Numerical Heat Transfer

(数值传热学)

Chapter 11 Application Examples of the General Code for 2D Elliptical FF & HT Problems

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11.1 2D steady heat conduction without source term in Cartesian coordinate

11.2 Steady heat conduction in a hollow cylinder

11.3 Fully-developed heat transfer in a square duct

11.4 Fully developed heat transfer in annular space with straight fin at inner wall

Conduction

11.5 Fluid flow and heat transfer in a 2-D sudden expansion

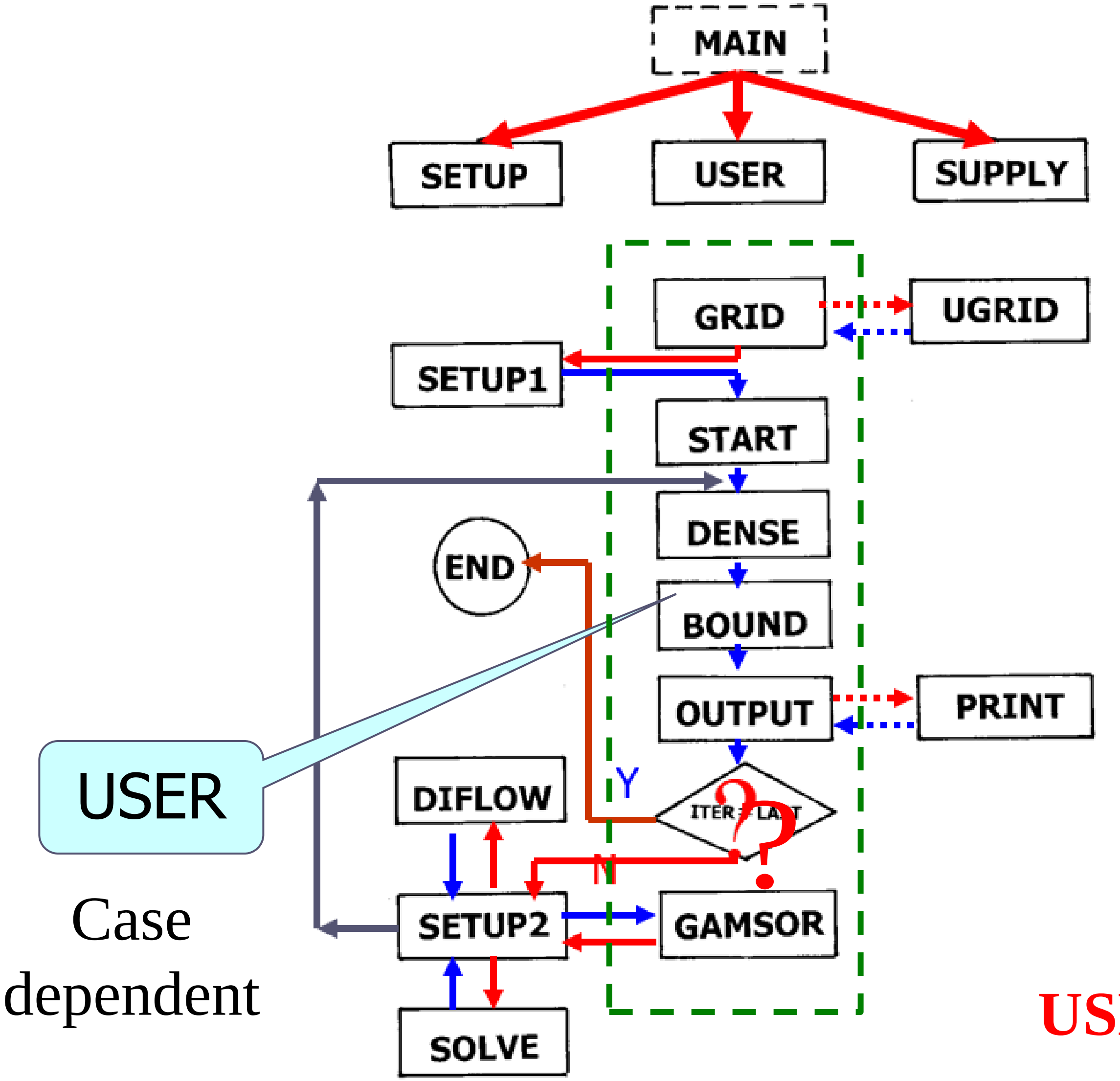
11.6 Complicated fully developed fluid flow and heat transfer in square duct

