Numerical Heat Transfer

(数值传热学)

Chapter 6 Primitive Variable Methods for Elliptic Flow and Heat Transfer（3）

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6.7 Boundary condition treatments for open system

6.7.1 Selections for outlet boundary

6.7.2 Treatment of outlet boundary condition without recirculation

1. Local one-way 2. Fully developed

6.7.3 Treatment of outlet boundary condition with recirculation

1. Example with recirculation ; 2. Suggestion

6.7.4 Methods for outlet normal velocity satisfying total mass conservation

1. Two cases; 2. Application

6.7 Boundary condition treatments for open system

6.7.1 Selections for outlet boundary position

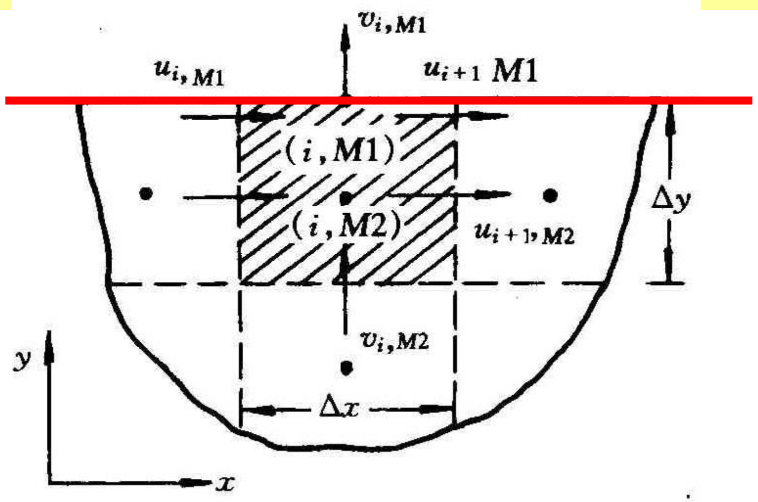
1. It should be at the location without recirculation **(回流)**---Suggested by Patankar

2 .If it is at the location with recirculation---special attention should be paid for boundary condition treatment

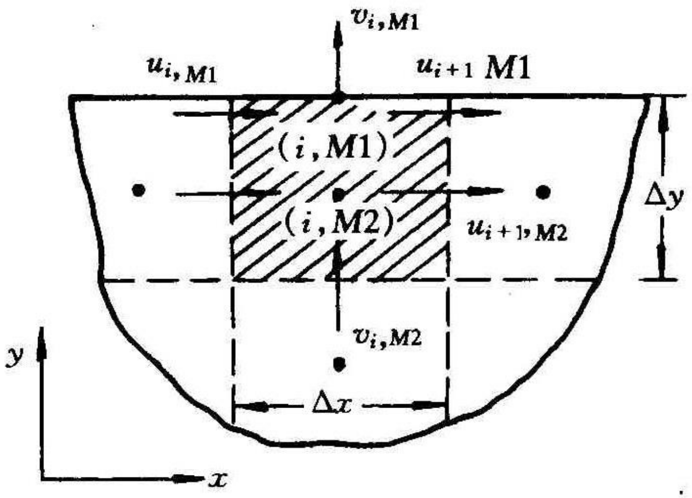
6.7.2 Treatment of B.C. without recirculation

1. Local one-way assumption **(局部单向化假设)**

$$\boxed{(a_N)_{i,M2} = 0}$$



2. Fully developed $\left.\frac{\partial \phi}{\partial n}\right)_{i,M1} = 0$



(1) Updating (**更新**)boundary value

$$\frac{\phi_{i,M1} - \phi_{i,M2}}{(\delta y)_B} = 0 \longrightarrow \phi_{i,M1} = \phi^*_{i,M2}$$

(2) ASTM：

Taking $\left.\frac{\partial \phi}{\partial n}\right)_{i,M1} = 0$ as given heat flux condition

For both methods， the outlet normal velocity must satisfy the total mass conservation condition.

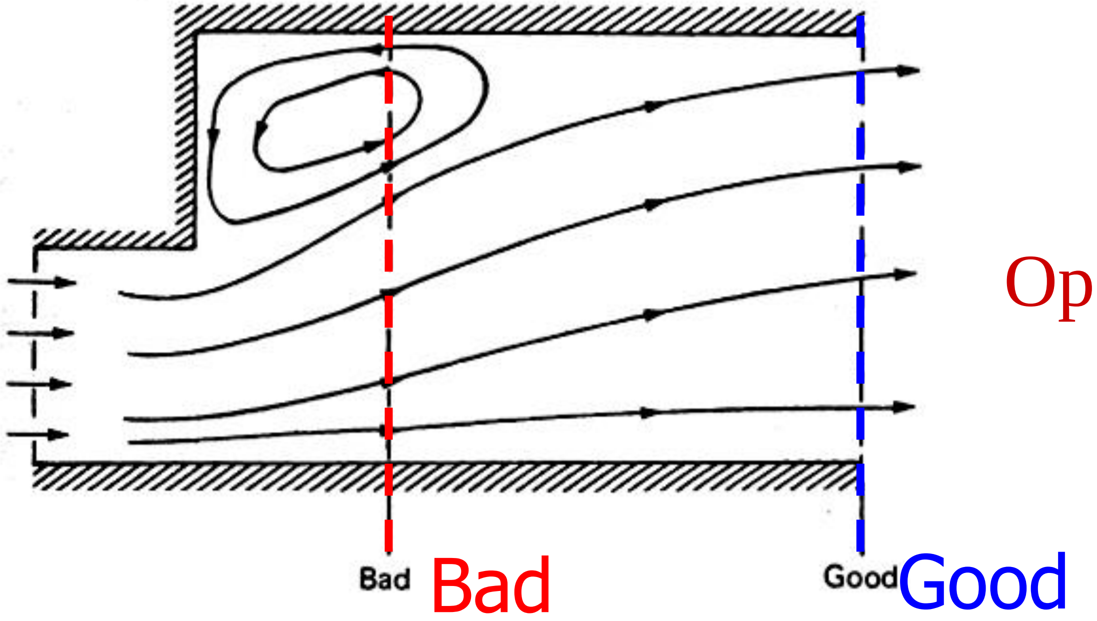
6.7.3 Treatment of outlet boundary condition with recirculation

1. Necessity (**必要性**) for such selection

Required from some practical problems.

According to Patankar， the outlet boundary of the sudden expansion case must be positioned at the location without recirculation (“Good” position）. It should not be positioned at the “Bad” location, otherwise the results are meaningless.

This suggestion not only needs more computer memory but is also not possible for some situations.



Open flow system

If the neglect of the diffusion at an outflow boundary appears, for some reason, to be serious, then we should conclude that the analyst has placed the outflow boundary at an inappropriate location. A repositioning of the boundary would normally make the outflow treatment acceptable. A particularly bad choice of an outflow-boundary location is the one in which there is an "inflow" over a part of it. An example of this is shown in Fig. 5.12. For su bad choice of the boundary, no meaningful solution can be obtained.

This may be a convenient place to review the boundary-condition practices for convection-diffusion problems. Whenever there is no fluid flow across the boundary of the calculation domain, the boundary flux is purely a diffusion flux, and the practices described in Chapter 4 apply. For those parts of the boundary where the fluid flows *into* the domain, usually the values of ϕ are known. (The problem is not properly specified if we do not know the value of ϕ that a fluid stream brings with it.) The parts of the boundary where the fluid *leaves* the calculation domain form the outflow boundary, which we have already discussed.

