Numerical Heat Transfer
(数值传热学)
Chapter 1 Introduction

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1.1 Mathematical formulation of heat transfer and fluid flow (HT & FF) problems

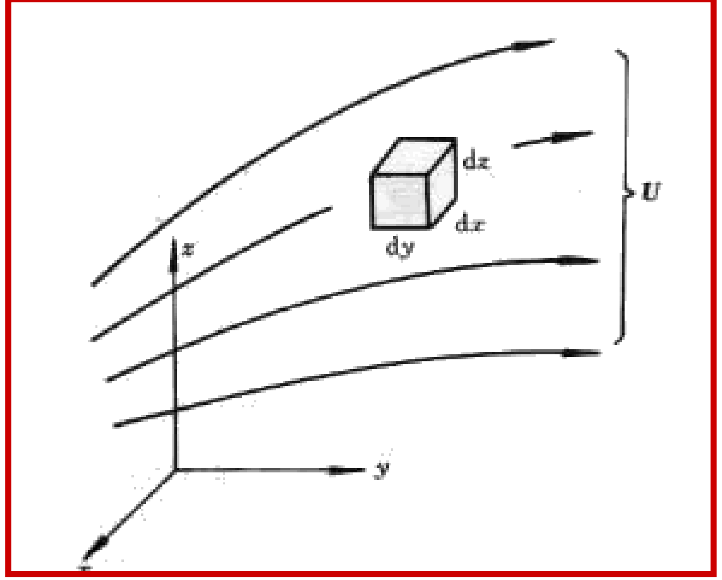
All macro-scale （宏观）HT & FF problems are governed （控制）by three conservation laws： mass, momentum and energy conservation law (守恒定律).

The differences between different problems are in: conditions for the unique solution（唯一解）：（1）initial （初始的）& boundary conditions, （2）physical properties and （3）source terms （源项）.

1.1.1 Governing equations and their general form

1. Mass conservation

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$



u, v, w are three components (分量) of $\vec{U}$.

$$\frac{\partial \rho}{\partial t} + \mathbf{div}(\rho \vec{U}) = 0$$

"**div**"（bold type,粗体） is the mathematical symbol for divergence（散度）：

In Cartesian coordinate (直角 坐标系)

$$\mathbf{div}(\rho \vec{U}) = \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}$$

For incompressible fluid（不可压缩流体）， density is constant,

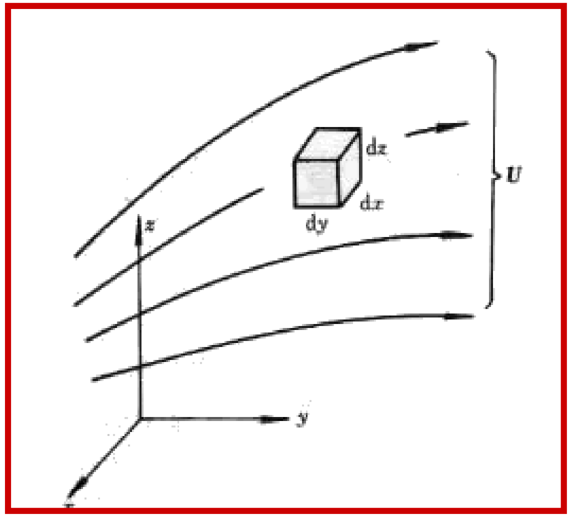
$$\mathbf{div}(\vec{U}) = 0 \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

flow without source and sink (没有源与汇的流动. For example, flow of water in a pipe)。

2. Momentum conservation

Applying the 2nd law of Newton （$F=ma$) to an elemental control volume（控制容积） in the three-dimensional coordinates:

[Increasing rate of momentum of the CV(控制容积中动量的增加率）]= [Summation of external forces applying on the CV（作用在控制容积中的外力之和）]



Adopting Stokes assumption(采用斯托克斯假设）： stress is linearly proportional to strain (应力与应变成线性关系)， we have following governing equation for component u in x-direction：

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}+\frac{\partial(\rho uw)}{\partial z}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}(\bar{\lambda}\mathrm{div}\vec{U}+2\eta\frac{\partial u}{\partial x})$$

Transient term（瞬态项） **Convection term（对流项）** **Diffusion term（扩散项）** **Source term（源项）**

Equation of the variation of fluid momentum in x-direction （ρu）.

$$+\frac{\partial}{\partial y}[\eta(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})]+\frac{\partial}{\partial z}[\eta(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x})]+\rho F_x$$

Diffusion term（扩散项）

η dynamic viscosity(动力粘度），

$\bar{\lambda}$ fluid 2nd molecular viscosity（第2分子粘度）.For gas, $\bar{\lambda}=-\frac{2}{3}\eta$

It can be shown (see the notes) that the above equation can be reformulated as （改写为） following general form of Navier-Stokes equation for *u* component of fluid momentum:

$$\frac{\partial(\rho u)}{\partial t} + \mathbf{div}(\rho u \vec{U}) = \mathbf{div}(\eta\ \mathbf{grad} u) + S_u$$

Transient term 非稳态项	Convection term对流项	Diffusion term扩散项	Source term源项

u, v, w ----velocity components in x,y,z three directions, respectively, they are the dependent variable（因变量） to be solved;

$\vec{U}$ ----fluid velocity vector; $\vec{U} = u\vec{i} + v\vec{j} + w\vec{k}$

S_u ----source term.

For *v* , *w* components, similar equations can be detived.

Source term in x-direction:

For incompressible fluid

$$S_u = \frac{\partial}{\partial x}(\eta \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(\eta \frac{\partial v}{\partial x}) + \frac{\partial}{\partial z}(\eta \frac{\partial w}{\partial x}) + \frac{\partial}{\partial x}(\bar{\lambda} \mathrm{div} \vec{U}) + \rho F_x - \frac{\partial p}{\partial x}$$

Similarly:

$$S_v = \frac{\partial}{\partial x}(\eta \frac{\partial u}{\partial y}) + \frac{\partial}{\partial y}(\eta \frac{\partial v}{\partial y}) + \frac{\partial}{\partial z}(\eta \frac{\partial w}{\partial y}) + \frac{\partial}{\partial y}(\bar{\lambda} \mathrm{div} \vec{U}) + \rho F_y - \frac{\partial p}{\partial y}$$

$$S_w = \frac{\partial}{\partial x}(\eta \frac{\partial u}{\partial z}) + \frac{\partial}{\partial y}(\eta \frac{\partial v}{\partial z}) + \frac{\partial}{\partial z}(\eta \frac{\partial w}{\partial z}) + \frac{\partial}{\partial z}(\bar{\lambda} \mathrm{div} \vec{U}) + \rho F_z - \frac{\partial p}{\partial z}$$

For incompressible fluid (不可压流体) with constant properties the source term does not contain velocity-related part.

3. Energy conservation

[Increasing rate of internal energy in the CV（控制容积内能的增加率）]= [Net heat transfer rate going into the CV]+[Power conducted by body forces and surface forces（进入控制容积的净传热速率+体积力与表面力作用在控制容积上的功率）]

Introducing Fourier's law of heat conduction and neglecting the work conducted by forces； Introducing enthalpy（焓）

$h = c_p T$, assuming c_p = constant,

We have:

$$\frac{\partial(\rho T)}{\partial t} + \mathbf{div}(\rho T \vec{U}) = \mathbf{div}(\frac{\lambda}{c_p}\mathbf{grad}(T)) + S_T$$

$$\mathbf{grad}(T) = \frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k}$$（温度梯度）

$$\frac{\lambda}{c_p} \rightarrow \frac{\lambda\eta}{c_p\eta} \rightarrow (\frac{\lambda}{c_p\eta})\eta \rightarrow \frac{\eta}{\Pr}$$

4. General form of the governing equations

$$\frac{\partial(\rho\phi)}{\partial t}+\mathbf{div}(\rho\phi\vec{U})=\mathbf{div}(\Gamma^{*}_{\phi}\mathbf{grad}(\phi))+S^{*}_{\phi}$$

Transient **Convection** **Diffusion** **Source**

ϕ is a general dependent variable: u, v, w and T ;

The differences between different problems:

(1) Different boundary and initial conditions;

(2) Different nominal source（名义源项） terms;

(3) Different physical properties (nominal diffusion coefficients, 名义扩散系数, λ/Pr)

5. Some remarks（说明）

1. The derived transient 3D Navier-Stokes equations can be applied for both laminar and turbulent flows （湍流）.

2. When a HT & FF problem is in conjunction with (与…有关) mass transfer process, the component（组份） conservation equation should be included in the governing equations.

3. Although c_p is assumed constant, the above governing equation can also be applied to cases with weakly changed c_p (比热略有变化）.

4. Radiative heat transfer （辐射换热） is governed by a differential-integral （微分-积分）equation, and its numerical solution will not be dealt with (处理) here.

1.1.2 Conditions for unique solution(taking energy eq. as example)

1. Initial condition (初始条件) $t = 0, \quad T = f(x, y, z)$

2. Boundary condition (边界条件)

（1） First kind (Dirichlet)： $T_B = T_{given}$

（2） Second kind (Neumann)： $q_B = -\lambda(\frac{\partial T}{\partial n})_B = q_{given}$

（3） Third kind (Rubin)：Specifying（规定） the relationship between boundary value and its first-order normal derivative（法向导数）：

$q = h(T_w - T_\infty)$ or $q = h(T_\infty - T_w)$

$$-\lambda(\frac{\partial T}{\partial n})_B = h(T_B - T_f)$$

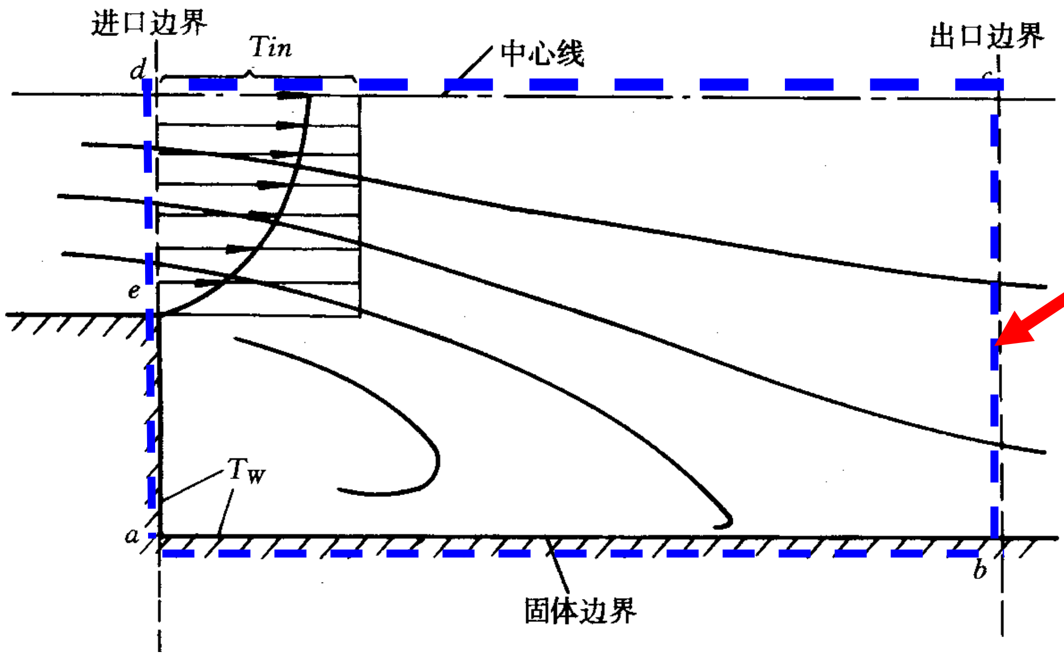
For the 3rd kind boundary condition heat flux at the boundary is not known!