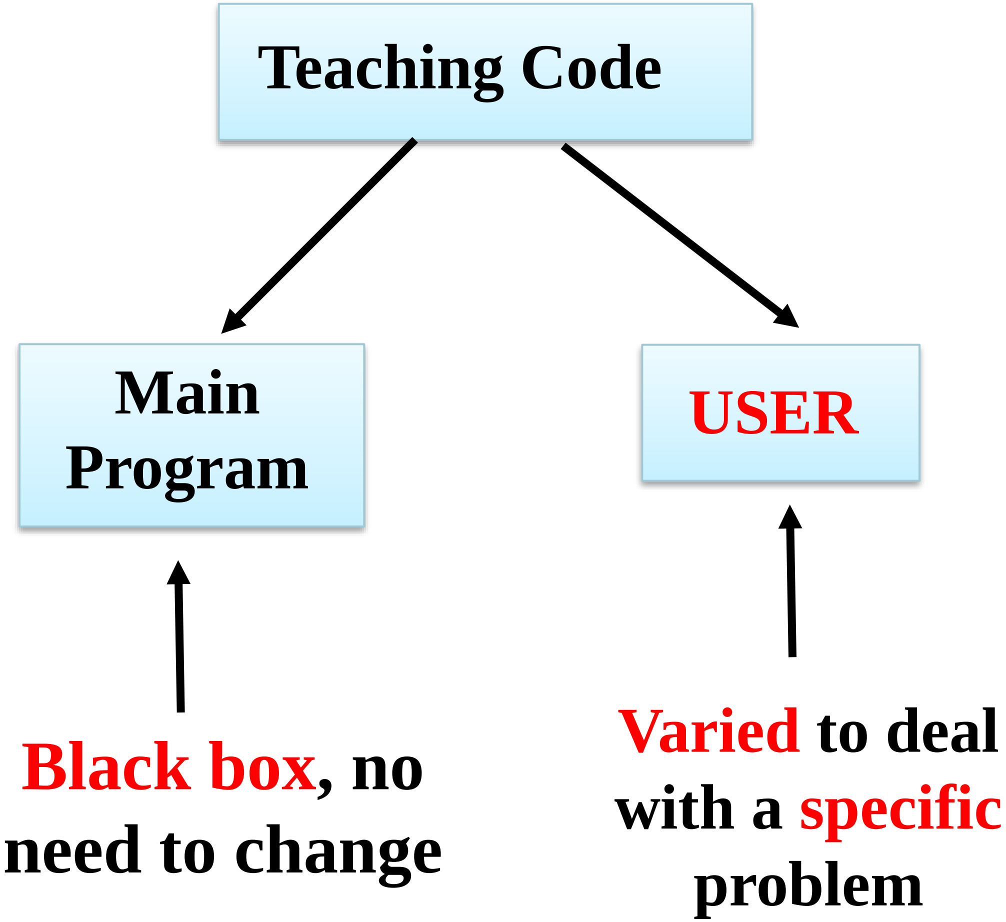
11.7 Impinging flow on a rotating disc

11.8 Turbulent flow and heat transfer in duct with a central jet

Convective

Review of Teaching Code





USER: Includes modules GRID, START, DENSE, BOUND, OUTPUT, GAMSOR

11-1 2D steady heat conduction without source term in Cartesian coordinate －Knowing USER structure

11-1-1 Physical problem and its math formulation

Known: Steady heat conduction of constant properties without source term shown in Fig. 1 has following temperature distribution on four boundaries：

$$T = x + y + xy$$

Find: Temp. distribution within the region.

