \frac{1}{2}\left(\frac{T_{i+1}^{n+1}-2T_i^{n+1}+T_{i-1}^{n+1}}{\Delta x^2}+\frac{T_{i+1}^n-2T_i^n+T_{i-1}^n}{\Delta x^2}\right)$$

2. Explicit scheme of 1-D model equation

Analytical form

$$\rho\frac{\phi(i,n+1)-\phi(i,n)}{\Delta t}+\rho u\frac{\phi(i+1,n)-\phi(i-1,n)}{2\Delta x}=$$
$$\Gamma\frac{\phi(i+1,n)-2\phi(i,n)+\phi(i-1,n)}{\Delta x^2}+S(i,n)+HOT$$

HOT---Sum of higher order terms. $\phi(\ ,\ )$ represents analytical solution.

Finite difference form

Explicit in space derivatives

$$\rho\frac{\phi_i^{n+1}-\phi_i^n}{\Delta t}+\rho u\frac{\phi_{i+1}^n-\phi_{i-1}^n}{2\Delta x}=\Gamma\frac{\phi_{i+1}^n-2\phi_i^n+\phi_{i-1}^n}{\Delta x^2}+S_i^n,\ O(\Delta t,\Delta x^2)$$

Forward in time, (Δt)

Central in space, (Δx^2)

Central in space, (Δx^2)

TE. of FD equation $O(\Delta t,\Delta x^2)$

Forward time & central space--FTCS

2.3 Control Volume and Heat Balance Methods for Equation Discretization

2.3.1 Procedures for implementing (实行) CV method

2.3.2 Two conventional profiles(型线)

2.3.3 Discretization of 1-D model eq. by CV method

2.3.4 Discussion on profile assumptions in FVM

2.3.5 Discretization equation by balance(平衡) method

2.3.6 Comparisons between two methods

2.3 Control Volume and Heat Balance Methods for Equation Discretization

2.3.1 Procedures for implementing CV method

1. Integrating (积分) the conservative PDE over a CV

2. Selecting (选择) profiles for dependent variable (因变量) and its 1st –order derivative (一阶导数)

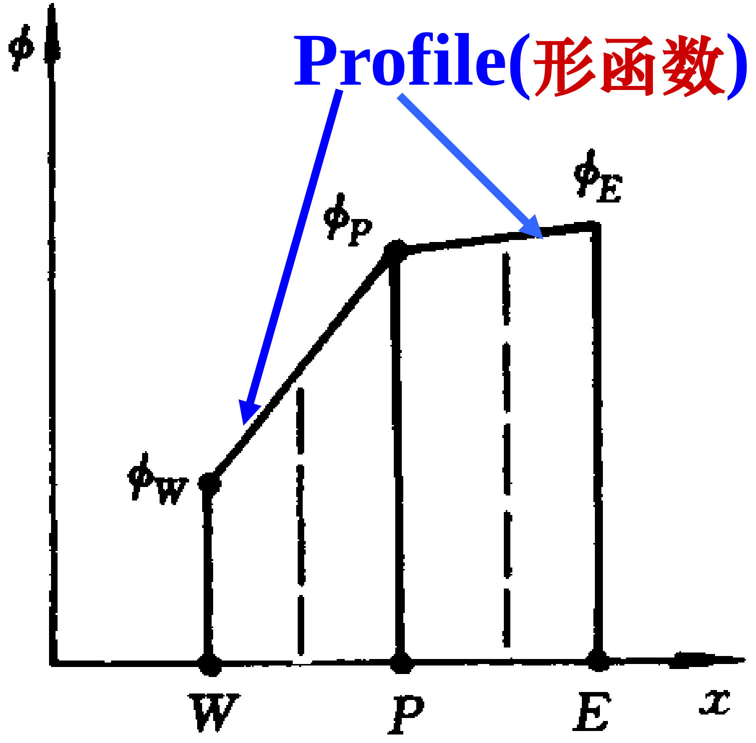
Profile is a local variation pattern of dependent variables with space coordinate, or with time.

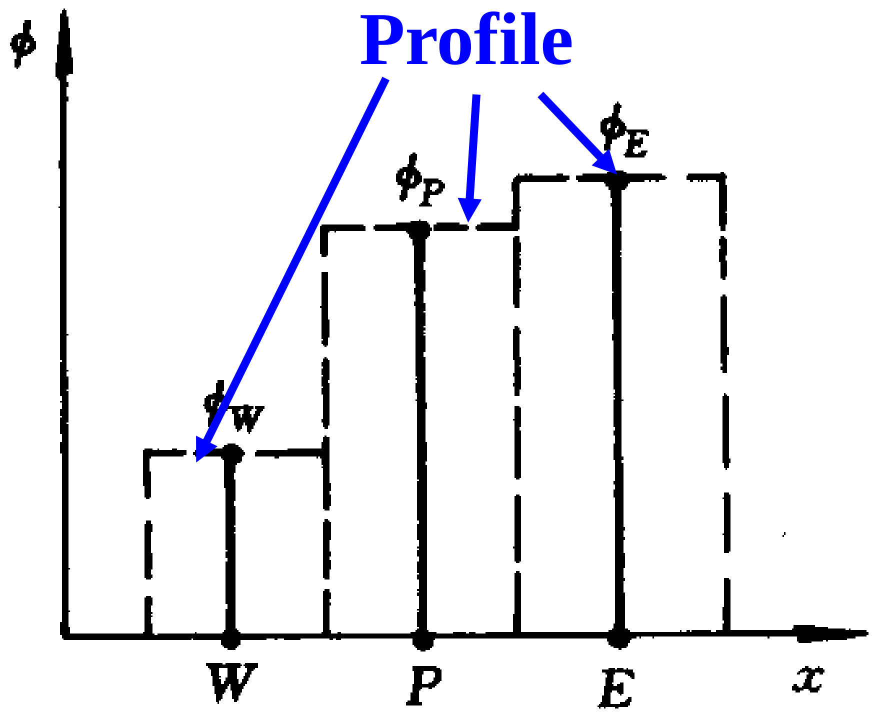
3. Completing integral and rearranging algebraic equations

2.3.2 Two conventional （常见）profiles (shape function)

Originally (本来) shape function (形函数） is to be solved; here it is to be assumed!----Approximation made in the numerical simulation!