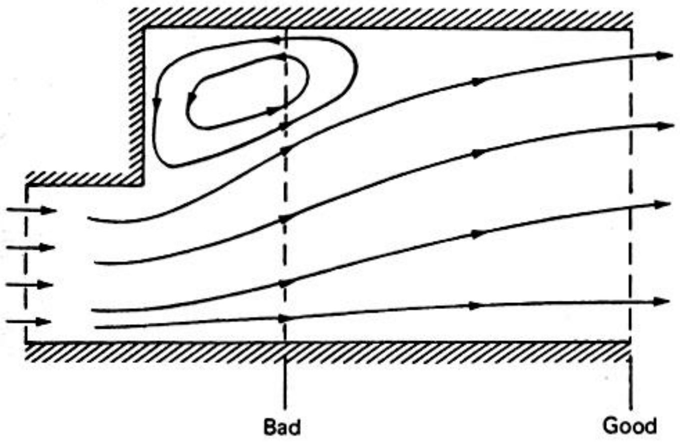
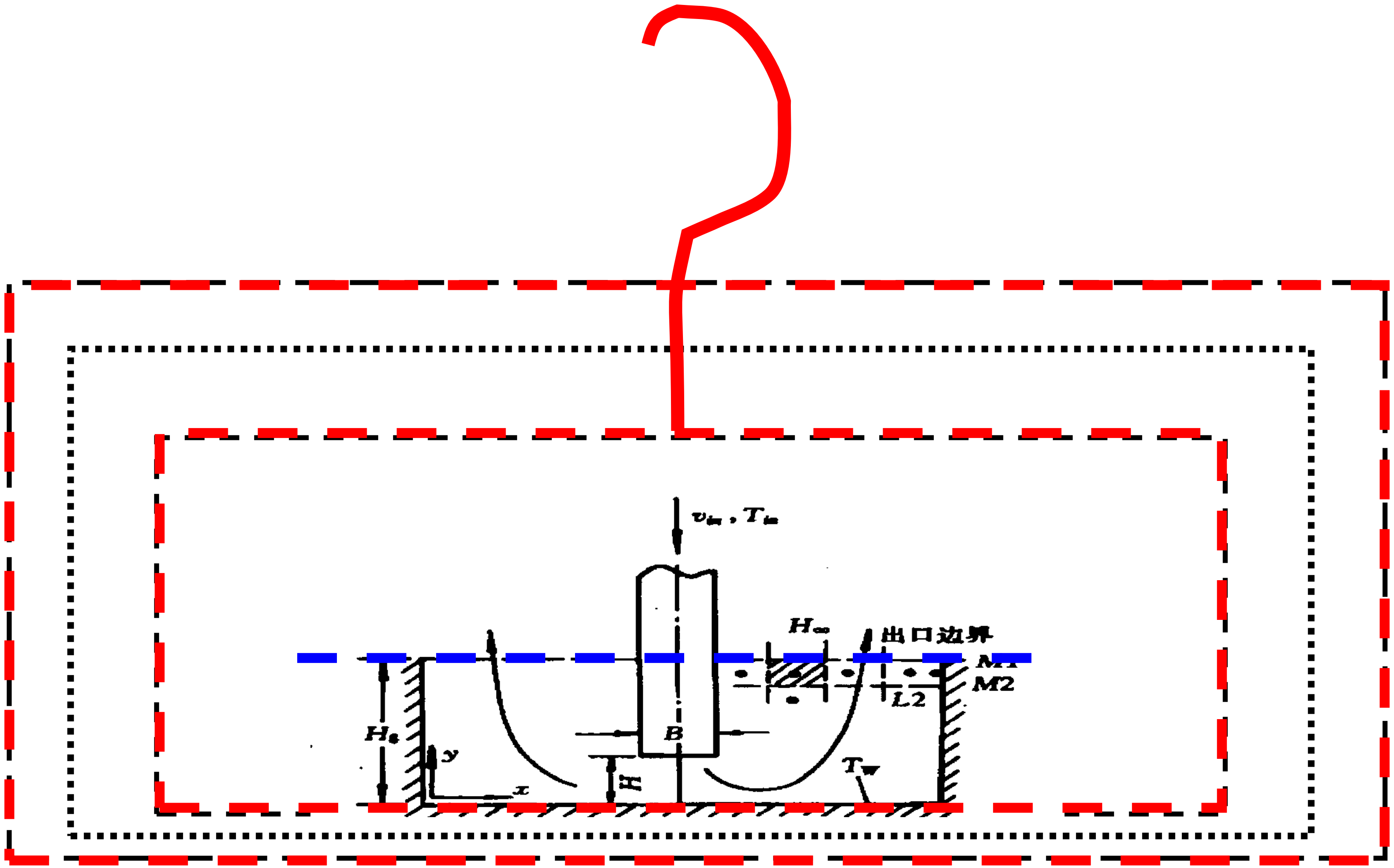


Figure 5.12 Good and bad choices of the location of the outflow boundary.

A particular bad choice of an outflow-boundary location is the one in which there is an "inflow" over a part of it. … For such a bad choice of the boundary, no meaningful solution can be obtained .(1980)



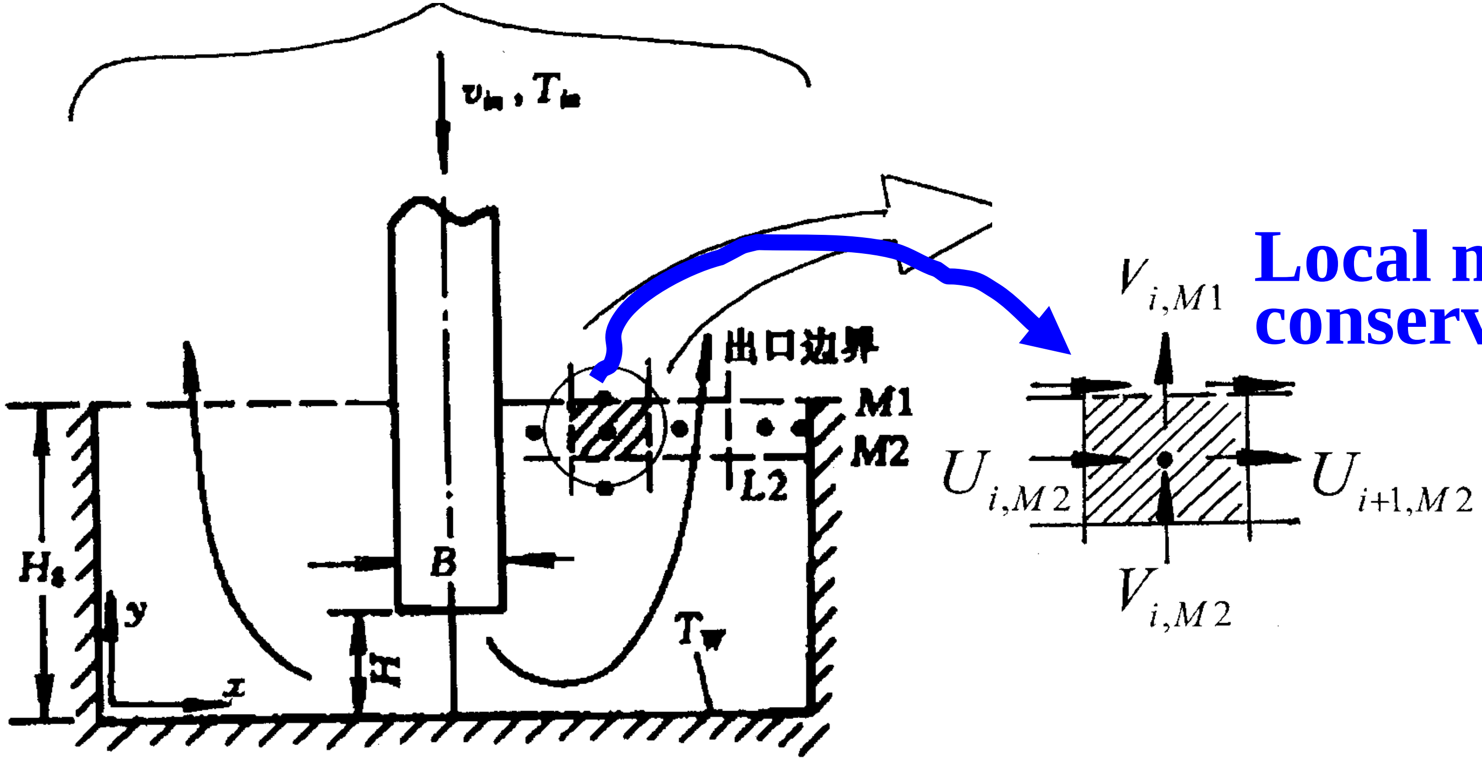
Cooling of plate TV screen

2. Suggestion

(1) Outlet normal velocity---treated according to local mass conservation

(2) Outlet parallel velocity---treated by homogeneous Neumann condition (**齐次诺曼条件,一阶导数 为零**)