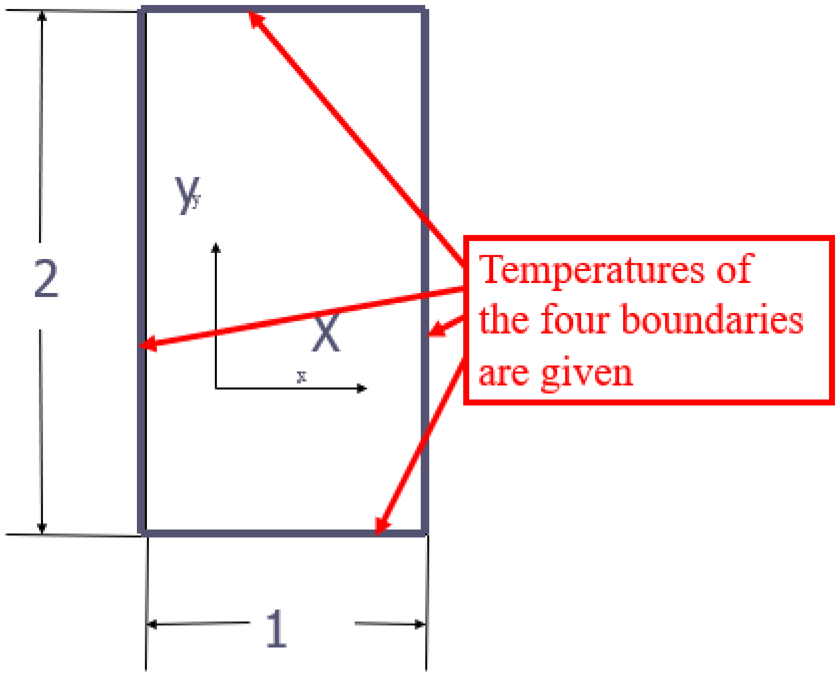


Fig.1 Computational domain

Remarks: In all examples, physical quantities are given by their numerical values without units. It is assumed that all units are homogeneous (单位和谐).

Solution: GGE $\frac{\partial(\cancel{\rho^* \Phi})}{\cancel{\partial t}} + \cancel{div(\rho^* \vec{u}\Phi)} = div(\Gamma_\Phi grad\Phi) + \cancel{S_\phi^*}$

2D, steady state, conduction

constant property, no source term

Laplace equation: $\boxed{\nabla \cdot \left(\Gamma_\Phi \nabla \Phi\right) = 0}$

Compared with the standard form, it is a diffusion problem with **Γ and source term** as follows:

$$\Gamma_\phi = \lambda = 1, S_\phi^* = 0$$

Boundary conditions: $\boldsymbol{T = x + y + xy}$ at four boundaries

11-1-2 Program reading

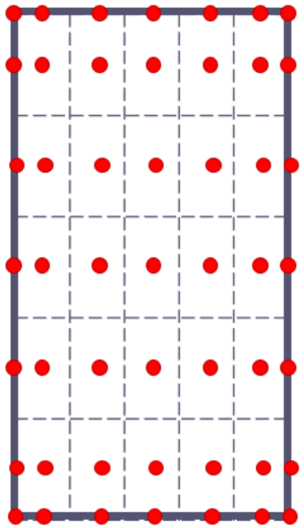
Define new variables

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C***************************************
      INTEGER*4 I,J
C***************************************
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
     SUBROUTINE USER
C***************************************
     USE START_L
     USE USER_L
     IMPLICIT NONE
C***************************************
C--------------------------------PROBLEM ONE---------------------------------
C                          Example of USER structure
C*********************************************************************
     ENTRY GRID
        LAST=10              ! Numbers of iteration
        LSOLVE(4)=.TRUE.     ! 4th variable for solving temperature equation
        LPRINT(4)=.TRUE.     ! Print out the temperature filed
        TITLE(4)='  .TEMP.  ' ! Title to output temperature field.
```

