Numerical Heat Transfer

(数值传热学)

Chapter 11 Application Examples of the General Code for 2D Elliptical FF & HT Problems (2)

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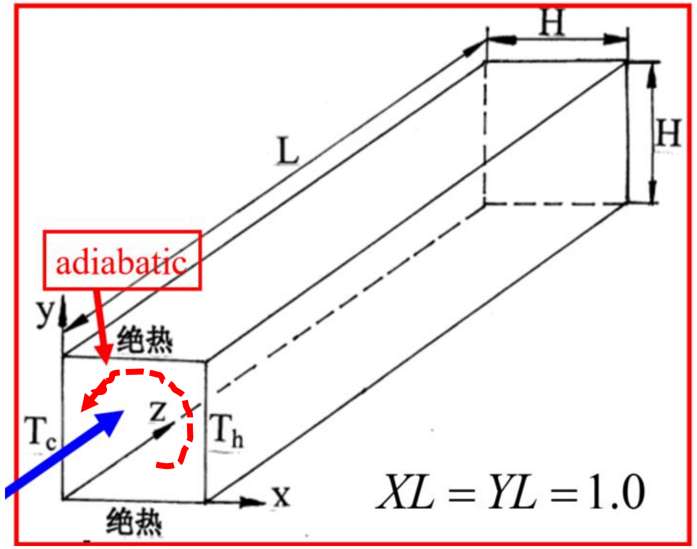
11-6 Complicated fully developed fluid flow and heat transfer in a square duct

---Velocity is regarded as a ϕ variable

11-6-1 Physical problem and its math formulation

Known: Fully developed heat transfer in a square duct shown in Fig. 1. The effect of gravitation is taken into account by Boussinesq assumption. Duct top and bottom walls are adiabatic, while left and right walls are kept at constant and uniform temperatures: T_c=0, T_h=1; Pr=0.7, η=1.0, dp/dz=-3000, and $\rho g \beta = 10^4$.

Find: Cross sectional distributions of u, v, and w, temperature distribution and fRe.



Natural convection due to buoyancy

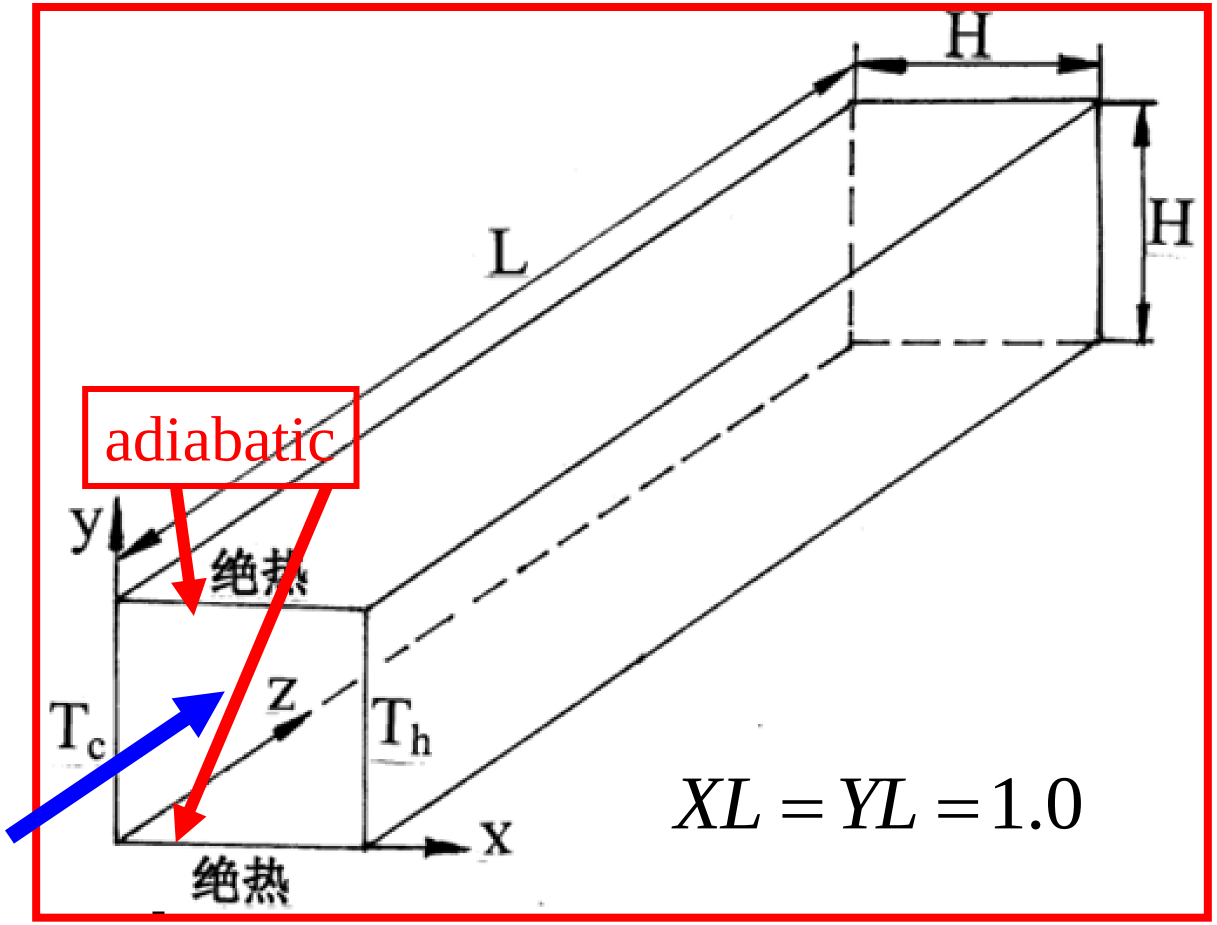


Fig. 1 Physical model of Problem 6

Main feature: When heat transfer goes into the fully developed region, the heat leaves the hot wall goes into the cold wall, *i.e.*, the heat transfer rate is determined by the flow at the cross-section, and the axial flow does not make any contribution to this heat transfer.

Analysis of the governing eq.：

- According to the fully developed condition

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\cancel{\frac{\partial u}{\partial z}}\right)=-\frac{\partial p}{\partial x}+\eta\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\cancel{\frac{\partial v}{\partial z}}\right)=-\frac{\partial p}{\partial y}+\eta\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\cancel{\frac{\partial^2 v}{\partial z^2}}\right)-\rho g$$

$$\rho\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\cancel{\frac{\partial w}{\partial z}}\right)=-\frac{\partial p}{\partial z}+\eta\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\cancel{\frac{\partial^2 w}{\partial z^2}}\right)$$

- The axial flow does not make contributions to heat transfer:

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\cancel{\frac{\partial T}{\partial z}}\right)=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\cancel{\frac{\partial^2 T}{\partial z^2}}\right)$$

Analysis for the computational domain：

This problem looks like Problem 3 where we take 1/4 of the cross section as the computational domain. Can we still take such practice for this case?

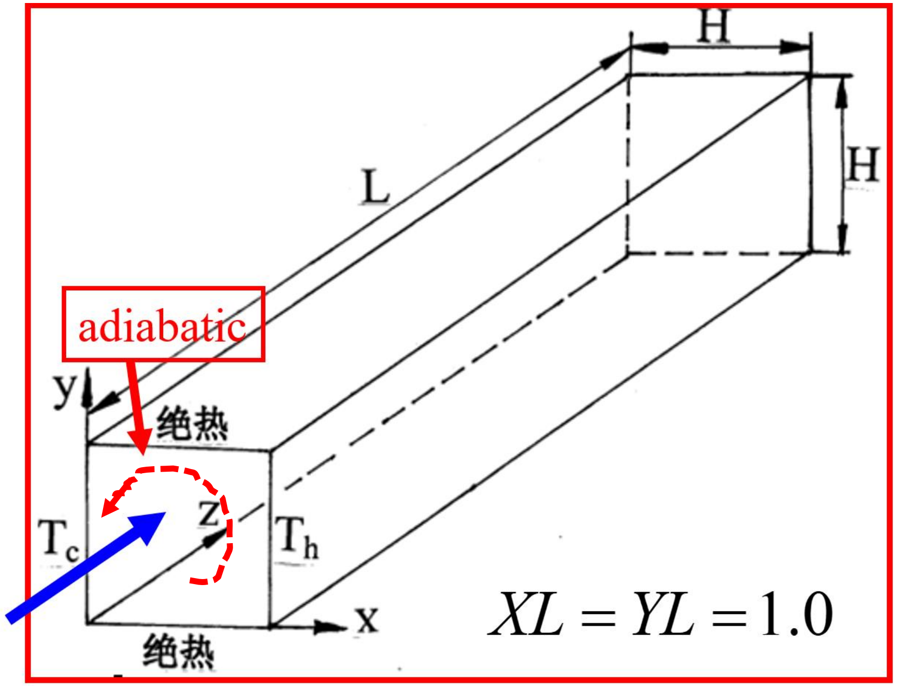
No! Because of the cross sectional natural convection, the entire region must be taken as the computational domain.

Boundary conditions：

At x=0, $T=T_c$: $x=XL$, $T=T_h$

At y=0 and $y=YL$: adiabatic

At four walls: $u=v=w=0$.



Major features of the problem

（1）There are three velocity components：*u*, *v*, *w* ; However *u*, *v* are not coupled with *w* ；

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+\eta(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}-\rho g+\eta(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})$$

$$\rho(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y})=-\frac{\partial p}{\partial z}+\eta(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2})$$

(2) *T* is coupled with velocity *u*, *v*. The variation of ρg term with *T* causes the buoyancy, driving the natural convection in cross section.

$$\rho c_p(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})=\lambda(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})$$

11-6-2 Numerical methods

(1) Boussinesq assumption is adopted for the density in the source term of *v*-equation: $\rho = \rho_{ref}[1-\beta(T-T_{ref})]$

Treatment of pressure gradient and gravitation term for *v*-equation

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}) = -\frac{\partial p}{\partial y} - \rho g + \eta(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})$$

$$-\frac{\partial p}{\partial y} - \rho g = -\frac{\partial p}{\partial y} - \rho_{ref}[1-\beta(T-T_{ref})]g \quad = -\frac{\partial p}{\partial y} - \rho_{ref}(1+\beta T_{ref})g + g\rho_{ref}\beta T$$

$$= -\frac{\partial}{\partial y}\underline{[p+\rho_{ref}(1+\beta T_{ref})gy]} + g\rho_{ref}\beta T \quad = -\frac{\partial p_{eff}}{\partial y} + g\rho_{ref}\beta T$$

$$p_{eff} \approx p$$

The source term in *v*-equation is a function of temperature

(2) How to use 2-D code for solving three velocity components?

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+\eta(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}+\rho g\beta T+\eta(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})$$

$$\rho c_p(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})=\lambda(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})$$

$$\rho(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y})=-\frac{\partial p}{\partial z}+\eta(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2})$$

- *u*, *v*, *T* are coupled and should be solved simultaneously;
- *u*, *v*, *T* are not coupled with *w*, while *w* is coupled with *u* and *v*; *u*, *v*, *T* are solved first, then *w* is solved. Thus *w* is regarded as a scalar variable.

(3) The problem studied can be separated into two sub-problems：

(a) Natural convection in a 2-D square cavity: *u*, *v*, *T* are solved;

(b) Fully developed axial flow for solving *w*, with a pre-specified source term of $-dp/dz$.

Governing equations of the problem studied:

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p_{eff}}{\partial x}+\eta(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p_{eff}}{\partial y}+\eta(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})+\rho g\beta T$$

$$\rho c_p(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})=\lambda(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})$$

Natural convection

Solved first to get u, v and T

$$\rho(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y})=-\frac{dp}{dz}+\eta(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2})$$

Solved 2nd with known u, v and specified pressure gradient!

dp/dz (<0) can be assumed and is specified as -3000.

11-6-3 Program reading

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
       MODULE USER_L
C*******************************************************************
       INTEGER*4 I,J
       REAL*8  GBR, DPDZ, PR, AMU, FRE, WBAR, TM
       END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE USER
C********************************************************************
      USE START_L
      USE USER_L
      IMPLICIT NONE
C*********************************************************************
C----------------------------------------PROBLEM SEVEN--------------------------------- -----
C            Complex fully developed laminar fluid flow and heat transfer in a
C                                        horizontal square duct
C********************************************************************
```

```
ENTRY GRID
TITLE(1)=‘ .VEL  U.'
TITLE(2)=‘ .VEL  V.'
TITLE(3)=‘ .STR FN.'
TITLE(4)=‘ . TEMP .'
TITLE(5)=‘ .W/WBAR.'
TITLE(11)='PRESSURE'
RELAX(1)=0.8
RELAX(2)=0.8
LSOLVE(1)=.TRUE.
LSOLVE(4)=.TRUE.
LPRINT(1)=.TRUE.
LPRINT(2)=.TRUE.
LPRINT(3)=.TRUE.
LPRINT(4)=.TRUE.
LPRINT(5)=.TRUE.
LPRINT(11)=.TRUE.
LAST=25
XL=1.
YL=1.
L1=7
M1=7
CALL UGRID
RETURN
```

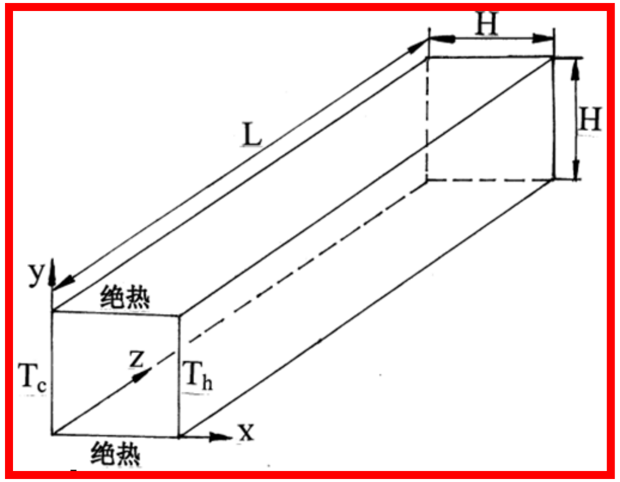
! *w* is treated as fifth variable!

! Not for *w*; With known *u*, *v*, the *w* eq is linear.

! *u*, *v*, *p*, *T* are solved first

! In SIMPLER code, when the 1st variable is set to be solved, the 2nd and 3rd ones (v and p_c) are automatically regarded as variables to be solved.

! Computation for the entire region



```
      ENTRY START
      GBR=1.E4
      DPDZ=-3000.
      DO 100 J=1,M1
      DO 101 I=1,L1
      U(I,J)=0.
      V(I,J)=0.
      T(I,J)=0.
      T(L1,J)=1.
      F(I,J,5)=100.
      IF (I==1.OR.I==L1)  F(I,J,5)=0.
      IF (J==1.OR.J==M1)  F(I,J,5)=0.
101 ENDDO
100 ENDDO
      PR=0.7
      AMU=1.
      AMUP=AMU*CPCON/PR
      RETURN

      ENTRY DENSE
      RETURN
```

! $\rho g \beta$

!Initial temperature and some boundary conditions

! Initial field for axial velocity *w*

!Boundary cond. of *w* at four walls *w*=0

! CPCON=1, default value

$$!\Pr = \mu c_p / \lambda; \quad \lambda = \mu c_p / \Pr$$

! λ for temperature

Bossinesq assumption: It ignores density differences except where they appear in terms ρg

```
      ENTRY BOUND
      FRE=0.
      IF(ITER<20) RETURN
      IF(.NOT.LSOLVE(5)) THEN
      LSOLVE(1)=.FALSE.
      LSOLVE(5)=.TRUE.
      ENDIF
      WBAR=0.
      DO 302 J=2,M2
      DO 303 I=2,L2
      WBAR=WBAR+F(I,J,5)*XCV(I)*YCV(J)
  303 ENDDO
  302 ENDDO
      FRE=-DPDZ*2.*4.*(XL*YL)**3/(XL+YL)**2/(WBAR*AMU)
      RETURN
```

! *w* is not solved when ITER<20

! Switch of the solved variables, only executed once. The default value of LSOLVE is FALSE. When ITER=20, .NOT.LSOLVE(5) is TRUE; When ITER>=21, it is .FALSE.

! Once 5th variable is solved, these two statements are not needed to executed any more.

! For computing average velocity

! Computing (*fRe*) according to definition; Shown in the next page.

! WBAR=WBAR/(XL*YL) Because both XL and YL =1, this calculation is ignored!

```
FRE=-DPDZ*2.*4.*(XL*YL)**3/(XL+YL)**2/(WBAR*AMU)
```

$$f\ \mathrm{Re} = -[(dp/dz)D_h / \frac{1}{2}\rho w_m^2]\frac{\rho w_m D_h}{\eta}$$

$$f\ \mathrm{Re} = -2[(dp/dz)D_h^2 / w_m\eta] = \frac{-2dp/dz}{\eta(\sum w_{i,j}\Delta A_{i,j}/A)}\bullet(\frac{4A}{P})^2$$

$$= \frac{-2dp/dz}{\eta\sum w_{i,j}\Delta A_{i,j}}\bullet(\frac{4A}{P})^2 A$$

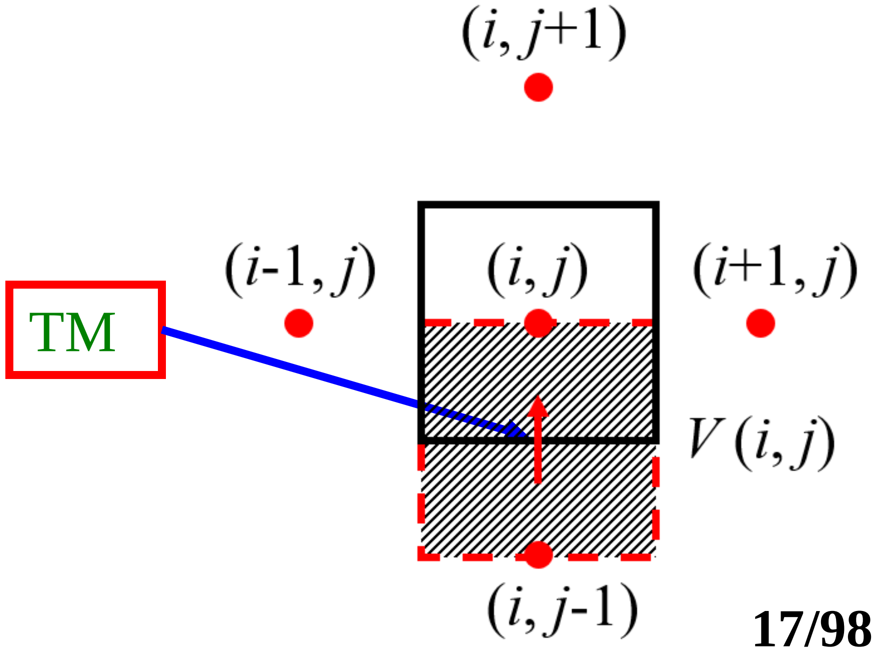
$$= \frac{-2dp/dz}{\eta\sum w_{i,j}\Delta A_{i,j}}\bullet(\frac{4XL*YL}{2(XL+YL)})^2\bullet XL*YL$$

$$= \frac{-2dp/dz}{\eta\sum w_{i,j}\Delta A_{i,j}}\bullet\frac{4(XL*YL)^3}{(XL+YL)^2}$$

```
      ENTRY OUTPUT
      IF(ITER==0) THEN
      PRINT401
      WRITE(8,401)
401 FORMAT(1X,'  ITER',6X,'SMAX',8X,'SSUM',7X,'V(6,4)',
     &  6X,'T(2,6)',6X,'F.RE')
       ELSE
       PRINT 403, ITER, SMAX, SSUM, V(6,4), T(2,6), FRE
       WRITE(8,403) ITER,SMAX,SSUM,V(6,4),T(2,6),FRE
403 FORMAT(1X,I6,1P5E12.3)
       ENDIF
      IF(ITER/=LAST) RETURN
      DO 410 J=1,M1
      DO 411 I=1,L1
      F(I,J,5)=F(I,J,5)/WBAR      !Dimensionless output for w
411 ENDDO
410 ENDDO
      CALL PRINT
      RETURN
```

```
      ENTRY GAMSOR
      DO 500 J=1,M1
      DO 501 I=1,L1
      GAM(I,J)=AMU          !  Γ for velocity
      IF(NF== 4) THEN
      GAM(I,J)=COND         !  Γ for temp.
      GAM(I,1)=0.           !  Adiabatic for south and north boundaries
      GAM(I,M1)=0.
      ENDIF
501ENDDO
500ENDDO
      DO 510 J=2,M2
      DO 511 I=2,L2
      IF(NF==2) THEN
      IF(J/=2) THEN                !  JST = 3 for v-eq
      TM=(T(I,J)+T(I,J-1))*0.5
      CONT(I,J)=TM*GBR             !  Source term of v-eq.
      ENDIF
      ENDIF
      IF(NF==5)  CON(I,J)=-DPDZ
511 ENDDO                      !  Source term of w-eq.
510 ENDDO
      RETURN
      END
```

$GBR = g\rho_{ref}\beta T$

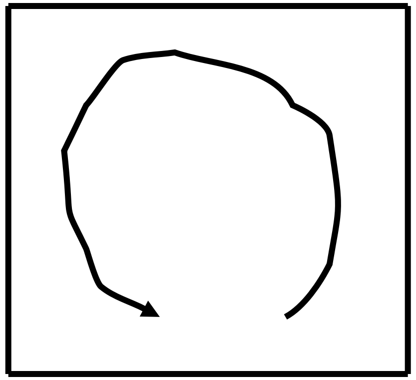


11-6-4 Results analysis