3. Fluid thermo-physical properties and source term of the process.

1.1.3 Example of mathematical formulation

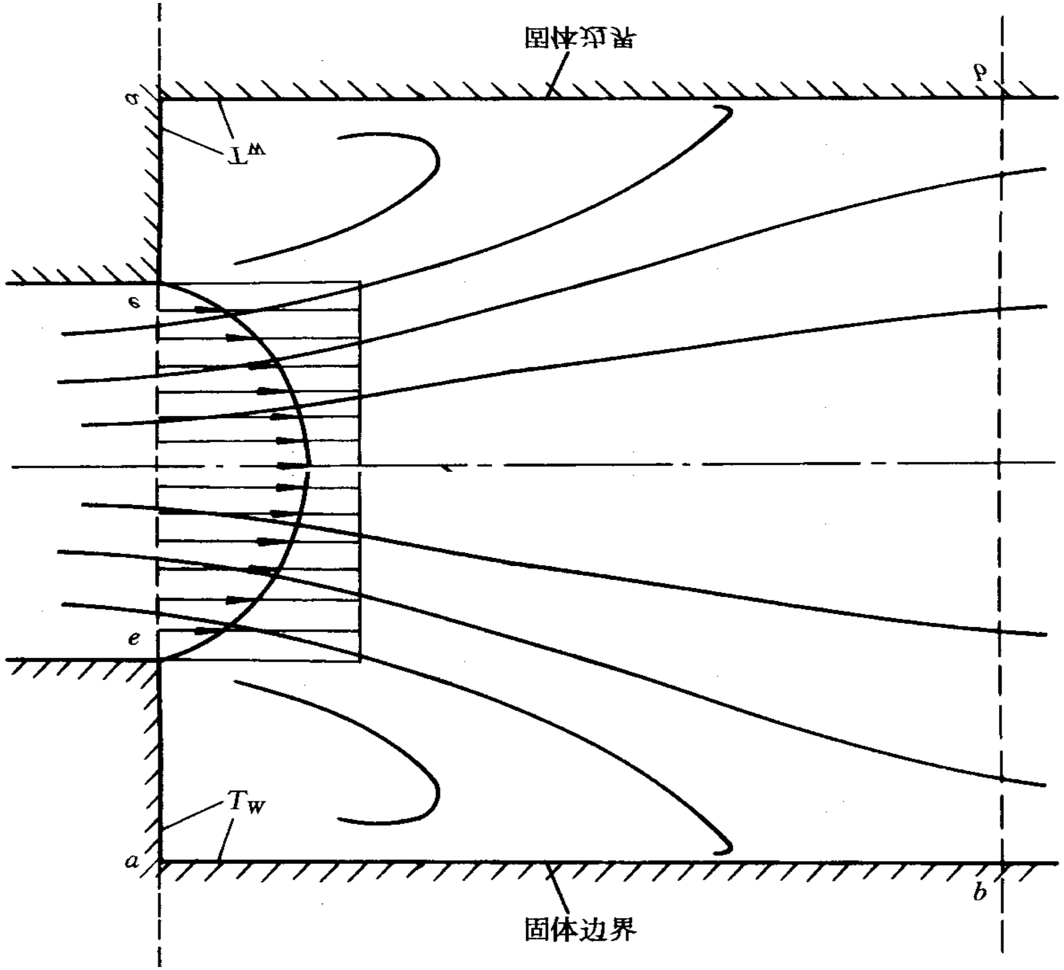
1. Problem and assumptions

Convective heat transfer in a sudden expansion region（突扩区域）： 2D, steady-state, incompressible fluid, constant properties, neglecting gravity and viscous dissipation（粘性耗散）.



Solution domain（求解区域）

2D Sudden expansion region



2. Governing equations

Complete set of governing equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial (uu)}{\partial x}+\frac{\partial (uv)}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial (vu)}{\partial x}+\frac{\partial (vv)}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial (uT)}{\partial x}+\frac{\partial (vT)}{\partial y}=a\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

$$a=\frac{\lambda}{\rho c_p}$$

thermal diffusivity（热扩散率）

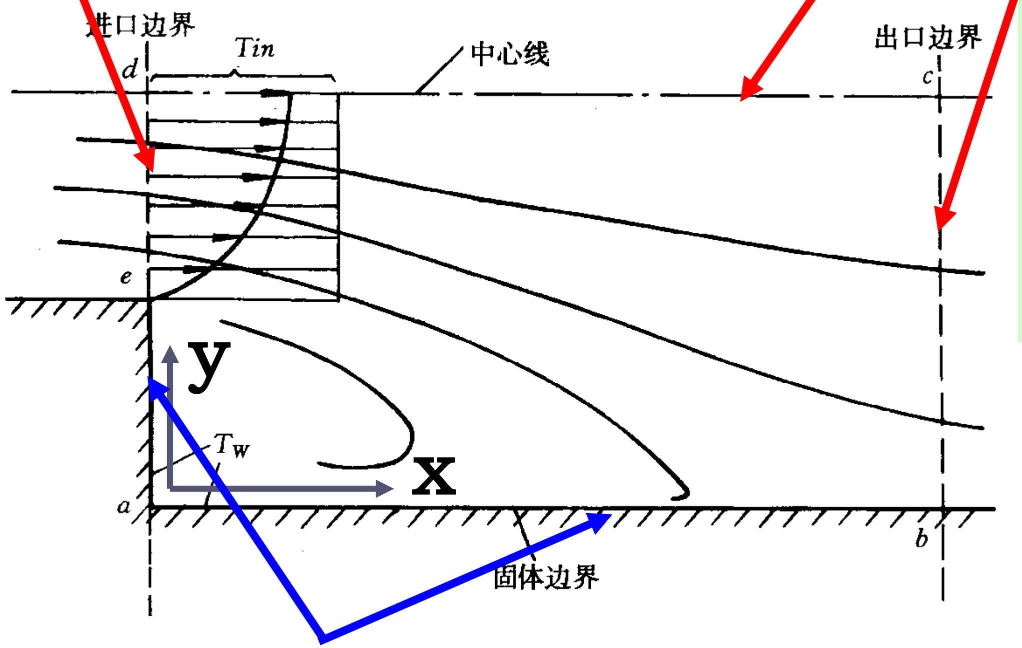
3. Boundary conditions

（1）Inlet：specifying (说明) variations of u,v,T with y；

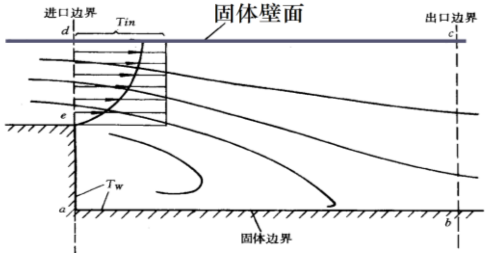
（3）Center line：

$$\frac{\partial u}{\partial y}=\frac{\partial T}{\partial y}=0;\quad v=0$$

（4）Outlet：Mathematically the distributions of u,v,T or their first-order derivatives(一阶导数) are required. Actually，numerical approximations must be made.

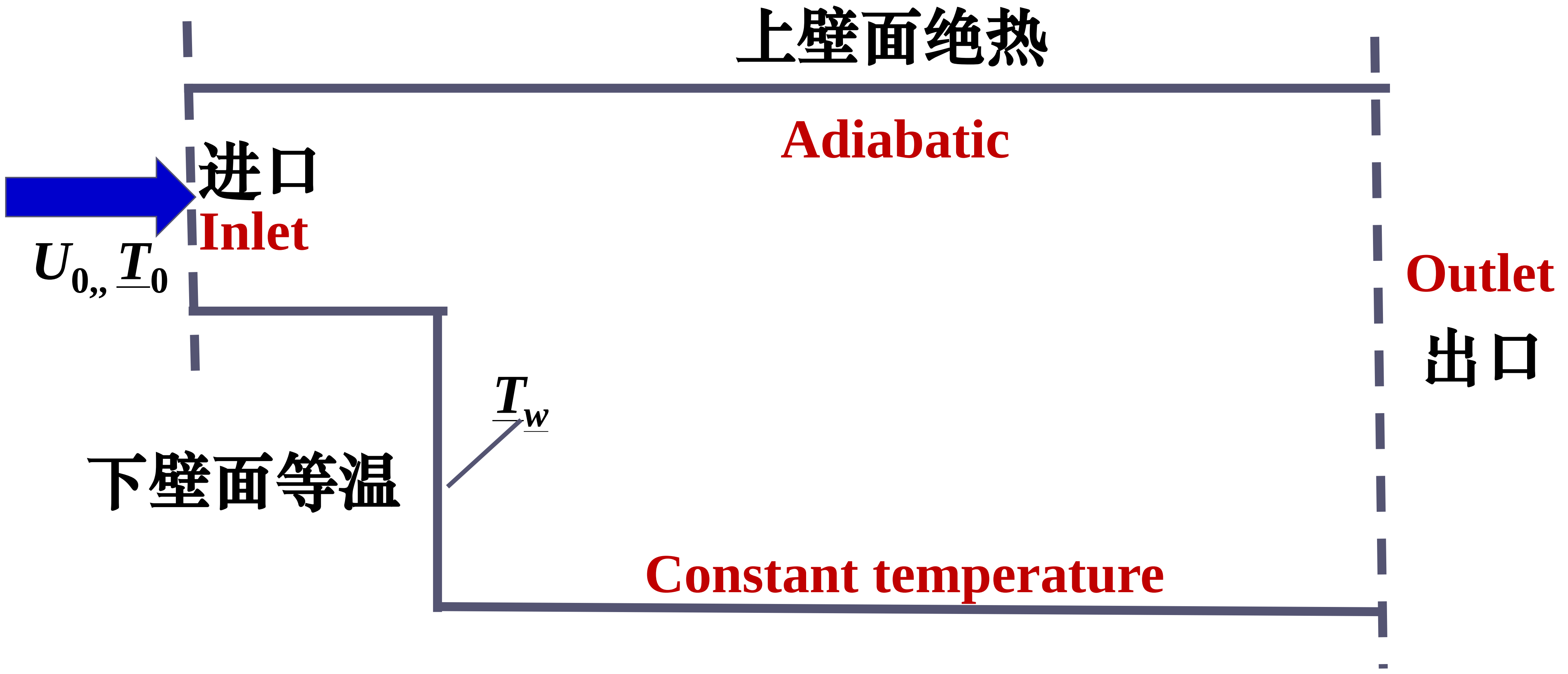


Can we regard this boundary cons as HT & FF **over a backward step?**



（2）Solid B.C.：No slip（滑移） in velocity, no jump（跳跃） in temp.