```
MODULE START_L
 PARAMETER(NI=100,NJ=200,NIJ=NI,NFMAX=10,NFX4=NFMAX+4)
*****************************************************************
CHARACTER*8 TITLE(NFX4)
LOGICAL LSOLVE(NFX4),LPRINT(NFX4),LBLK(NFX4),LSTOP
REAL*8,DIMENSION(NI,NJ,NFX4)::F
REAL*8,DIMENSION(NI,NJ,6)::COF,COFU,COFV,COFP
REAL*8,DIMENSION(NI,NJ)::P,RHO,GAM,CP,CON,AIP,AIM,AJP,AJM,AP
REAL*8,DIMENSION(NI,NJ)::U,V,PC,T,DU,DV,UHAT,VHAT
REAL*8,DIMENSION(NI)::X,XU,XDIF,XCV,XCVS,XCVI,XCVIP
REAL*8,DIMENSION(NJ)::Y,YV,YDIF,YCV,YCVS,YCVR,YCVRS,ARX,ARXJ,
1    ARXJP,R,RMN,SX,SXMN
REAL*8,DIMENSION(NI)::FV,FVP,FX,FXM
REAL*8,DIMENSION(NJ)::FY,FYM
REAL*8,DIMENSION(NIJ)::PT,QT
REAL*8 RELAX(NFX4),TIME,DT,XL,YL,RHOCON,CPCON
INTEGER*4 NF,NP,NRHO,NGAM,NCP,L1,L2,L3,M1,M2,M3,
1    IST,JST,ITER,LAST,MODE,NTIMES(NFX4),IPREF,JPREF
REAL*8 SMAX,SSUM
REAL*8 FLOW,DIFF,ACOF
```

Module GRID

```
XL=1.     ! Computation domain
YL=2.                         ! MODE=1 is a default (Cartesian)
L1=7      ! Grid number
M1=7
CALL UGRID    ! Generate interface position of CV
RETURN
```



Module START

```
ENTRY START
DO 100 J=1,M1
DO 101 I=1,L1
T(I,J)=0.          ! Initial temperature values.
IF(I==1.OR.I== L1)  T(I,J)=(X(I)+Y(J)+X(I)*Y(J))  ! Unchanged B.C.
IF(J==1.OR.J== M1) T(I,J)=(X(I)+Y(J)+X(I)*Y(J))  are given here
101 ENDDO
100 ENDDO
RETURN
*
```

$T = x + y + xy$

Module DENSE

```
ENTRY DENSE   ! Empty, but keep it
RETURN
*
```

Module BOUND

```
ENTRY BOUND   ! Empty, B.C. has been set up in START
RETURN
```

```
PROGRAM MAIN
USE START_L
IMPLICIT NONE
****************************
OPEN(8,FILE='RESULT.TXT')
CALL GRID
CALL SETUP1
CALL START
DO WHILE(.NOT.LSTOP)
CALL DENSE
CALL BOUND
CALL OUTPUT
CALL SETUP2
ENDDO
```

```
PROGRAM MAIN
USE START_L
IMPLICIT NONE
**************************
OPEN(8,FILE='RESULT.TXT')
```

Module OUTPUT

```
      ENTRY OUTPUT
      IF(ITER == 0) THEN            ! The title only needs output once
      PRINT 401          ! Output to screen
      WRITE(8,401)       ! Output through file
  401 FORMAT(1X,'  ITER',13X,'T(4,4)',14X,'T(5,3)')   ! ITER   T(4,4)   T(5,3)
      ELSE
      PRINT 403,    ITER,  T(4,4), T(5,3)    ! Print out two temps. in each
      WRITE(8,403) ITER,  T(4,4), T(5,3)     iteration for observation
  403 FORMAT(1X,I5,2F20.6)
      ENDIF
      IF(ITER == LAST) CALL PRINT    ! Output 2D field after
      RETURN                         getting converged solution.
*
```

Module GAMSOR

```
      ENTRY GAMSOR
      IF(ITER == 0) THEN        ! constant thermo-properties, call once only
      DO 500 J=1,M1
      DO 501 I=1,L1
      GAM(I,J)=1.
  501 ENDDO
  500 ENDDO
      ELSE
      ENDIF
      RETURN
      END
```

$$\boxed{\nabla \cdot \left(\nabla T \right) = 0}$$

! The zero initial values of S_c, S_p have been set in "RESET". Only Γ is set up here.

11-1-3 Analysis of results

COMPUTATION IN CARTESIAN COORDINATES

****

ITER	T(4,4)	T(5,3)
0	0.000000	0.000000
1	1.999978	1.720364
2	2.000000	1.720001
3	2.000000	1.720000
4	2.000000	1.720000
5	2.000000	1.720000
6	2.000000	1.720000
7	2.000000	1.720000
8	2.000000	1.720000
9	2.000000	1.720000
10	2.000000	1.720000