Variation with spatial coordinate

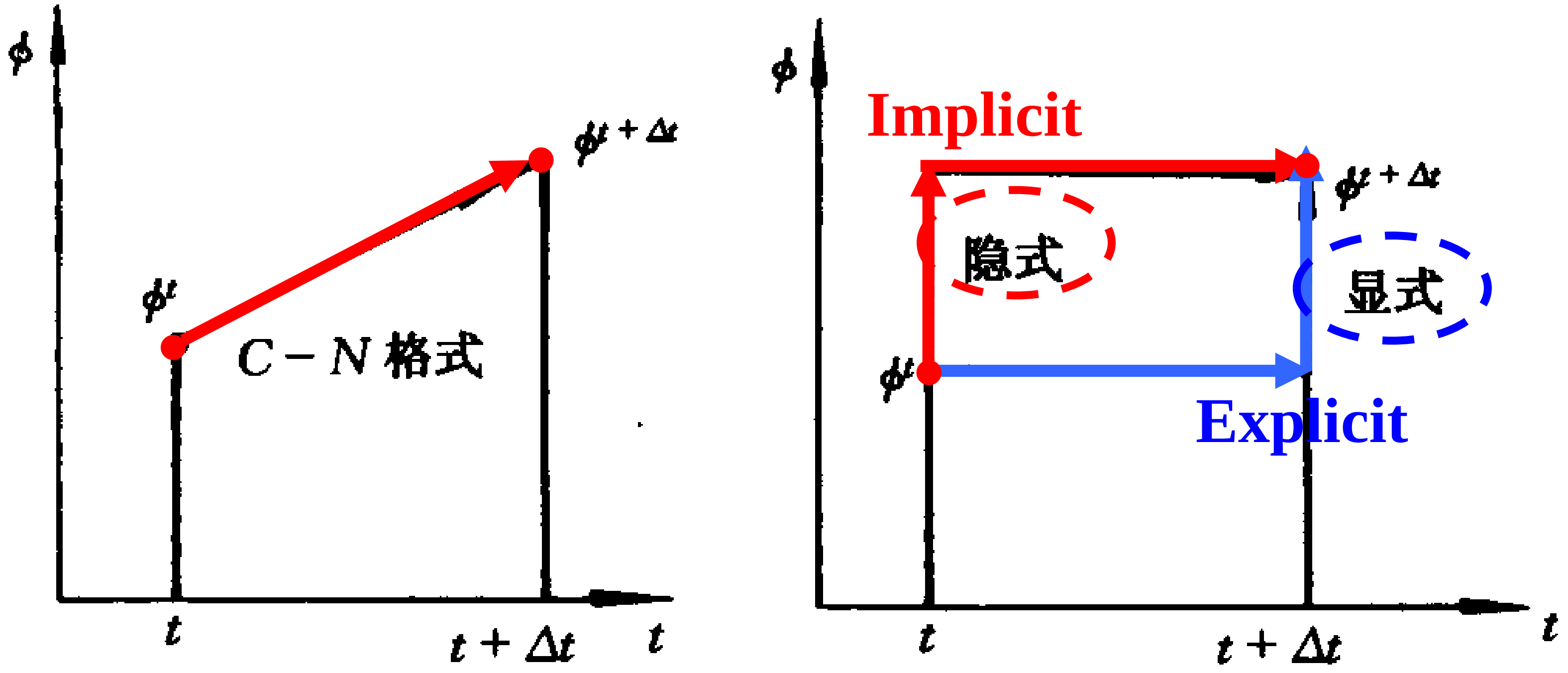




piece-wise linear step-wise approximation

分段线性 阶梯逼近

Variation with time



piece-wise linear **step-wise approximation**

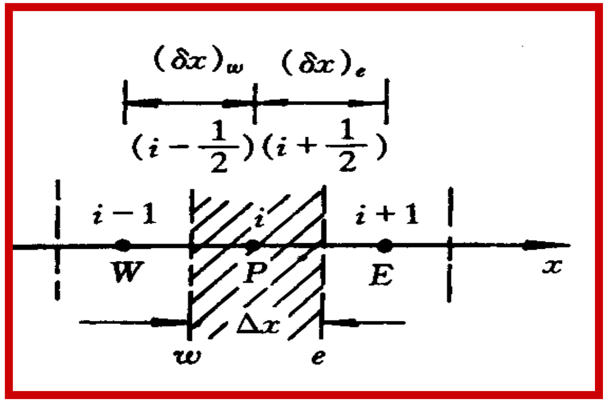
分段线性 **阶梯逼近**

2.3.3 Discretization of 1-D model eq. by CV method

Integrating conservative GE over a CV within $[t, t+\Delta t]$,

$$\frac{\partial(\rho\phi)}{\partial t}+\frac{\partial(\rho u\phi)}{\partial x}=\frac{\partial}{\partial x}(\Gamma\frac{\partial\phi}{\partial x})+S_{\phi}$$

yields:



$$\rho\int_{w}^{e}(\phi^{t+\Delta t}-\phi^{t})dx+\rho\int_{t}^{t+\Delta t}[(u\phi)_{e}-(u\phi)_{w}]dt=$$

$$\Gamma\int_{t}^{t+\Delta t}[(\frac{\partial\phi}{\partial x})_{e}-(\frac{\partial\phi}{\partial x})_{w}]dt+\int_{t}^{t+\Delta t}\int_{w}^{e}S_{\phi}dxdt \qquad (1)$$

To complete the integration we need the profiles of the dependent variable and its 1st derivative.