```
        COMPUTATION IN CARTESIAN COORDINATES
        **********************************************

ITER   SMAX        SSUM        V(6,4)      T(2,6)      F.RE
   0   0.000E+00   0.000E+00   0.000E+00   0.000E+00   0.000E+00
   1   0.000E+00   0.000E+00   0.000E+00   1.000E-01   0.000E+00
   2   1.273E+01  -1.907E-06   1.016E+01   2.848E-01   0.000E+00
   3   6.308E+00   1.073E-06   1.926E+01   3.445E-01   0.000E+00
   4   2.978E+00   7.153E-07   2.076E+01   3.826E-01   0.000E+00
   5   1.237E+00  -5.960E-07   2.284E+01   3.854E-01   0.000E+00
   6   6.454E-01  -4.768E-07   2.304E+01   3.889E-01   0.000E+00
   7   2.911E-01   7.153E-07   2.342E+01   3.894E-01   0.000E+00
   8   1.338E-01  -3.278E-07   2.346E+01   3.900E-01   0.000E+00
   9   6.046E-02  -5.364E-07   2.352E+01   3.900E-01   0.000E+00
  10   2.868E-02  -5.364E-07   2.352E+01   3.900E-01   0.000E+00
  11   1.286E-02  -4.321E-07   2.353E+01   3.900E-01   0.000E+00
```

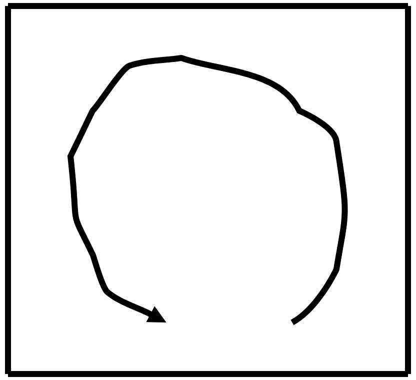
```
! ITER   SMAX         SSUM         V(6,4)        T(2,6)        F.RE
  12   6.224E-03   2.850E-07   2.353E+01   3.901E-01   0.000E+00
  13   3.349E-03  -3.660E-07   2.353E+01   3.901E-01   0.000E+00
  14   1.544E-03   1.974E-07   2.353E+01   3.901E-01   0.000E+00
  15   8.407E-04  -2.626E-07   2.353E+01   3.901E-01   0.000E+00
  16   3.686E-04  -1.118E-08   2.353E+01   3.901E-01   0.000E+00
  17   1.961E-04   1.043E-07   2.353E+01   3.901E-01   0.000E+00
  18   7.963E-05   2.775E-07   2.353E+01   3.901E-01   0.000E+00
  19   4.327E-05   3.166E-08   2.353E+01   3.901E-01   0.000E+00
  20   2.098E-05  -1.825E-07   2.353E+01   3.901E-01   6.000E+01   ! Solving w-eq.
  21   2.098E-05  -1.825E-07   2.353E+01   3.901E-01   5.323E+01
  22   2.098E-05  -1.825E-07   2.353E+01   3.901E-01   5.238E+01
  23   2.098E-05  -1.825E-07   2.353E+01   3.901E-01   5.236E+01
  24   2.098E-05  -1.825E-07   2.353E+01   3.901E-01   5.236E+01
  25   2.098E-05  -1.825E-07   2.353E+01   3.901E-01   5.236E+01
```

```
***************************   .VEL  U.   ***************************

 I =       2         3         4         5         6         7
J
 7    0.00E+00 0.00E+00  0.00E+00 0.00E+00  0.00E+00  0.00E+00
 6    0.00E+00-1.52E+01-1.78E+01-1.77E+01 -1.31E+01  0.00E+00
 5    0.00E+00-8.36E+00-1.40E+01-1.40E+01 -9.70E+00  0.00E+00
 4    0.00E+00 7.76E-01  8.31E-02 -8.31E-0 2 -7.76E-01  0.00E+00
 3    0.00E+00 9.70E+00 1.40E+01 1.40E+01   8.36E+00  0.00E+00
 2    0.00E+00 1.31E+01 1.77E+01 1.78E+01   1.52E+01  0.00E+00
 1    0.00E+00 0.00E+00 0.00E+00 0.00E+00   0.00E+00  0.00E+00
```

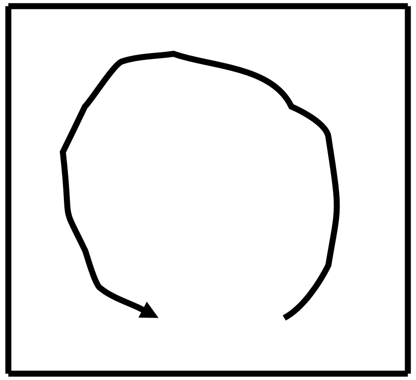


Natural convection
in cross section

```
****************************    .VEL  V.    ****************************

 I =       1          2          3          4          5          6          7
J
 7    0.00E+00 0.00E+00  0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00
 6    0.00E+00-1.52E+01-2.64E+00 1.01E-01  4.66E+00 1.31E+01 0.00E+00
 5    0.00E+00-2.35E+01-8.26E+00 8.31E-02  8.96E+00 2.28E+01 0.00E+00
 4    0.00E+00-2.28E+01-8.96E+00-8.31E-02  8.26E+00 2.35E+01 0.00E+00
 3    0.00E+00-1.31E+01-4.66E+00-1.01E-01  2.64E+00 1.52E+01 0.00E+00
 2    0.00E+00 0.00E+00  0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00
```



```
****************************   .STR FN.   ****************************

 I =      2          3          4          5          6          7
J
 7    0.00E+00-3.91E-07  2.60E-07  1.16E-07  1.26E-08  0.00E+00
 6    0.00E+00 3.03E+00  3.56E+00  3.54E+00 2.61E+00 0.00E+00
 5    0.00E+00 4.71E+00  6.36E+00  6.34E+00 4.55E+00 0.00E+00
 4    0.00E+00 4.55E+00  6.34E+00  6.36E+00 4.71E+00 0.00E+00
 3    0.00E+00 2.61E+00  3.54E+00  3.56E+00 3.03E+00 0.00E+00
 2    0.00E+00 0.00E+00  0.00E+00  0.00E+00 0.00E+00 0.00E+00
```



Stream functions of the four walls are zero

No decoration

```
***************************     . TEMP .    ***************************

I =       1          2          3          4          5          6          7
J
7   0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00 1.00E+00
6   0.00E+00 3.90E-01  7.20E-01  7.86E-01  8.33E-01  9.11E-01  1.00E+00
5   0.00E+00 3.25E-01  6.21E-01  6.77E-01  7.15E-01  8.43E-01  1.00E+00
4   0.00E+00 2.41E-01  4.58E-01  5.00E-01  5.42E-01  7.59E-01  1.00E+00
3   0.00E+00 1.57E-01  2.85E-0 1 3.23E-01  3.79E-01  6.75E-0 1 1.00E+00
2   0.00E+00 8.92E-02  1.67E-01  2.14E-01  2.80E-01  6.10E-0 1 1.00E+00
1   0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00 1.00E+00
```

Hot wall

Cold wall

No decoration

```
**************************    .W/WBAR.    **************************

 I =      1         2         3         4         5         6         7
J
 7   0.00E+00 0.00E+00 0.00E+00 0.00E+00  0.00E+00 0.00E+00 0.00E+00
 6   0.00E+00 4.96E-01  7.74E-01  7.73E-01  6.99E-01  4.72E-01  0.00E+00
 5   0.00E+00 7.89E-01  1.50E+00 1.54E+00 1.34E+00  7.52E-01 0.00E+00
 4   0.00E+00 8.21E-01  1.63E+00 1.85E+00 1.63E+00  8.21E-01 0.00E+00
 3   0.00E+00 7.52E-01  1.34E+00 1.54E+00 1.50E+00  7.89E-01 0.00E+00
 2   0.00E+00 4.72E-01  6.99E-01  7.73E-01  7.74E-01   4.96E-01 0.00E+00
 1   0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00  0.00E+00 0.00E+00
```

w velocity of the four walls are zero

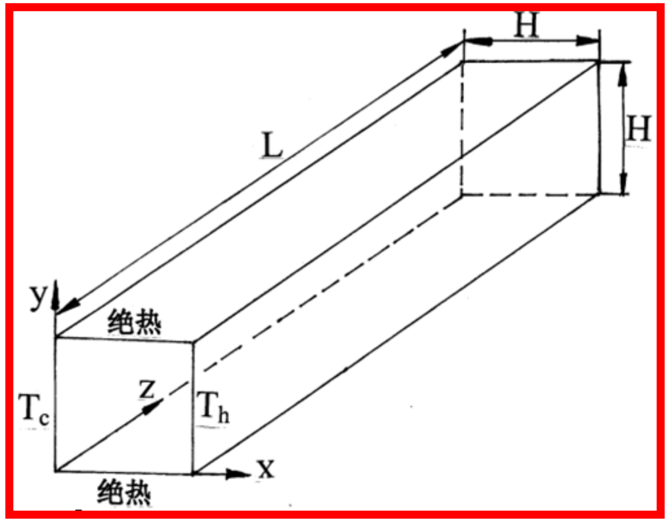
Pmax

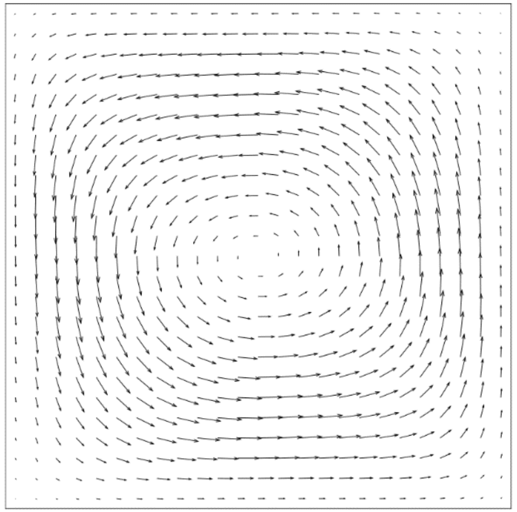
```
*************************** PRESSURE ***************************