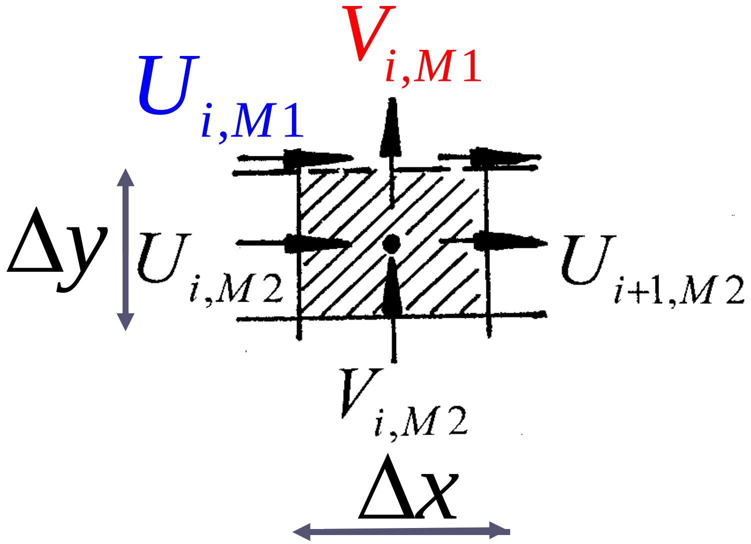
Total mass conservation



Local mass conservation

$$\frac{v_{i,M1}-v_{i,M2}}{\Delta y}+\frac{u_{i+1,M2}-u_{i,M2}}{\Delta x}=0 \rightarrow$$

$$v_{i,M1}=v^*_{i,M2}+\frac{\Delta y}{\Delta x}(u^*_{i+1,M2}-u^*_{i,M2})$$



The resulted $v_{i,M1}$ has to be corrected by total mass conservation condition.

Tangential velocity $\boxed{\frac{\partial U}{\partial y})_{i,M1}=0} \rightarrow \boxed{U_{i,M1}=U^*_{i,M2}}$

6.7.4 Methods for outlet normal velocity to satisfy total mass conservation

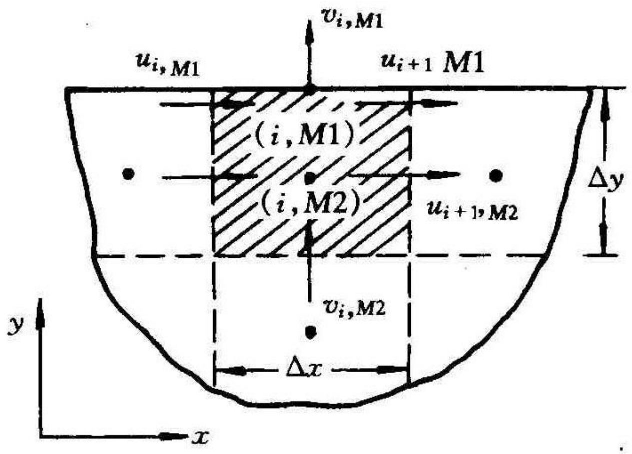
1. Two situations

1) Outlet without recirculation

(1) Relative changes of outlet normal velocity =constant

$$\frac{v_{i,M1}-v_{i,m2}}{v_{i,M2}}=k=const$$

$$v_{i,M1}=v_{i,M2}(1+k)=f\,v_{i,M2}$$



f is determined according to total mass conservation :

$$\sum_{i=2}^{L2}\rho_{i,M1}v_{i,M1}\Delta x_i=\sum_{i=2}^{L2}\rho_{i,M1}f\,v_{i,M2}\Delta x_i=FLOWIN$$

$$f=\frac{FLOWIN}{\sum_{2}^{L2}\rho_{i,M1}v_{i,M2}\Delta x_i}$$

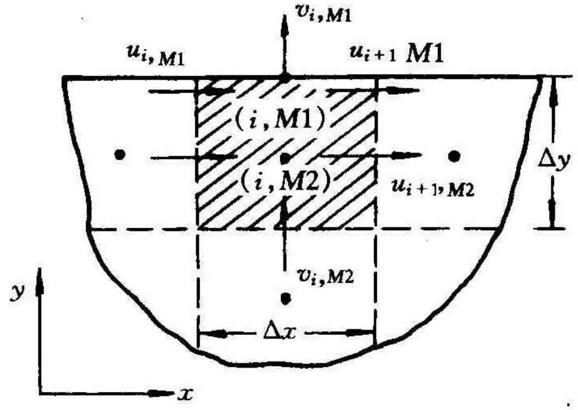
$$\boxed{v_{i,M1}=f\bullet v^{*}_{i,M2}}$$

It is taken as the boundary condition for next iteration.

(2) The 1st derivatives at outlet =const.

$$\frac{v_{i,M1}-v_{i,M2}}{\Delta y}=k=const$$

$$v_{i,M1}=v_{i,M2}+k\Delta y=v_{i,M2}+C$$



C is determined according to total mass conservation.

$$\sum_{i=2}^{L2}\rho_{i,M1}(v_{i,M2}+C)\Delta x_i=FLOWIN$$

$$C=\frac{FLOWIN-\sum\rho_{i,M1}v_{i,M2}\Delta x_i}{\sum\rho_{i,M1}\Delta x_i}$$

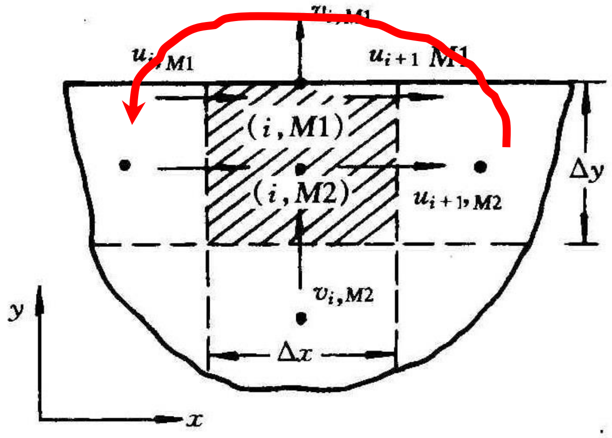
$v_{i,M1}=v^*_{i,M2}+C$ is taking as boundary condition for next iteration.

When flow is fully developed at outlet,: $f=1, C=0$;

Otherwise there is some differences between the two treatments, but such difference is not important.