1. Transient term

Assuming the step-wise approximation for ϕ with space:

$$\rho\int_w^e(\phi^{t+\Delta t}-\phi^t)dx=\rho(\phi_P^{t+\Delta t}-\phi_P^t)\Delta x \qquad (2)$$

2. Convective term

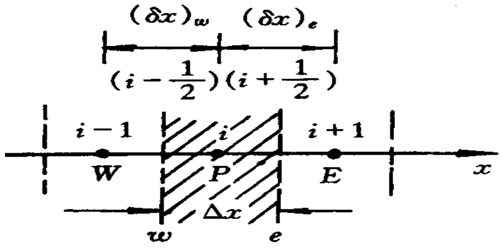
Assuming the explicit step-wise approximation for ϕ with time:

$$\rho\int_t^{t+\Delta t}[(u\phi)_e-(u\phi)_w]dt=\rho[(u\phi)_e^t-(u\phi)_w^t]\Delta t$$

In the FVM simulation all information (u,v,p,t, properties） is stored at grids. The interface value （界面值） should inter**er**polated (插值) by node values.

Further, assuming linear-wise variation of ϕ with space

$$\rho[(u\phi)_e^t-(u\phi)_w^t]\Delta t=\rho u\Delta t\left(\frac{\phi_E+\phi_P}{2}-\frac{\phi_W+\phi_P}{2}\right)=\rho u\Delta t\frac{\phi_E-\phi_W}{2} \qquad (3)$$



Uniform grid

Superscript "*t*" is temporary(暂时) neglected!

3. Diffusion term

Taking explicit step-wise variation of $\frac{\partial\phi}{\partial x}$ with time, yields:

$$\Gamma\int_t^{t+\Delta t}\left[\left(\frac{\partial\phi}{\partial x}\right)_e-\left(\frac{\partial\phi}{\partial x}\right)_w\right]dt=\Gamma\left[\left(\frac{\partial\phi}{\partial x}\right)_e^t-\left(\frac{\partial\phi}{\partial x}\right)_w^t\right]\Delta t$$

Further, assuming linear-wise variation of ϕ with space

$$\Gamma[(\frac{\partial\phi}{\partial x})_e^t-(\frac{\partial\phi}{\partial x})_w^t]\Delta t=\Gamma\Delta t[\frac{\phi_E-\phi_P}{(\delta x)_e}-\frac{\phi_P-\phi_W}{(\delta x)_w}] \quad (4)$$

Uniform grid

$$=\Gamma\Delta t\frac{\phi_E-2\phi_P+\phi_W}{\Delta x}$$

Super-script "t" is temporary neglected!

4. Source term

Temporary （暂时） assuming explicit step-wise with time and step-wise variation with space:

$$\int_t^{t+\Delta t}\int_w^e S dx dt=\overline{S}^t(\Delta x)_P\Delta t \quad (5);\ \overline{S}$$ ---averaged one over space.

Substituting Eqs.(2),(3), (4) and (5) into Eq. (1), and dividing both sides by $\Delta t\Delta x$ for uniform grids, yielding:

$$\rho\frac{\phi_P^{t+\Delta t}-\phi_P^t}{\Delta t}+\rho u\frac{\phi_E^t-\phi_W^t}{2\Delta x}=\Gamma\frac{\phi_E^t-2\phi_P^t+\phi_W^t}{\Delta x^2}+\overline{S}^t,O(\Delta t,\Delta x^2)$$

For the uniform grid system, the results are the same as that from Taylor expansion, which reads:

$$\rho\frac{\phi_i^{n+1}-\phi_i^n}{\Delta t}+\rho u\frac{\phi_{i+1}^n-\phi_{i-1}^n}{2\Delta x}=\Gamma\frac{\phi_{i+1}^n-2\phi_i^n+\phi_{i-1}^n}{\Delta x^2}+S_i^n,O(\Delta t,\Delta x^2)$$

FDM and FVM are a kind of brothers: with FDM being mathematically more rigorous (严格） and FVM being physically more meaningful（有意义）; They usually have the same TE and can help each other!

2.3.4 Discussion on profile assumptions in FVM

1. In FVM the only purpose (目的) of profile is to derive the discretization equations; Once they have been established, the function of profile is fulfilled (完成）.

2. The selection criterion (准则） of profile is easy to be implemented and good numerical characteristics; Consistency (协调) among different terms is not required.