```
    ENTRY OUTPUT
    IF(ITER==0) THEN
    PRINT 401
    WRITE(8,401)
401 FORMAT(1X,'  ITER',13X,'T(4,4)',14X,'T(5,3)')
    ELSE
    PRINT 403, ITER,T(4,4),T(5,3)
    WRITE(8,403) ITER,T(4,4),T(5,3)
403 FORMAT(1X,I5,2F20.6)
    ENDIF
    IF(ITER==LAST) CALL PRINT
    RETURN
```

401 FORMAT(1X,' ITER',13X,'T(4,4)',14X,'T(5,3)')

403 FORMAT(1X,I5,2F20.6)

2F20.6

2F-two floating-point number

20.6-Every data take 20 places;
after decimal (小数点) there are 6 digits

```
LPRINT(4)=.TRUE.
TITLE(4)=' TEMP '
```

Node numbers: 7 * 7

```
IF(ITER==LAST) CALL PRINT
RETURN
```

**************************** .TEMP. ****************************

I =	1	2	3	4	5	6	7
J							
7	2.00E+00	2.30E+00	2.90E+00	3.50E+00	4.10E+00	4.70E+00	5.00E+00
6	1.80E+00	2.08E+00	2.64E+00	3.20E+00	3.76E+00	4.32E+00	4.60E+00
5	1.40E+00	1.64E+00	2.12E+00	2.60E+00	3.08E+00	3.56E+00	3.80E+00
4	1.00E+00	1.20E+00	1.60E+00	2.00E+00	2.40E+00	2.80E+00	3.00E+00