2) Outlet with recirculation

$\tilde{v}_{i,M1}$ is the normal velocity determined by local mass conservation， then



$$\sum_{i=2}^{L2} \rho_{i,M1}(f \bullet \tilde{v}_{i,M1})\Delta x = FLOWIN$$

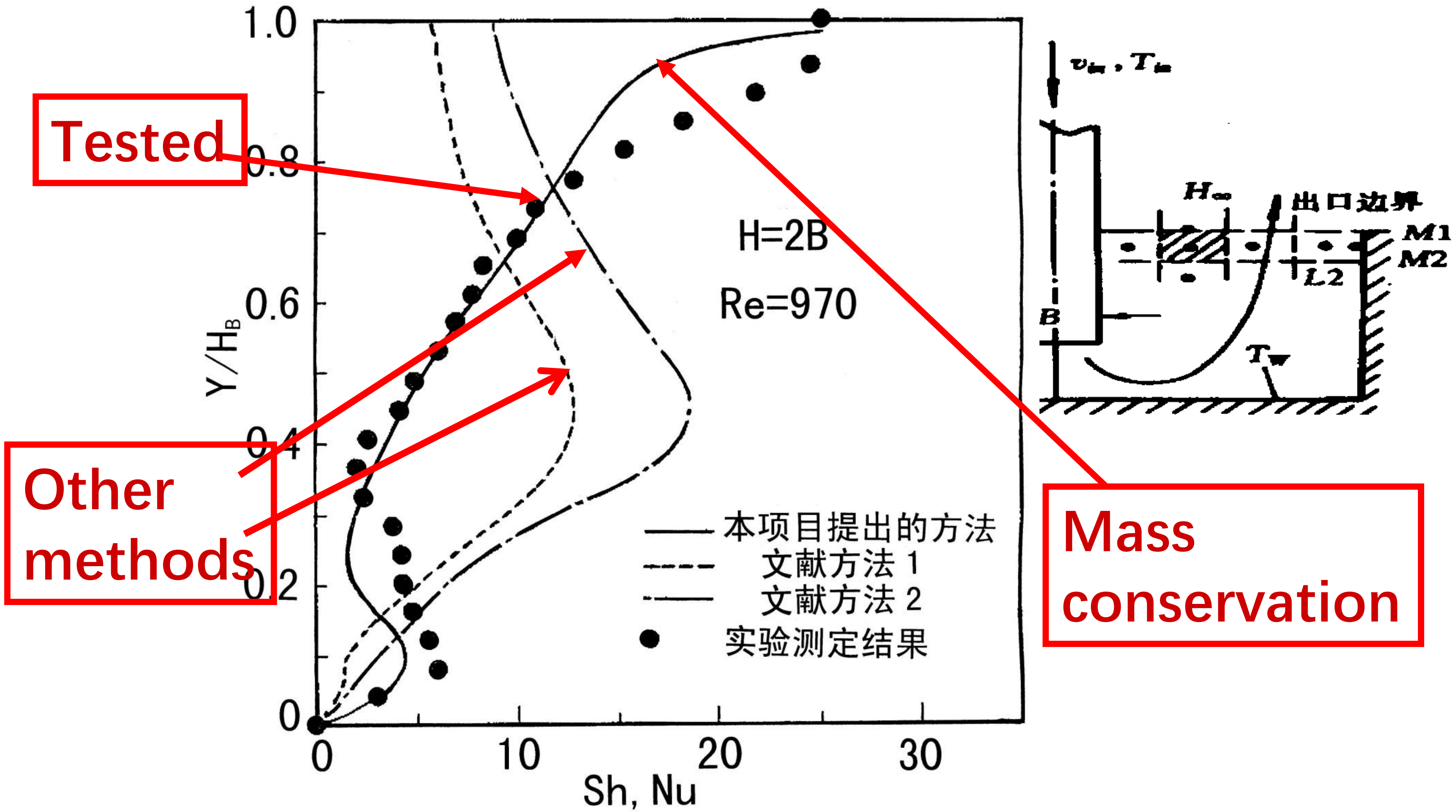
$$\boxed{f = FLOWIN \Big/ \left(\sum_{i=2}^{L2} \rho_{i,M1} \tilde{v}_{i,M1} \Delta x_i\right)}$$

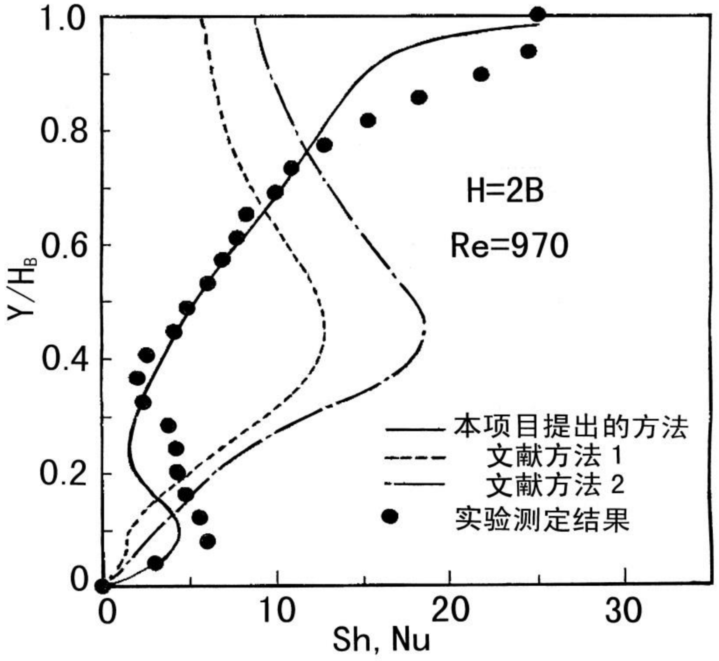
$v_{i,M1} = f \bullet \tilde{v}_{i,M1}$ is taking as the boundary condition for next iteration.

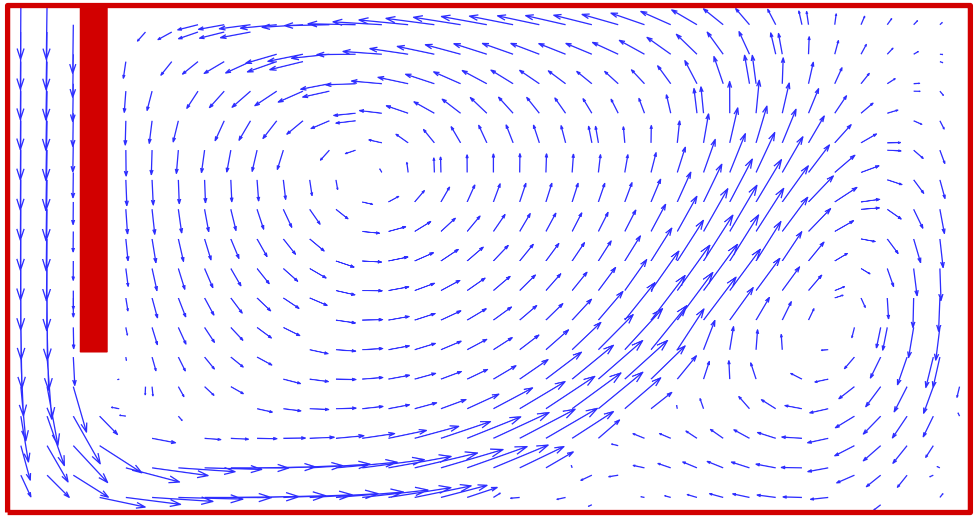
2. Applications

Li PW, Tao WQ. Effects of outflow boundary condition on convective heat transfer with strong recirculation flow. Warme- Stoffubertrag,1994, 29 (8): 463-470