 I =      1         2         3         4         5         6         7
J
 7   3.64E+03 3.73E+03 4.05E+03  4.33E+03 4.67E+03  4.89E+03 5.00E+03
 6   3.09E+03 3.18E+03 3.36E+03  3.56E+03 3.84E+03  4.05E+03 4.16E+03
 5   2.14E+03 2.09E+03 1.99E+03  2.02E+03 2.17E+03  2.36E+03 2.46E+03
 4   1.10E+03 1.02E+03 8.42E+02  7.85E+02 8.42E+02  1.02E+03 1.10E+03
 3   4.58E+02 3.63E+02 1.73E+02  2.45E+01-7.31E+00  9.20E+01 1.42E+02
 2   1.56E+02 5.04E+01-1.61E+02-4.37E+02-6.35E+02 -8.17E+02-9.08E+02
 1   0.00E+00-1.06E+02-3.28E+02-6.67E+02-9.49E+02 -1.27E+03-1.36E+03
```

Pressure reference point

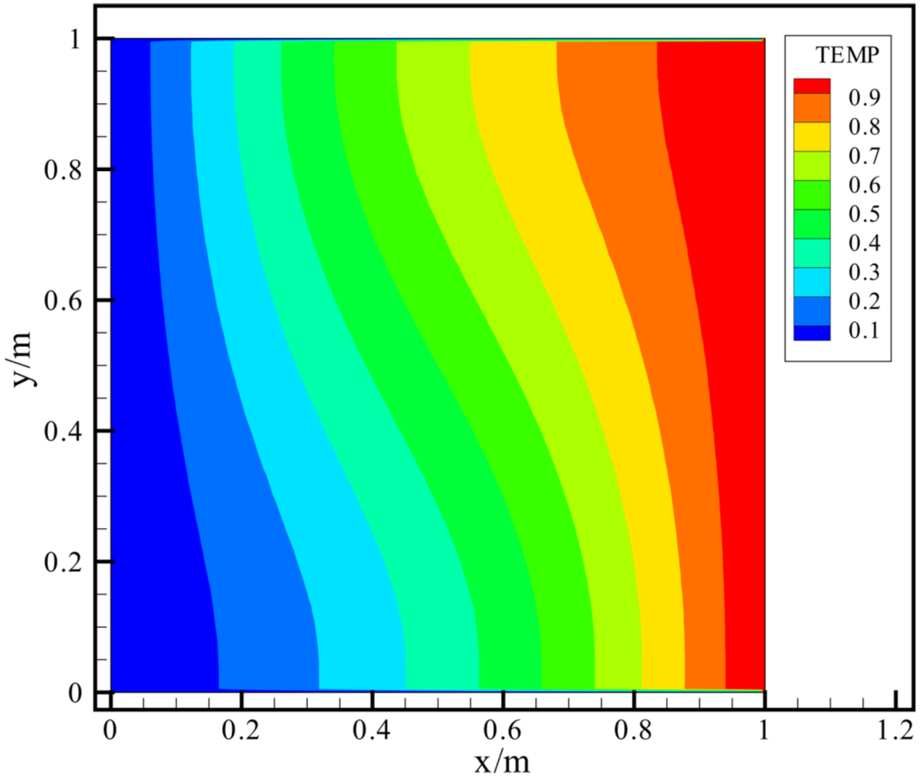
Pmin

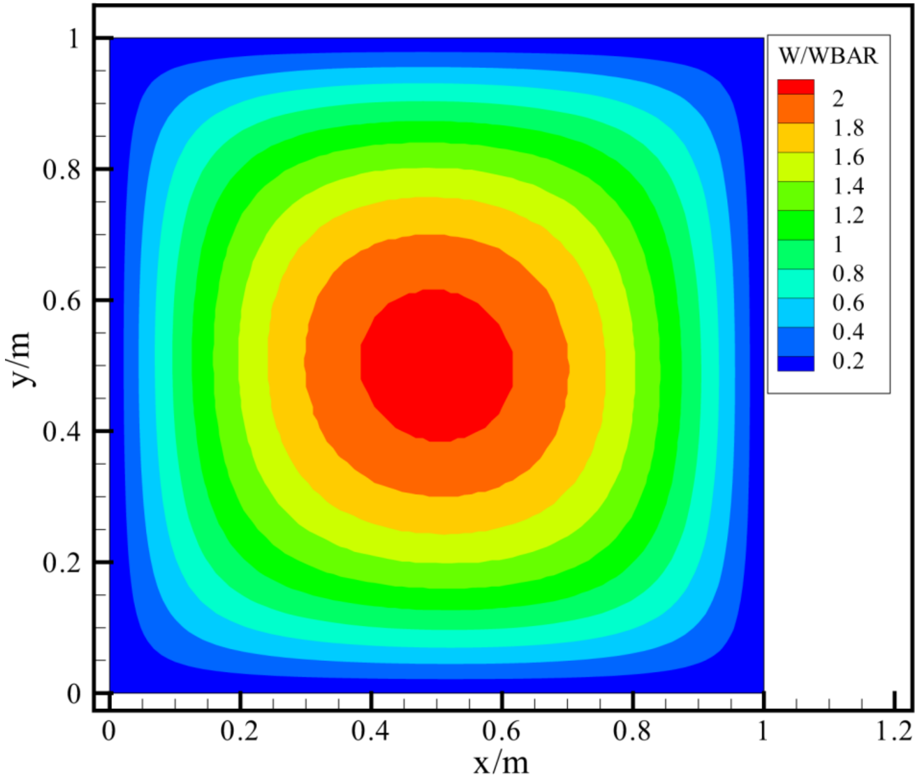
Fig. 2 Results of Problem 6





Temp.





Axial velocity

11-7 Impinging flow on a rotating disc ---Discretization of source term of momentum equation in cylindrical coordinate

11-7-1 Physical problem and its math formulation

Known: A rotating disc with ω=100 is partially covered by a shell (壳体). Fluid flows into the shell through the central inlet of the shell with inlet velocity $U_{in}=100$; impinges onto the disc and then leaves the disc (盘) through the gap between the shell and the disc. Fluid viscosity $\eta=1$.

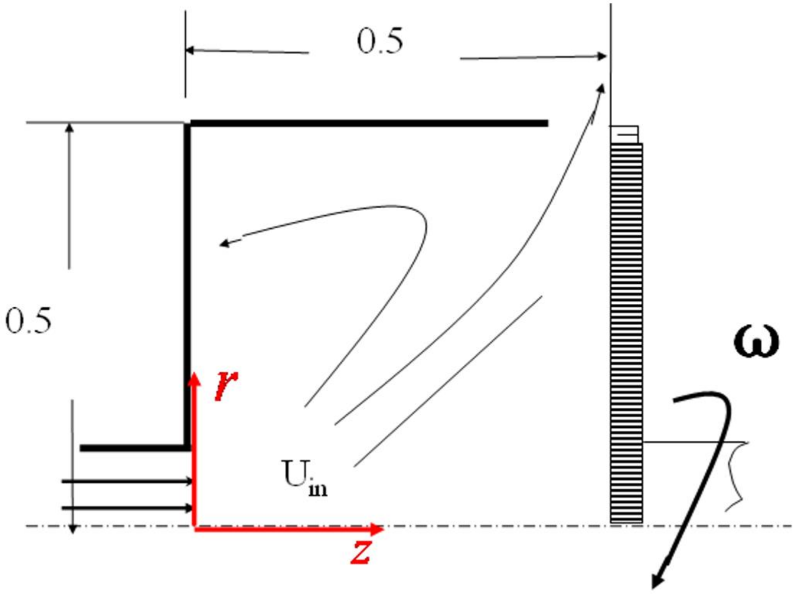
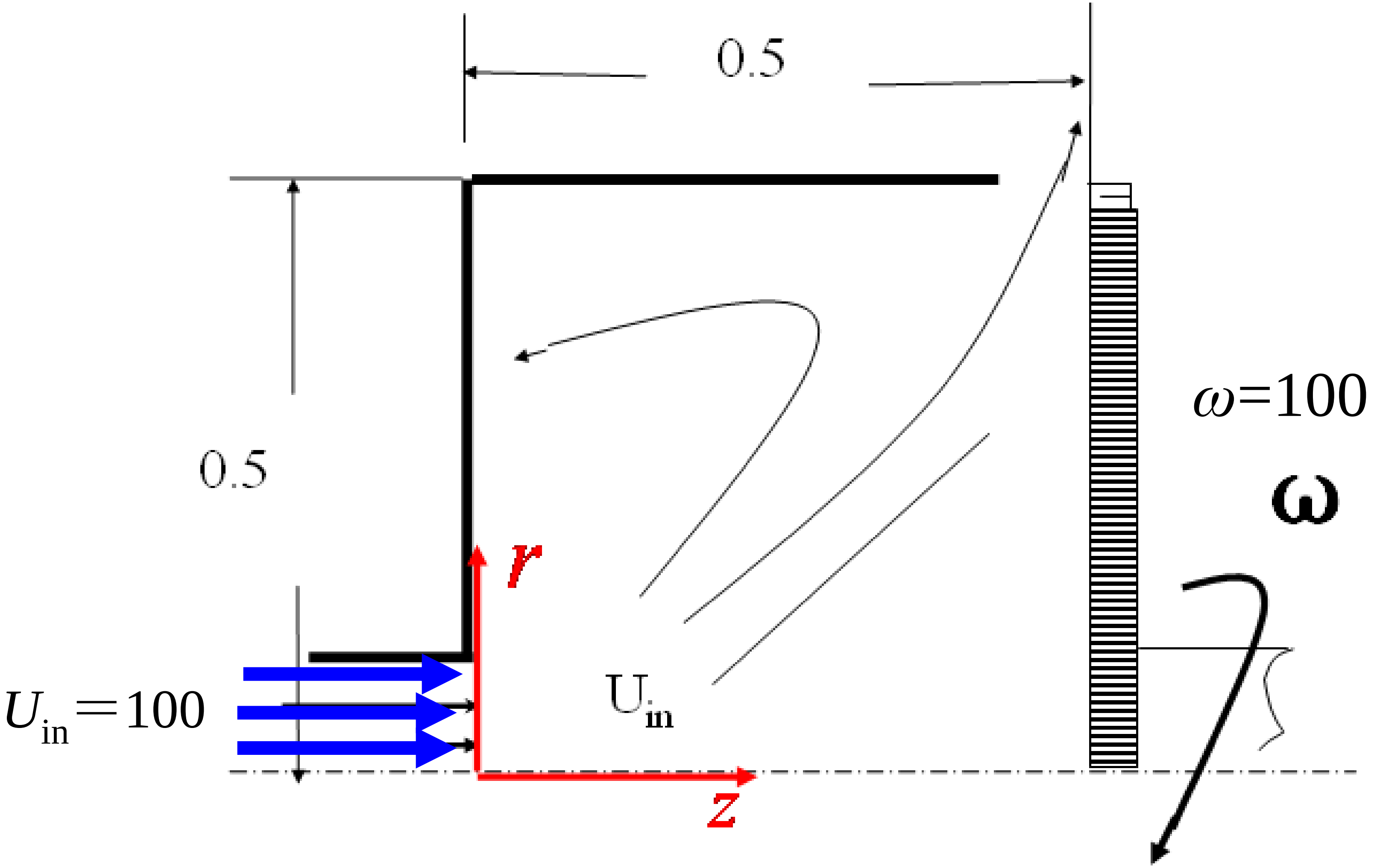
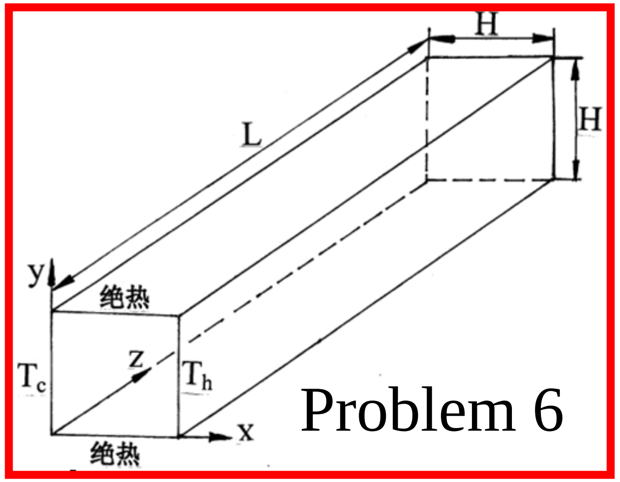
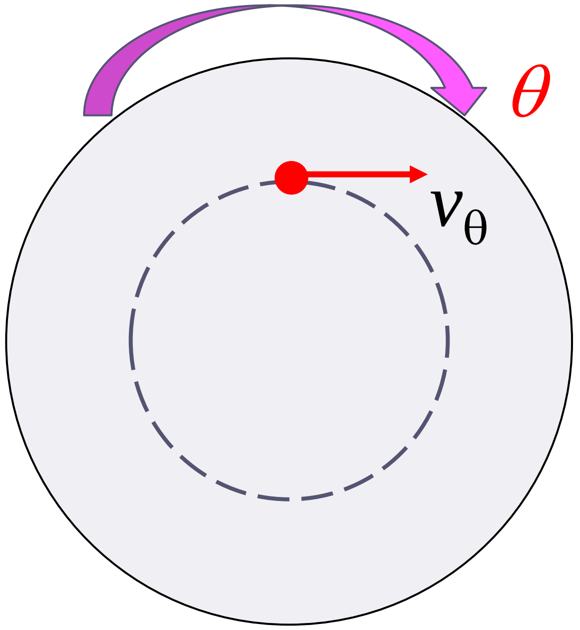


Fig.1 Schematic diagram of problem 7



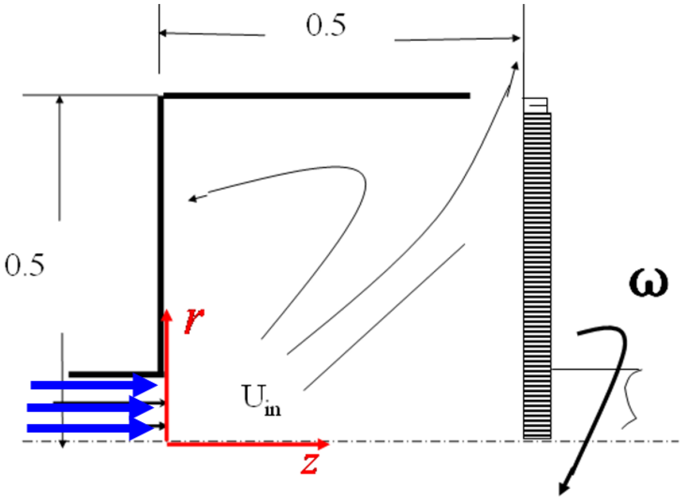




No change along the circumferential (θ) direction（圆周方向）

Find： Velocity and pressure distribution in the cavity.

Solution： This is a fluid flow problem in three-dimensional cylindrical coordinate. The fluid flow is caused by the impingement of the inlet flow and the rotating effect of the disc. The circumferential velocity, v_θ, is purely caused by the rotating disc. Thus, there exists v_θ, but no circumferential pressure drop. The velocity along the θ direction is uniform when in steady state.



Circumferential θ

- Original N-S eqs. in cylindrical coordinate are:

$$z\ \text{direction}: \rho(v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}+\cancel{\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}})=-\frac{\partial p}{\partial z}+\eta(\frac{\partial^2 v_z}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_z}{\partial r})+\cancel{\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}})$$

$$r\ \text{direction}: \rho(v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}+\cancel{\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}})=-\frac{\partial p}{\partial r}+\eta(\frac{\partial^2 v_r}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_r}{\partial r})+\cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}}-\cancel{\frac{2}{r}\frac{\partial v_\theta}{\partial \theta}})$$

$$+\rho\frac{v_\theta^2}{r}-\eta\frac{v_r}{r^2} \longleftarrow \textbf{Source term}$$

$$\theta\ \text{direction}: \rho(v_r\frac{\partial v_\theta}{\partial r}+v_z\frac{\partial v_\theta}{\partial z}+\cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}})=0+\eta(\frac{\partial^2 v_\theta}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_\theta}{\partial r})+\cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}}+\cancel{\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}})$$

$$\textbf{Source term} \longrightarrow -\rho\frac{v_r v_\theta}{r}-\eta\frac{v_\theta}{r^2}$$

Zero pressure gradient!

There exists v_θ, but $\frac{\partial}{\partial\theta}$ should be zero for this problem.

- Thus, governing equations of the three velocities are:

$$z \text{ direction}: \rho(v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z}) = -\frac{\partial p}{\partial z} + \eta(\frac{\partial^2 v_z}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_z}{\partial r}))$$

$$r \text{ direction}: \rho(v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z}) = -\frac{\partial p}{\partial r} + \eta(\frac{\partial^2 v_r}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_r}{\partial r}))$$

$$+\rho\frac{v_\theta^2}{r} - \eta\frac{v_r}{r^2}$$

Source term

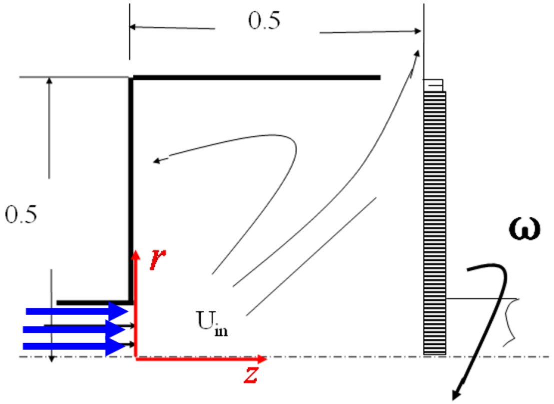
$$\theta \text{ direction}: \quad \rho(v_r \frac{\partial v_\theta}{\partial r} + v_z \frac{\partial v_\theta}{\partial z}) = 0 + \eta(\frac{\partial^2 v_\theta}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_\theta}{\partial r}))$$

$$-\rho\frac{v_r v_\theta}{r} - \eta\frac{v_\theta}{r^2}$$

Source term

11-7-2 Numerical method

(1) There are **three** velocity components, but **no terms contain** $\partial/\partial\theta$, such as no terms with $\partial/\partial z$ in Example 6.



$$v_z:\quad \rho\left(v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\eta\left(\frac{\partial^2 v_z}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right)$$

$$v_r:\quad \rho\left(v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\eta\left(\frac{\partial^2 v_r}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right)\right)+\rho\frac{v_\theta^2}{r}-\eta\frac{v_r}{r^2}$$

$$v_\theta:\quad \rho\left(v_r\frac{\partial v_\theta}{\partial r}+v_z\frac{\partial v_\theta}{\partial z}\right)=\eta\left(\frac{\partial^2 v_\theta}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right)\right)-\rho\frac{v_r v_\theta}{r}-\eta\frac{v_\theta}{r^2}$$

scalar variable

(2) v_θ is not in convection terms of v_z, v_r, but it is included in source term of v_r. Thus, v_θ can be viewed as a **scalar variable** (such as *T*) coupled with v_r, v_z; it is a 2-D cylindrical case with MODE＝2.

(3) In v_θ eq., rv_θ can be taken as the variable to be solved to enhance solution stability.

The original v_θ momentum equation is:

$$\rho\left(v_r\frac{\partial v_\theta}{\partial r}+v_z\frac{\partial v_\theta}{\partial z}\right)=\ \eta\left(\frac{\partial^2 v_\theta}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right)\right)-\rho\frac{v_r v_\theta}{r}-\eta\frac{v_\theta}{r^2}$$

It is transformed to： $\rho\left(v_r\frac{\partial(rv_\theta)}{\partial r}+v_z\frac{\partial(rv_\theta)}{\partial z}\right)=$

$$\eta\left(\frac{\partial^2(rv_\theta)}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial(rv_\theta)}{\partial r}\right)\right)\underline{-\frac{2\eta}{r}\frac{\partial(rv_\theta)}{\partial r}}$$

rv_θ taken as variable

（4）Numerical treatment of source term in v_{r} :

$$\rho\left(v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\eta\left(\frac{\partial^2 v_r}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right)\right)+\rho\frac{v_\theta^2}{r}-\eta\frac{v_r}{r^2}$$

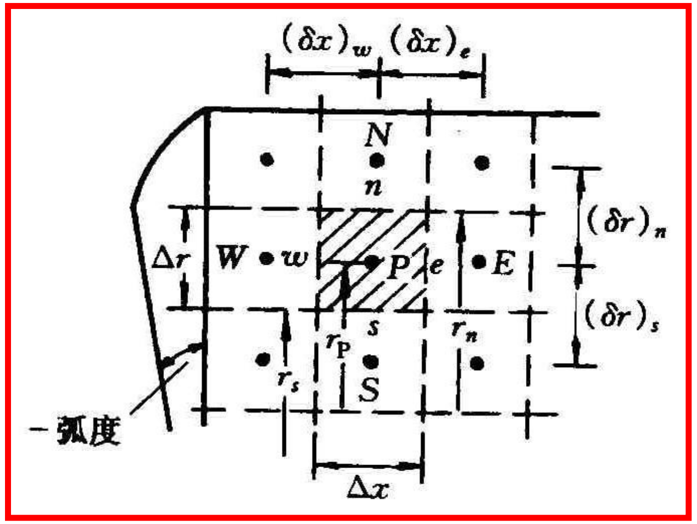
$$S_{v_r}=\rho\frac{v_\theta^2}{r}-\eta\frac{v_r}{r^2}=\rho\frac{(rv_\theta)^2}{r^3}\underbrace{-\eta\frac{1}{r^2}}_{S_{\mathrm{P}}}v_r$$

$$S_\phi=S_{\mathrm{c}}+S_{\mathrm{p}}\phi$$

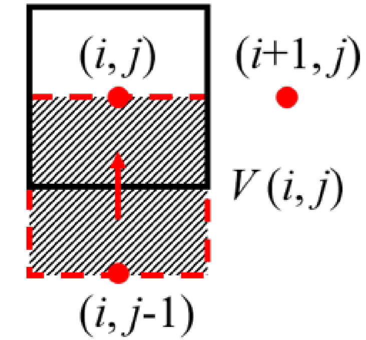
Numerical treatment of source term of rv_θ

$$\rho(v_r\frac{\partial(rv_\theta)}{\partial r}+v_z\frac{\partial(rv_\theta)}{\partial z})=\eta(\frac{\partial^2(rv_\theta)}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial(rv_\theta)}{\partial r}))-\frac{2\eta}{r}\frac{\partial(rv_\theta)}{\partial r}$$

$$S_{(rv_\theta)}=-\frac{2\eta}{r}\frac{\partial(rv_\theta)}{\partial r}=-\frac{2\eta}{r_P}\frac{(rv_\theta)_n-(rv_\theta)_s}{YCV(j)}$$



Idea of FUD

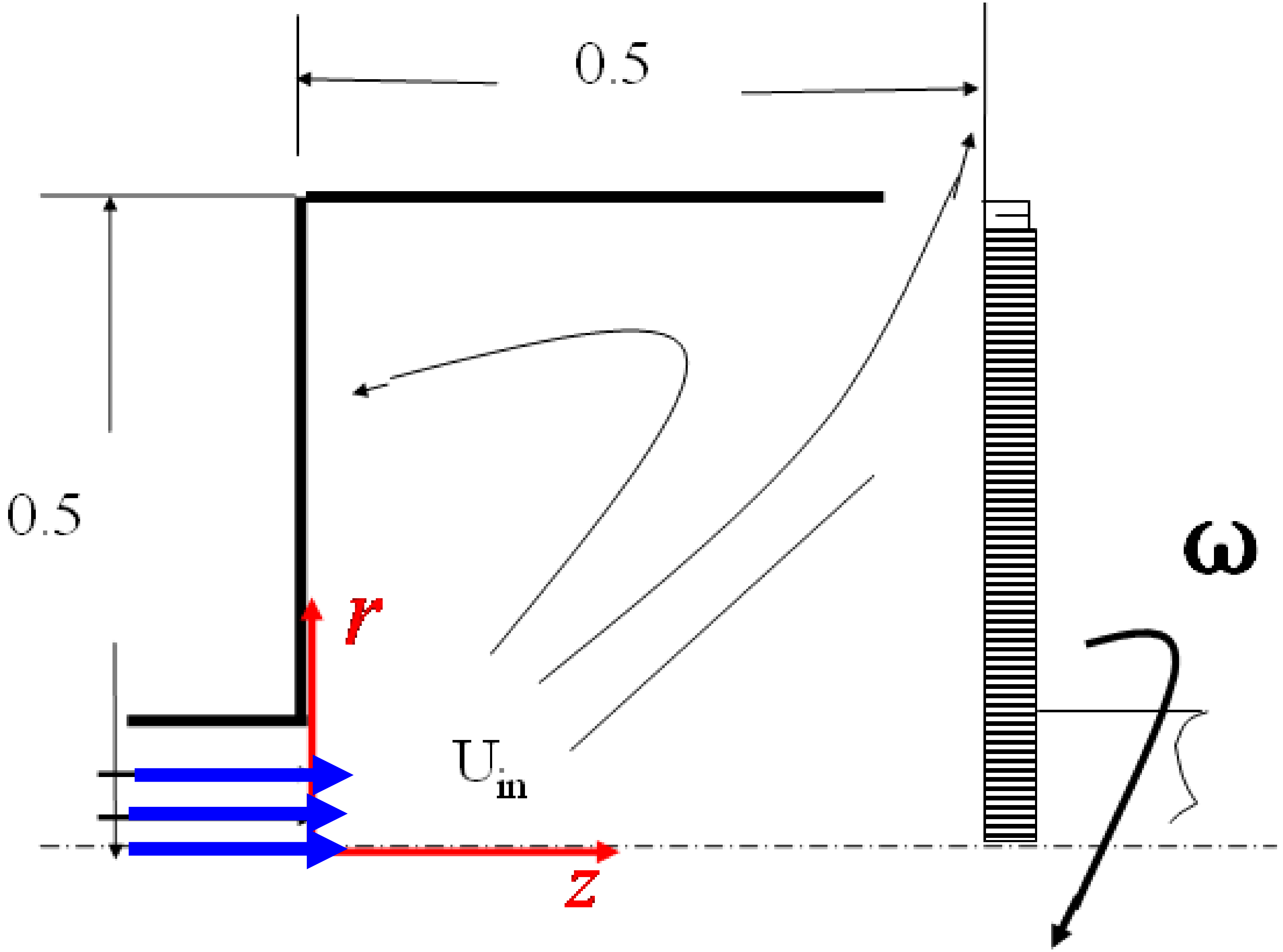


$$-\frac{2}{r_P}\frac{\eta}{YCV(j)}[(rv_\theta)_P-(rv_\theta)_S]$$

$$=\underbrace{\frac{2}{r_P}\frac{\eta(rv_\theta)_S}{YCV(j)}}_{S_C}\underbrace{-\frac{2}{r_P}\frac{\eta}{YCV(j)}}_{S_P}(rv_\theta)_P$$

Introducing a minor value of S_p to enhance solution stability.

Fig.1 Schematic diagram of Example 7



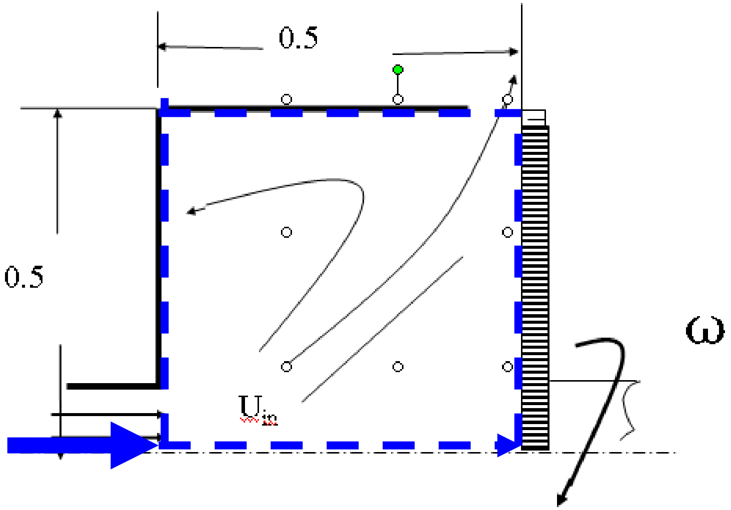
11-7-3 Program reading

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C*************************************************************************
       INTEGER*4 I,J
       REAL*8 OMEGA, UIN, AMU, FLOWIN, AR, ADD, FL,
     1 RSWM, RHOM, FLT
       END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
       SUBROUTINE USER
C**************************************************************************
       USE START_L
       USE USER_L
       IMPLICIT NONE
C***************************************************************************
C--------------------------PROBLEM EIGHT------------------------------
C                  Laminar impinging flow over a rotating disk
C***************************************************************************
```

```
ENTRY GRID
 TITLE(1)=‘ .VEL  U.'
 TITLE(2)=‘ .VEL  V.'
 TITLE(3)=‘ .STR FN.‘
 TITLE(5)=‘ .R.VTH.'
 TITLE(11)='PRESSURE'
 RELAX(1)=0.8
 RELAX(2)=0.8
 LSOLVE(1)=.TRUE.
 LSOLVE(5)=.TRUE.
 LPRINT(1)=.TRUE.
 LPRINT(2)=.TRUE.
 LPRINT(3)=.TRUE.
 LPRINT(5)=.TRUE.
 LPRINT(11)=.TRUE.
 LAST=25
 MODE=2
 R(1)=0.
 XL=0.5
 YL=0.5
 L1=7
 M1=7
 CALL UGRID
 RETURN
```

Regarding (rv_θ) as 5th variable

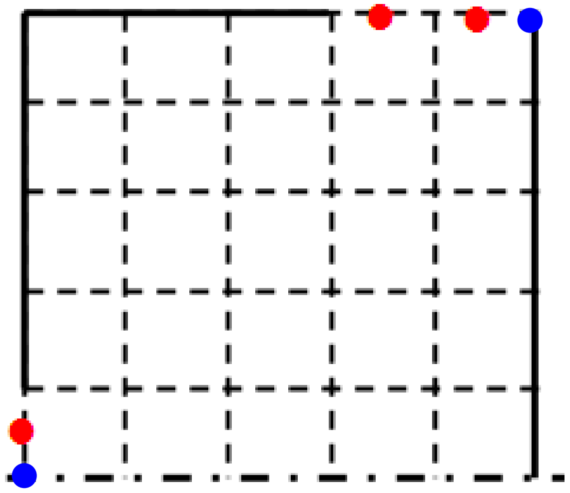
In SIMPLER code, when the 1st variable is set to be solved, the 2nd and 3rd ones are automatically solved.

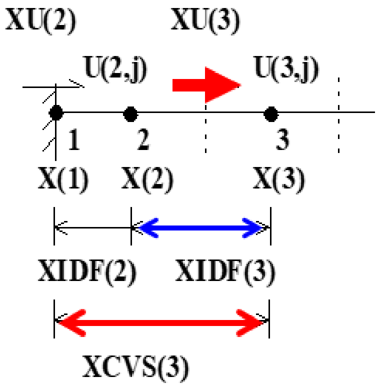


$$r \bullet v_{\theta} = r \cdot \omega r = \omega r^{2}$$

```
      ENTRY START
      OMEGA=100.
      UIN=100.
      DO 100 J=1,M1
      DO 101 I=1,L1
      U(I,J)=0.
      V(I,J)=0.
      F(I,J,5)=0.
      F(L1,J,5)=R(J)**2*OMEGA
101   ENDDO          ! Velocity on disc, causing circumferential flow
100   ENDDO
      U(2,2)=UIN
      AMU=1.
      RETURN
*
      ENTRY DENSE
      RETURN
```

5th variable is R.VTheta





One way for obtaining outlet velocity of open system:

Assuming that the 1st derivatives at outlet =constant

$$\frac{v_{i,M1}-v_{i,M2}}{\Delta y}=k=const \longrightarrow v_{i,M1}=v_{i,M2}+k\Delta y=v_{i,M2}+C$$

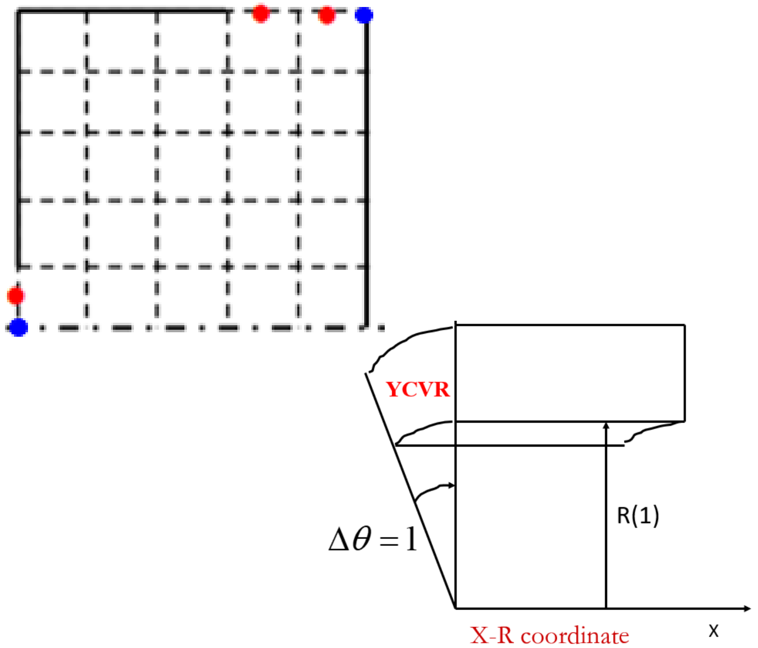
***C* is determined according to total mass conservation**

$$\sum_{i=2}^{L2}\rho_{i,M1}(v_{i,M2}+C)\Delta x_i=FLOWIN \longrightarrow$$

$$C=\frac{FLOWIN-\sum\rho_{i,M1}v_{i,M2}\Delta x_i}{\sum\rho_{i,M1}\Delta x_i}$$

$\underline{v_{i,M1}=v^*_{i,M2}+C}$ is taking as boundary condition for next iteration.