Can we regard this boundary formulation as **heat transfer and fluid flow over a backward step?**



Notes to Section 1.1

In the left hand side

The right hand side :

$$\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}+\frac{\partial(\rho uw)}{\partial z}=\mathrm{div}(\rho u\vec{U})$$

$$\frac{\partial}{\partial x}(\bar{\lambda}\mathrm{div}\vec{U}+2\eta\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}[\eta(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})]+\frac{\partial}{\partial z}[\eta(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x})]+\rho F_x-\frac{\partial p}{\partial x}=$$

$$\underbrace{\frac{\partial}{\partial x}(\eta\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(\eta\frac{\partial u}{\partial y})+\frac{\partial}{\partial z}(\eta\frac{\partial u}{\partial z})}_{\mathrm{div(grad}(u))}+\underbrace{\frac{\partial}{\partial x}(\eta\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(\eta\frac{\partial v}{\partial x})+\frac{\partial}{\partial z}(\eta\frac{\partial w}{\partial x})+\frac{\partial}{\partial x}(\bar{\lambda}\mathrm{div}\vec{U})}_{S_u}$$

$$\rho F_x-\frac{\partial p}{\partial x} \quad =\mathrm{div}(\eta\,\mathrm{grad}u)+S_u$$

$$grad(u)=\frac{\partial u}{\partial x}\boldsymbol{i}+\frac{\partial u}{\partial y}\boldsymbol{j}+\frac{\partial u}{\partial z}\boldsymbol{k}$$

$$\mathrm{div(grad}(u))=\frac{\partial}{\partial x}(\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(\frac{\partial u}{\partial y})+\frac{\partial}{\partial z}(\frac{\partial u}{\partial z})$$

Thus we have:

$$\frac{\partial(\rho u)}{\partial t}+\mathrm{div}(\rho u\vec{U})=\mathrm{div}(\eta\,\mathrm{grad}u)+S_u$$

Navier-Stokes

Gradient of a scalar (标量的梯度) is a vector:

$$\mathrm{grad}(u) = \frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k}$$

Divergence of a vector (矢量的散度) is a scalar:

$$\mathrm{div}(\mathrm{grad}(u)) = \mathrm{div}\left(\frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k}\right)$$

$$\mathrm{div}(\mathrm{grad}(u)) = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right)$$

$$\mathrm{div}(\eta\,\mathrm{grad}(u)) = \frac{\partial}{\partial x}\left(\eta\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\eta\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\eta\frac{\partial u}{\partial z}\right)$$

End of Notes to Section 1.1

1.2 Basic concepts of NHT, its importance and application examples

1.2.1 Three fundamental approaches （方法） of scientific research and their relationships

1.2.2 Basic concepts of numerical solutions based on continuum(连续） assumption

1.2.3 Classification of numerical solution methods based on continuum assumption

1.2.4 Importance and application examples

1.2.5 Stories of two celebrities (名人) and related international conferences

1.2.6 Some suggestions

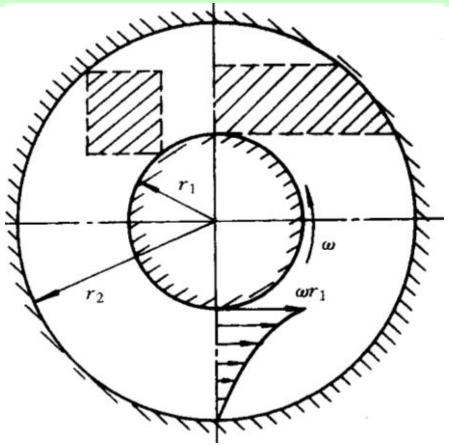
1.2 Basic concepts of NHT , importance and its application examples

1.2.1 Three fundamental approaches （方法）of scientific research and their relationships

1. Theoretical analysis (Analytical solution)

Its importance should not be underestimated（低估). It provides comparison for verifying(验证) numerical solutions.

Examples: The analytic solution (分析解） of velocity from NS eq. for following case：



$$\frac{u}{u_1} = \frac{r_1/r_2}{1-(r_1/r_2)^2} \bullet \frac{1-(r/r_2)^2}{r/r_2}$$

$$u_1 = \varpi r_1$$

2. Experimental study

A basic research method: observation(观察); properties measurement; verification (验证) of numerical results

3. Numerical simulation

Numerical simulation is an inter-discipline (交叉学科), and plays an important and un-replaceable role in exploring (探索)unknowns, promoting (促进) the development of science & technology, and for the safety of national defense（国防安全）.

With the rapid development of computer hardware（硬件), the importance and function of the numerical simulation become greater and greater.

1.2.2 Basic concepts of numerical solutions based on continuum assumption（连续性假设）

Replacing the fields of continuum variables (velocity, temp. etc.) by sets (集合） of values at discrete（离散的） points (nodes, grids节点) (Discretization of domain,区域离散);

Establishing algebraic equations for these values at the discrete points by some principles (Discretization of equations，方程离散);

Solving the algebraic equations by computers to get approximate solutions of the continuum variables (Solution of equation，方程求解).

NHT

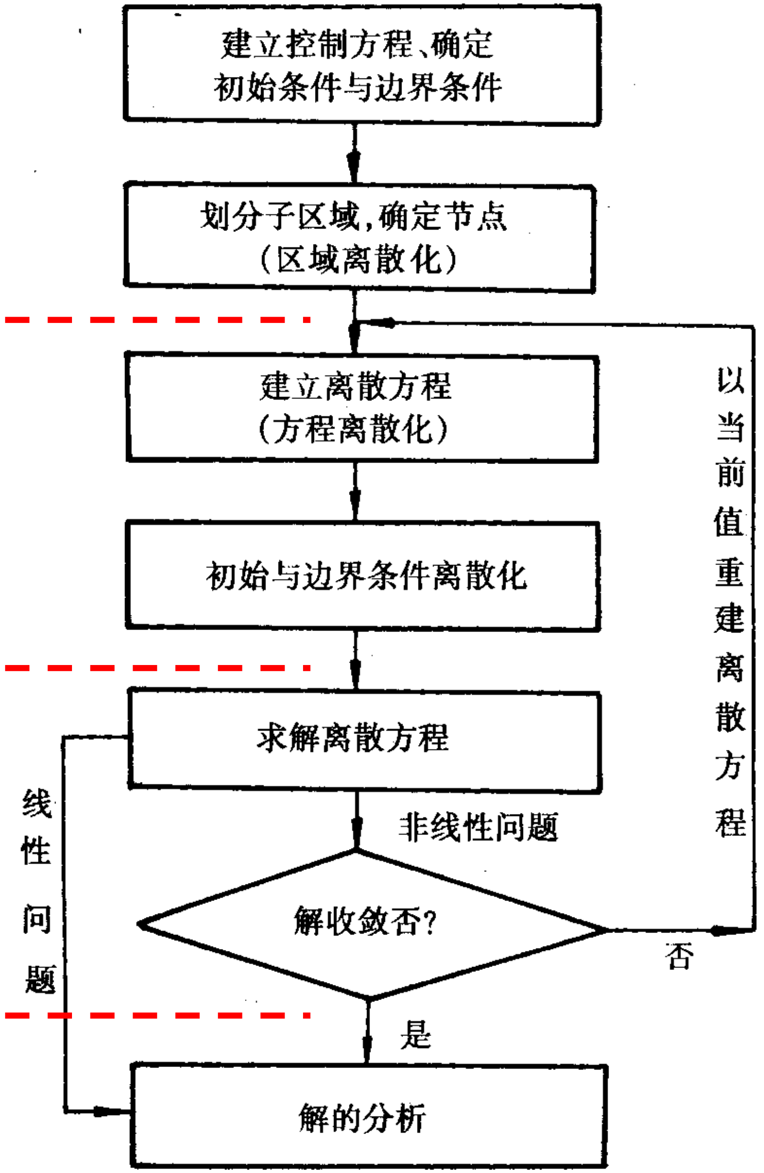
Math formulation
(建立数学描写)

Discretizing Domain(区域离散)

Discretizing Equations
(方程离散)

Solving algebraic equations
(方程求解)

Analyzing numerical results
(结果分析)



Flow chart （流程图）

1.2.3 Classification of numerical solution methods based on continuum assumption

1. Finite difference method （**FDM**）	有限差分法：L F Richardson (1910), A Thome（1940s）
2. Finite volume method （**FVM**）	有限容积法：D B Spalding；S V Patankar
3. Finite element method （**FEM**）	有限元法：O C Zienkiewicz；冯康 (Kang Feng)
4. Finite analytic method （**FAM**）	有限分析法：陈景仁(Ching Jen Chen)
5. Boundary element method （**BEM**）	边界元法：D B Brebbia
6. Spectral analysis method （**SAM**）	谱分析法

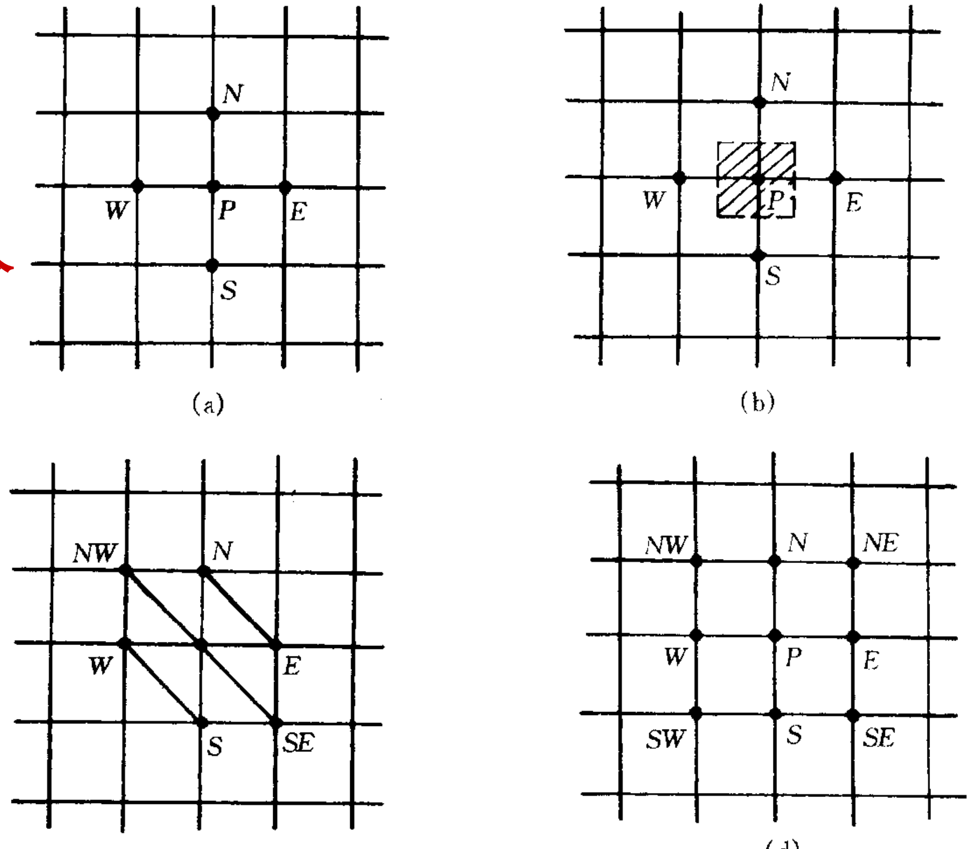
Comparisons of FDM(a),FVM(b),FEM(c),FAM(d)