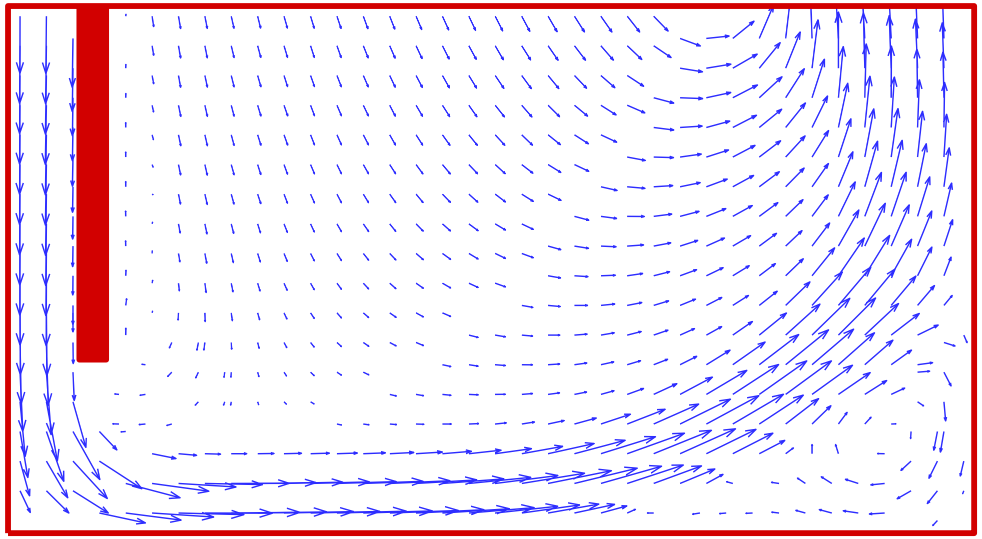
Comparison of predicted and measured local heat transfer coefficients along vertical wall







(a) Mass conservation



(b) Reference

Effect of outflow boundary condition on flow pattern

6.8 Fluid Flow and Heat Transfer in a Closed System

6.8.1 Natural convection in an enclosure

1. Boussinesq assumption
2. Governing equs. of natural convection in enclosure
3. Effective pressure in natural convection in enclosure
4. Governing eqs. with Boussinesq assumption and effective pressure

6.8.2 Numerical treatments of island (孤岛)

1. Method for setting zero velocity in island
2. Method for setting given temperature in island

6.8 Fluid Flow and Heat Transfer in a Closed system

6.8.1 Natural convection in enclosure

1. Boussinesq assumption

1) Viscous dissipation(耗散) is neglected;

2) Thermo-physical properties are constant except density;

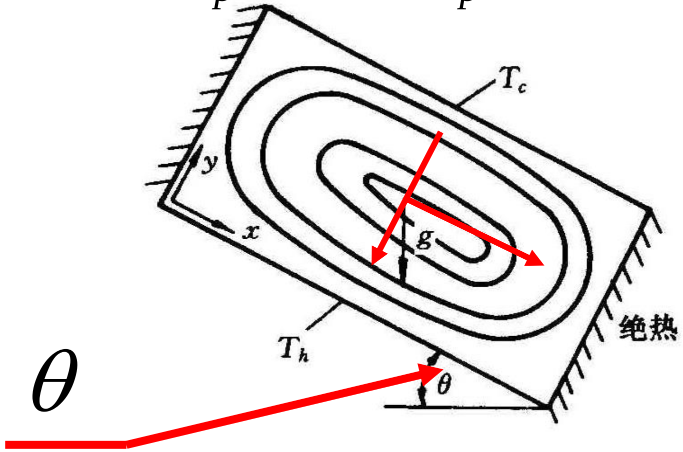
3) Only the density in the gravitational term is considered varying with temperature as follows:

$$\rho = \rho_r[1-\alpha(T-T_r)] \qquad \alpha \text{ -expansion coefficient}$$

2. Governing equations of natural convection in an enclosure

Governing equations for steady natural convection in a rectangular cavity:

$$
\begin{cases}
\dfrac{\partial(\rho u^2)}{\partial x}+\dfrac{\partial(\rho uv)}{\partial y}=-\dfrac{\partial p}{\partial x}+\dfrac{\partial}{\partial x}(\eta\dfrac{\partial u}{\partial x})+\dfrac{\partial}{\partial y}(\eta\dfrac{\partial u}{\partial y})+\rho g\sin\theta \\[2ex]
\dfrac{\partial(\rho uv)}{\partial x}+\dfrac{\partial(\rho v^2)}{\partial y}=-\dfrac{\partial p}{\partial y}+\dfrac{\partial}{\partial x}(\eta\dfrac{\partial v}{\partial x})+\dfrac{\partial}{\partial y}(\eta\dfrac{\partial v}{\partial y})-\rho g\cos\theta \\[2ex]
\dfrac{\partial(\rho uT)}{\partial x}+\dfrac{\partial(\rho vT)}{\partial y}=\dfrac{\partial}{\partial x}(\dfrac{\lambda}{c_p}\dfrac{\partial v}{\partial x})+\dfrac{\partial}{\partial y}(\dfrac{\lambda}{c_p}\dfrac{\partial v}{\partial y})+\dfrac{S_T}{c_p} \\[2ex]
\dfrac{\partial(\rho u)}{\partial x}+\dfrac{\partial(\rho v)}{\partial y}=0
\end{cases}
$$



3.Effective pressure in natural convection in enclosure

$$\frac{\partial(\rho u^2)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}(\eta\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(\eta\frac{\partial u}{\partial y})+\rho g\sin\theta$$

$$-\frac{\partial p}{\partial x}+\rho g\sin\theta=-\frac{\partial p}{\partial x}+\rho_c g\sin\theta[1-\alpha(T-T_c)]$$

$$\boxed{\rho=\rho_r[1-\alpha(T-T_r)]}$$

$$=-\frac{\partial p}{\partial x}+g\rho_c\sin\theta-g\rho_c\alpha(T-T_c)\sin\theta$$

$$\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(\rho v^2)}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}(\eta\frac{\partial v}{\partial x})+\frac{\partial}{\partial y}(\eta\frac{\partial v}{\partial y})-\rho g\cos\theta$$

$$-\frac{\partial p}{\partial y}-\rho g\cos\theta=-\frac{\partial p}{\partial y}-\rho_c g\cos\theta[1-\alpha(T-T_c)]$$

$$\boxed{\rho=\rho_r[1-\alpha(T-T_r)]}$$

$$=-\frac{\partial p}{\partial y}-g\rho_c\cos\theta+g\rho_c\alpha(T-T_c)\cos\theta$$

From $-\frac{\partial p}{\partial x}+g\rho_c \sin\theta, \quad -\frac{\partial p}{\partial y}-g\rho_c\cos\theta$ an effective pressure is introduced:

$$p_{eff}=p-(g\rho_c\sin\theta)x+(g\rho_c\cos\theta)y$$

Then

$$\frac{\partial p_{eff}}{\partial x}=\frac{\partial p}{\partial x}-g\rho_c\sin\theta \qquad \frac{\partial p_{eff}}{\partial y}=\frac{\partial p}{\partial y}+g\rho_c\cos\theta$$

The gradient results are the same as in the moment. eqs.

Order of magnitude estimation(**数量级估计**) for $g\rho y$

For air: set y=1m, $g=9.8ms^{-2}$, $\rho=1.2kg\bullet m^{-3}$

Then pressure introduced by natural convection is: $9.8m{\bullet}s^{-2}\times 1.2kg{\bullet}m^{-3}\times 1m=11.76\boxed{kg{\bullet}m{\bullet}s^{-2}}/m^2$

$11.76\,N/m^2=11.76Pa$

4. Mathematical formulation with Boussinesq assumption and effective pressure

Re-write ρ_c in the buoyancy term as ρ

$$\left\{\begin{aligned}
&\frac{\partial(\rho u^2)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}=-\frac{\partial p_{eff}}{\partial x}+\frac{\partial}{\partial x}(\eta\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(\eta\frac{\partial u}{\partial y})-\rho g\alpha(T-T_c)\sin\theta\\
&\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(\rho v^2)}{\partial y}=-\frac{\partial p_{eff}}{\partial y}+\frac{\partial}{\partial x}(\eta\frac{\partial v}{\partial x})+\frac{\partial}{\partial y}(\eta\frac{\partial v}{\partial y})+\rho g\alpha(T-T_c)\cos\theta\\
&\frac{\partial(\rho uT)}{\partial x}+\frac{\partial(\rho vT)}{\partial y}=\frac{\partial}{\partial x}(\frac{\lambda}{c_p}\frac{\partial v}{\partial x})+\frac{\partial}{\partial y}(\frac{\lambda}{c_p}\frac{\partial v}{\partial y})+\frac{S_T}{c_p}\\
&\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0
\end{aligned}\right.$$

buoyancy term

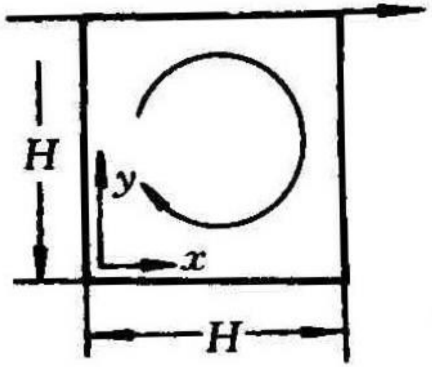
With correspondent boundary condition.