In this example this method is used



ENTRY BOUND

```
 IF(ITER.NE.0)   FLOWIN=RHO(1,2)*U(2,2)*YCVR(2)
 FL=0.
 AR=0.
 DO 301 I=L3,L2
 FLT=R(M1)*XCV(I)*RHO(I,M1)
 AR=AR+FLT  ! Denominator
 FL=FL+FLT*V(I,M2)
                      ! 2nd part of the Numerator
301 ENDDO
 ADD=(FLOWIN-FL)/AR
 DO 302 I=L3,L2
 V(I,M1)=V(I,M2)+ADD
                           ! C---ADD
302 ENDDO
 RETURN
```

! FLOWIN =

$$\sum \rho(I,M1)\bullet XCV(I,M1)\bullet R(M1)\bullet (V(I,M2)+C)$$

$$! \quad C = \frac{\text{FLOWIN-}\sum \rho(i,M1)\bullet XCV(i)\bullet R(M1)\bullet V(i,M2)}{\underline{\sum \rho(i,M1)\bullet XCV(i)\bullet R(M1)}}$$

! C-method is adopted to guarantee the total mass conservation condition

```
      ENTRY OUTPUT
       IF(ITER==0) THEN
       PRINT 401
       WRITE(8,401)
  401 FORMAT(1X,'  ITER',7X,'SMAX',11X,'SSUM',10X,'U(4,4)',
      & 9X,'V(4,4)')
        ELSE
        PRINT 403
        WRITE(8,403) ITER,SMAX,SSUM,U(4,4),V(4,4)
  403 FORMAT(1X,I6,1P5E15.4)
        ENDIF
        IF(ITER==LAST) CALL PRINT
        RETURN
```

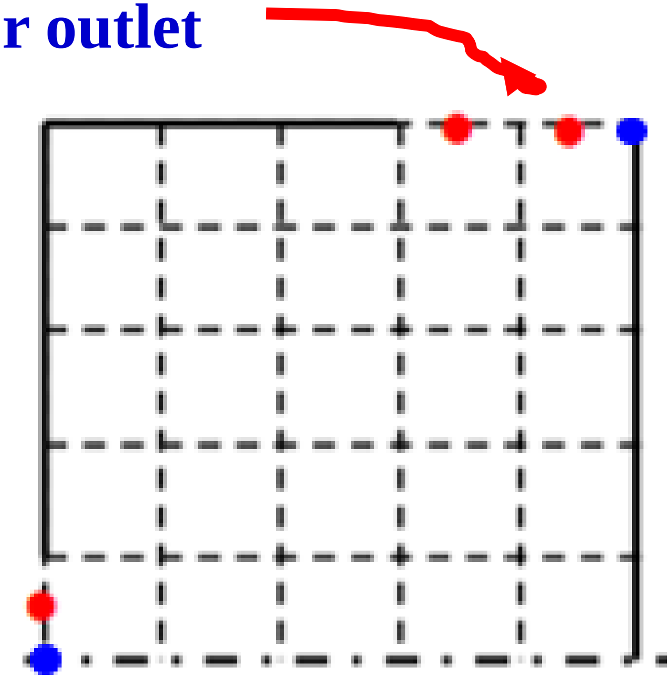
```
      ENTRY GAMSOR
      IF(ITER== 0) THEN
      DO 500 J=1,M1
      DO 501 I=1,L1
      GAM(I,J)=AMU
  501 ENDDO
  502 ENDDO
      GAM(L3,M1)=0.
      GAM(L2,M1)=0.
      ENDIF
```

! Constant viscosity, calculation once is enough

! Local one-way for outlet

! GAM(1:L1, 1)=0 ??

No needed in cylindrical coordinate



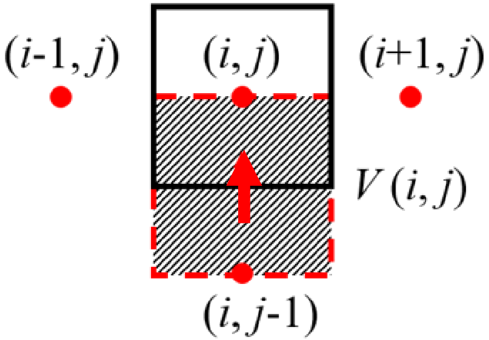
! Source term of v_r –eq.

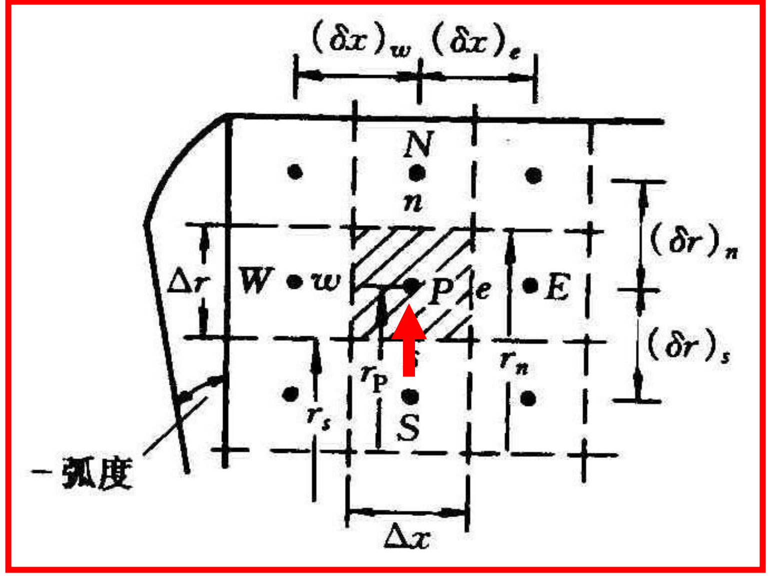
! rv_θ Is interpolated from main nodes

```
IF(NF== 2) THEN
 DO 502 J=3,M2
 DO 503 I=2,L2
 RSWM=FY(J)*F(I,J,5)+FYM(J)*F(I,J-1,5)
 RHOM=FY(J)*RHO(I,J)+FYM(J)*RHO(I,J-1)
 CON(I,J)=RHOM*RSWM**2/RMN(J)**3
  AP(I,J)=-AMU/RMN(J)**2
503 ENDDO
502 ENDDO
  ENDIF
```

! Interface density is interpolated from node density for the source term of v_r

$$S_{v_r} = \rho\frac{v_\theta^2}{r} - \eta\frac{v_r}{r^2} = \underline{\rho\frac{(rv_\theta)^2}{r^3}}\,\underline{-\eta\frac{1}{r^2}}\,v_r$$





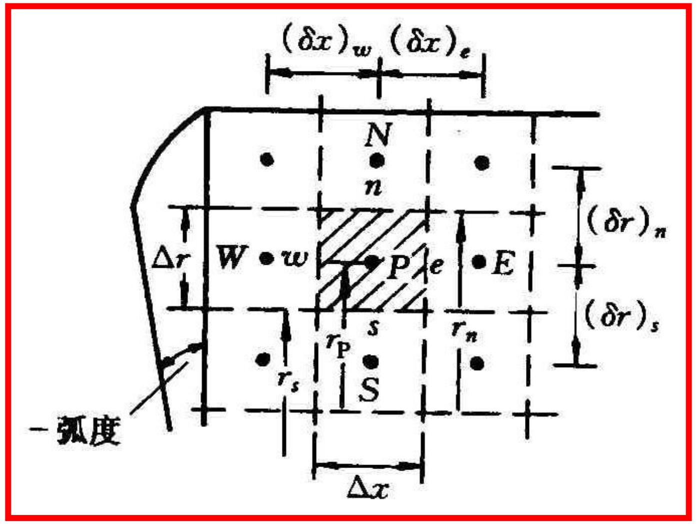
```
510 IF(NF/=5) RETURN
    DO 512 J=2,M2
    DO 513 I=2,L2
    AR=2.*AMU/YCVR(J)
    CON(I,J)=AR*F(I,J-1,5)
    AP(I,J)=-AR
512 ENDDO
513 ENDDO
    RETURN
    END
```

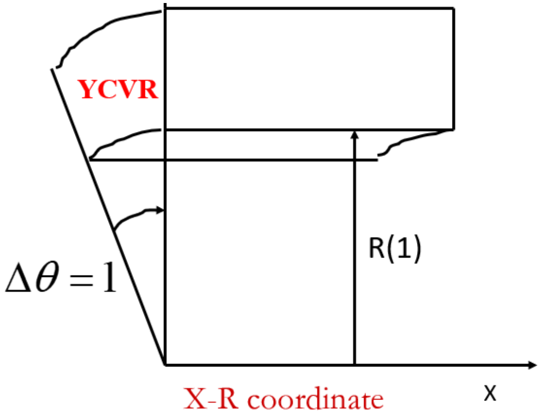
! Source term of rv_θ is calculated at main node

$$S_{(rv_\theta)} = \frac{2}{r_P}\frac{\eta (rv_\theta)_S}{YCV(j)} - \frac{2}{r_P}\frac{\eta}{YCV(j)}(rv_\theta)_P$$

$$= \underline{\frac{2\eta}{YCVR(j)}(rv_\theta)_S} - \underline{\frac{2\eta}{YCVR(j)}}(rv_\theta)_P$$

CON(I,J)=AR*F(I,J-1,5) **AP(I,J)=-AR**



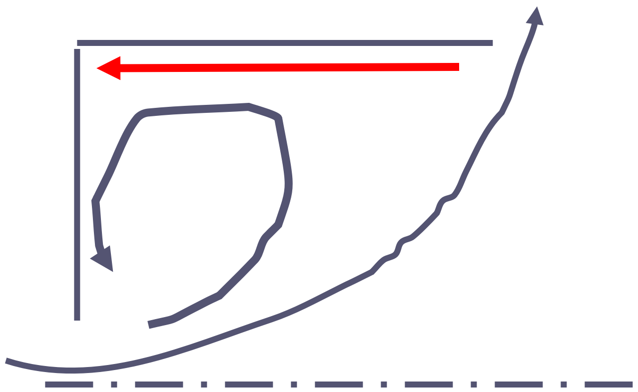


11-7-4 Results analysis

```
        COMPUTATION FOR AXISYMMETRICAL SITUATION
        ****************************************************
ITER    SMAX         SSUM          U(4,4)        V(4,4)
 0    0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00
 1    3.1852E-01   0.0000E+00   3.3742E+00   4.8158E+00
 2    3.6224E-01   1.1921E-07   2.9314E+00   7.6065E+00
 3    1.1265E-01   7.4506E-09   1.8755E+00   8.5863E+00
 4    6.1974E-02  -3.7253E-08   1.5199E+00   8.8029E+00
 5    3.2279E-02  -3.1665E-08   1.2971E+00   8.4019E+00
 6    1.7869E-02  -4.0280E-08   1.2738E+00   7.6836E+00
 7    1.2370E-02   5.1223E-09   1.3363E+00   6.8852E+00
 8    1.0312E-02  -1.1176E-08   1.4400E+00   6.1421E+00
 9    7.9294E-03  -2.9569E-08   1.5480E+00   5.5244E+00
10    5.9429E-03   4.8894E-08   1.6437E+00   5.0452E+00
11    4.6140E-03  -1.6531E-08   1.7207E+00   4.6926E+00
12    3.3741E-03   3.1199E-08   1.7787E+00   4.4432E+00
13    2.6291E-03  -5.1106E-08   1.8202E+00   4.2728E+00
```

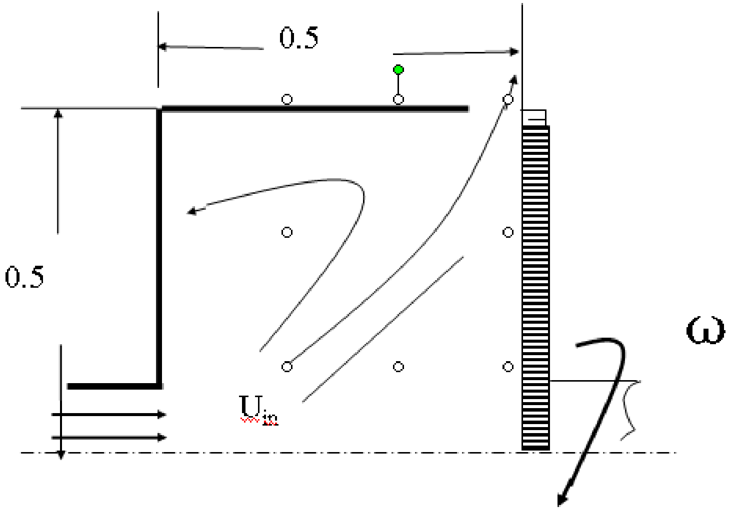
			!U(4,4)	!V(4,4)
14	1.9695E-03	-2.6543E-08	1.8486E+00	4.1597E+00
15	1.4364E-03	6.2981E-08	1.8674E+00	4.0867E+00
16	1.0142E-03	-4.5111E-08	1.8792E+00	4.0409E+00
17	6.9815E-04	8.9640E-09	1.8864E+00	4.0129E+00
18	4.6667E-04	3.8388E-08	1.8906E+00	3.9963E+00
19	3.0389E-04	3.3469E-09	1.8929E+00	3.9868E+00
20	1.9290E-04	-1.1176E-08	1.8941E+00	3.9816E+00
21	1.1830E-04	5.2169E-09	1.8946E+00	3.9790E+00
22	7.0846E-05	4.6941E-08	1.8947E+00	3.9778E+00
23	4.0823E-05	5.4388E-08	1.8947E+00	3.9773E+00
24	2.2590E-05	-8.0094E-08	1.8945E+00	3.9772E+00
25	1.1003E-05	-3.8743E-08	1.8944E+00	3.9773E+00

```
****************************   .VEL  U.   **************************
 I =      2          3          4           5          6         7
J
 7   0.00E+00 0.00E+00 0.00E+00  0.00E+00 0.00E+00  0.00E+00
 6   0.00E+00-1.33E+00-2.67E+00-2.12E+00-8.37E-01   0.00E+00
 5   0.00E+00-1.86E+00-2.70E+00-1.86E+00-6.39E-01   0.00E+00
 4   0.00E+00-2.17E-01  1.89E+00  2.90E+00 1.65E+00  0.00E+00
 3   0.00E+00 1.33E+01 1.97E+01  1.92E+01  1.04E+01  0.00E+00
 2   1.00E+02 8.63E+01 7.43E+01  5.99E+01  3.27E+01  0.00E+00
 1   0.00E+00 0.00E+00 0.00E+00  0.00E+00  0.00E+00  0.00E+00
```



```
***************************   .VEL  V.   ***************************

 I =      1         2         3         4         5         6         7
J
 7    0.00E+00 0.00E+00 0.00E+00 0.00E+00 3.99E+00 6.01E+00 0.00E+00
 6    0.00E+00-1.50E+00-1.50E+00 6.18E-01 6.44E+00 8.45E+00 0.00E+00
 5    0.00E+00-4.17E+00-2.98E+00 1.81E+00 1.00E+01 1.20E+01 0.00E+00
 4    0.00E+00-6.53E+00-1.84E+00 3.98E+00 1.34E+01 1.60E+01 0.00E+00
 3    0.00E+00 6.87E+00 5.96E+00 7.21E+00 1.36E+01 1.63E+01 0.00E+00
 2    0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00
```

```
**************************   .STR FN.   ************************

 I =     2         3         4         5         6         7
J
 7   5.00E-01  5.00E-01  5.00E-01  5.00E-01  3.00E-01  0.00E+00
 6   5.00E-01  5.60E-01  6.20E-01  5.95E-01  3.38E-01  0.00E+00
 5   5.00E-01  6.25E-01  7.15E-01  6.60E-01  3.60E-01  0.00E+00
 4   5.00E-01  6.31E-01  6.67E-01  5.88E-01  3.19E-01  0.00E+00
 3   5.00E-01  4.31E-01  3.72E-01  3.00E-01  1.63E-01  0.00E+00
 2   0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00
```

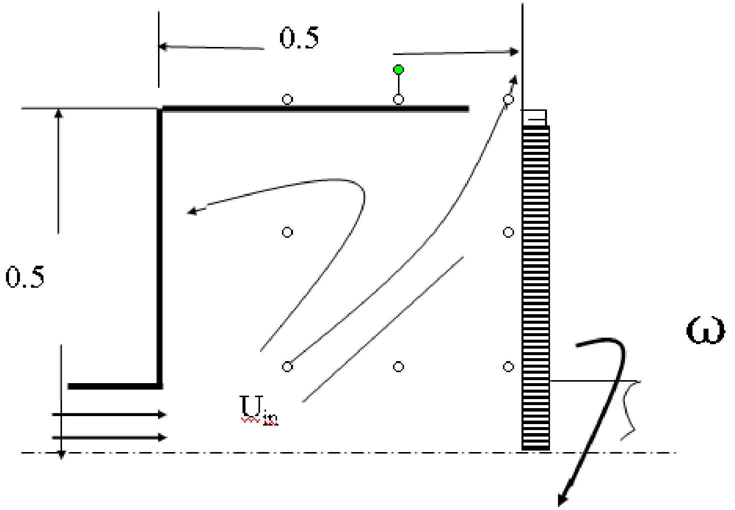


At the shell flow rate is constant

Zero flow rate on disc

```
*************************** R. VTH     ***************************

 I =     1         2         3         4         5         6         7
J
 7   0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00 2.50E+01
 6   0.00E+00 1.24E-01 5.24E-01 1.64E+00 5.76E+00 1.26E+01 2.02E+01
 5   0.00E+00 2.02E-01 7.28E-01 1.69E+00 3.66E+00 7.75E+00 1.23E+01
 4   0.00E+00 1.40E-01 4.46E-01 8.49E-01 1.53E+00 3.54E+00 6.25E+00
 3   0.00E+00 5.15E-02 1.49E-01 2.47E-01 3.84E-01 1.09E+00 2.25E+00
 2   0.00E+00 4.66E-03 1.84E-02 3.72E-02 5.53E-02 1.55E-01 2.50E-01
 1   0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00 0.00E+00
```

$\varpi * r^2$



```
****************************    PRESSURE    ****************************

 I =       1          2          3           4          5          6          7
J
 7   -4.93E+02-4.81E+02-4.57E+02-3.68E+02-3.47E+02-3.61E+02-3.61E+02
 6   -5.08E+02-4.96E+02-4.72E+02-3.94E+02-3.61E+02-3.61E+02-3.61E+02
 5   -5.38E+02-5.26E+02-5.02E+02-4.46E+02-3.89E+02-3.61E+02-3.47E+02
 4   -6.85E+02-6.47E+02-5.72E+02-4.92E+02-3.60E+02-2.41E+02-1.81E+02
 3   -1.15E+03-9.63E+02-5.97E+02-4.57E+02-1.85E+02 1.02E+02  2.46E+02
 2   -3.01E+02-3.62E+02-4.84E+02-3.04E+02 1.83E+02 6.20E+02  8.39E+02
 1    0.00E+00-6.11E+01-4.27E+02-2.28E+02 3.67E+02 8.79E+02  1.10E+03
```