FDM
有限差分

FVM
有限容积

FEM
有限元

FAM
有限分析

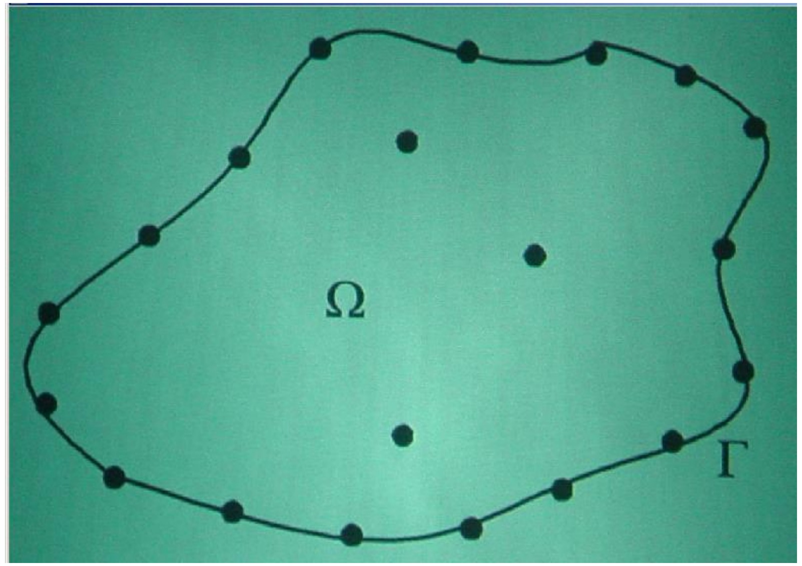


(c)

All these methods need a grid system (网格系统):

1）Determination of grid positions；2）Establishing the influence relationships between grids.

BEM



BEM （边界元）requires a basic solution(基准解), which greatly limits its applications in convective problems.

SAM can only be applied to geometrically simple cases.

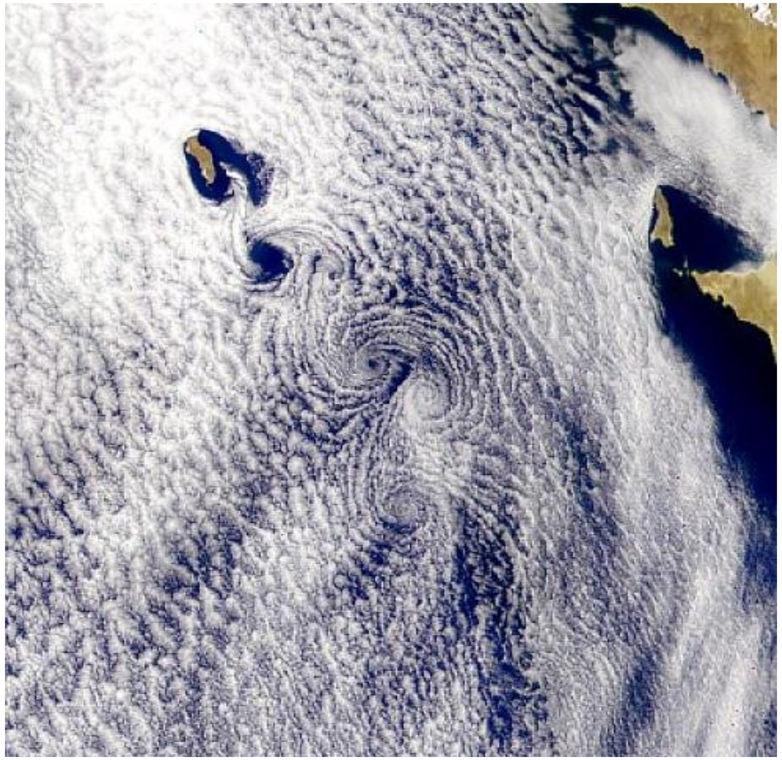
Manole, Lage 1990－1992 statistics （统计)：FVM ---47％; adopted by most commercial software; Our statistics of NHT in 2007 even much higher.

1.2.4 Importance and application examples

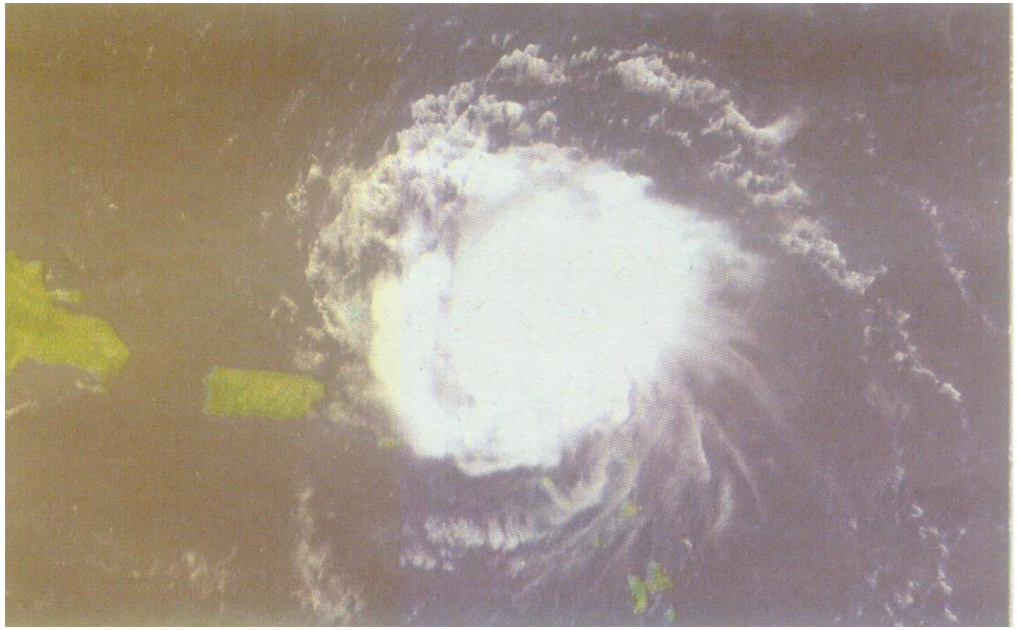
1. Application examples

Example 1: Weather forecast—

Numerical solution is the only way.

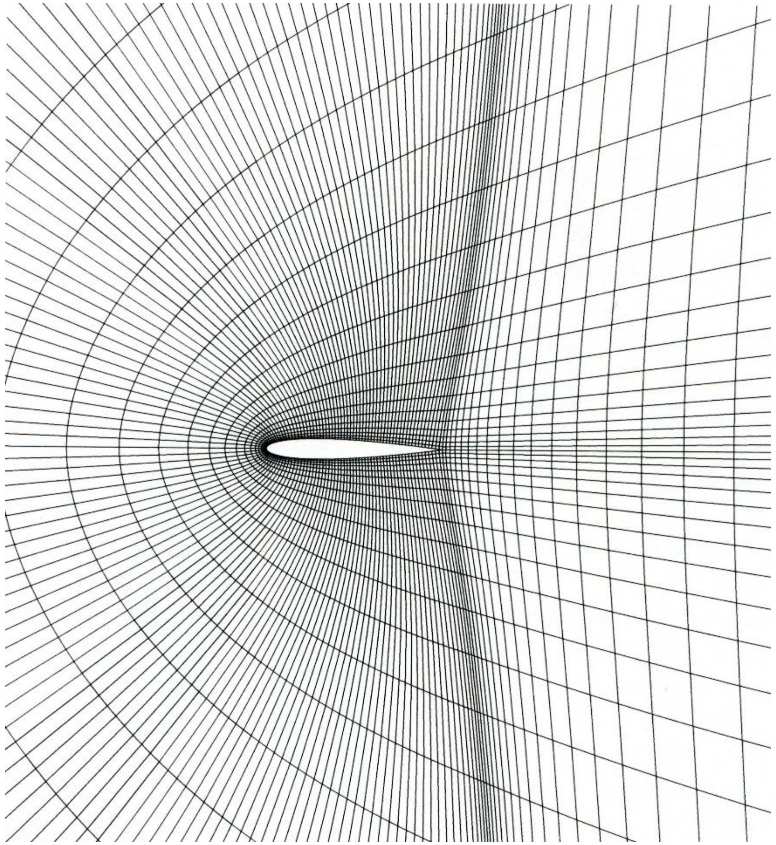


Large scale vortex

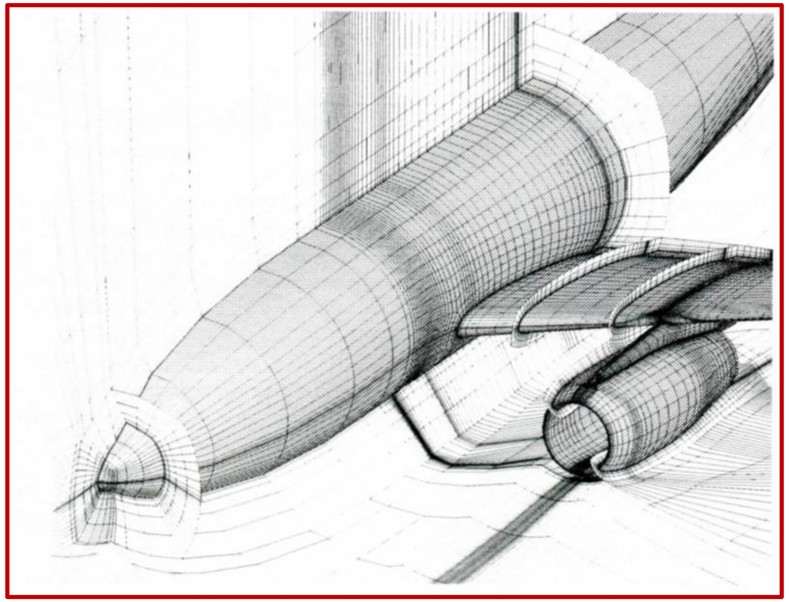


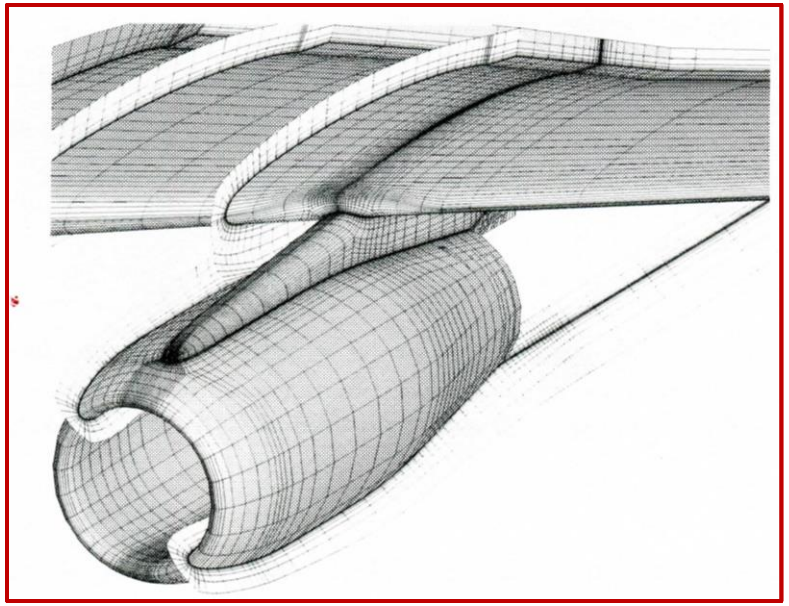
Cloud Atlas sent back by a meteorological satellite