3. In FVM profile is indeed the scheme（差分格式）.

2.3.5 Discretization equation by balance method

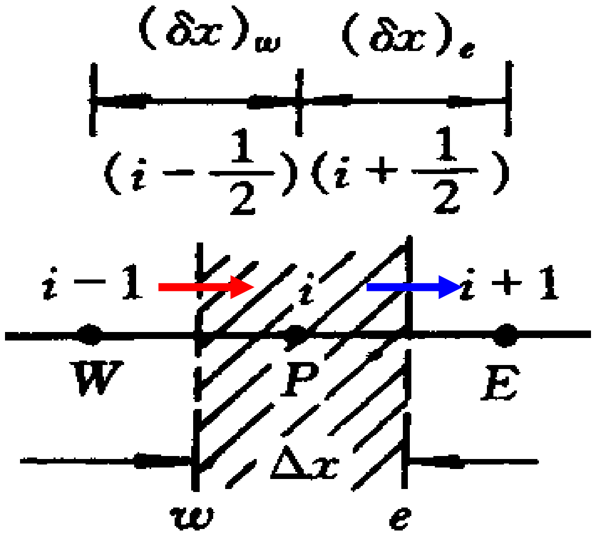
1. Major concept： **Applying the conservative law directly to a CV, viewing the node as its representative（代表）**

2. 1-D diffusion-convection problem with source term

Writing down balance equation for Δx and Δt

$$\underbrace{\rho c_p(\phi_P^{t+\Delta t}-\phi_P^t)\Delta x}_{\text{Transient}}=\underbrace{\rho c_p[(u\phi)_w^t-(u\phi)_e^t]\Delta t}_{\text{Convection}}$$

$$+\underbrace{\Gamma[(\frac{\partial\phi}{\partial x})_e^t-(\frac{\partial\phi}{\partial x})_w^t]\Delta t}_{\text{Diffusion}}+\underbrace{\overline{S}^t\Delta x\Delta t}_{\text{Source}}$$



By selecting the profile of dependent variable ϕ with space, the discretization equation can be obtained.

If the same profiles of the variable ϕ of FVM are assumed, the final results are the same:

$$\rho\frac{\phi_i^{n+1}-\phi_i^n}{\Delta t}+\rho u\frac{\phi_{i+1}^n-\phi_{i-1}^n}{2\Delta x}=\Gamma\frac{\phi_{i+1}^n-2\phi_i^n+\phi_{i-1}^n}{\Delta x^2}+S_i^n,O(\Delta t,\Delta x^2)$$

The heat balance method is actually adopting the conservation law directly to a CV, and is very useful.

2.3.6 Comparisons of two ways

Content	FDM	FVM
1. Error analysis	Easy	Not easy; via FDM
2. Physical concept	Not clear	Very clear
3. Variable length step(变步长)	Not easy	Easy
4. Conservation feature of algebraic Eqs.	Not guaranteed	May be guaranteed

FVM has been the 1st choice of most commercial software.

2-3-7 Boundary condition treatment and grid-independent solutions

1. Numerical treatment of boundary Condition：A complete discretization of a conduction problem includes the numerical treatment of its boundary condition.

First kind boundary condition can be directly adopted in the solution of the algebraic eqs.

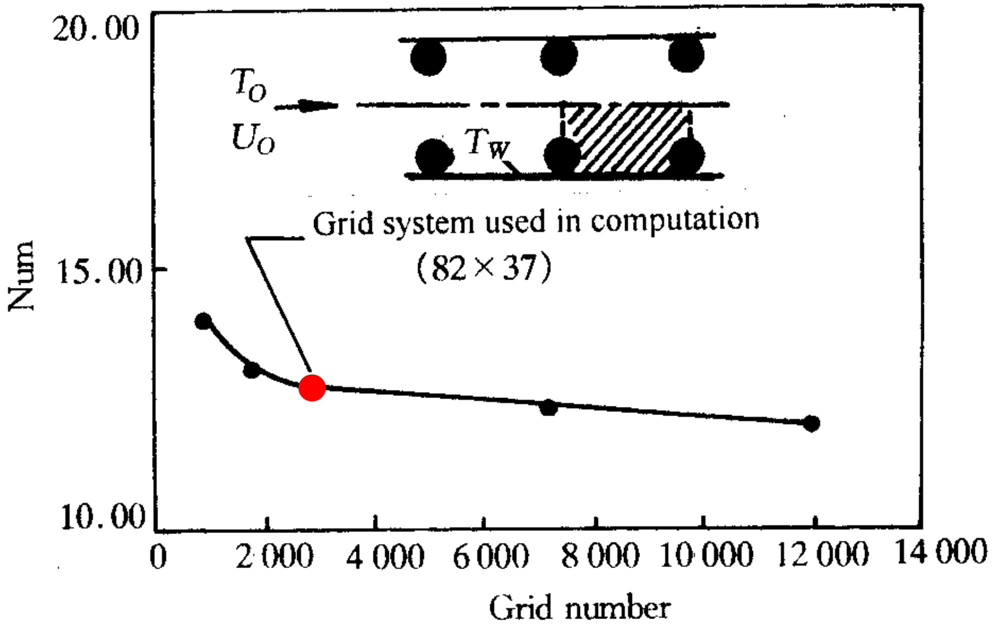
For the second and third kinds of boundary condition, the boundary temperatures are unknown , hence the algebraic eqs of the inner nodes are not complete. Numerical treatments of the 2nd and third kind boundary conditions will be presented in Chapter 3.