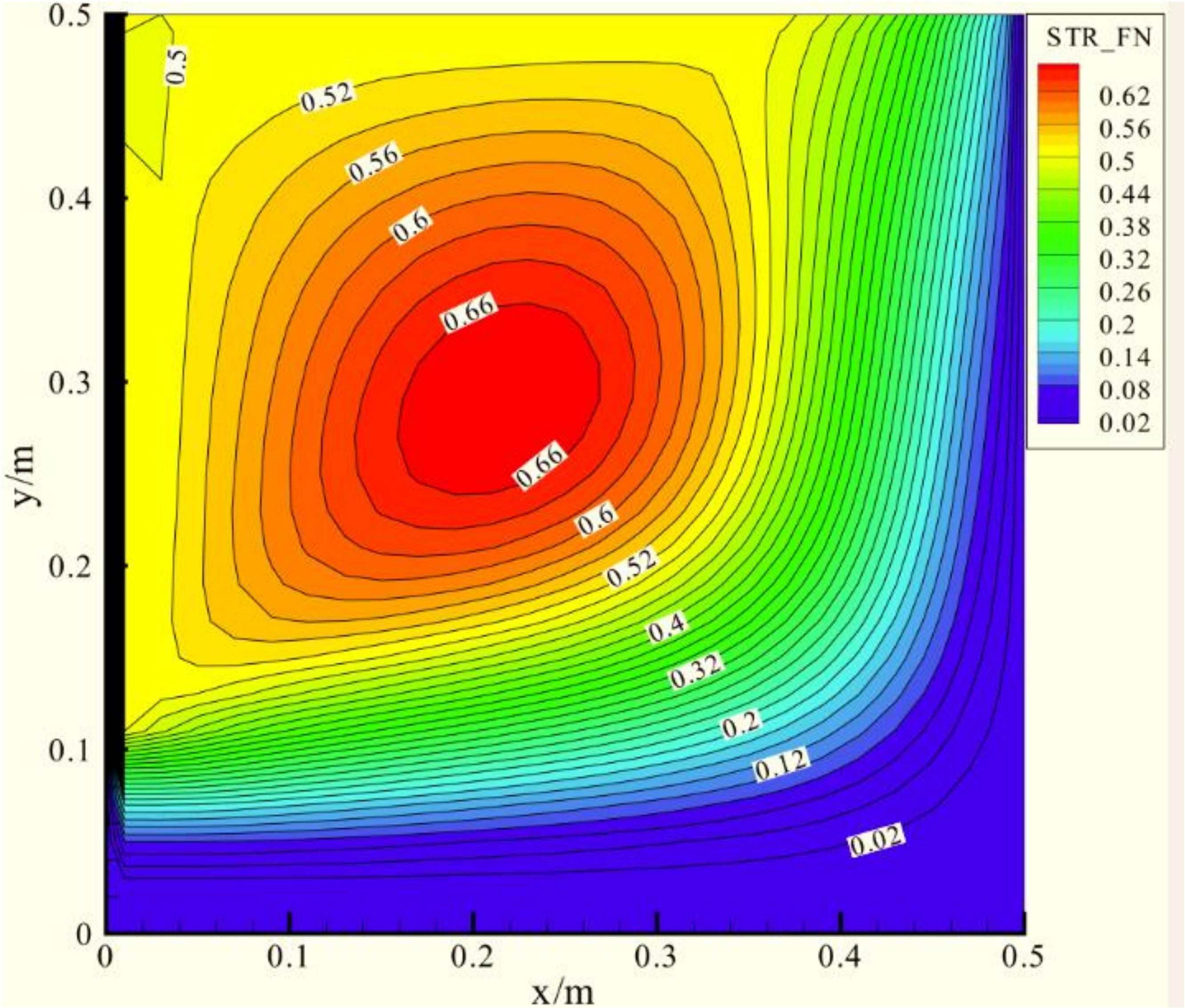


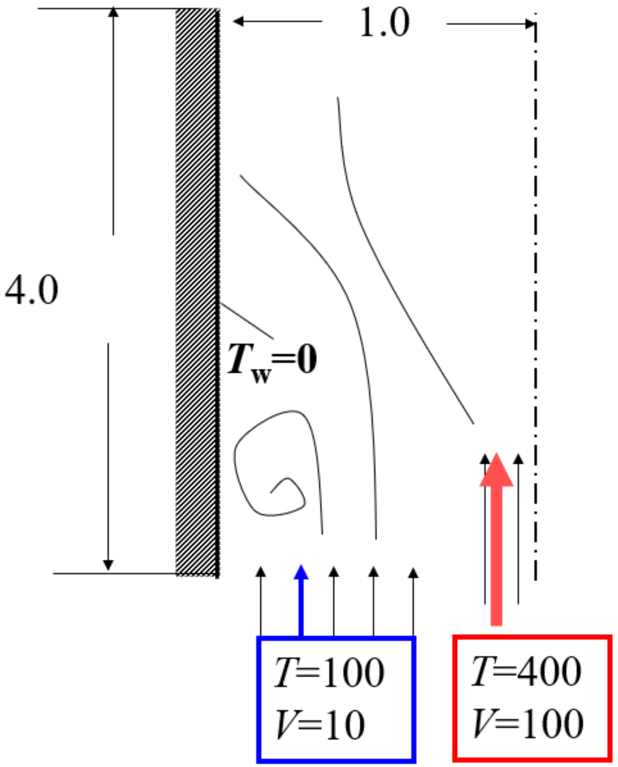
Fig.2 Schematic diagram of Section 7

11-8 Turbulent flow and heat transfer in duct with a stepwise inlet velocity distribution ---k-epsilon turbulence model with WFM

11-8-1 Physical problem and its math formulation

Known： A stream with a central jet goes into a parallel channel; Flow is in turbulent state, $\eta = 10^{-6}$ and $Pr = 0.7$.

Find: Adopt the standard k-ε model and the wall function method to determine velocity and temperature fields in the channel.



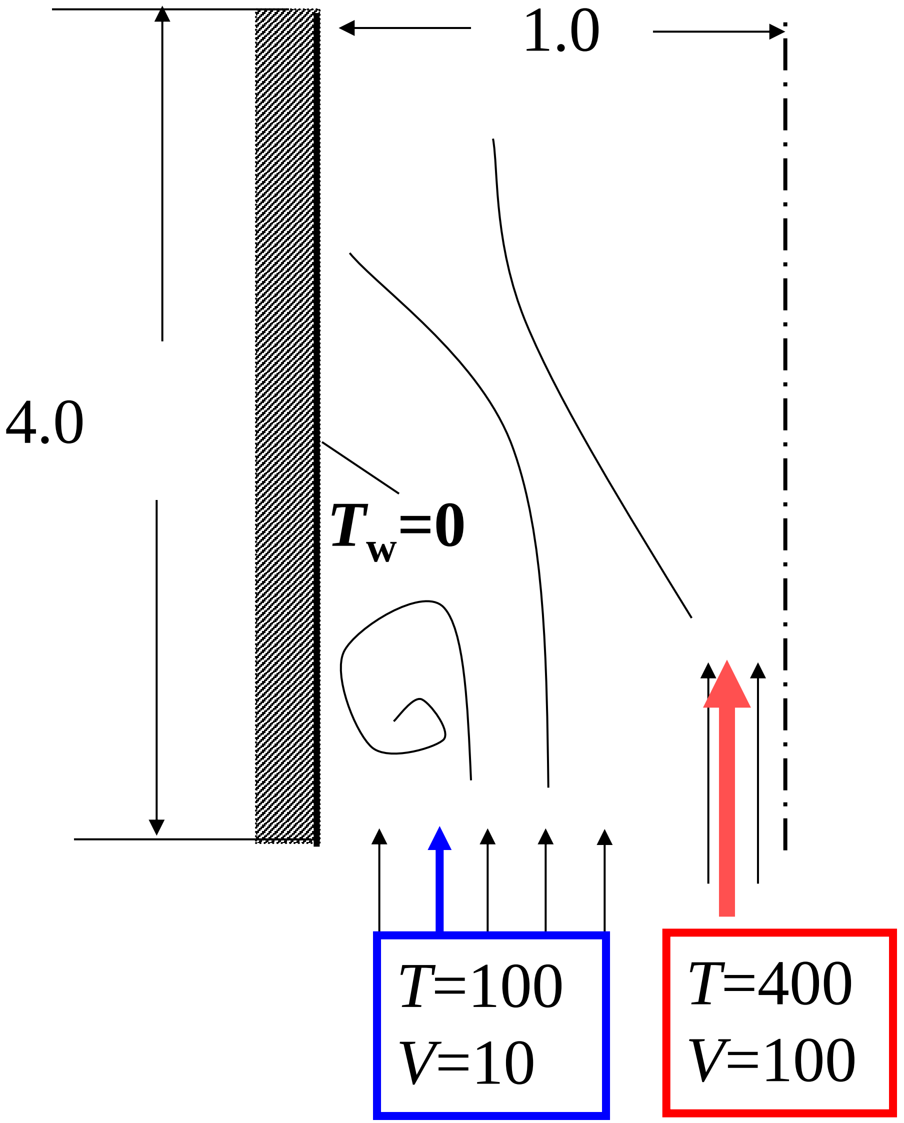


Fig. 1 of Example 8

Flow is in turbulent state, $\eta = 10^{-6}$ and Pr = 0.7

Governing equations

$$\begin{cases} \dfrac{\partial u_k}{\partial x_k}=0 \\ \dfrac{\partial u_i}{\partial t}+\dfrac{\partial(\rho u_k u_i)}{\partial x_k}=-\dfrac{\partial p_{eff}}{\partial x_i}+\dfrac{\partial}{\partial x_k}[\overset{\eta_{\mathrm{eff}}}{\underline{(\eta_l+\eta_t)}}\dfrac{\partial u_i}{\partial x_k}]+S_i \quad ;p_{eff}=p+p_t \\ \dfrac{\partial(\rho^* \phi)}{\partial t}+\dfrac{\partial(\rho^* u_k \phi)}{\partial x_k}=\dfrac{\partial}{\partial x_k}[\underset{\Gamma_{eff}}{\underline{(\Gamma_l+\Gamma_t)}}\dfrac{\partial \phi}{\partial x_k}]+S_\phi \end{cases}$$

$$u:\ S=\frac{\partial}{\partial x}\left(\eta_{\mathrm{eff}}\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\eta_{\mathrm{eff}}\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial z}\left(\eta_{\mathrm{eff}}\frac{\partial w}{\partial x}\right)$$

$$v:\ S=\frac{\partial}{\partial x}\left(\eta_{\mathrm{eff}}\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial y}\left(\eta_{\mathrm{eff}}\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(\eta_{\mathrm{eff}}\frac{\partial w}{\partial y}\right)$$

$$w:\ S=\frac{\partial}{\partial x}\left(\eta_{\mathrm{eff}}\frac{\partial u}{\partial z}\right)+\frac{\partial}{\partial y}\left(\eta_{\mathrm{eff}}\frac{\partial v}{\partial z}\right)+\frac{\partial}{\partial z}\left(\eta_{\mathrm{eff}}\frac{\partial w}{\partial z}\right)$$

Additional Eqs. are needed to determine turbulent viscosity η_t, so as to close model

Using k-ε model to determine η_t

k equation

$$\boxed{\rho G} \qquad \boxed{G=\frac{\eta_t}{\rho}\frac{\partial u_i}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}$$

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho u_j k)}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\eta_l+\frac{\eta_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+\underbrace{\eta_t\frac{\partial u_j}{\partial x_i}\left(\frac{\partial u_j}{\partial x_i}+\frac{\partial u_i}{\partial x_j}\right)-\rho\varepsilon}_{\text{Source term}}$$

ε equation:

$$\underbrace{\frac{\partial(\rho\varepsilon)}{\partial t}}_{\text{transient}}+\underbrace{\frac{\partial(\rho u_j\varepsilon)}{\partial x_j}}_{\text{convection}}=\underbrace{\frac{\partial}{\partial x_j}\left[\left(\eta_l+\frac{\eta_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial x_j}\right]}_{\text{diffusion}}+\underbrace{C_1\frac{\varepsilon}{k}\eta_t\frac{\partial u_j}{\partial x_i}\left(\frac{\partial u_j}{\partial x_i}+\frac{\partial u_i}{\partial x_j}\right)-C_2\rho\frac{\varepsilon^2}{k}}_{\text{source}}$$

σ_ε Prandtl number of ε; C_1, C_2 are empirical coefficients

turbulent viscosity: $$\eta_t=C_\mu'\rho k^{1/2}l=\underline{C_\mu'C_D}\rho k^{1/2+3/2}\frac{l}{C_D k^{3/2}}=C_\mu\rho k^2/\varepsilon$$

$$C_\mu'C_D\rightarrow C_\mu \qquad \varepsilon=C_D\frac{k^{3/2}}{l}$$

Governing equation is:

$$div(\rho \vec{u}\phi) = div(\Gamma_{\phi} grad\phi) + S_{\phi}$$

where $\phi = u, v, T, k, \varepsilon, p, p^{'}$

- The diffusion coefficients are:

NF=	1	2	3	4	5	6	7	8	11
Variable	U	V	P_C	T	k	ε	η_t	G	P
Γ_{ϕ}	η_t	η_t	/	$\frac{\eta_t c_p}{\Pr_t}$	$\frac{\eta_t}{\sigma_k}$	$\frac{\eta_t}{\sigma_\varepsilon}$			
α	0.8	0.8		1.0	0.6	0.6	0.6		

$$\eta_{\text{eff}} = \eta + \eta_t \approx \eta_t$$

For our new temperature G.E.: $\Gamma_t = \lambda_t = \eta_t c_p / \Pr_t$

- The source terms are:

$$u: \quad S_u = \frac{\partial}{\partial x}(\eta_{\text{eff}} \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(\eta_{\text{eff}} \frac{\partial v}{\partial x})$$

$$v: \quad S_v = \frac{\partial}{\partial x}(\eta_{\text{eff}} \frac{\partial u}{\partial y}) + \frac{\partial}{\partial y}(\eta_{\text{eff}} \frac{\partial v}{\partial y})$$

$$k: \quad S_k = \eta_t G - \rho\varepsilon$$

$$\varepsilon: \quad S_\varepsilon = \frac{c_1 \varepsilon \eta_t G}{k} - \frac{c_2 \rho \varepsilon^2}{k}$$

$$G = \frac{\eta_t}{\rho}\left\{2\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right\}$$

- Boundary conditions are：

(1) **Inlet：**

Fluctuation kinetic energy *k* – taking 1% of kinetic energy of oncoming flow；

Dissipation rate ε – determined by following eq.

$$\varepsilon = \frac{c_\mu \rho k^2}{\eta_t}$$

where η_t is determined by $\mathrm{Re}_t = \frac{\rho V(2L_{in})}{\eta_t} = 100$

(2) **Wall**: adopting Wall Function Method；

(3) **Outlet**: taking local one–way;

（4）**At symmetric line**: normal velocity component (u) = 0, all others have their first order normal derivatives equal to zero!

11-8-2 Numerical method

（1） Source term treatment for $k-\varepsilon$

$$S_\phi = S_c + S_p\phi$$

$$S_k = \eta_t G - \rho\varepsilon = \underbrace{\eta_t G}_{\mathbf{S_C}} \underbrace{-\left(\frac{\rho\varepsilon}{k^*}\right)}_{\mathbf{S_P}} k$$

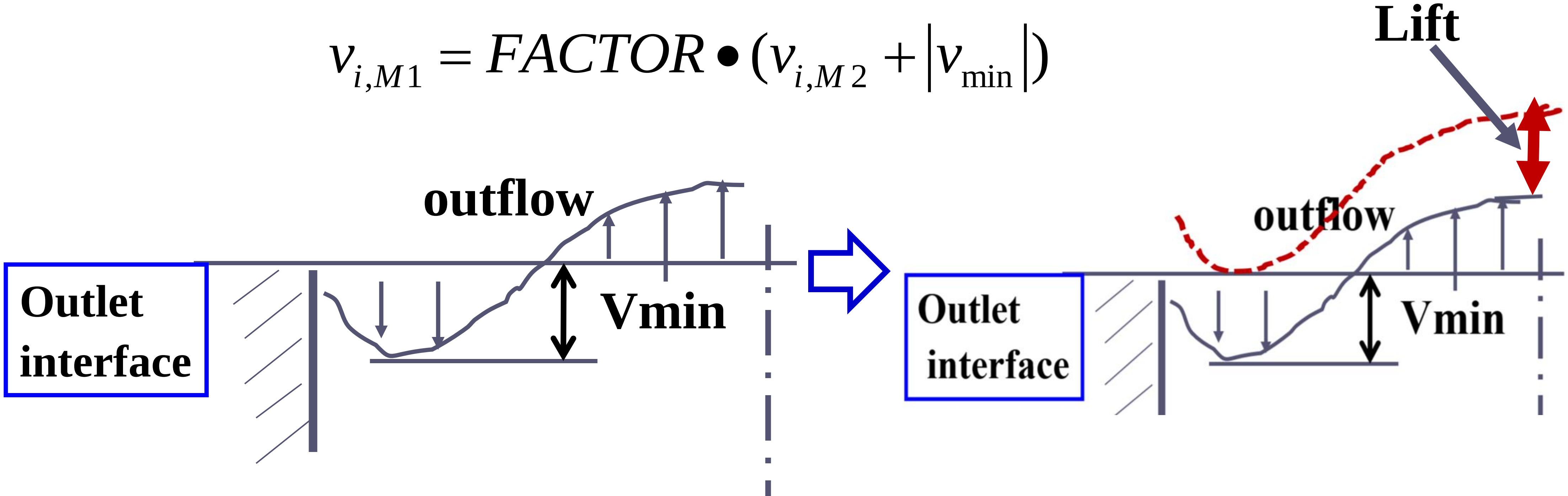
$$S_\varepsilon = \frac{c_1\varepsilon\eta_t G}{k} - \frac{c_2\rho\varepsilon^2}{k} = \underbrace{\frac{c_1\varepsilon\eta_t G}{k}}_{\mathbf{S_C}} \underbrace{-\left(\frac{c_2\rho\varepsilon^*}{k}\right)}_{\mathbf{S_P}}\varepsilon$$

（2）Lift (提升）of outlet velocity

In order to avoid negative outlet velocity during iteration, we may adopt method for lifting temporary (暂时的) outlet velocity:

$$FACTOR = \frac{FLOWIN}{\sum_{i=2}^{L2}[(V_{i,M2} + |V_{\min}|) * RHO_{i,M1} * XCV(i)]}$$

$$v_{i,M1} = FACTOR \bullet (v_{i,M2} + |v_{\min}|)$$



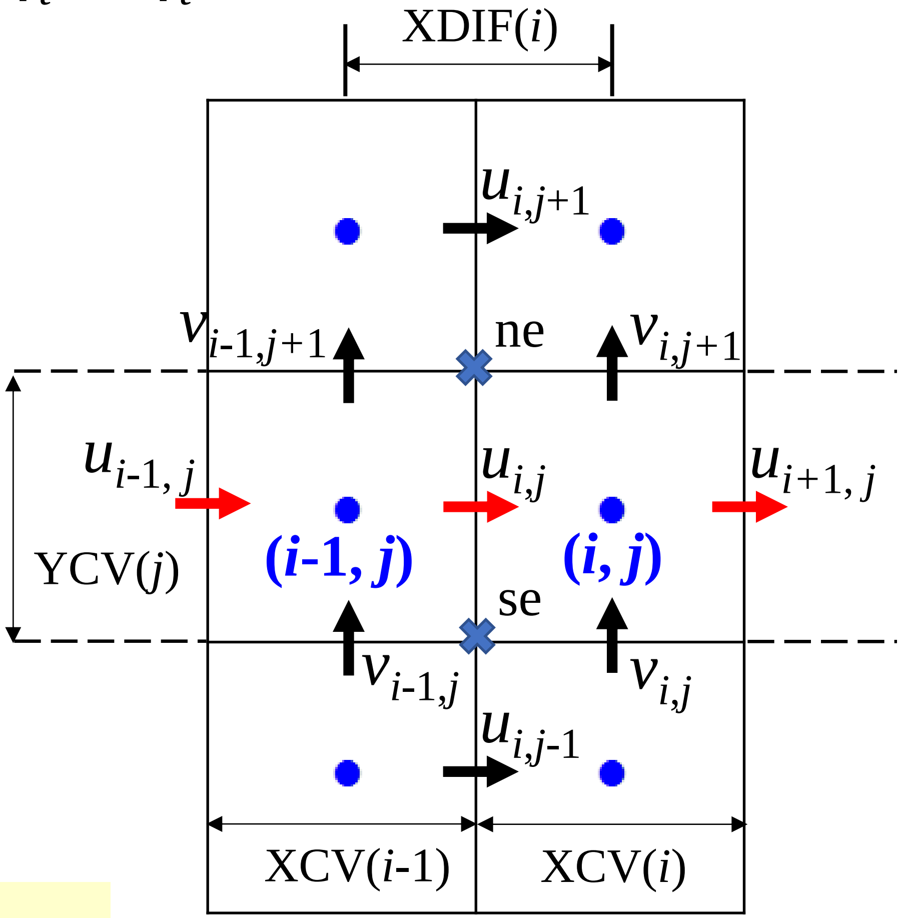
(3) Treatment of source term in *u*-momentum equation

$$S_u = \boxed{\frac{\partial}{\partial x}(\eta_t \frac{\partial u}{\partial x})} + \frac{\partial}{\partial y}(\eta_t \frac{\partial v}{\partial x}) \qquad \eta_{\text{eff}} = \eta + \eta_t \approx \eta_t$$

$$\frac{\partial}{\partial x}(\eta_t \frac{\partial u}{\partial x}) = \frac{1}{XDIF(i)}$$

$$\{GAM(i,j)\frac{u(i+1,j)-u(i,j)}{xcv(i)} -$$

$$GAM(i-1,j)\frac{u(i,j)-u(i-1,j)}{xcv(i-1)}\}$$



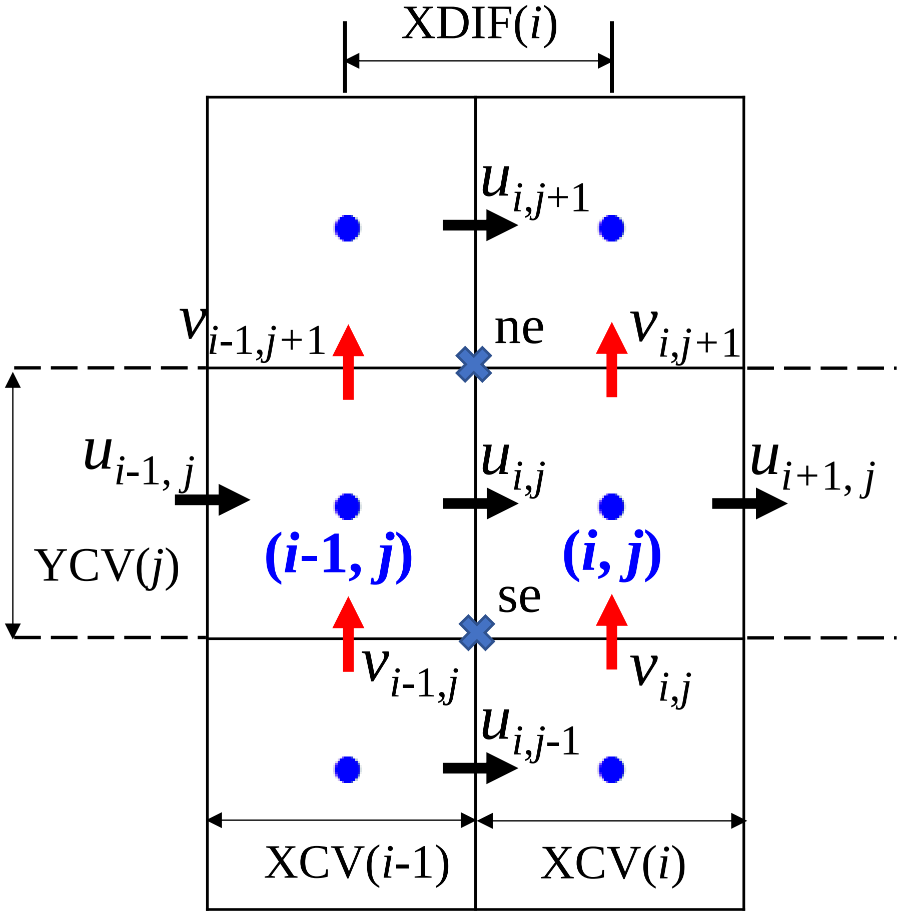
The above term is taken as S_c of *u*-equation!