Example 2：Aeronautical & aerospace（航空航天）engineering



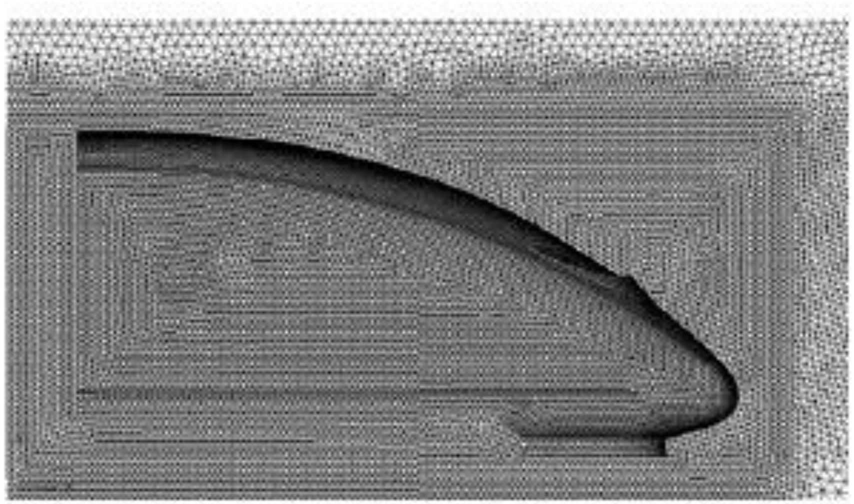
Partial view of grid system around NACA 0012 airfoil（机翼）

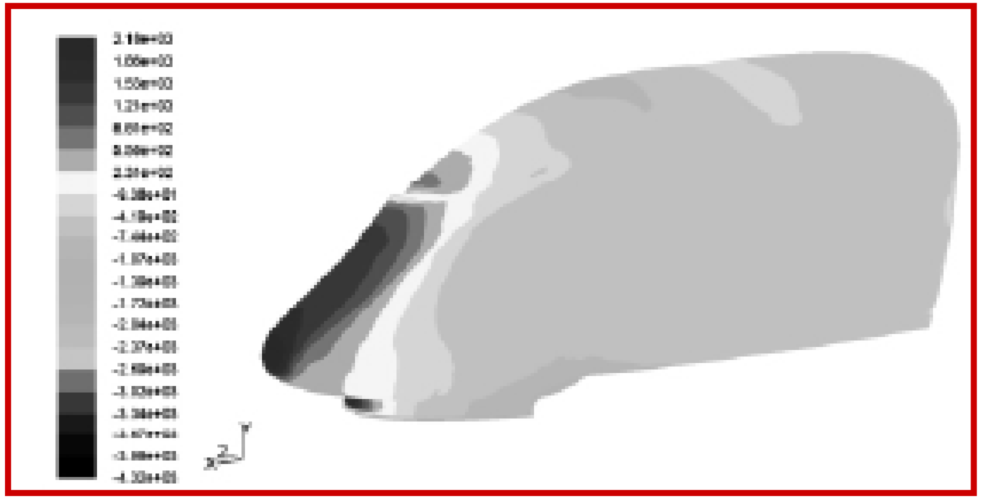




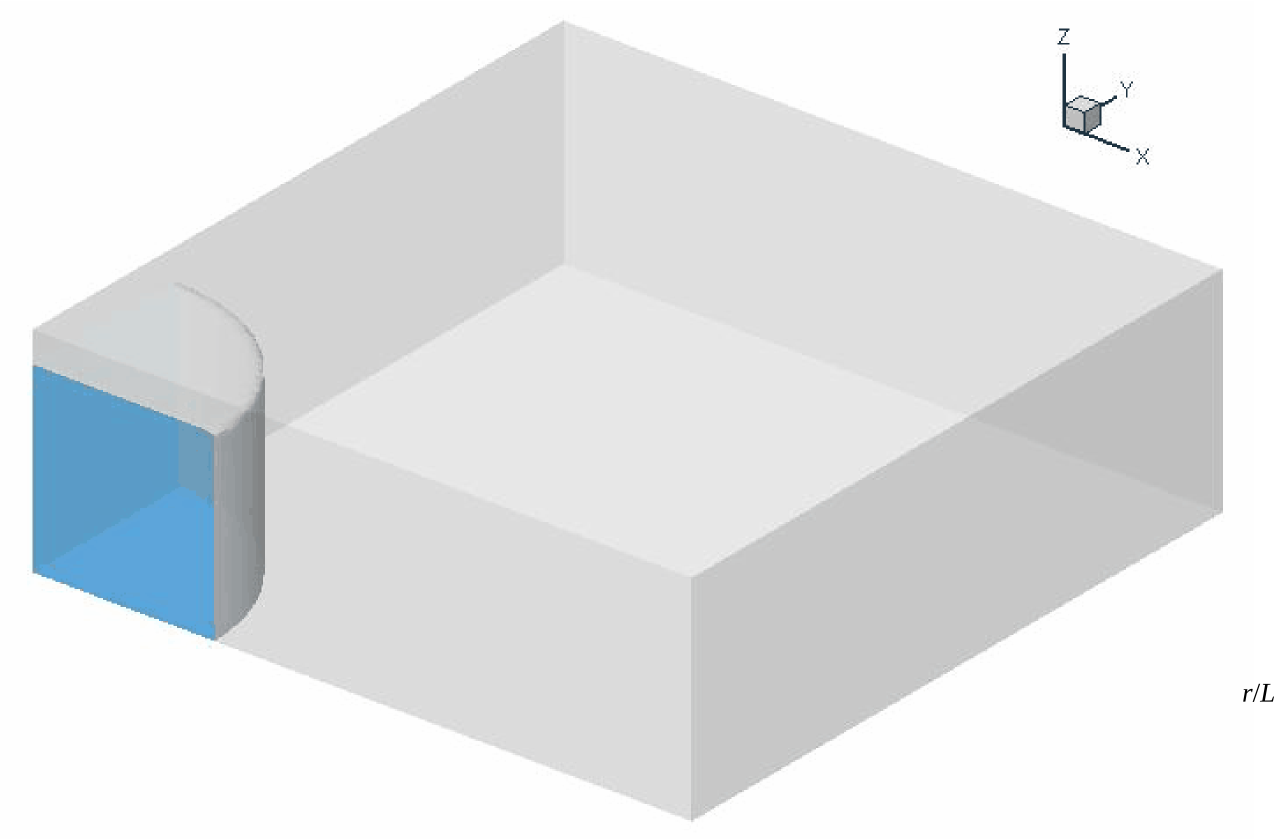
Example 3：Design of head shape of high-speed train（

The front head shape of the high speed train is of great importance for its aerodynamic performance（空气动力学特性). Numerical wind tunnel is widely used to optimize the front head shape.

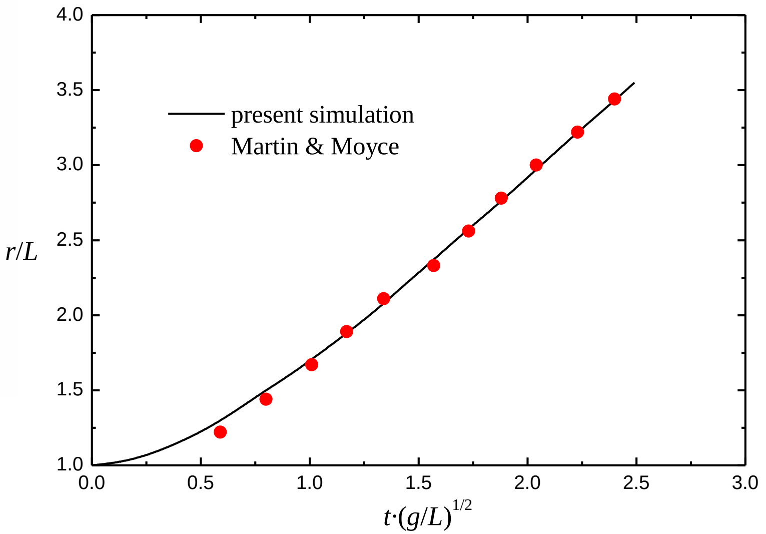




Example 4：Simulation of breaking dam (溃坝)