2. Grid-independent solution

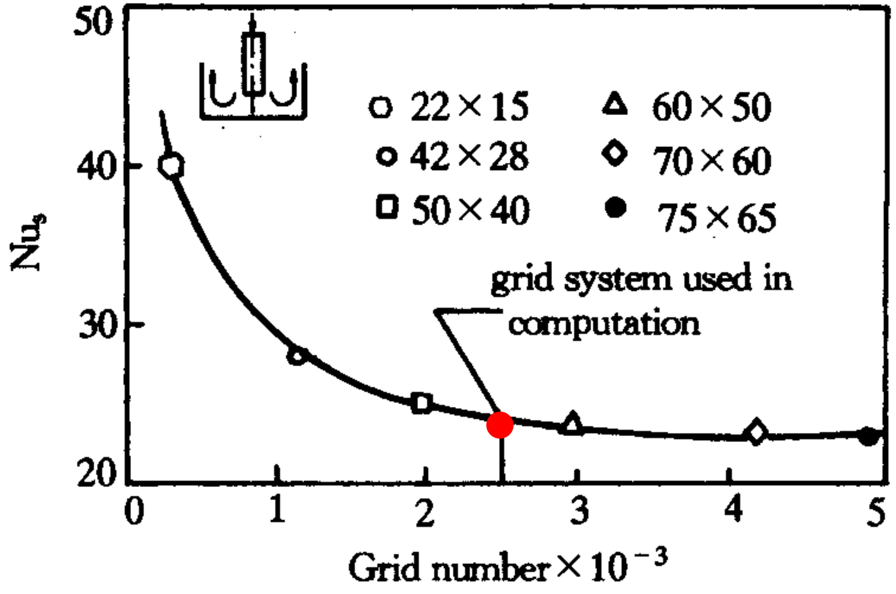
Grid generation is an iterative procedure（迭代过程）; Debugging (调试） and comparison are often needed. For a complicated geometry grid generation may take a major part of total computational time.

The appropriate grid fineness (细密程度) is determined in the solution process. It should be such that the numerical solutions are nearly independent on the grid numbers. Such numerical solutions are called grid-independent solutions (网格独立解). They are required for publication of a paper.

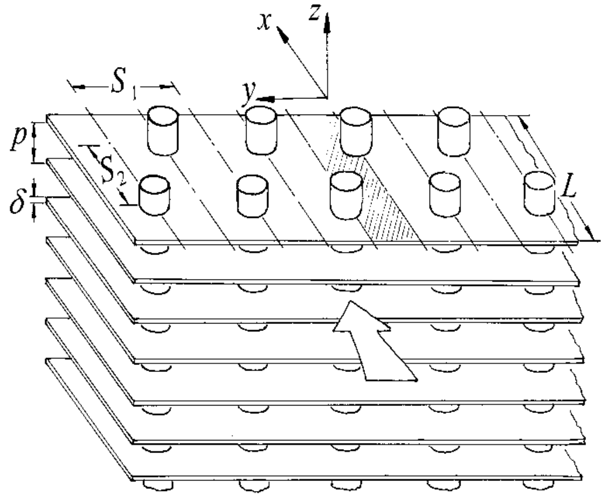
As the represent of numerical solution, it can be dependent variable or some other interested parameter determined from numerical solution: Nusselt number, friction factor, etc.

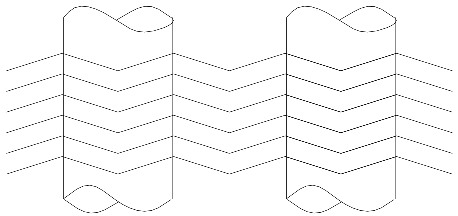


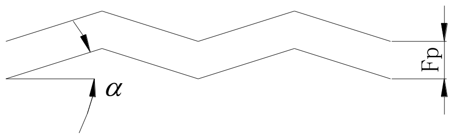
Int. Journal Heat & Fluid Flow, 1993, 14(3):246-253

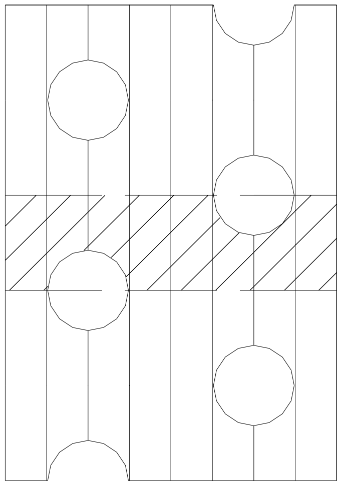


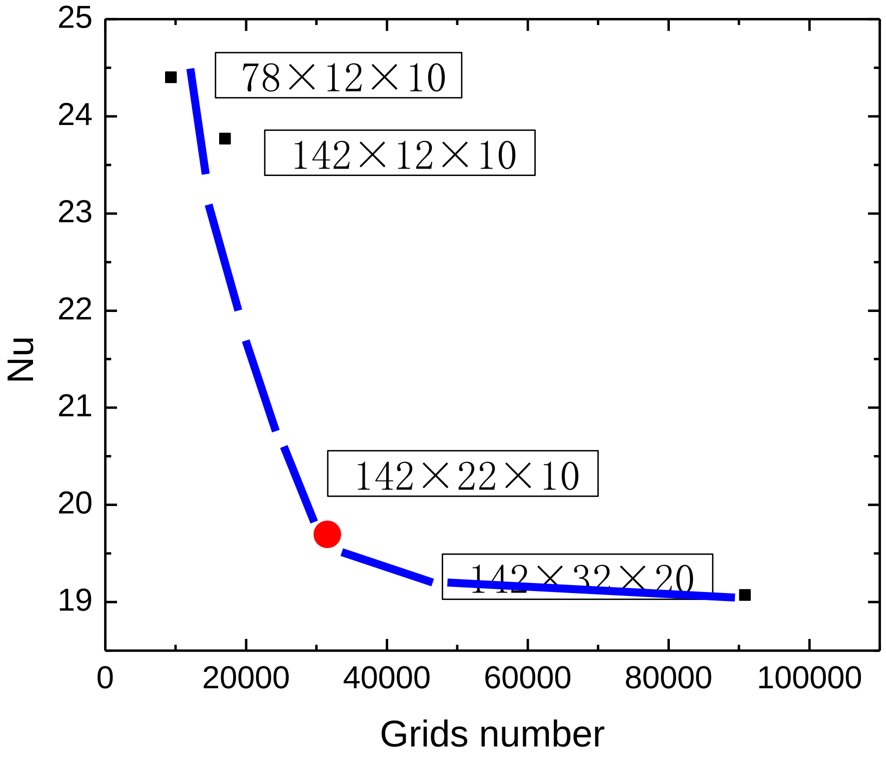
Int. Journal Numerical Methods in Fluids, 1998, 28: 1371-1387