➢ source term in *u*-equation

$$\frac{\partial}{\partial y}(\eta_t \frac{\partial v}{\partial x}) = \frac{1}{YCV(j)}$$

$$\{\eta_{t,ne} \frac{v(i, j+1) - v(i-1, j+1)}{XDIF(i)} -$$

$$\eta_{t,se} \frac{v(i, j) - v(i-1, j)}{XDIF(i)}\}$$



Also, taken as S_c of *u*-equation!

(4) Flow field and temperature are solved separately

Because velocities are not coupled with temperature, the turbulent flow field can be solved first, then the fluid temperature.

11-8-3 Program reading

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C************************************************************************************
      INTEGER*4 I,J
      REAL*8 CMU, C1, C2, PRT, PRK, PRD, PRPRT, PFN, CMU4,
     1 AFL, VMIN, REL, AMT, ALOG, GAP, GAMM, DUDX, DUDY, DVDX,
     1 DVDY, DISS, AMU, PR, FLOWIN，FL, FACTOR
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE USER
C*************************************************************************************
      USE START_L
      USE USER_L
      IMPLICIT NONE
C*************************************************************************************
C------------------------------PROBLEM TEN--------------------------------
C      Turbulent fluid flow and heat transfer in a parallel duct with stepwise
C                                 inlet velocity distribution
C*************************************************************************************
```

*

```
ENTRY GRID
TITLE(1)=‘ .VEL  U.'
TITLE(2)=‘ .VEL  V.'
TITLE(3)=‘ .STR  FN.'
TITLE(4)=‘ . TEMP .'
TITLE(5)='KIN  ENE'
TITLE(6)=‘ .DISIPA.’
TITLE(7)='TURB  VI'
TITLE(11)='PRESSURE'
TITLE(12)=‘ DENSITY'
```

!All are titles for printing

```
RELAX(1)=0.8
RELAX(2)=0.8
RELAX(5)=0.6
RELAX(6)=0.6
RELAX(7)=0.6
LSOLVE(1)=.TRUE.
LSOLVE(5)=.TRUE.
LSOLVE(6)=.TRUE.
LPRINT(1)=.TRUE.
LPRINT(2)=.TRUE.
LPRINT(3)=.TRUE.
LPRINT(4)=.TRUE.
LPRINT(5)=.TRUE.
LPRINT(6)=.TRUE.
LPRINT(7)=.TRUE.
LPRINT(11)=.TRUE.
LAST=100
XL=1.
YL=4.
L1=7
M1=9
CPCON=1000.
CALL UGRID
RETURN
```

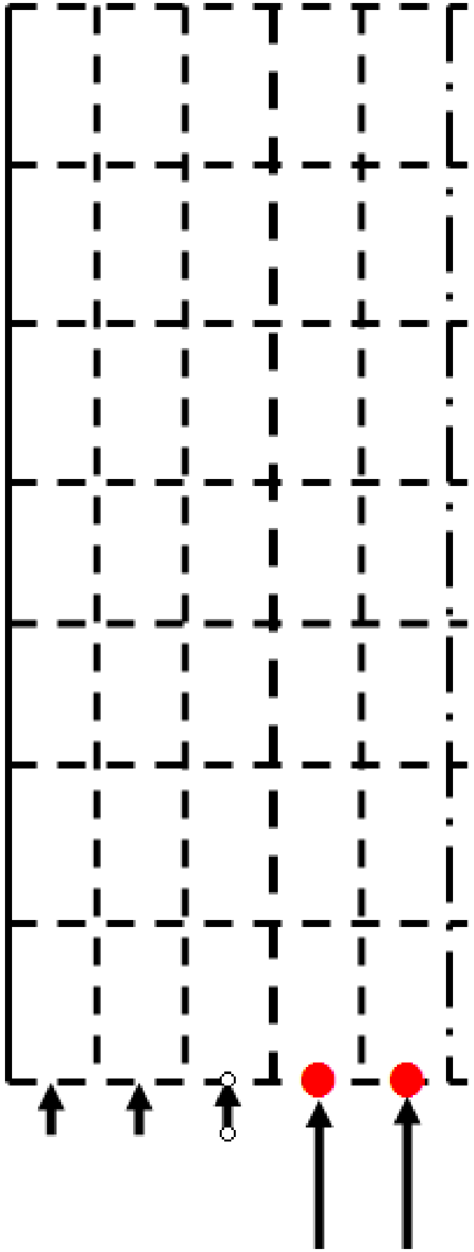
! NF=7 for turbulent viscosity η_t

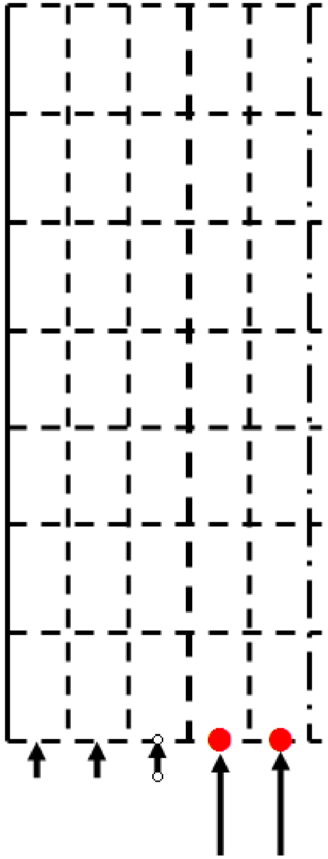
!All logical values for solving and printing

Regarding η_t as the 7th element of F(i, j, NF)

! C_p in the Γ expression for temperature

$$\Gamma_t = \lambda_t = \eta_t c_p / \Pr_t$$





```
     ENTRY START
     DO 100 J=1,M1
     DO 101 I=1,L1
     U(I,J)=0.
     V(I,J)=10.
     V(1,J)=0.
     V(I,2)=10.
     IF(I.GT.4) V(I,2)=100.
     T(I,J)=100.
     T(1,J)=0.
     IF(I.GT.4) T(I,1)=400.
     AKE(I,J)=0.005*V(I,2)**2
     DIS(I,J)=0.1*AKE(I,J)**2
101 ENDDO
100 ENDDO
```

1% of inlet kinetic energy, initial value, also B.C. for inlet

η_t : determied from

$$\mathrm{Re}_t = \frac{\rho V (2L_{in})}{\eta_t} = 100$$

$$100 = \frac{1 \times 100 \times 1.0}{\eta_t}, \eta_t = 1.0$$

$$\boxed{\varepsilon = C_\mu \rho k^2 / \eta_t = 0.09 \times 1 \times k^2 \approx 0.1k^2}$$

Initial value, also B.C. for inlet !

```
AMU=1.E-6   ! Attention, very small value, turbulent flow
CMU=0.09
C1=1.44
C2=1.92
PRT=0.9
PRK=1.0
PRD=1.3
PR=0.7
PRPRT=PR/PRT
PFN=9.*(PRPRT-1.)/PRPRT**.25   ! P function of WFM for T
CMU4=CMU**.25
RETURN

ENTRY DENSE
RETURN
```

! Constants of Standard *k*-ε

Most widely accepted values of model constants

C_1	C_2	C_μ	σ_k	σ_ε	σ_T
1.44	1.92	0.09	1.0	1.3	0.9-1.0

! P function of WFM for *T*

$$T^+ = \frac{\sigma_t}{\kappa}\ln(Ey^+) + P\sigma_t$$

$$\boxed{P = 8.96\left(\frac{\sigma_l}{\sigma_t} - 1\right)\left(\frac{\sigma_l}{\sigma_t}\right)^{-1/4}}$$

```
      ENTRY BOUND
      IF(ITER == 0) THEN
      FLOWIN=0.
      DO 310 I=2,L2
      FLOWIN=FLOWIN+RHO(I,1)*V(I,2)*XCV(I)   ! Flow rate at inlet
310 ENDDO
      ELSE
      FL=0.
      AFL=0.
      VMIN=0.
      ENDIF
      DO 301 I=2,L2
      IF(V(I,M2)< 0.) VMIN=DMAX1(VMIN,-V(I,M2))   ! Search for Vmin
      AFL=AFL+RHO(I,M1)*XCV(I)
      FL=FL+RHO(I,M1)*V(I,M2)*XCV(I)
      FACTOR=FLOWIN/(FL+AFL*VMIN)
301 ENDDO
      DO 302 I=2,L2
      V(I,M1)=(V(I,M2)+VMIN)*FACTOR
```

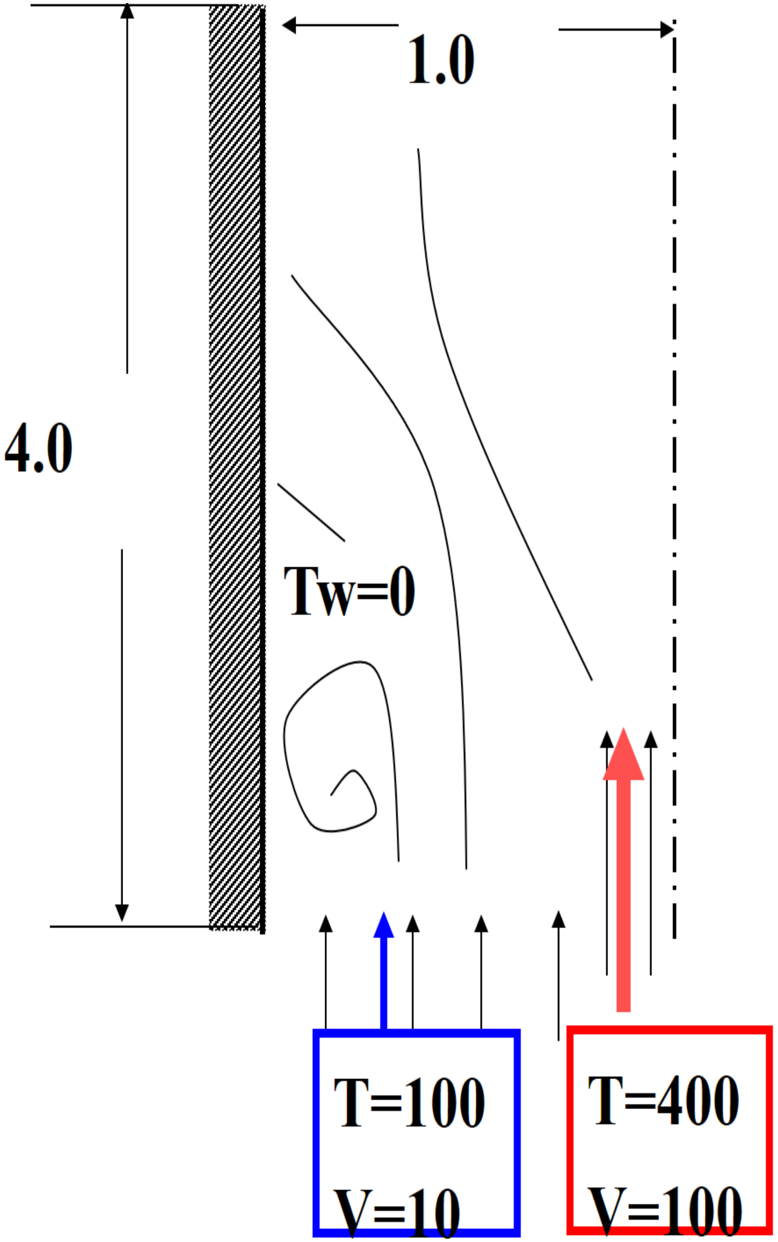
$$!v_{i,M1} = FACTOR \bullet (v_{i,M2} + |v_{\min}|)$$

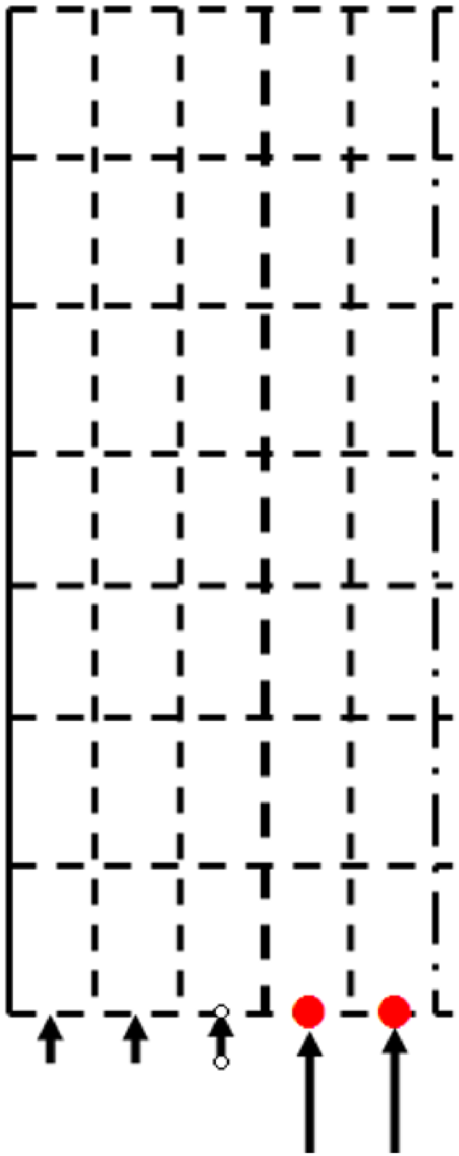
```
302 ENDDO
      DO 303 J=2,M2
      AKE(L1,J)=AKE(L2,J)   ! symmetry; decoration for print out
      DIS(L1,J)=DIS(L2,J)
303 ENDDO
      RETURN
```

$$FACTOR = \frac{FLOWIN}{\sum_{i=2}^{L2}[(V_{i,M2} + |V_{\min}|) * RHO_{i,M1} * XCV(i)]}$$

```
      ENTRY OUTPUT
      IF(ITER==0) THEN
      PRINT 401
      WRITE(8,401)
  401 FORMAT(1X,'   ITER',6X,'SMAX',6X,'SSUM',5X,'V(6,6)',
    1 4X,'T(5,6)',4X,'KE(5,6)')
      ELSE
      PRINT 403, ITER, SMAX, SSUM, V(6,6),T(5,6), AKE(5,6)
      WRITE(8,403) ITER,SMAX,SSUM,V(6,6),T(5,6),AKE(5,6)
 403 FORMAT(1X,I6,1P5E11.3)
      ENDIF
      IF(ITER>=55) THEN
      LSOLVE(4)=.TRUE.
      LSOLVE(1)=.FALSE.
      LSOLVE(5)=.FALSE.
      LSOLVE(6)=.FALSE.
      ENDIF
      IF（ITER==LAST) CALL PRINT
      RETURN
```

! Switch off the solution variables: Flow is not coupled with temperature！ After obtaining converged flow field, temperature is solved


1.0
4.0
Tw=0
T=100
V=10
T=400
V=100



```
     ENTRY GAMSOR
     IF(NF== 3) RETURN
     IF(NF== 1) THEN
     REL=1.-RELAX(7)           !  NF=7  for turbulent viscosity
     DO 500 J=1,M1
     DO 501 I=1,L1
     AMT=CMU*RHO(I,J)*AKE(I,J)**2/(DIS(I,J)+1.E-30)
     IF(ITER==0) AMUT(I,J)=AMT    !  Initial values
     AMUT(I,J)=RELAX(7)*AMT+REL*AMUT(I,J)
501 ENDDO                      !  Underrelaxation  for turbulent viscosity
500 ENDDO
     FACTOR=1.
     ELSE
     IF(NF== 4) FACTOR=CPCON/PRT
     IF(NF== 5) FACTOR=1./PRK
     IF(NF== 6) FACTOR=1./PRD
     DO 520 J=1,M1
     DO 521 I=1,L1              !  Laminar part is omitted.
     GAM(I,J)=AMUT(I,J)*FACTOR
     IF(NF/= 1) GAM(L1,J)=0.  !  Symmetric line, u=0
     GAM(I,M1)=0. !  Local one way for outlet
521 ENDDO
520 ENDDO
```

$$\eta_t = \frac{c_\mu \rho k^2}{\varepsilon}$$

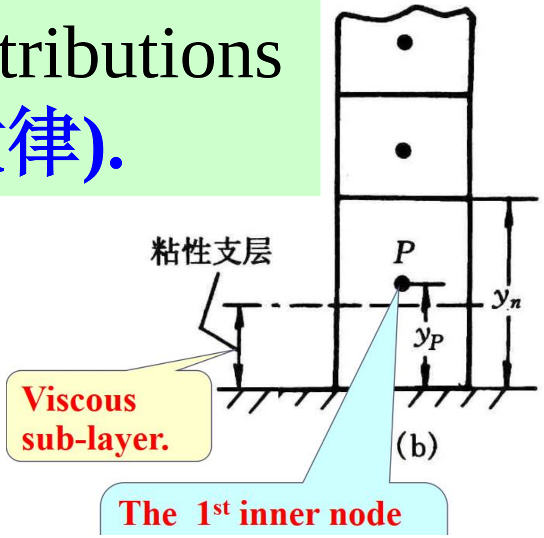
$$\boxed{\mathrm{Pr}_\mathrm{t} = \eta_\mathrm{t} c_p / \lambda_\mathrm{t}, \;\; \lambda_\mathrm{t} = \eta_\mathrm{t} c_p / \mathrm{Pr}_\mathrm{t}}$$

$$\boxed{(\eta_l + \frac{\eta_t}{\sigma_k}) - \text{for } k; \quad (\eta_l + \frac{\eta_t}{\sigma_\varepsilon}) - \text{for } \varepsilon}$$

Fundamentals of Wall Function Method

1) Assuming that the dimensionless velocity and temp. distributions outside the viscous sub-layer are of **logarithmic law(对数律).**

For $x_P^+ > 11.0$ $\begin{cases} u^+ = \dfrac{1}{\kappa}\ln(Ex_P^+), & \dfrac{1}{\kappa}\ln(E) = 5.0 \sim 5.5 \\ T^+ = \dfrac{\sigma_t}{\kappa}\ln(Ex_P^+) + P\sigma_t; & P = 8.96(\dfrac{\sigma_l}{\sigma_t} - 1)(\dfrac{\sigma_l}{\sigma_t})^{-1/4} \end{cases}$



2) Placing the **1st inner node P** outside the viscous sub-layer, where logarithmic law is valid ($x_P^+ > 11$).

3) The **effective turbulent viscosity and thermal conductivity** between the 1st inner node and wall determined by :

$$\eta_B = (\frac{x_P^+}{u_P^+})\eta_l \quad \lambda_B = (\frac{x_P^+}{T_P^+})\Pr_l \lambda_l \quad x_P^+ = \frac{\rho x(C_\mu^{1/4}k^{1/2})}{\eta_l}$$

Fundamentals of Wall Function Method

4) The boundary condition of k equation $\partial k / \partial n = 0$

5) The $\boldsymbol{\varepsilon}$ **at 1st inner node** is determined by: $\varepsilon = C_\mu^{3/4} k_P^{3/2} / (\kappa y_P)$

- For Solid wall: adopting wall function method

(1)Velocity — normal to wall $\frac{\partial \phi}{\partial n})_w = 0$;

Velocity — parallel to wall $\phi_w = 0$, and η_B determined by WFM

$$\eta_B = (\frac{x_P^+}{u_P^+})\eta_l \qquad x_P^+ = \frac{\rho x (C_\mu^{1/4} k^{1/2})}{\eta_l}$$

(2) Temperature

λ_B determined by WFM $\lambda_B = (\frac{x_P^+}{T_P^+})\,\mathrm{Pr}_l\, \lambda_l$

WFM implementation!