3	6.00E-01	7.60E-01	1.08E+00	1.40E+00	1.72E+00	2.04E+00	2.20E+00
2	2.00E-01	3.20E-01	5.60E-01	8.00E-01	1.04E+00	1.28E+00	1.40E+00
1	0.00E+00	1.00E-01	3.00E-01	5.00E-01	7.00E-01	9.00E-01	1.00E+00

From initial field

$$T = x + y + xy$$

7×7 nodes

27×27 nodes

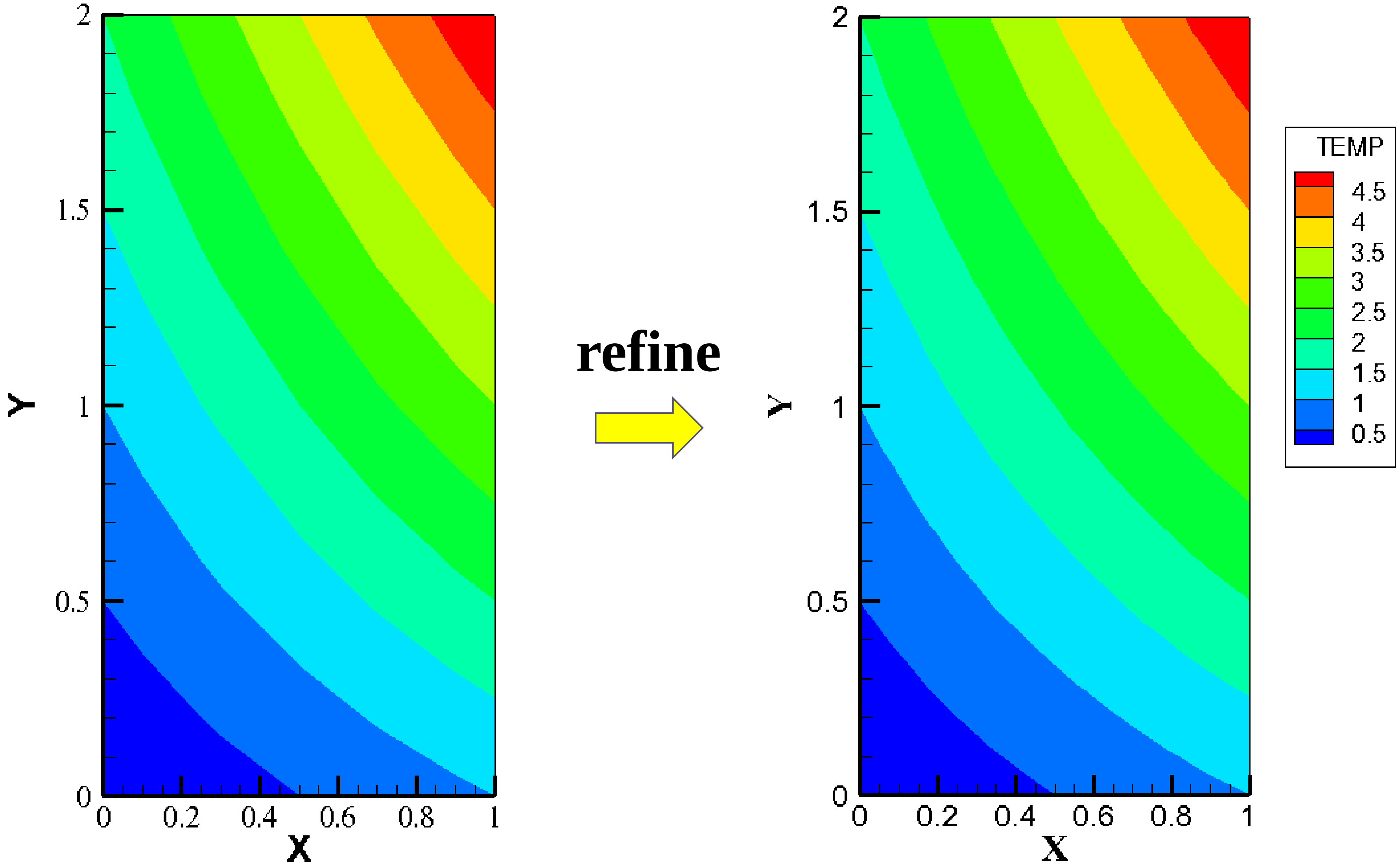


Fig. 2 Isotherms from TECPLOT

11-2 Steady heat conduction in a hollow cylinder ---ASTM for 2nd and 3rd boundary conditions

11-2-1 Physical problem and its math formulation

Known: Steady heat conduction in a hollow cylinder with variable property and source term shown in Fig. 1 has following boundary conditions:

Left boundary---given temperature:

$$T = 100(1+y)$$

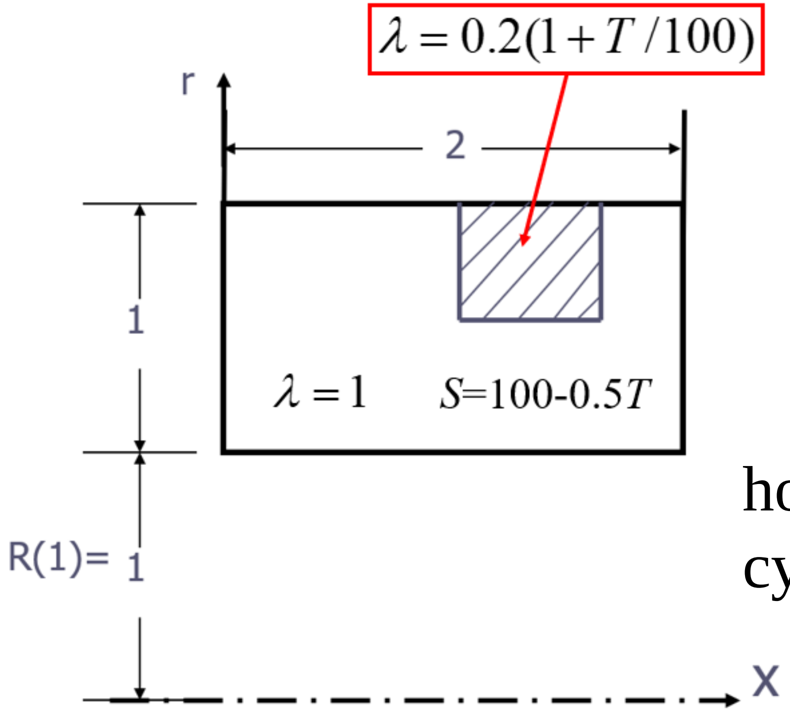
Right boundary---convective heat transfer:

Heat transfer coefficient $h = 5$;

Fluid temperature $T_f = 100$.

Top boundary---adiabatic;

Bottom boundary---given heat flux: $q = 50$

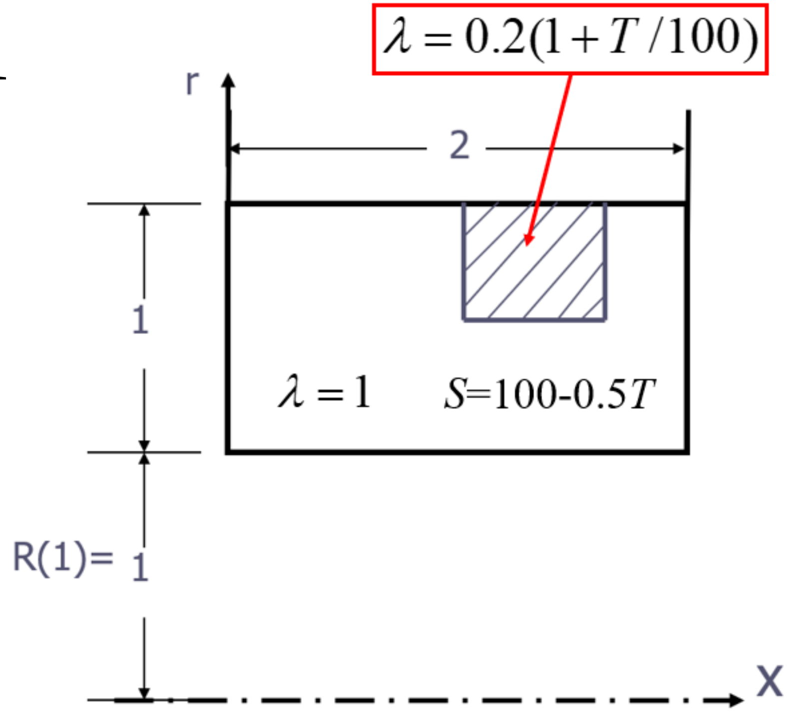


hollow cylinder