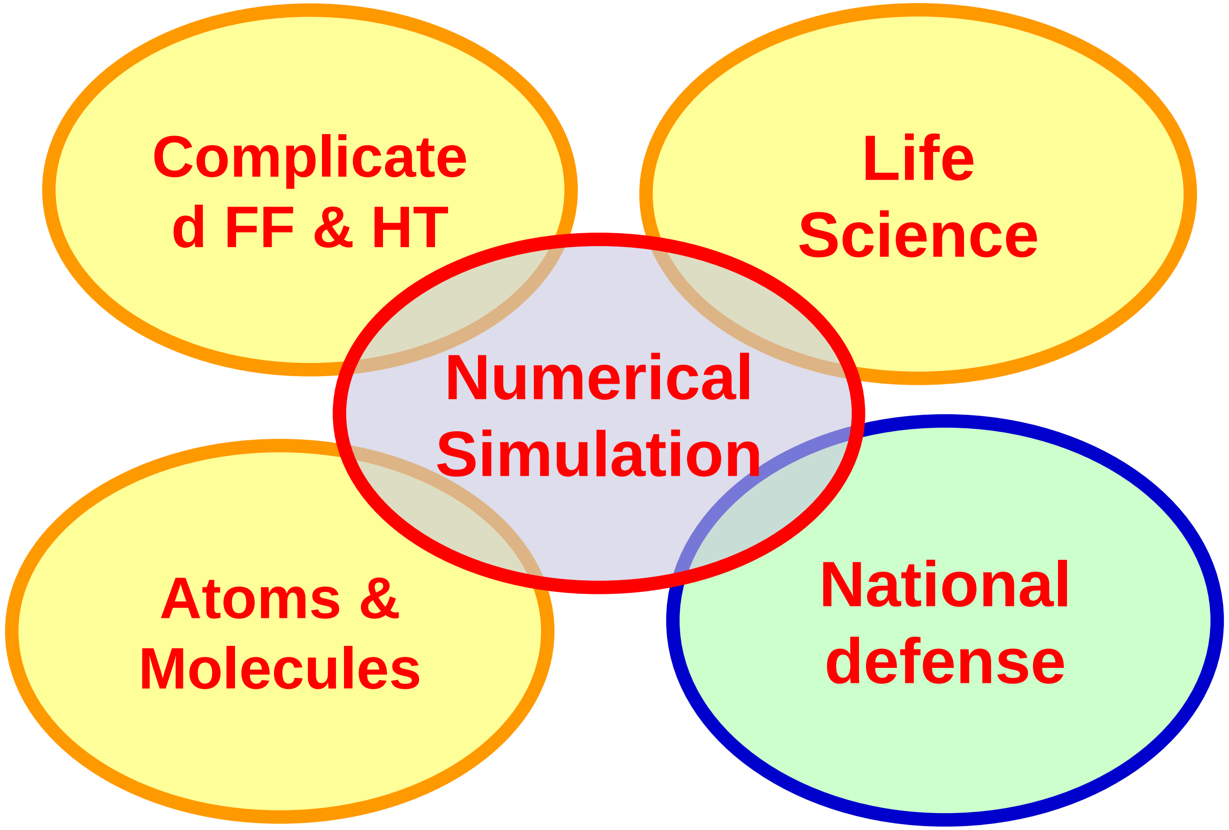
Evolution（演变） process of interface



Base radius vs. time

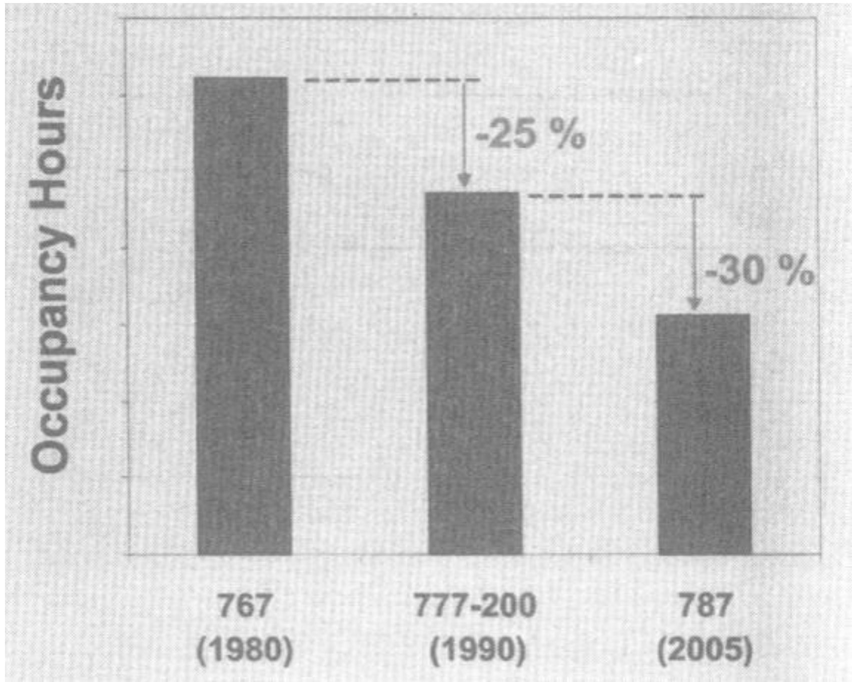
2. Importance of numerical simulation

Historically, in 1985 the West Europe listed the first commercial software-PHEONICS as the one which was not allowed to sell to the communist countries. The prohibition (禁令) was cancelled （取消） in 1990s.



In 2005 the USA President Advisory Board put forward a suggestion to the president that in order to keep competitive power （竞争力） of USA in the world scientific computation should be developed.

In the year of 2006 the director of design department of Boeing , M. Grarett , reported to the US Congress （国会） indicating that the high performance computers have completely changed the way of designing Boeing airplane.

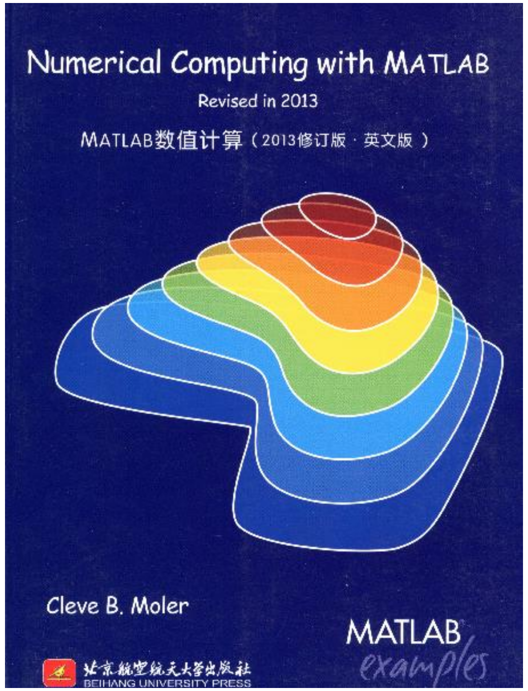


Numerical simulation plays an important role in the design of Boeing airplane

Years ago, the Trump administration in USA has banned (禁止) Harbin Institute of Technology and Harbin University of Engineering from using MATLAB. MATLAB is an important tool for engineering design and research.

Therefore independently developed software or home-made software(自主研发的软件） is very important for a country’s development.

Graduate students at a research-led university（研究导向的大学）should have the capability（能力） to independently develop a software.



To meet such requirement this course is composed of following three major parts:

Part 1: Fundamental theories of numerical heat transfer, You will learn basic numerical solution methods for incompressible fluid flow and heat transfer.(40 hours)

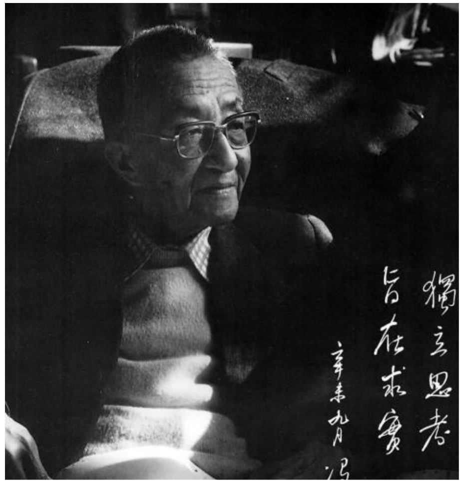
Part 2: 2D-teaching code by FORTRAN-95, which contains only about 700 sentences while is able to simulate fluid flow and heat transfer problems in three 2D coordinates; This part cultivates (培养) students' ability to write programs for themselves. (8 hours)

Part 3: Commercial software FLUENT, including fundamentals and applications. This part cultivates students' ability to apply commercial software to solve complicated engineering problems . (12 hours).

1.2.5 Stories of two celebrities (名人) and related int. conference

1. Kang FENG (冯康)

叶笃正，气象学家
中科院院士
(Meteorology)



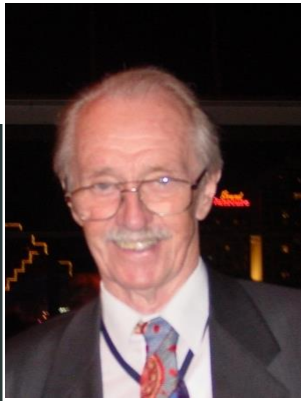
冯康，数学家
中科院院士

冯端，物理学家
中科院院士

Professor K.FENG developed very strict and beautiful mathematical theory of finite element method(FEM). **He was not married for his whole life, and devoted himself to the innovation of science & technology in China .**

The year of 2020 was the 100th birthday of Feng KANG. A solemn（隆重） commemoration（纪念） was held in the Mathematical Institute in Beijing.

2. D.B. Spalding (UK)



3. Related international conferences

（1）ICCHMT---Initiated by Prof Mohamad in Canada

(1) ASCHT---Initiated by Prof W Q Tao

In 2007 I initiated The Asian Symposium on Computational Heat Transfer (ASCHT). Professor Spalding was invited to attend the conference.

It has become an international conference and is held every two years.

The 2023-ASCHT was held in Saudi. 22 graduate students of my group attended the conference. 2025-ASCHT will be held in Wuhan.

1.3 Mathematical and physical classification (分类)of HT & FF problems and its effects on numerical solution

1.3.1 From mathematical viewpoint (观点)

1. General form of 2nd-order PDE (偏微分方程) with two independent variables (二元)
2. Basic features (特点) of three types of PDEs
3. Relationship to numerical solution method

1.3.2 From physical viewpoint

Conservative (守恒型) and non-conservative

1.3 Mathematical and physical classification of FF & HT problems and its effects on numerical solutions

1.3.1 From mathematical viewpoint

1. General formulation of 2nd order PDEs with two independent variables

$$a\phi_{xx} + b\phi_{xy} + c\phi_{yy} + d\phi_x + e\phi_y + f\phi = g(x,y)$$

a, b, c, d, e, f can be function of x, y, ϕ

$$b^2 - 4ac \begin{cases} < 0, & \text{Elliptic} \\ = 0, & \text{Parabolic} \\ > 0, & \text{Hyperbolic} \end{cases}$$

- Elliptic: 椭圆型（回流型）
- Parabolic: 抛物型（边界层）
- Hyperbolic: 双曲型

2. Basic feature of three types of PDEs

$b^2 - 4ac < 0,$ having no real characteristic line (elliptic type);

(没有实的特征线)

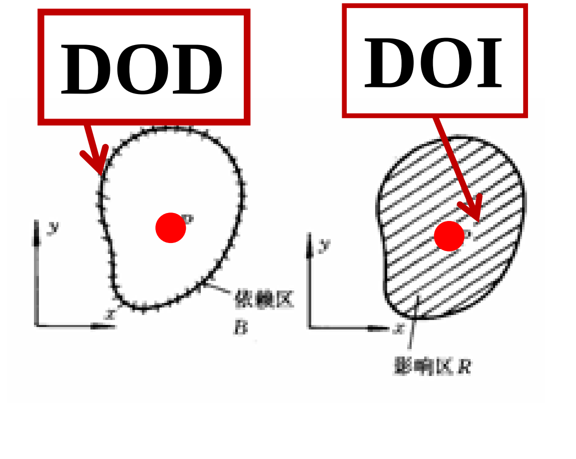
$b^2 - 4ac = 0,$ having one real characteristic line (parabolic type) ;

$b^2 - 4ac > 0,$ having two real characteristic lines (hyperbolic)

leading to the difference in domain of dependence (DOD,依赖区) and domain of influence (DOI,影响区);

For 2-D case, DOD of a node is a line which determines the value of a dependent variable at the node; DOI of a node is an area within which the values of dependent variable are affected by the node.