```
DO 530 J=2,M2
SELECT CASE (NF) ! For u, p’, k, ε
CASE (1,3,5,6)
GAM(1,J)=0.
CASE (2)             !  For velocity v, WFM should be used!
GAM(1,J)=AMU  !  First, laminar viscosity is given for the left wall
XPLUS(J)=RHO(2,J)*SQRT(AKE(2,J))*CMU4*XDIF(2)/AMU
IF(XPLUS(J)>11.5)   GAM(1,J)=AMU*XPLUS(J)/
& (ALOG(9.*XPLUS(J))*2.5)! Turbulence viscosity
CASE (4)  !  For temperature, WFM for temperature
GAM(1,J)=AMU*CPCON/PR!  First, laminar thermal conductivity
IF(XPLUS(J)>11.5) GAM(1,J)=AMU*CPCON/PRT*XPLUS(J)
& /(2.5*ALOG(9.*XPLUS(J))+PFN)  ! Turbulence thermal conductivity
ENDSELECT
530 ENDDO
```

For *u*, *p*’and *k*--adiabatic; For *ε*, set up its value of 1st inner node**, then cut the connection to its boundary. All lead to *Γ*=0**

$$\eta_B = \left(\frac{x_P^+}{u_P^+}\right)\eta_l$$

$$\boxed{x^+ = \frac{\rho x(C_\mu^{1/4}k^{1/2})}{\eta_l}}$$

$$\lambda_B = \left(\frac{x_P^+}{T_P^+}\right)\mathrm{Pr}_l\ \lambda_l$$

WFM ImpLementation

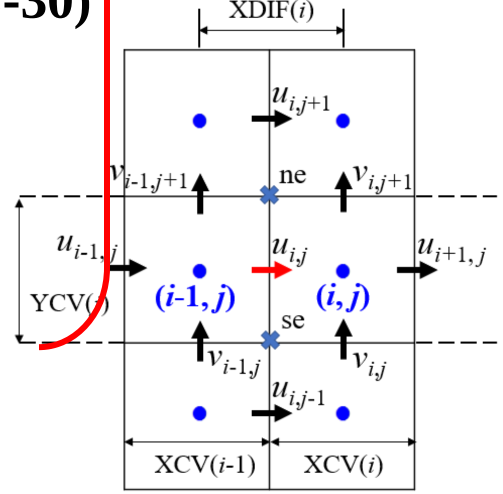
$$S_u = \frac{\partial}{\partial x}\left(\eta_t \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\eta_t \frac{\partial v}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\eta_t \frac{\partial u}{\partial x}\right) = \frac{1}{XDIF(i)}\left\{GAM(i,j)\frac{u(i+1,j)-u(i,j)}{xcv(i)} - GAM(i-1,j)\frac{u(i,j)-u(i-1,j)}{xcv(i-1)}\right\}$$

```
IF(NF==1) THEN
DO 590 J=2,M2
DO 591 I=3,L2
CON(I,J)=(GAM(I,J)*(U(I+1,J)-U(I,J))/XCV(I)
1 -GAM(I-1,J)*(U(I,J)-U(I-1,J))/XCV(I-1))/XDIF(I)
GAMP=GAM(I,J+1)*GAM(I-1,J+1)/(GAM(I,J+1)+GAM(I-1,J+1)+1.E-30)
GAMP=GAMP+GAM(I,J)*GAM(I-1,J)/(GAM(I,J)+GAM(I-1,J)+1.E-30)
GAMM=GAM(I,J-1)*GAM(I-1,J-1)/(GAM(I,J-1)+GAM(I-1,J-1)+1.E-30)
GAMM=GAMM+GAM(I,J)*GAM(I-1,J)/(GAM(I,J)+GAM(I-1,J)+1.E-30)
CON(I,J)=CON(I,J)+(GAMP*(V(I,J+1)-V(I-1,J+1))
1 -GAMM*(V(I,J)-V(I-1,J)))/(YCV(J)*XDIF(I))
AP(I,J)=0.
591 ENDDO
590 ENDDO
RETURN
```

$$\eta_{t,ne} = \frac{\eta_t(i-1,j)\eta_t(i,j)}{\eta_t(i-1,j)+\eta_t(i,j)} + \frac{\eta_t(i-1,j+1)\eta_t(i,j+1)}{\eta_t(i-1,j+1)+\eta_t(i,j+1)}$$

Source term for u-eq.



Refer to textbook page 358

```
509 IF(NF==2) THEN
      DO 594 J=3,M2
      DO 595 I=2,L2
      CON(I,J)=(GAM(I,J)*(V(I,J+1)-V(I,J))/YCV(J)-
    1  GAM(I,J-1)*(V(I,J)-V(I,J-1))/YCV(J-1))/(YDIF(J))
      GAMP=GAM(I+1,J)*GAM(I+1,J-1)/(GAM(I+1,J)+GAM(I+1,J-1)+1.E-30)
      GAMP=GAMP+GAM(I,J)*GAM(I,J-1)/(GAM(I,J)+GAM(I,J-1)+1.E-30)
      GAMM=GAM(I-1,J)*GAM(I-1,J-1)/(GAM(I-1,J)+GAM(I-1,J-1)+1.E-30)
      GAMM=GAMM+GAM(I,J)*GAM(I,J-1)/(GAM(I,J)+GAM(I,J-1)+1.E-30)
      CON(I,J)=CON(I,J)+(GAMP*(U(I+1,J)-U(I+1,J-1))
    1  -GAMM*(U(I,J)-U(I,J-1)))/(XCV(I)*YDIF(J))
      AP(I,J)=0.
595 ENDDO
594 ENDDO
      RETURN
      ENDIF
```

Source term calculation for *v*- eq.

```
  IF(NF==4) THEN
      DO 596 J=2,M2
      DO 597 I=2,L2
      CON(I,J)=0.
      AP(I,J)=0.
597 ENDDO
586 ENDDO
      RETURN
```

Source term calculation for *T*- eq.

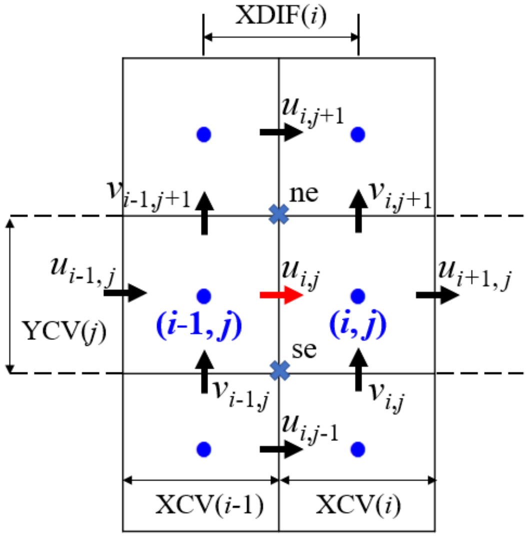
! Following part is for the source term of *k*- eq.:

$$S_k = \eta_t G - \rho\varepsilon = \eta_t G - \left(\frac{\rho\varepsilon}{k^*}\right)k$$

$$G = \frac{\partial u_i}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

! Most part of the code is for calculation of GEN term

```
ELSE IF(NF==5) THEN
DO 598 J=2,M2
DO 599 I=2,L2
DUDX=(U(I+1,J)-U(I,J))/XCV(I)
DVDY=(V(I,J+1)-V(I,J))/YCV(J)
IF(J==2) DUDY=(0.5*(U(I,J+1)-U(I,J))+0.5*(U(I+1,J+1)-
C U(I+1,J)))/YDIF(J+1)
```



```
       IF(J==M2) DUDY=(0.5*(U(I,J)-U(I,J-1))+0.5*(U(I+1,J)-U(I+1,J-1))) /YDIF(J)
       IF(J/=2.AND.J/=M2) DUDY=(0.5*(U(I,J+1)-U(I,J-1))+0.5*(U(I+1,J+1)-
      1 U(I+1,J-1)))/(YDIF(J)+YDIF(J+1))
       IF(I==2) DVDX=(0.5*(V(I+1,J)-V(I-1,J))+0.5*(V(I+1,J+1)
      1 -V(I-1,J+1)))/(XDIF(I)+XDIF(I+1))
       IF(I==L2)  DVDX=(0.5*(V(I,J)-V(I-1,J))+0.5*(V(I,J+1)
      1 -V(I-1,J+1)))/XDIF(I)
       IF(I/=2.AND.I/=L2) DVDX=(0.5*(V(I+1,J)-V(I,J))+0.5*(V(I+1,J+1)
      1 -V(I,J+1)))/XDIF(I+1))
        GEN(I,J)=2.*(DUDX**2+DVDY**2)+(DUDY+DVDX)**2      !
        CON(I,J)=GEN(I,J)*AMUT(I,J)
        AP(I,J)=-RHO(I,J)*DIS(I,J)/(AKE(I,J)+1.E-30)
  598 ENDDO
  599 ENDDO
        RETURN
        ENDIF
```

$$G=\frac{\partial u_i}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$

Sp of *k*-eq.

$$S_k=\eta_t G-\rho\varepsilon=\underline{\eta_t G}-\left(\frac{\rho\varepsilon}{k^*}\right)k$$

$$S_\varepsilon = \frac{c_1 \varepsilon \eta_t G}{k} - \frac{c_2 \rho \varepsilon^2}{k} = \frac{c_1 \varepsilon \eta_t G}{k} - \left(\frac{c_2 \rho \varepsilon^*}{k}\right)\varepsilon$$

$$S_c = A\phi_{given}, S_P = -A,$$

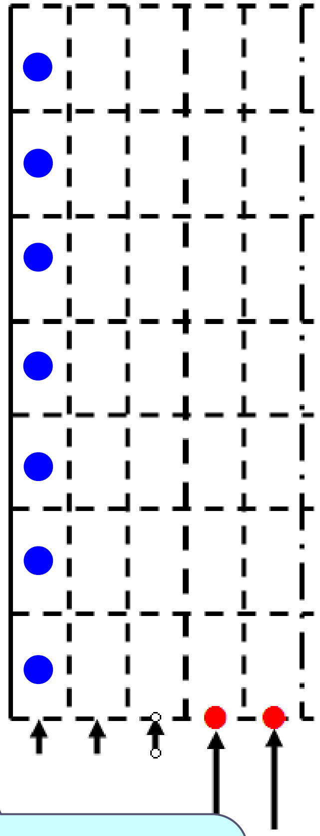
$$A = 10^{20} \sim 10^{30}$$

Large source term method

```
     DO 600 J=2,M2
     DO 601 I=2,L2
     CON(I,J)=C1*GEN(I,J)*CMU*RHO(I,J)*AKE(I,J)
     AP(I,J)=-C2*RHO(I,J)*DIS(I,J)/(AKE(I,J)+1.E-30)
601 ENDDO
600 ENDDO
     DO 602 J=2,M2
     DISS=CMU*AKE(2,J)**1.5/(0.4*CMU4*XDIF(2))
     CON(2,J)=1.E30*DISS
     AP(2,J)=-1.E30
602 ENDDO
     RETURN
     END
```

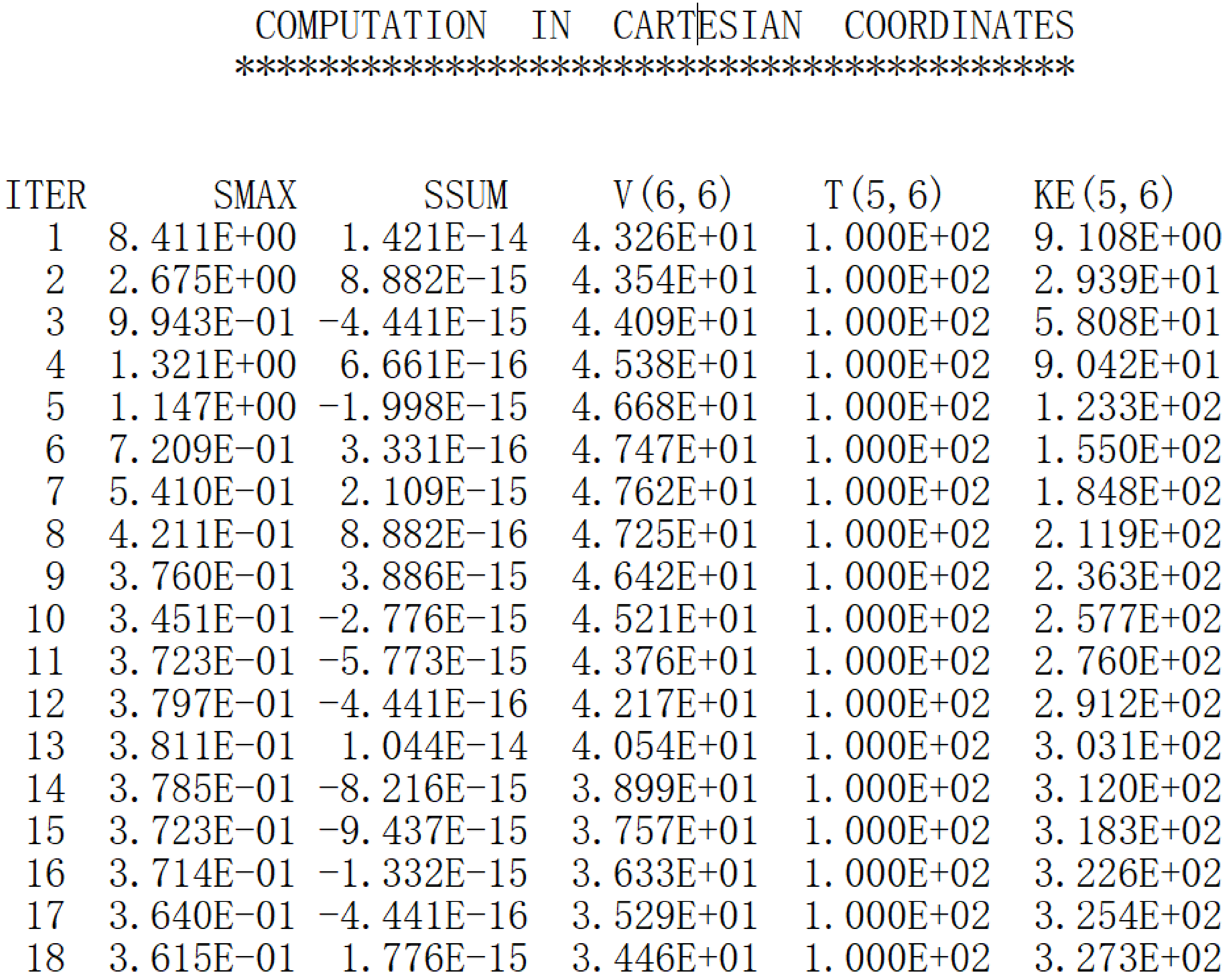
$$\varepsilon = \frac{C_\mu^{3/4} k_P^{3/2}}{\kappa y_P}$$

Adopt large source term method for 1st inner node where i=2;



Source term calculation for Epsilon eq.

9.8.4 Results analysis



```
                 COMPUTATION  IN  CARTESIAN  COORDINATES
                ****************************************


ITER      SMAX        SSUM       V(6,6)      T(5,6)     KE(5,6)
  1   8.411E+00   1.421E-14  4.326E+01  1.000E+02  9.108E+00
  2   2.675E+00   8.882E-15  4.354E+01  1.000E+02  2.939E+01
  3   9.943E-01  -4.441E-15  4.409E+01  1.000E+02  5.808E+01
  4   1.321E+00   6.661E-16  4.538E+01  1.000E+02  9.042E+01
  5   1.147E+00  -1.998E-15  4.668E+01  1.000E+02  1.233E+02
  6   7.209E-01   3.331E-16  4.747E+01  1.000E+02  1.550E+02
  7   5.410E-01   2.109E-15  4.762E+01  1.000E+02  1.848E+02
  8   4.211E-01   8.882E-16  4.725E+01  1.000E+02  2.119E+02
  9   3.760E-01   3.886E-15  4.642E+01  1.000E+02  2.363E+02
 10   3.451E-01  -2.776E-15  4.521E+01  1.000E+02  2.577E+02
 11   3.723E-01  -5.773E-15  4.376E+01  1.000E+02  2.760E+02
 12   3.797E-01  -4.441E-16  4.217E+01  1.000E+02  2.912E+02
 13   3.811E-01   1.044E-14  4.054E+01  1.000E+02  3.031E+02
 14   3.785E-01  -8.216E-15  3.899E+01  1.000E+02  3.120E+02
 15   3.723E-01  -9.437E-15  3.757E+01  1.000E+02  3.183E+02
 16   3.714E-01  -1.332E-15  3.633E+01  1.000E+02  3.226E+02
 17   3.640E-01  -4.441E-16  3.529E+01  1.000E+02  3.254E+02
 18   3.615E-01   1.776E-15  3.446E+01  1.000E+02  3.273E+02
```

```
19  3.499E-01  5.773E-15  3.380E+01  1.000E+02  3.285E+02
20  1.993E-01  0.000E+00  3.331E+01  1.000E+02  3.293E+02
21  1.916E-01  7.327E-15  3.294E+01  1.000E+02  3.298E+02
22  1.632E-01 -3.275E-15  3.267E+01  1.000E+02  3.299E+02
23  1.494E-01 -5.773E-15  3.248E+01  1.000E+02  3.299E+02
24  1.283E-01 -3.220E-15  3.234E+01  1.000E+02  3.295E+02
25  1.071E-01 -8.327E-16  3.224E+01  1.000E+02  3.290E+02
26  8.615E-02 -1.024E-14  3.218E+01  1.000E+02  3.282E+02
27  7.442E-02  5.301E-15  3.213E+01  1.000E+02  3.273E+02
28  7.219E-02 -3.969E-15  3.210E+01  1.000E+02  3.261E+02
29  6.907E-02 -1.638E-15  3.207E+01  1.000E+02  3.248E+02
30  6.246E-02 -5.704E-15  3.205E+01  1.000E+02  3.234E+02
31  5.292E-02 -6.689E-15  3.202E+01  1.000E+02  3.218E+02
32  4.163E-02 -3.039E-15  3.199E+01  1.000E+02  3.201E+02
33  3.782E-02  6.467E-15  3.196E+01  1.000E+02  3.183E+02
34  3.624E-02  1.332E-15  3.193E+01  1.000E+02  3.165E+02
35  3.316E-02 -7.938E-15  3.189E+01  1.000E+02  3.145E+02
36  2.901E-02  1.693E-15  3.185E+01  1.000E+02  3.126E+02
37  2.497E-02 -1.303E-14  3.181E+01  1.000E+02  3.105E+02
38  2.160E-02 -1.010E-14  3.177E+01  1.000E+02  3.085E+02
39  1.930E-02  1.041E-16  3.173E+01  1.000E+02  3.064E+02
40  1.730E-02  1.774E-14  3.168E+01  1.000E+02  3.043E+02
41  1.535E-02 -9.714E-16  3.164E+01  1.000E+02  3.022E+02
42  2.275E-02  5.967E-16  3.160E+01  1.000E+02  3.002E+02
```

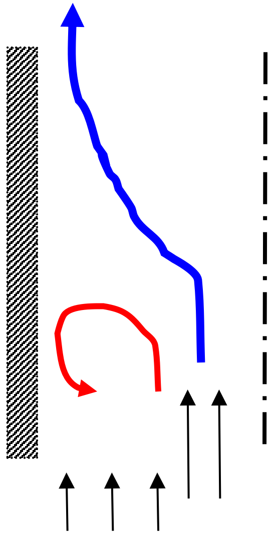
```
                                   V          T          KE
43  4.093E-02 -4.635E-15  3.156E+01  1.000E+02  2.981E+02
44  4.235E-02 -1.457E-15  3.152E+01  1.000E+02  2.961E+02
45  3.395E-02  8.327E-16  3.148E+01  1.000E+02  2.941E+02
46  2.645E-02  1.388E-16  3.144E+01  1.000E+02  2.921E+02
47  2.060E-02  8.188E-16  3.140E+01  1.000E+02  2.901E+02
48  1.581E-02  4.718E-15  3.136E+01  1.000E+02  2.882E+02
49  1.193E-02 -6.939E-16  3.133E+01  1.000E+02  2.863E+02
50  8.833E-03 -2.772E-15  3.130E+01  1.000E+02  2.845E+02
51  6.423E-03  7.556E-15  3.127E+01  1.000E+02  2.827E+02
52  6.119E-03 -2.288E-15  3.124E+01  1.000E+02  2.810E+02
53  6.003E-03 -3.456E-15  3.121E+01  1.000E+02  2.793E+02
54  5.891E-03 -5.551E-15  3.118E+01  1.000E+02  2.776E+02
55  5.779E-03 -7.527E-15  3.116E+01  1.000E+02  2.760E+02
56  5.779E-03 -7.527E-15  3.116E+01  2.126E+02  2.760E+02
57  5.779E-03 -7.527E-15  3.116E+01  2.170E+02  2.760E+02
58  5.779E-03 -7.527E-15  3.116E+01  2.174E+02  2.760E+02
59  5.779E-03 -7.527E-15  3.116E+01  2.174E+02  2.760E+02
60  5.779E-03 -7.527E-15  3.116E+01  2.174E+02  2.760E+02
61  5.779E-03 -7.527E-15  3.116E+01  2.174E+02  2.760E+02
62  5.779E-03 -7.527E-15  3.116E+01  2.174E+02  2.760E+02
63  5.779E-03 -7.527E-15  3.116E+01  2.174E+02  2.760E+02
64  5.779E-03 -7.527E-15  3.116E+01  2.174E+02  2.760E+02
65  5.779E-03 -7.527E-15  3.116E+01  2.174E+02  2.760E+02
66  5.779E-03 -7.527E-15  3.116E+01  2.174E+02  2.760E+02
```

Changing solution variables

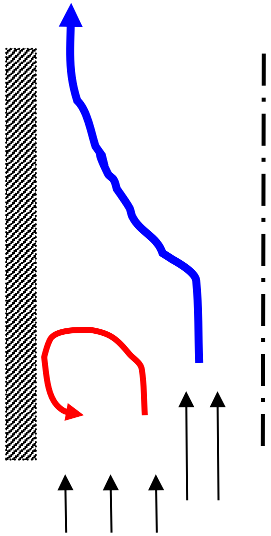
Seven iterations of *T* reach converged solution