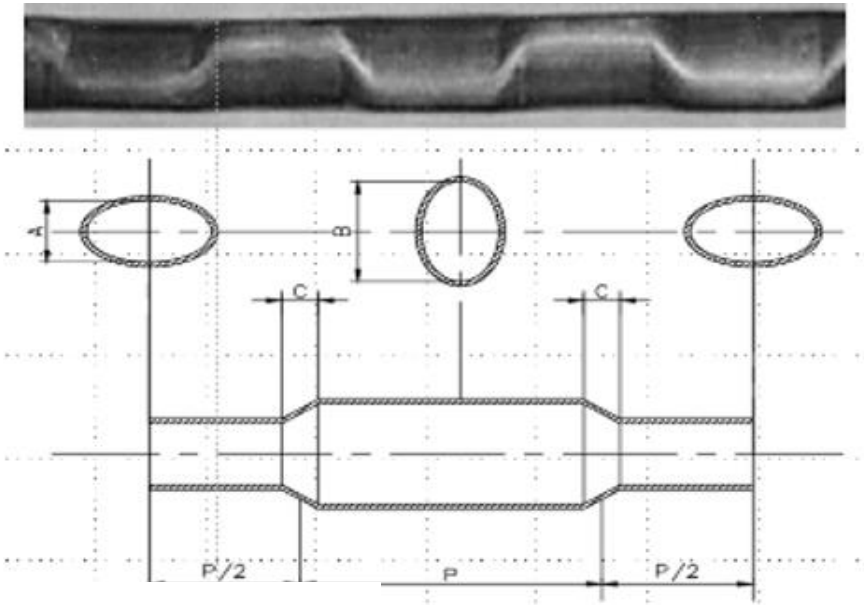


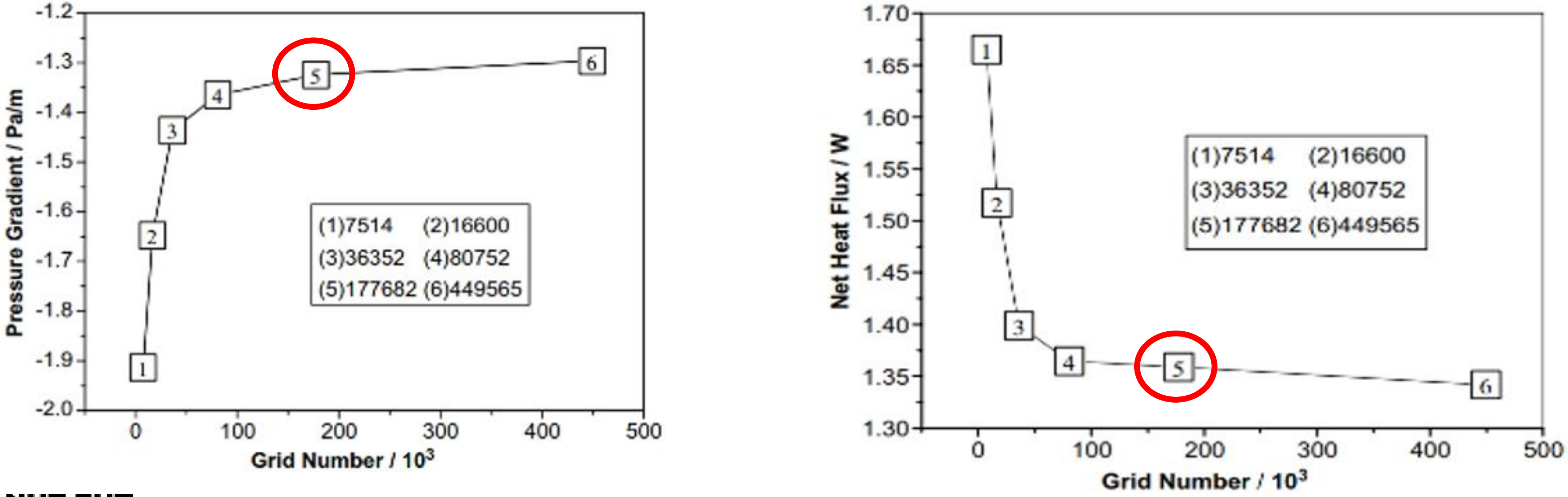






International Journal of Heat Mass Transfer, 2007, 50:1163-1175


A
B
C
C
P/2
P
P/2


Pressure Gradient / Pa/m
Grid Number / 10^3
(1)7514 (2)16600
(3)36352 (4)80752
(5)177682 (6)449565
Net Heat Flux / W
Grid Number / 10^3
(1)7514 (2)16600
(3)36352 (4)80752
(5)177682 (6)449565

Example 2-2 Discretization and numerical solution of 1-D steady heat conduction without source term

For a 1-D steady no source term heat conduction with constant properties and 1st kind boundary condition at two ends, adopting FVM to discretize the equation, expressing the algebraic equations in matrix form. Take L1 = 7

Analysis

To be more general let us work for the following 1-D steady state heat conduction of variable heat conduction area without source term:

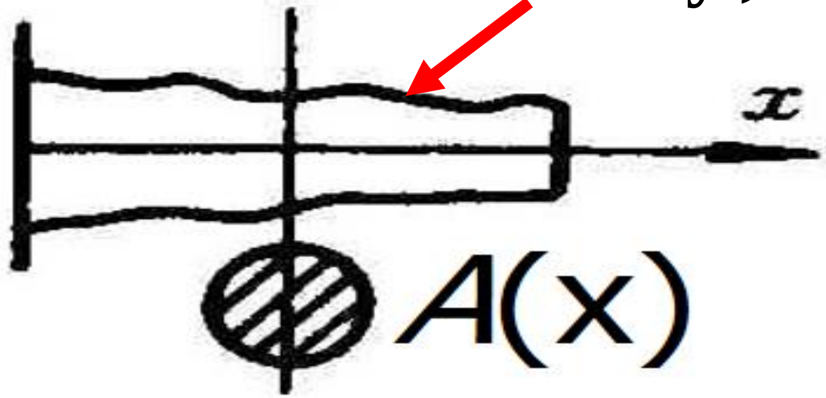
$$\frac{d}{dx}[\underbrace{\lambda A(x)\frac{dT}{dx}}_{}]=0$$

$A(x)$----Area normal to heat conduction direction;

Heat transfer rate ϕ through area A(x)

Heat transfer rate ϕ through area A(x) is a constant for 1-D problem, hence its 1st-order derivative equal zero!

Geometrical boundary , not physical boundary!

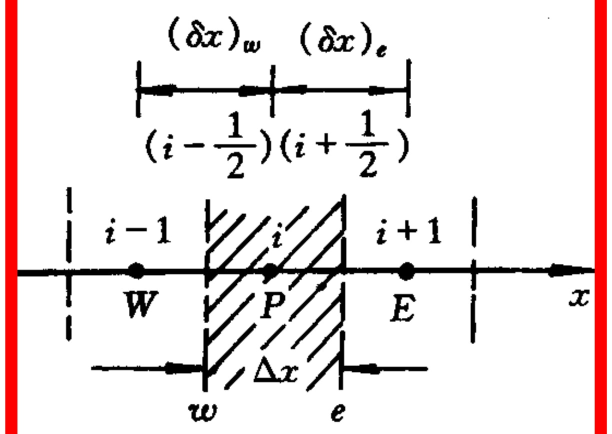


When temperature varies only in x-direction , the problem can be regarded one-dimensional.

Solution

Integrating over control volume P:

$$\frac{d}{dx}[\lambda A(x)\frac{dT}{dx}]=0$$



yielding(得)

$$[\lambda A(x)\frac{dT}{dx}]_e - [\lambda A(x)\frac{dT}{dx}]_w = 0$$

Using the piecewise linear profile for temperature:

$$\lambda_e A_e(x)\frac{T_E - T_P}{(\delta x)_e} - \lambda_w A_w(x)\frac{T_P - T_W}{(\delta x)_w} = 0$$

Moving terms with T_P to left side while those with T_E, T_W to right side

$$T_P[\frac{A_e(x)\lambda_e}{(\delta x)_e} + \frac{A_w(x)\lambda_w}{(\delta x)_w}] = T_E[\frac{A_e(x)\lambda_e}{(\delta x)_e}] + T_W[\frac{A_w(x)\lambda_w}{(\delta x)_w}]$$

We adopt following well-accepted form for discretized eqs.:

$$a_P T_P = a_E T_E + a_W T_W + b$$

$$a_E = \frac{\lambda_e A(x)_e}{(\delta x)_e},\ a_W = \frac{\lambda_w A(x)_w}{(\delta x)_w};$$

$$a_P = a_E + a_W;\ b = 0$$

Substituting the following values to above equation, $A(x) = 1, \lambda = 1$, $L1 = 7$, $X(7) = 1$ following matrix can be obtained

$$\begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -10.00 & 15.00 & -5.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -5.00 & 10.00 & -5.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -5.00 & 10.00 & -5.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -5.00 & 10.00 & -5.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -5.00 & 15.00 & -10.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \end{bmatrix} = \begin{bmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 1.00 \end{bmatrix}$$

Tridiagonal (三对角)

$$a_E = \frac{\lambda_e A(x)_e}{(\delta x)_e},\ a_W = \frac{\lambda_w A(x)_w}{(\delta x)_w};$$

We temporary remain the interface thermal conductivities in the above equation ,and will discuss how to interpolate them by node thermal conductivities in Chapter 3.

How to develop the code for solving the matrix equation will be introduced in the after-class tutoring.

Example 2-3 Numerical solution for 1-D no source term transient heat conduction with third kind of boundary condition.