Elliptic



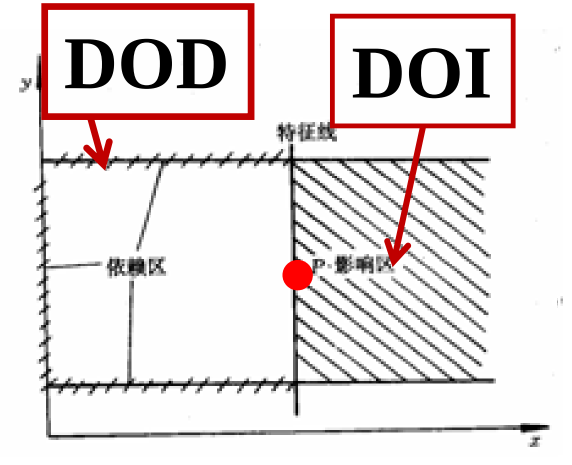
$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$$ Steady HC

$$(a=1, b=0, c=1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$ 2D N.S. Eq.

$$b^2-4ac<0$$

Parabolic



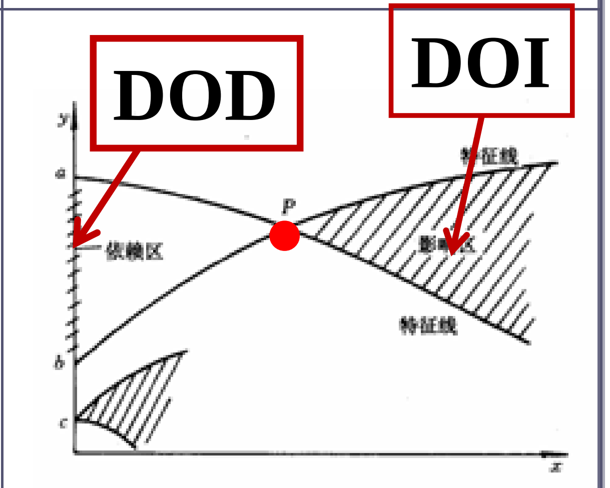
$$\frac{\partial T}{\partial t}=a\frac{\partial^2 T}{\partial y^2}$$ Un-Steady HC

$$(a=0, b=0, c=a)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}$$ 2D B. L. Eq.

$$b^2-4ac=0$$

Hyperbolic



$$\frac{1}{a}\frac{\partial T}{\partial t}+\frac{1}{c^2}\frac{\partial^2 T}{\partial t^2}=\frac{\partial^2 T}{\partial y^2}$$ Non-Fourier HC

$$(a=1/c^2, b=0, c=-1)$$

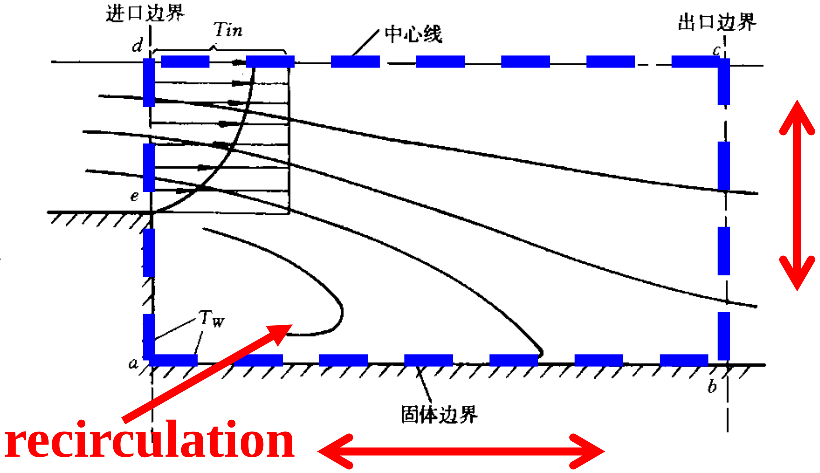
$$\frac{\partial^2 \phi}{\partial t^2}=C^2\frac{\partial^2 \phi}{\partial y^2}$$ Wave（波动）equation

$$(a=1, b=0, c=-C^2)$$

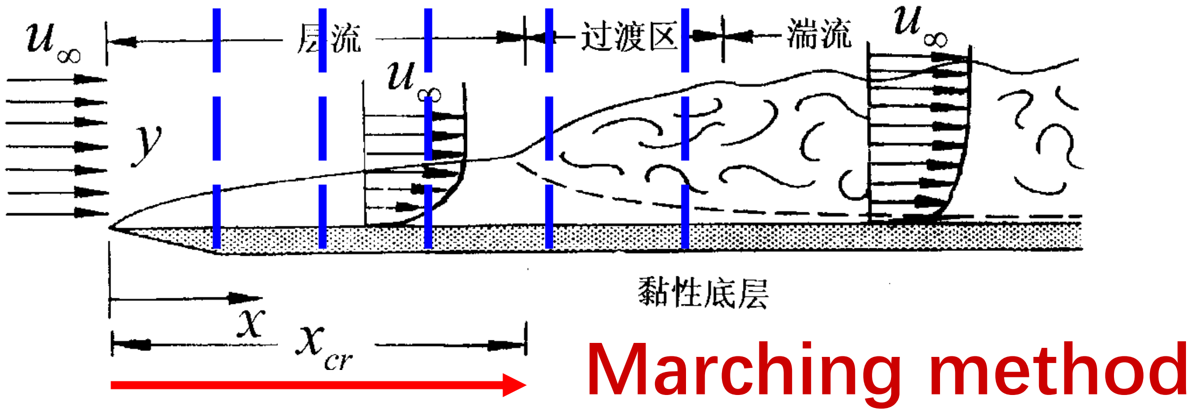
$$b^2-4ac>0$$

3. Relationship to numerical methods

（1）Elliptic： flow with recirculation （回流）， solution should be conducted simultaneously(同时) for the whole domain；



（2）Parabolic： flow without recirculation ， solution can be conducted by marching method（步进方法）， greatly saving computing time!



1.3.2 From physical viewpoint

1. Conservative (守恒型) vs. non-conservative:

Non-conservative: those governing equations whose convective terms are not expressed by divergence form are called non-conservative governing equation . For 2D energy eq.:

$$u\frac{\partial(\rho c_p T)}{\partial x}+v\frac{\partial(\rho c_p T)}{\partial y}$$ is not divergence form

Conservative: those governing equations whose convective terms are expressed by divergence form(散度形式) are called conservative governing equation .

$$\frac{\partial(\rho u c_p T)}{\partial x}+\frac{\partial(\rho v c_p T)}{\partial y} \Rightarrow \frac{\partial(\rho c_p T u)}{\partial x}+\frac{\partial(\rho c_p T v)}{\partial y}=\mathbf{div}(\rho c_p T\vec{U})$$

These two concepts are only for numerical solution.

2. Conservative GE. can guarantee the conservation of physical quantity (mass, momentum ,energy , etc.) within a finite (有限大小) volume.

$$\frac{\partial(\rho c_p T)}{\partial t} + \mathbf{div}(\rho c_p T\vec{U}) = \mathbf{div}(\lambda \mathbf{grad} T) + S_T c_p$$

$$\frac{\partial(\rho T)}{\partial t} + \mathbf{div}(\rho T\vec{U}) = \mathbf{div}(\frac{\lambda}{c_p}\mathbf{grad}(T)) + S_T$$

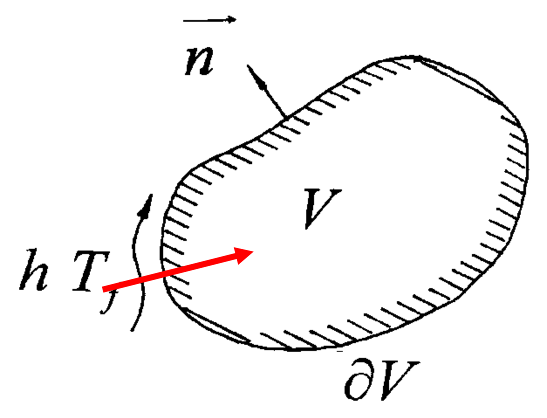
$$\frac{\partial}{\partial t}\int_V (\rho c_p T)dV = -\int_V \mathbf{div}(\rho c_p T\vec{U})dV + \int_V \mathbf{div}(\lambda \mathbf{grad} T)dV + \int_V S_T c_p dV$$

From Gauss theorem(高斯定律)

$$\int_V \mathbf{div}(\rho c_p T\vec{U})dV = \int_{\partial V} (\rho c_p T\vec{U}) \bullet \vec{n}dA$$

$$\int_V \mathbf{div}(\lambda \mathbf{grad} T)dV = \int_{\partial V} (\lambda \mathbf{grad}(T)) \bullet \vec{n}dA$$

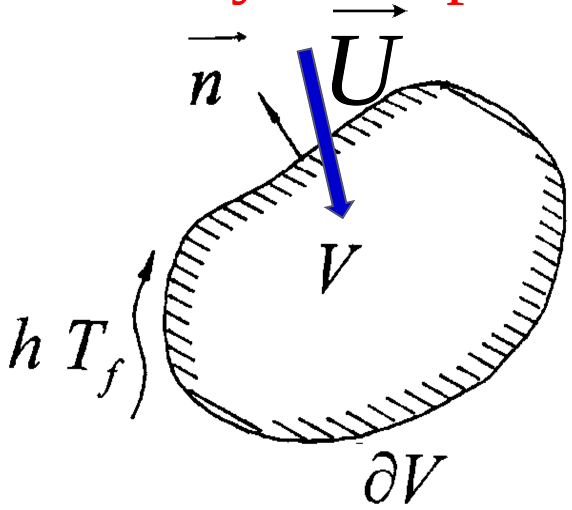
Dot product（矢量的点积）

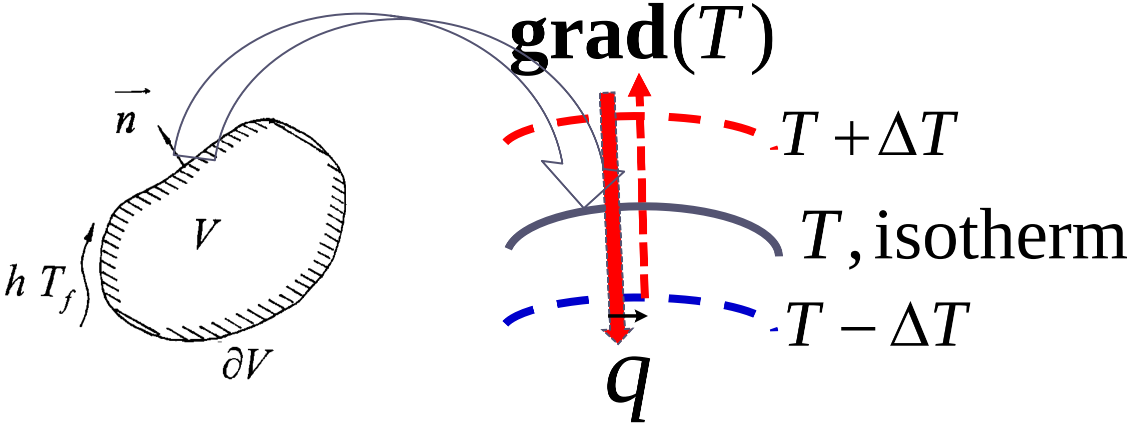


$$\frac{\partial}{\partial t}\int_V (\rho c_p T)dV = -\int_{\partial V} (\rho c_p T\vec{U})\bullet\vec{n}dA + \int_{\partial V} (\lambda\,\mathbf{grad}(T))\bullet\vec{n}dA + \int_V (S_T c_p)dV$$

Increment （增量） of internal energy	**Energy into the region by fluid flow**	**Energy into the region by conduction**	**Energy generated by source**

Exactly an expression of energy conservation!