```
67   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
68   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
69   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
70   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
71   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
72   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
73   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
74   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
75   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
76   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
77   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
78   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
79   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
80   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
81   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
82   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
83   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
84   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
85   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
86   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
87   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
88   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
89   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
90   5.779E-03  -7.527E-15   3.116E+01   2.174E+02   2.760E+02
```

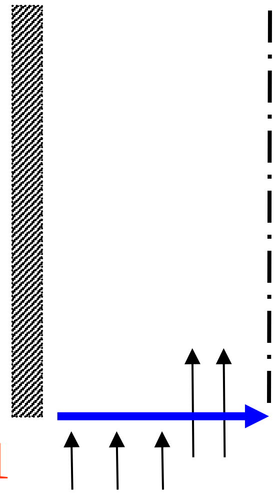
```
**************************** .VEL  U.   ****************************

 I =      2         3         4          5          6          7
J
 9   0.00E+00 0.00E+00 0.00E+00 0.00E+00  0.00E+00 0.00E+00
 8   0.00E+00 1.56E-02  3.75E-02  3.84E-02   2.04E-02 0.00E+00
 7   0.00E+00-1.65E+00-2.68E+00-2.78E+00-1.33E+00 0.00E+00
 6   0.00E+00-2.37E+00-3.56E+00-3.57E+00-1.63E+00 0.00E+00
 5   0.00E+00-2.38E+00-3.88E+00-3.98E+00-1.66E+00 0.00E+00
 4   0.00E+00-1.39E+00-3.33E+00-3.86E+00-1.45E+00 0.00E+00
 3   0.00E+00 3.74E+00-3.47E-01 -2.75E+00-8.62E-01  0.00E+00
 2   0.00E+00 4.44E+00 6.55E+0 0-2.87E+00-6.77E-01  0.00E+00
 1   0.00E+00 0.00E+00 0.00E+00  0.00E+00  0.00E+00 0.00E+00
```



```
****************************    .VEL  V.   ****************************

 I =      1         2          3         4          5         6          7
J
 9    0.00E+00 8.87E+00 3.18E+01 4.59E+01 6.52E+01 7.82E+01 1.00E+01
 8    0.00E+00 8.87E+00 3.18E+01 4.59E+01 6.52E+01 7.82E+01 1.00E+01
 7    0.00E+00 4.16E+00 2.89E+01 4.56E+01 6.93E+01 8.20E+01 1.00E+01
 6    0.00E+00-2.61E+00 2.55E+01 4.56E+01 7.48E+01 8.67E+01 1.00E+01
 5    0.00E+00-9.41E+00 2.12E+01 4.53E+01 8.15E+01 9.14E+01 1.00E+01
 4    0.00E+00-1.34E+01 1.56E+01 4.38E+01 8.83E+01 9.56E+01 1.00E+01
 3    0.00E+00-2.70E+00 3.98E+00 3.69E+01 9.37E+01 9.81E+01 1.00E+01
 2    1.00E+01 1.00E+01 1.00E+01 1.00E+01 1.00E+02 1.00E+02 1.00E+02
```

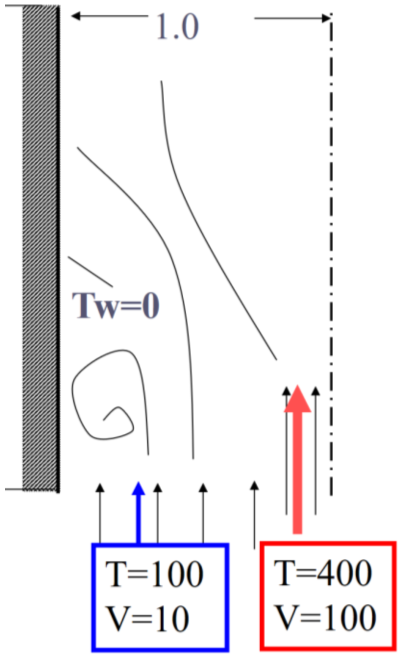


```
****************************    .STR  FN   ****************************

 I =       2        3          4          5          6          7
J
 9   0.00E+00-1.77E+00 -8.12E+00-1.73E+01-3.03E+01-4.60E+01
 8   0.00E+00-1.77E+00 -8.14E+00-1.73E+01-3.04E+01-4.60E+01
 7   0.00E+00-8.31E-01  -6.61E+00-1.57E+01-2.96E+01-4.60E+01
 6   0.00E+00 5.21E-01  -4.58E+00-1.37E+01 -2.87E+01-4.60E+01
 5   0.00E+00 1.88E+00 -2.36E+0 0-1.14E+01-2.77E+01-4.60E+01
 4   0.00E+00 2.68E+00 -4.55E-01 -9.22E+00 -2.69E+01-4.60E+01
 3   0.00E+00 5.39E-01  -2.57E-01 -7.64E+00 -2.64E+01-4.60E+01
 2   0.00E+00-2.00E+00 -4.00E+00-6.00E+00 -2.60E+01-4.60E+01
```



Stream function increase along this direction

```
*****************************   . TEMP .   *****************************

 I =       1         2         3          4         5          6         7
J
 9    0.00E+00 1.00E+02 1.00E+02 1.00E+02 1.00E+02 1.00E+02 1.00E+02
 8    0.00E+00 3.01E+02 3.26E+02 3.39E+02 3.60E+02 3.80E+02 1.00E+02
 7    0.00E+00 3.00E+02 3.21E+02 3.35E+02 3.63E+02 3.85E+02 1.00E+02
 6    0.00E+00 2.93E+02 3.10E+02 3.26E+02 3.64E+02 3.89E+02 1.00E+02
 5    0.00E+00 2.80E+02 2.93E+02 3.11E+02 3.67E+02 3.92E+02 1.00E+02
 4    0.00E+00 2.65E+02 2.69E+02 2.88E+02 3.72E+02 3.95E+02 1.00E+02
 3    0.00E+00 2.52E+02 2.36E+02 2.53E+02 3.79E+02 3.97E+02 1.00E+02
 2    0.00E+00 1.29E+02 1.16E+02 2.01E+02 3.90E+02 3.99E+02 1.00E+02
 1    0.00E+00 1.00E+02 1.00E+02 1.00E+02 4.00E+02 4.00E+02 4.00E+02
```



Given wall temp

Given inlet temp.

```
***************************    KIN  ENE   ***************************

 I =      1         2          3          4          5          6          7
J
 9   5.00E-01 5.00E-01 5.00E-01   5.00E-01  5.00E+01 5.00E+01 5.00E+01
 8   5.00E-01 1.59E+02 4.93E+02 4.65E+02 3.53E+02 2.15E+02 2.15E+02
 7   5.00E-01 1.90E+02 5.34E+02 4.85E+02 3.35E+02 1.74E+02 1.74E+02
 6   5.00E-01 2.20E+02 5.83E+02 5.22E+02 3.20E+02 1.37E+02 1.37E+02
 5   5.00E-01 2.39E+02 6.06E+02 5.46E+02 2.94E+02 1.06E+02 1.06E+02
 4   5.00E-01 2.15E+02 5.40E+02 5.31E+02 2.54E+02 8.23E+01 8.23E+01
 3   5.00E-01 1.15E+02 3.30E+02 4.69E+02 2.06E+02 6.62E+01 6.62E+01
 2   5.00E-01 1.88E+01 1.03E+01 3.22E+02 1.46E+02 5.55E+01 5.55E+01
 1   5.00E-01 5.00E-01  5.00E-01  5.00E-01  5.00E+01 5.00E+01 5.00E+01
```

Initial values, No decoration!

Initial values,
No decoration!

```
***************************    .DISIPA.    ***************************

 I =      1          2          3          4          5          6          7
 J
 9   2.50E-02 2.50E-02  2.50E-02  2.50E-02  2.50E+02 2.50E+02 2.50E+02
 8   2.50E-02 8.18E+03 1.25E+04 1.13E+04 7.78E+03 3.60E+03 3.60E+03
 7   2.50E-02 1.07E+04 1.44E+04 1.28E+04 7.79E+03 2.82E+03 2.82E+03
 6   2.50E-02 1.34E+04 1.71E+04 1.53E+04 7.94E+03 2.12E+03 2.12E+03
 5   2.50E-02 1.51E+04 1.93E+04 1.80E+04 7.66E+03 1.50E+03 1.50E+03
 4   2.50E-02 1.29E+04 1.79E+04 1.98E+04 6.81E+03 1.01E+03 1.01E+03
 3   2.50E-02 5.08E+03 1.04E+04 1.99E+04 5.46E+03 6.63E+02 6.63E+02
 2   2.50E-02 3.34E+02 1.53E+02 1.52E+04 3.43E+03 4.02E+02 4.02E+02
 1   2.50E-02 2.50E-02  2.50E-02  2.50E-02  2.50E+02 2.50E+02 2.50E+02
```

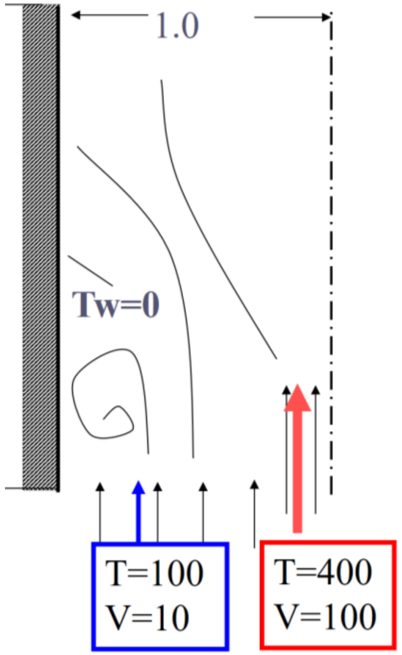
Initial values, No decoration!

Initial values,
No decoration!

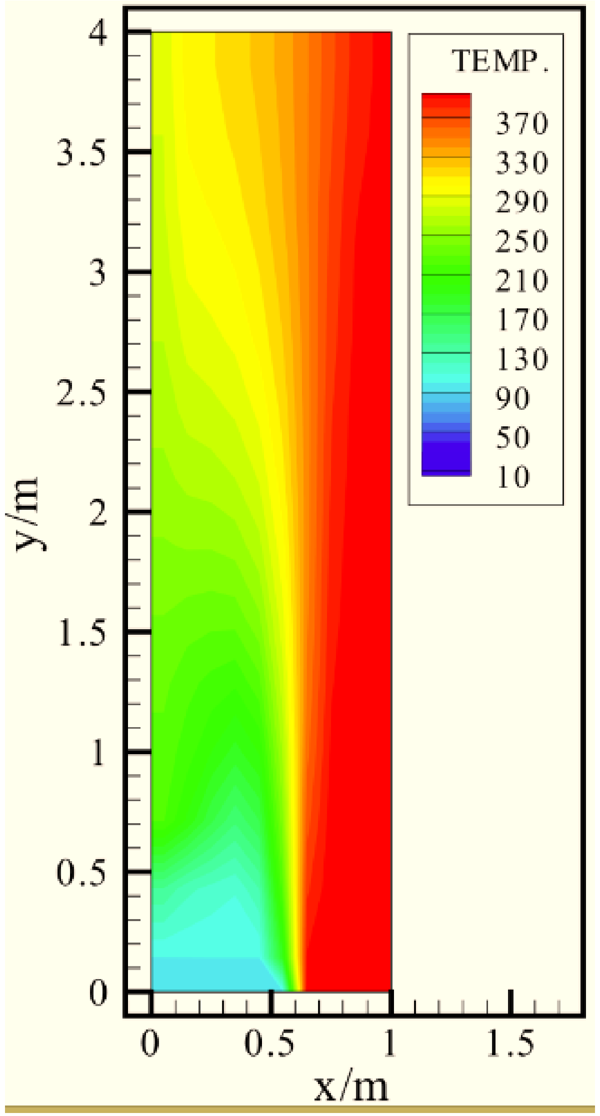
$$\varepsilon = \frac{C_\mu^{3/4} k_P^{3/2}}{\kappa y_P}$$

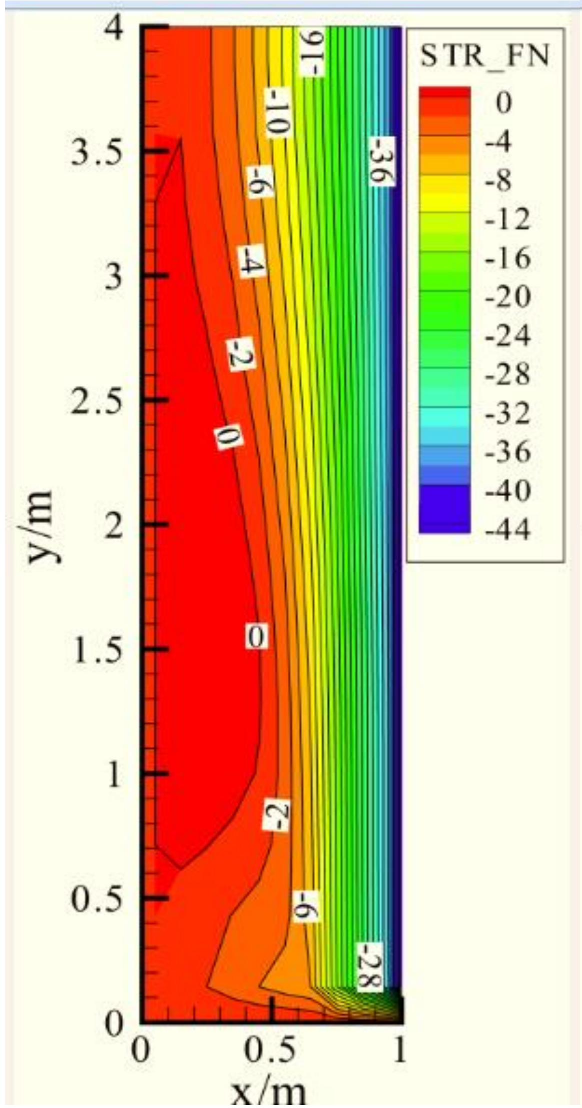
```
****************************    TURB  VI   ****************************

 I =      1         2          3         4          5          6         7
J
 9    9.00E-01 9.00E-01 9.00E-01  9.00E-01  9.00E-01  9.00E-01 9.00E-01
 8    9.00E-01 2.78E-01 1.72E+00 1.70E+00 1.42E+00 1.14E+00 1.14E+00
 7    9.00E-01 3.04E-01 1.76E+00 1.65E+00 1.29E+00 9.59E-01  9.59E-01
 6    9.00E-01 3.27E-01 1.77E+00 1.59E+00 1.16E+00 7.99E-01  7.99E-01
 5    9.00E-01 3.40E-01 1.71E+00 1.48E+00 1.01E+00 6.75E-01  6.75E-01
 4    9.00E-01 3.22E-01 1.46E+00 1.28E+00 8.54E-01  6.02E-01  6.02E-01
 3    9.00E-01 2.36E-01 9.39E-01  9.99E-01  7.00E-01  5.94E-01  5.94E-01
 2    9.00E-01 9.50E-02 6.24E-02  6.19E-01  5.58E-01  6.88E-01  6.88E-01
 1    9.00E-01 9.00E-01 9.00E-01  9.00E-01  9.00E-01  9.00E-01  9.00E-01
```

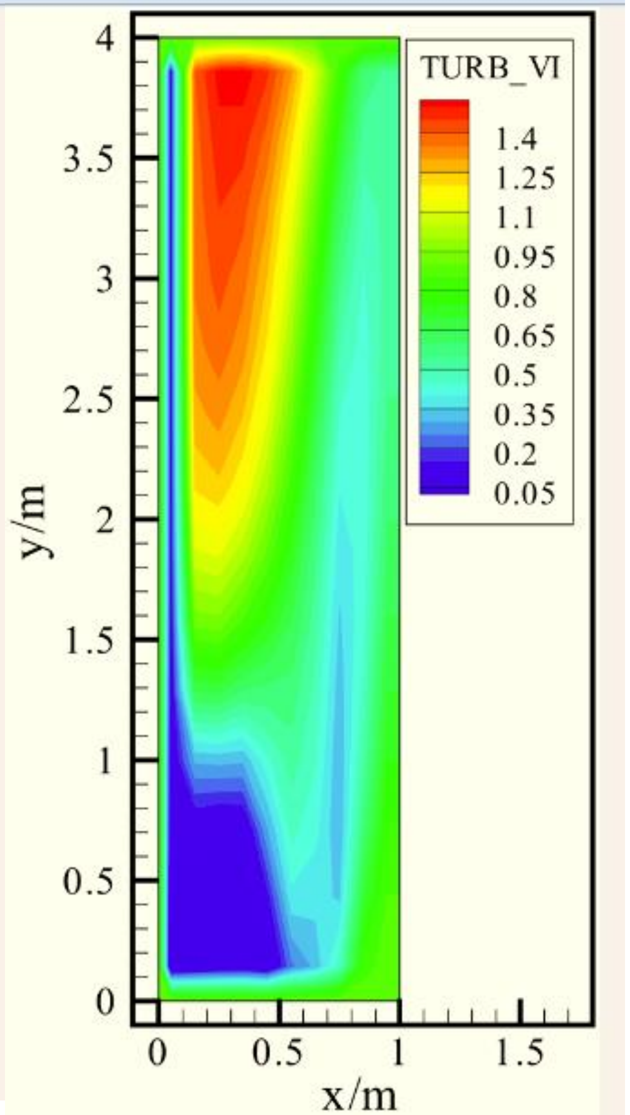
Molecular viscosity $\eta_l \approx 10^{-6}$



```
****************************   PRESSURE   ****************************

I =     1          2          3          4          5          6          7
J
9   1.44E+03  1.43E+03  1.41E+03 1.33E+03 1.21E+03  1.14E+03 1.12E+03
8   1.36E+03  1.35E+03  1.33E+03 1.28E+03 1.20E+03  1.15E+03 1.13E+03
7   1.20E+03  1.19E+03  1.17E+03 1.17E+03 1.17E+03  1.16E+03 1.16E+03
6   9.40E+02  9.31E+02  9.11E+02 9.19E+02 9.28E+02  9.26E+02 9.25E+02
5   6.02E+02  5.92E+02  5.72E+02 5.96E+02 6.22E+02  6.25E+02 6.27E+02
4   2.24E+02  2.16E+02  1.99E+02 2.54E+02 3.08E+02  3.24E+02 3.32E+02
3   4.20E+01  3.16E+01  1.09E+01 1.03E+02 1.39E+02  1.44E+02 1.46E+02
2   1.31E+01  5.48E+00-9.74E+00-6.55E+01  2.53E+01 4.85E+01 6.02E+01
1   0.00E+00-7.61E+00 -2.01E+01-1.50E+02-3.17E+01 1.07E+00 1.27E+01
```

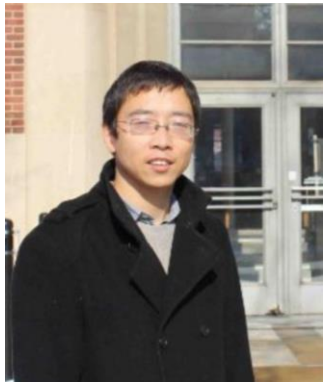

TEMP.
370
330
290
250
210
170
130
90
50
10
y/m
4
3.5
3
2.5
2
1.5
1
0.5
0
x/m
0
0.5
1
1.5


STR_FN
0
-4
-8
-12
-16
-20
-24
-28
-32
-36
-40
-44
-16
-10
-6
-4
-2
0
0
-2
-6
-36
-28
y/m
4
3.5
3
2.5
2
1.5
1
0.5
0
x/m
0
0.5
1


TURB_VI
1.4
1.25
1.1
0.95
0.8
0.65
0.5
0.35
0.2
0.05
y/m
4
3.5
3
2.5
2
1.5
1
0.5
0
x/m
0
0.5
1
1.5

Part I： Fundamentals of NHT and Teaching Code （11 chapters）

Part II of NHT：Study of FLUENT



冀文涛
（Wen-Tao Ji)

C 12 Basic contents
(6 hours)

任秦龙
(Qin-Long Ren)

Applications
(6 hours)

C 13a Fundamental Applications

C 13b Intermediate Applications

陈黎
（Li Chen）

Computer-Aided Project of 2024 Numerical Heat Transfer
Xi’an Jiaotong University

We present three computer-aided projects: one is to be solved by our teaching code (Project 1）, the 2nd and 3rd ones are to be solved by FLUENT (Fundamental , Project 2, Intermediate Project 3) . Every student can choose one project according to your interest and condition.

For the first project the self-developed computer code (USER) should attached in your final report. **Students are encouraged to take Project 1. The detail project will be released after class.**

For the second and third project Class F and Class I will have different projects. The instructors will assign the project at the end of the lecture.

Following students are invited for the office hour of this afternoon （2024.12.10）

Tuesday, Venue: 1-5080, 2:30 pm - 4:00 pm

3124103242	宋晔
3124103243	肖天佑
3124103244	房锭宸
3124103245	曾博文
3124103246	邓一飞
3124103248	余河江
3124103249	辜乐怡
3124103250	曹青
3124103251	修文恒
3124103255	薛欣宇

3124103256	李想想
3124103257	陈亮
3124103258	李文豪
3124103259	高言
3124103260	袁驰
3124103261	战金承
3124103262	刘晓龙
3124103263	谢宇轩
3124103264	付少琪
3124103265	张赛男

本组网页地址：http://nht.xjtu.edu.cn 欢迎访问！

Teaching PPT will be loaded on our website

同舟共济
渡彼岸！

People in the same boat help each other to cross to the other bank, where....