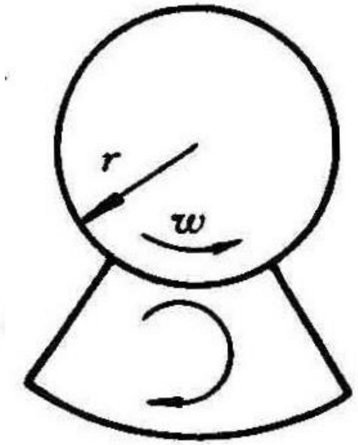
5. Typical results of 2-D natural convection in enclosure

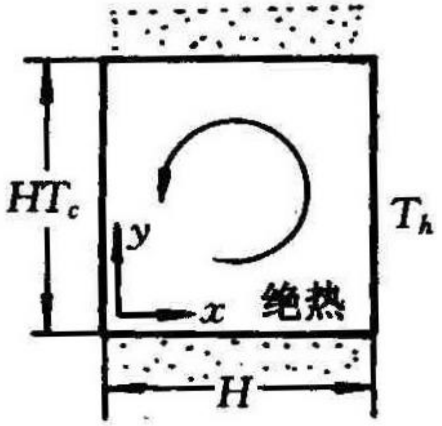
Table 6-8 2-D natural convection in enclosure of air

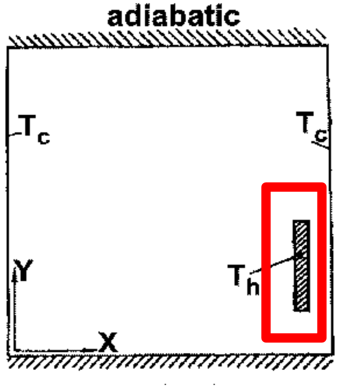
计算的量	$Ra=g\alpha(T_h-T_c)H^3/a\nu$			
	10^3	10^4	10^5	10^6
Nu	1.114	2.245	4.510	8.806
Nu_{max}	1.581	3.539	7.637	17.442
$(Y/H)_{max}$	0.099	0.143	0.085	0.036 8
Nu_{min}	0.670	0.583	0.773	1.001
$(Y/H)_{min}$	0.994	0.994	0.999	0.999
U_{max}	0.153	0.193	0.132	0.077
$(Y/H)u_{max}$	0.806	0.818	0.859	0.859
V_{max}	0.155	0.234	0.258	0.262
$(X/H)v_{max}$	0.181	0.119	0.066	0.039

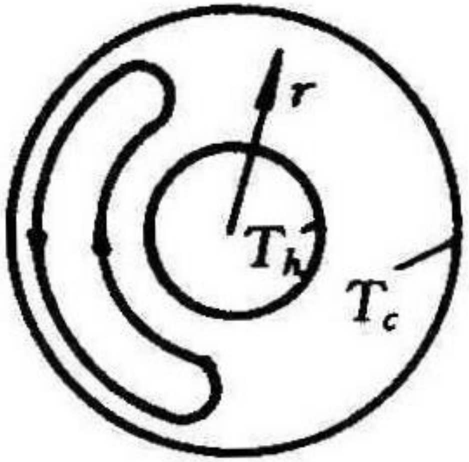
6. Other examples of 2D flow in enclosure

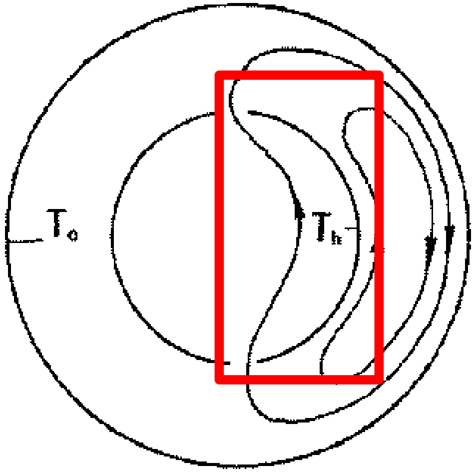












6.8.2 Numerical treatments for isolated island

Isolated island—solid region positioned within fluid region without connection with solid boundary.

An effective numerical method to deal with the island is regarding the island as a special fluid region with very large viscosity. The key issue in such method is to realize zero velicity in the isolated region.

1. Techniques guaranteeing zero velocity in island

(1) Setting zero initial values for u^0, v^0 in island at each iteration ---Pay attention to the features of staggered grid system;

(2) Setting very large coefficients (say 10^{25}) of the main-diagonal element at each iteration which leads to near-zero values of the intermidiate values of u^*, v^* in the island;

$$u_e = \sum \frac{a_{nb}u_{nb} + b}{a_e} + \frac{A_e}{a_e}(p_P - p_E); \quad \frac{A_e}{a_e} = d_e$$

(3) Setting near zero values for coefficient d in island at each iteration , say 10^{-25} which leads to near-zero values of u' and v';

$$u_e' = d_e \Delta p_e'$$

(4) Setting the solid diffusion coefficient very large (say 10^{25}) and adopting harmonic mean for interface interpolation. This will transferring near zero velocity in the island to its boundary.

2. Method for setting given temperature in island

(1) Large coefficient method—at each iteration resetting the coefficients in the correspondent discretized equations in island:

$$a_P\phi_P = \sum a_{nb}\phi_{nb} + b$$

$a_P = A$ (very large), and $b = A\phi_{given}$, $A = 10^{20} \sim 10^{30}$

(2) Large source term method (from Patanker) —at each iteration resetting source terms in island:

$$S_c = A\phi_{given}, S_P = -A, A = 10^{20} \sim 10^{30}$$

$$(\cancel{a_E + a_W + a_N + a_S} - S_P\Delta V)T_P = \cancel{a_E T_E + a_W T_W + a_N T_N + a_S T_S} + S_C\Delta V$$

This method is effective only when $\alpha = 1$

Remarks： In order to guarantee continuity of flux at the solid-fluid interface—the specific heat of solid region should takes the value of fluid region.

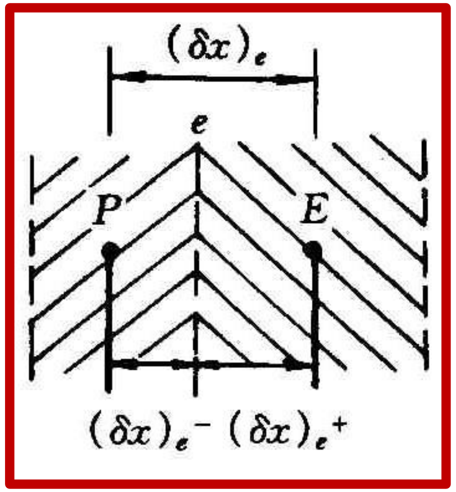
The harmonic mean for interface conductivity:

$$\frac{(\delta x)_e}{\lambda_e} = \frac{(\delta x)_{e^+}}{\lambda_E} + \frac{(\delta x)_{e^-}}{\lambda_P}$$

For interface conductivity!