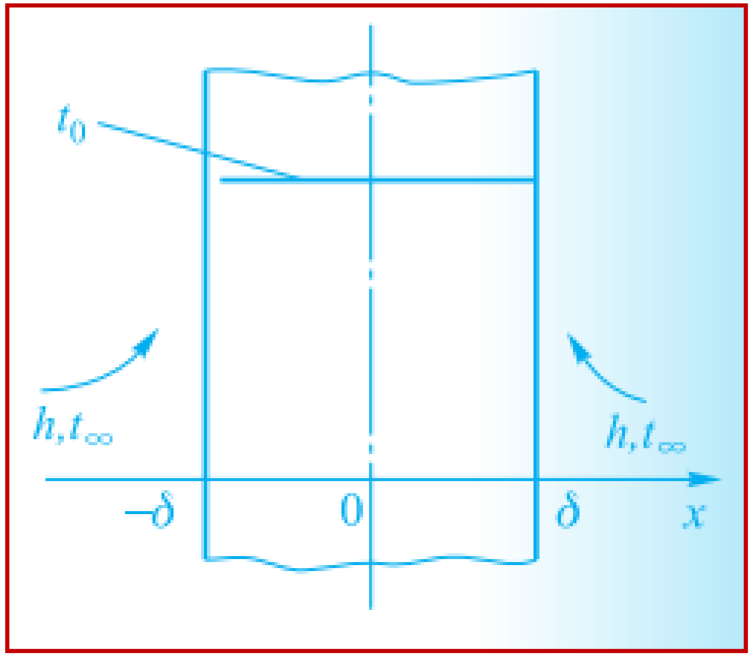
Given an infinite plate of thickness of 2δ with constant λ、a. At t=0, T= T_0. Suddenly it is placed into a large liquid volume with temperature T_∞ ; the heat transfer coefficients of its two surfaces is h. Adopting the fully implicit scheme to solve the temperature distribution at some typical instants.

Analysis

Adopting coordinate as shown, the math formulation is

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2}$$

Solution can be conducted for half domain because of **symmetry**.



$$t = 0, \quad T = T_0 \; ; x = 0, \quad \partial T/\partial x = 0$$

Discretizing the governing **equation** by FVM， following algebraic equation is obtained:

$$a_P T_P = a_E T_E + a_W T_W + a_P^0 T_P^0$$

where

$$a_E = \frac{\lambda_e A(x)_e}{(\delta x)_e},\ a_W = \frac{\lambda_w A(x)_w}{(\delta x)_w}$$

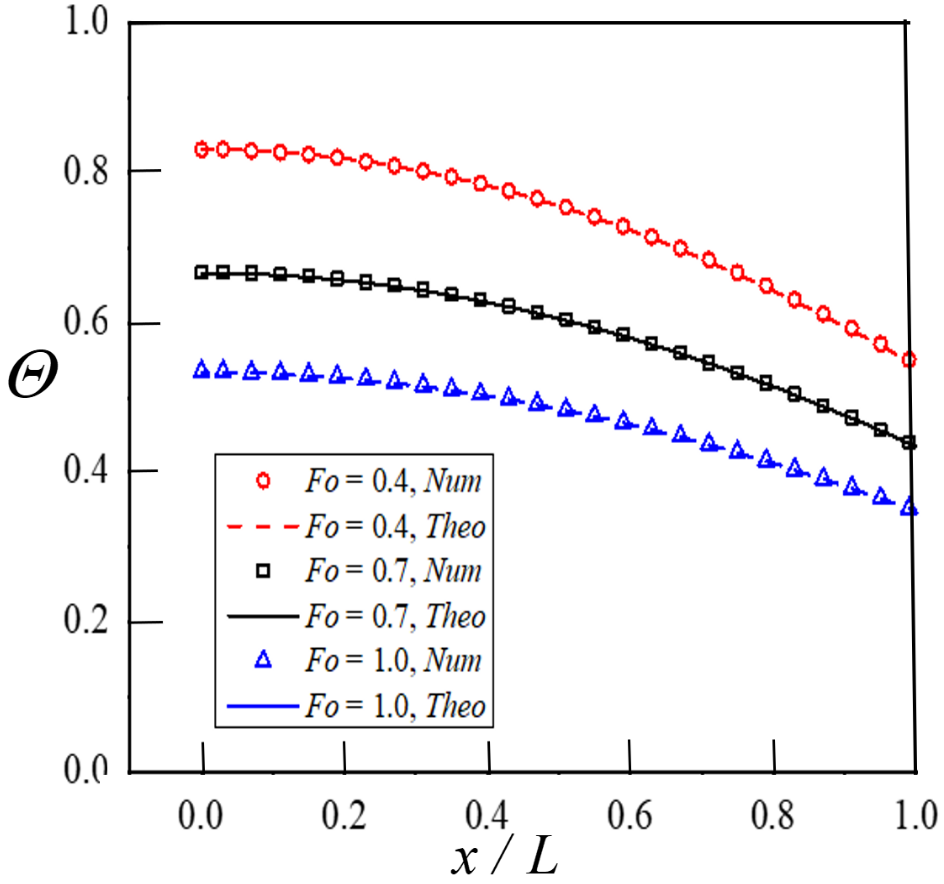
$$a_P = a_E + a_W + a_P^0$$

$$a_P^0 = \frac{\rho c A_P(x)\Delta x}{\Delta t} = \frac{\rho c \Delta V}{\Delta t}$$

Since the two boundary points are at second and _third boundary conditions, their temperature are unknown. Two more equations need to be added. The numerical treatment for second and third kinds of B.Cs. will be taught in Chapter 3.

The code to solve the algebraic equations will be presented in our after-class tutoring.

Part of the solution at three time instants in dimensionless temperature.



The following words are presented in the PPT of this chapter. If you are not familiar with them, please use AI to confirm their meanings and pronunciations (in the order of their appearance in PPT)

grid Generation	storage	forward	step-wise
construct	radian	backward	piece-wise
layout	Cartesian	stencil	interpolate
interface	biased	thumb	temporary
spatial	implement	dimension	rigorous
structured	expand	explicit	consistency
dashed line	truncation error	implicit	fineness
structured grid	exponent	profile	matrix
axe		debugging	tridiagonal

Teaching PPT will be loaded on our WeChat Group

本组网页地址：http://nht.xjtu.edu.cn 欢迎访问！

同舟共济
渡彼岸！

People in the same boat help each other to cross to the other bank, where....