$\vec{U}\bullet\vec{n}<0;$

$-\vec{U}\bullet\vec{n}>0$, heat flows in

$\mathbf{grad}(T)\bullet\vec{n}>0$

heat conducts in

Key to have a conservative form of governing equation：

convective term is expressed by divergence.

3. Generally conservation is expected. Discretization eqs. are suggested to be derived from conservative PDE.

4. Conservative and non-conservative are referred to （指） a finite space（有限空间）; For a differential volume（微分容积） they are identical（恒等的）：

$$u\frac{\partial(\rho c_p T)}{\partial x}+v\frac{\partial(\rho c_p T)}{\partial y}+\rho c_p T\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) \quad 0$$

$$=\frac{\partial(\rho u c_p T)}{\partial x}+\frac{\partial(\rho v c_p T)}{\partial y}$$

Summary of Section 1-3

1.The governing eqs. of HT and FF are of 2nd order PDE:

$$a\phi_{xx}+b\phi_{xy}+c\phi_{yy}+d\phi_{x}+e\phi_{y}+f\phi=g(x,y)$$

and depending on the value of (b^2-4ac), it can be **elliptic**, **parabolic** or **hyperbolic**;

The HT and FF problems of the incompressible fluid are either elliptic or parabolic;

2. If the convective term of a governing eq. is expressed by the divergence form it is **conservative**, otherwise it is **non-conservative**; Discretization eqs. are suggested to be derived from conservative PDE.

Summary of Chapter 1

Importance of the course

It is now widely accepted that an appropriate combination of theoretical analysis, experimental study and numerical simulation is the best approach for modern scientific research.

With the further development of computer hardware and numerical algorithm（算法）, the importance of numerical simulation will become more and more significant!

A new era of applying numerical simulation has already come with the emergence of the profound changes unseen in a century (随着百年未有之大变局的出现，数值模拟应用的新时代已经到来)!

Major contents of Chapter 1

1. A complete mathematic formulation of heat transfer and fluid flow problems is composed of two parts: governing eqs. and initial/boundary conditions
2. The basic procedure of numerical solution includes three steps: (1) Domain dicretization; (2) Equation discretization; (3) Solution of the algebraic eqs.
3. The governing eqs. can be divided into elliptic, parabolic and hyperbolic depending on the value of b^2-4ac; Elliptic and parabolic problems have different solution domains; When the convection terms are expressed by the divergence form, the governing eq. is called conservative; otherwise is non-conservative. Equation discretization is suggested to be conducted for conservative one.

Some Suggestions for learning the course

1. Understanding numerical methods from basic characteristics of physical process;

2. Mastering (掌握） complete picture and knowing every detail (明其全，析其微） for any numerical method；

3. Practicing simulation method by a computer； Working hard to develop your ability to write code for yourself;

4. Trying hard to analyze simulation results: rationality (合理性) and regularity (规律性)；

5. Adopting CSW(商业软件) in conjunction with （与….结合) self-developed code (与自编程序相结合).

The pdf file will be posted at our group website and WeChat group

Lecture of this week ---Chapter 1 of NHT textbook

Erratum（勘误表）

1. 第3页中间： $-2/3$ 应改为 $-2/3\,\eta$
2. 第3页倒数第3行： $-\frac{\partial p}{\partial x}$ 应改为 $-\frac{\partial p}{\partial x}+\rho F_x$
   倒数第1，2行仿此修改。
3. 第4页倒数第3行： $\lambda div\mathbf{U}$ 应改为 $\lambda(div\mathbf{U})^2$
4. 第7页 式(1-18)中右端： ρ 应改为 p
5. 第9页倒数第3、4行右端：扩散项前的系数应为 ν
6. 式(1-6),(1-8)中漏了重力项。

Home Work 1（2024-2025）

Please finish your homework independently（独立完成）!!!

Please hand in on Sept. 23, 2024

Problem 1-1

For the fluid flow and heat transfer in a 2-D situation shown in the following Fig., assuming:

(1) FF and HT are in steady state; (2) fluid is incompressible; (3) the gravity effect should be taken into account; (4) physical properties are constant; (5) viscosity dissipation can be neglected.

Try to write down:

1) The governing equations for the process in the computational domain;

2) The boundary conditions of the fluid flow and heat transfer.

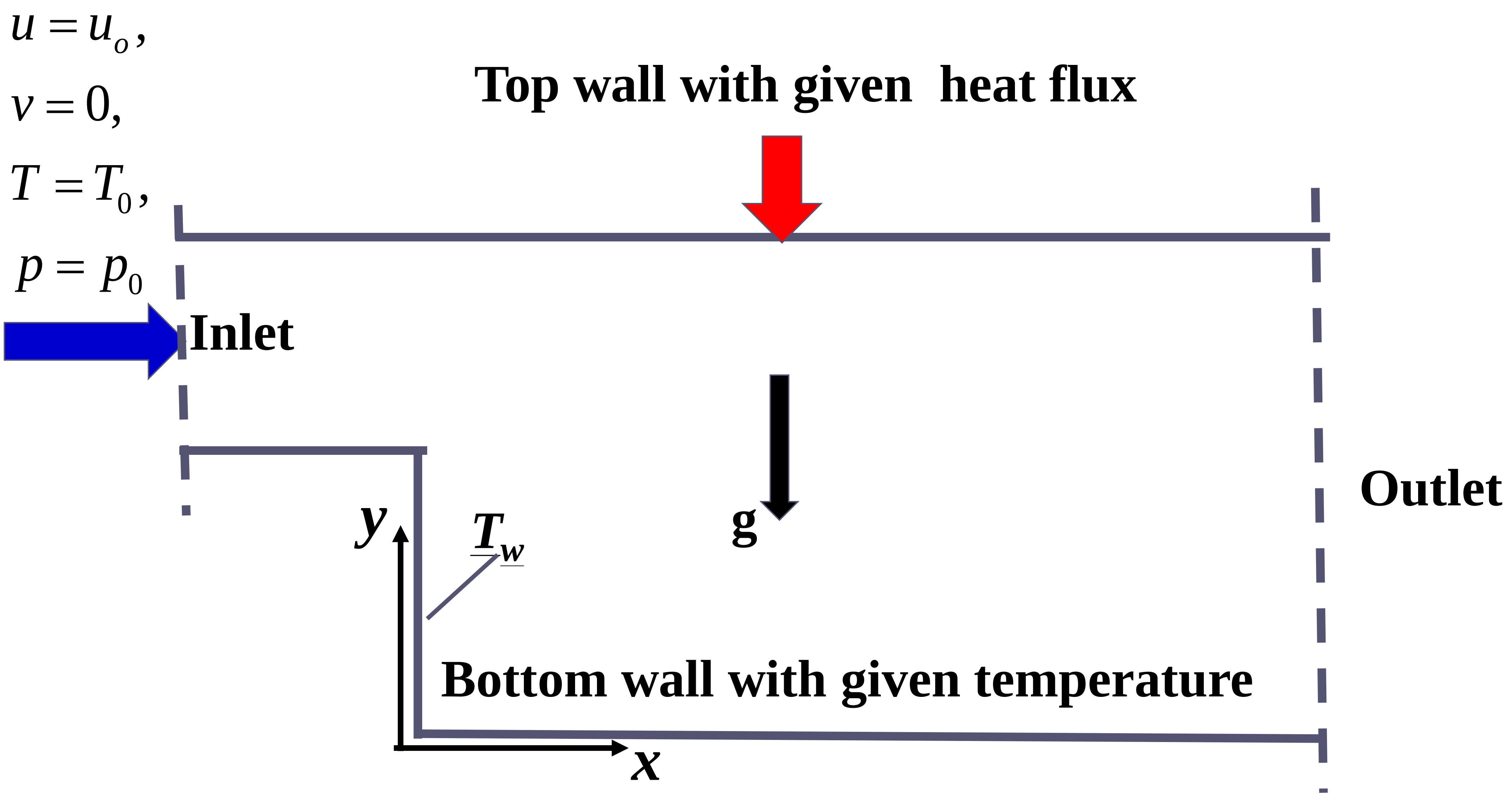


Figure of Prob. 1-1

Problem 1-2

Consider the following partial differential equation:

$$A\frac{\partial^2 T}{\partial x^2}+B\frac{\partial^2 T}{\partial x\partial y}+C\frac{\partial^2 T}{\partial y^2}=0$$

Determine the type of this equation for the following cases:

(1) A=1,B=3,C=2;
(2) A=1,B=-2,C=1;
(3) A=1,B=1,C=3.

Problem 1-3

(1) Determine the mathematical type of the following partial differential equation for two dimensional unsteady convective heat transfer:

$$\rho c_{\mathrm{p}}\left[\frac{\partial T}{\partial t}+\frac{\partial}{\partial x}(uT)+\frac{\partial}{\partial y}(vT)\right]=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+S$$

where T is the temperature, t is the time, x and y are the two coordinates, u and v are the two velocity components, and S is the source term. Note： There three independent valuables in this problem. Discussion should be conducted for (t,x) and (t,y) separately.

(2) If the heat transfer process becomes steady, what type is its governing equation for temperature?

Problem 1-4

The dimensionless energy equation of the slug flow（**段塞流**）in a circular tube is given by

$$\frac{\partial \Theta}{\partial X}=\frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial \Theta}{\partial R})+\frac{1}{Pe^2}\frac{\partial^2 \Theta}{\partial X^2})$$

where Θ, X, R are dimensionless temperature , axial distance and radius. Determine the mathematical type of this partial differential equation for following two cases:

(1) the values of *Pe* being finite (***Pe*的数值为有限大小**）；

(2) the values of *Pe* approaching infinite.

Important words you will study in next class:

Grid layout
节点布置

Overlap
重叠

Interface
界面

Spatial
空间的

Structured
结构化

Disorder
无序

Neighboring
相邻的

Vertex
顶点

Feasible
可行的

Accuracy
精确

Guarantee
保证

Debug
调试

Appropriate
合适的

fineness

truncation

exponent

forward

backward

stencil

distance

dimension

consistent

thumb

level

explicit

implicit

Please find their Chinese meanings by yourself

本组网页地址：http://nht.xjtu.edu.cn 欢迎访问！

Teaching PPT will be loaded on ou website

同舟共济

渡彼岸！

People in the same boat help each other to cross to the other bank, where….