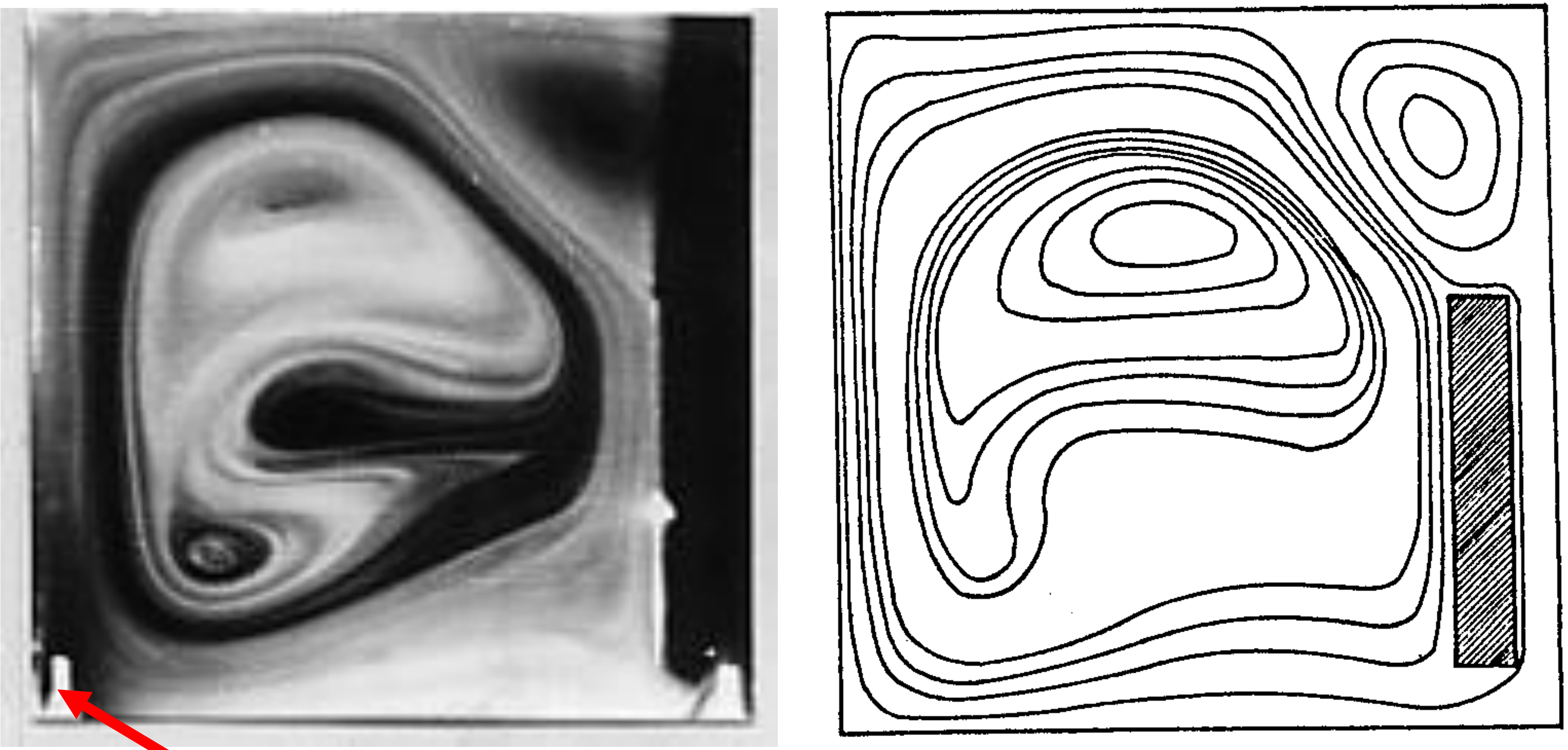
But here the normal diffusivity is used: $\Gamma = \lambda / c_p$

In order that harmonic mean is still valid for Γ, the specific heat must be the same at the two sides:



$$\frac{(\delta x)_e}{\lambda_e /(c_p)_f} = \frac{(\delta x)_{e^+}}{\lambda_E /(c_p)_f} + \frac{(\delta x)_{e^-}}{\lambda_P /(c_p)_f}$$

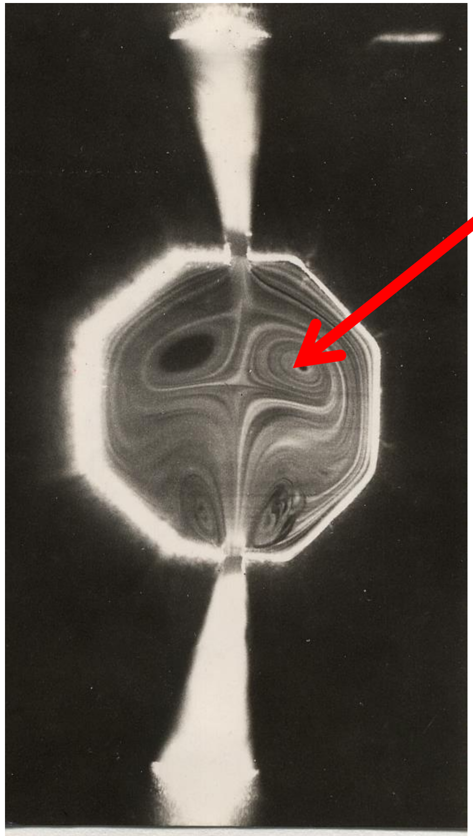
Such a practice is not convenient!



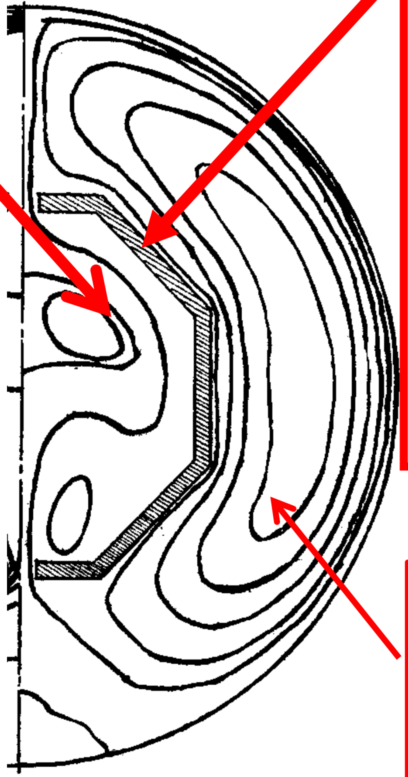
Gas inlet

Example of isolated island： comparison of numerical prediction and visualization （Hot island and cold enclosure wall）

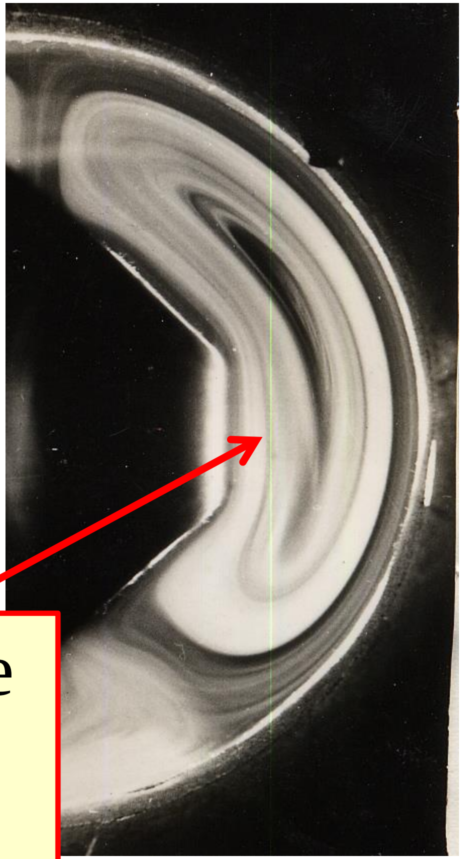
Wang QW, Yang M, Tao WQ. Natural convection in a square enclosure with an internal isolated vertical plate. Warme- Stoffubertrag , 1994, 29 (3): 161-169



Cat eye



Island—
2-D model
of bus bar
for large
electric
current



Kidney type vortex
（肾状涡）

Visualization for the core(心部)

Numerical results

Visualization for annulus(环)

Comparison of predicted and visualized natural convection in large electric current bus bar (大电流母线)

Zhang HL, Wu QJ, Tao WQ. ASME J Heat Transfer , 1991, 113 (1): 116-121

Home Work 6（2024-2025）

Please finish your homework independently !!!

Please hand in on Nov. 5

Problem 6-1 (Problem 6-1 of Textbook)

In the governing equations of incompressible fluid flow there is no governing equation for pressure. But a pressure equation can be derived from the momentum and continuity equations as follows:

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=2\left[\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial y}\right)-\left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right)\right]$$

Try (1) to derive this pressure Poisson equation; (2) to discuss that whether we can combine this equation with momentum and continuity equations to solve the flow fields, so that the special algorithm such as SIMPLE is not needed.

Problem 6-2

For the control volume shown in the figure, following conditions are known:

$$p_E = 15, p_N = 5 \quad u_w = 45, v_s = 25$$

$$u_e = 0.65(p_P - p_E) \; v_n = 0.5(p_P - p_N)$$

Find the values of p_P, u_e, v_n by using the concept of the SIMPLE algorithm.

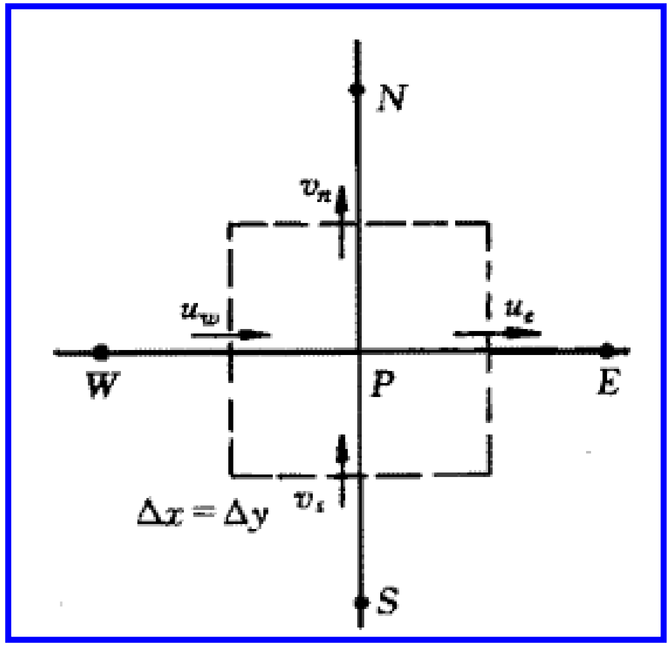


Figure of Prob.6-2

Problem 6-3 (Problem 6-3 of Textbook, modified)

For the following momentum discretized equation of steady incompressible flow,

$$a_e u_e = a_{nb} u_{nb} + b + A_e (p_P - p_E)$$

write the expressions for the coefficients at the location on the interface e, a_{nb} (i.e., a_E, a_W, a_N, a_S), a_e, A_e .

Adopt uniform grid system in x and y coordinates. Assume constant properties. See the figure shown. For the diffusion term the CD be used. For the convective term the 2nd-order central difference scheme be used.

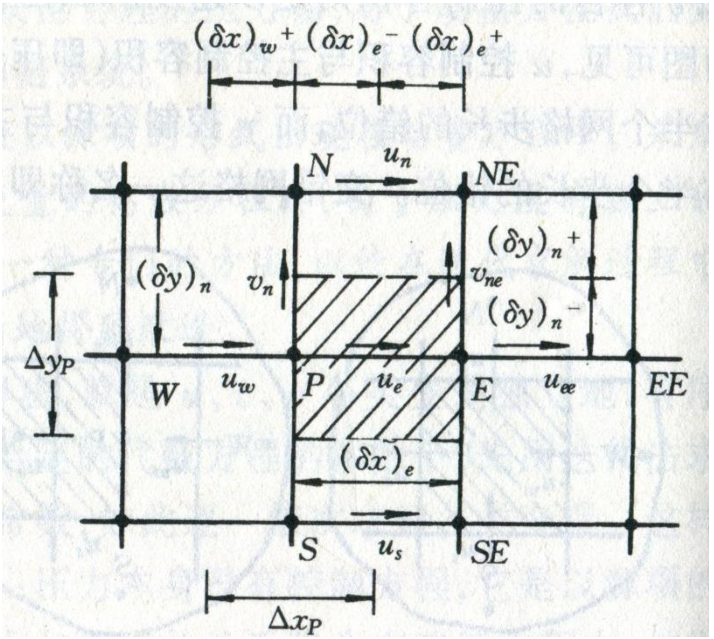


Figure of Prob.6-3

Problem 6-4

A piping system shown is applied to pump fluid from node 1 to node 2,3,4,5,6,and 7. The pressures of nodes 1,2,4,and 5 are given in the parentheses. The flow rate between two nodes can be calculated by $Q = C(\Delta p)$, where Δp is the pressure drop between the two neighboring nodes, and C is the hydraulic conductivity. For simplicity the conductivity of the two adjacent nodes is expressed by the letter between the two nodes. C_A=0.45, C_B=0.25, C_C=0.2,C_D=0.2, C_E=0.15, C_F=0.2。The flow rate between nodes 6 and 7 is Q_F=15. Try to find the values of p_3,p_6, p_7, Q_A,Q_B,Q_C,Q_D and Q_E by using the concept of the SIMPLE algorithm.(Hint: Reference to SIMPLE algorithm. Assume p_3^*, p_6^* to get the flow rate and calculate the pressure correction value using mass conservation relation of nodes 3 and 6.)

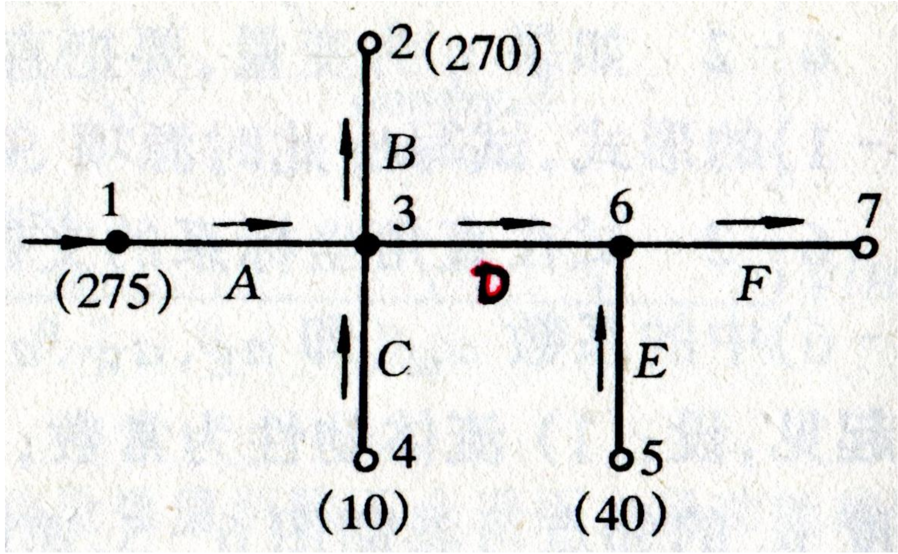


Figure of Prob.6-4

Problem 6-5 (Problem 6-12 of Textbook)

Try to combine the algorithms of SIMPLER and SIMPLEC, and write the solution steps for one level of iteration for the new algorithm.

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Teaching PPT will be loaded on ou website

同舟共济

渡彼岸！

People in the same boat help each other to cross to the other bank, where....