Variable thermo-properties:

Thermal conductivity---for most region, $\lambda = 1$

in a local region $\lambda = 0.2(1 + T/100)$



Sources term:

Entire region--- S =100 - 0.5T

Find: temperature distribution in the domain.

Solution:

Steady conduction problem with given Γ and source term: Γ_ϕ S_ϕ.

$$div(\Gamma_\phi grad\phi) + S_\phi = 0$$

Steady heat conduction in a hollow cylinder

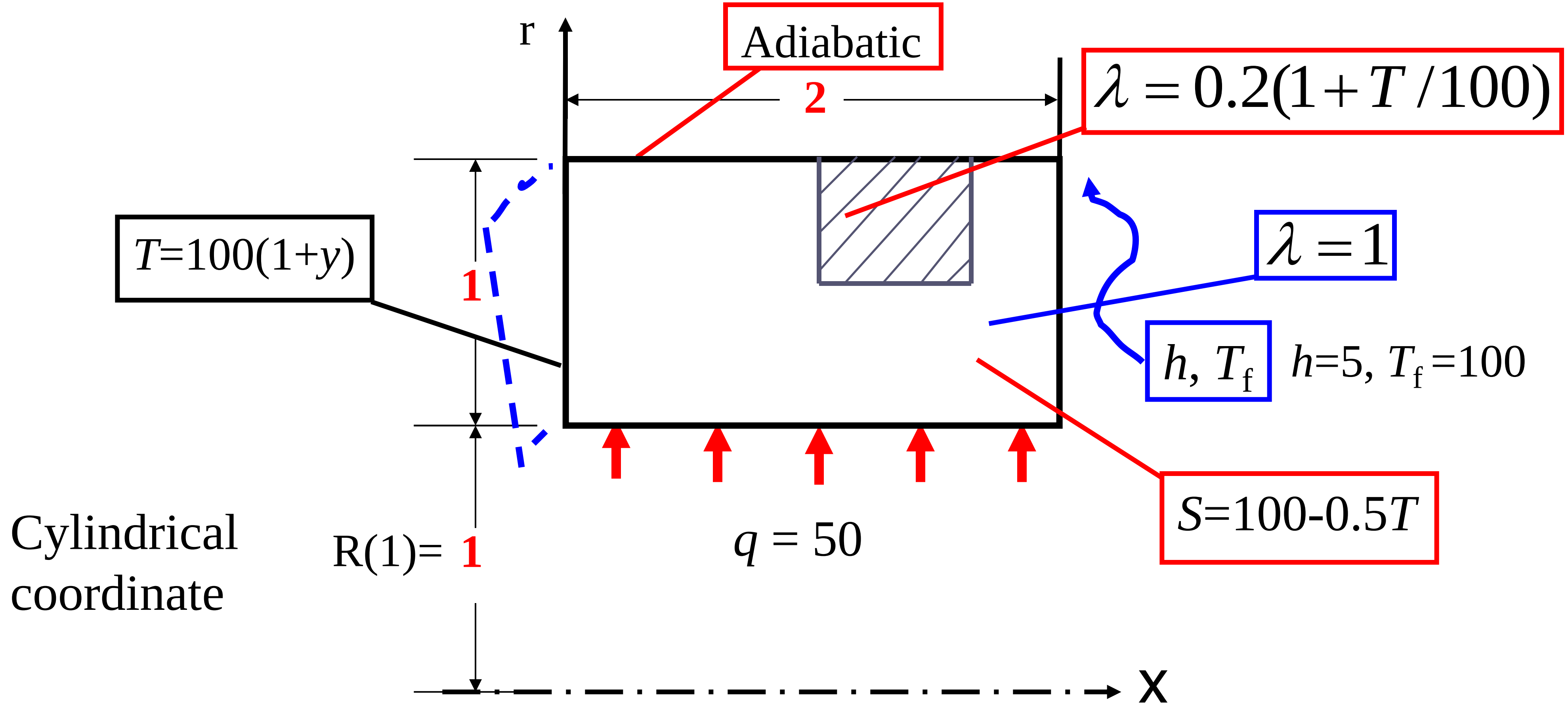
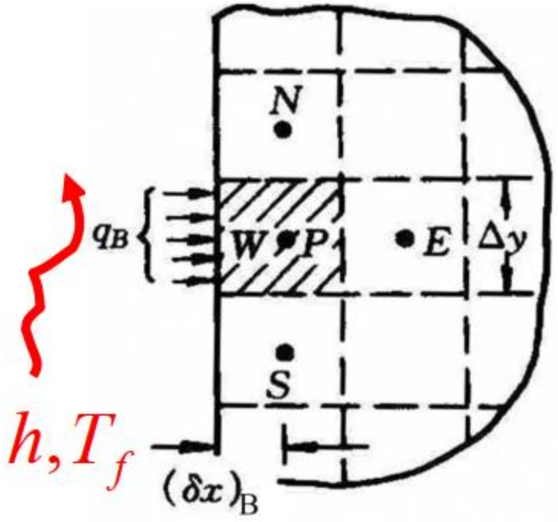


Fig.1 Diagram

ASTM (additional source term method)

Implementing procedure of ASTM



(1) Determining $S_{C,ad}$, $S_{P,ad}$ for CV neighboring to boundary

(2) Adding them into source term of related CV

$$S_C \leftarrow S_C + S_{C,ad} \qquad S_P \leftarrow S_P + S_{P,ad}$$

(3) Setting the conductivity of boundary node as zero

$$\lambda = 0 \rightarrow a_W = \frac{\lambda \Delta y}{\delta x} = 0$$

2nd BC. $S_{C,ad} = \frac{q_B \Delta y}{\Delta V}$ $S_{P,ad} = 0$

(4) Solving the algebraic Eqs. for inner nodes

3rd BC. $S_{C,ad} = \frac{\Delta y \bullet T_f}{\Delta V[\frac{1}{h} + \frac{(\delta x)_B}{\lambda_B}]}$

(5) Using Newton' law of cooling or Fourier eq. to get the boundary temperatures when converged

$$S_{P,ad} = -\frac{\Delta y}{\Delta V \bullet [1/h + (\delta x)_B / \lambda_B]}$$

11-2-2 Program reading

MODULE

USER_L

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C***********************************************************************
      INTEGER*4 METHOD, I, J
      REAL*8 HTC, TF, GAM1, GY, RES, ARES
C***********************************************************************
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
     SUBROUTINE USER
C************************************************************************
     USE START_L
     USE USER_L
     IMPLICIT NONE
C*************************************************************************
C---------------------------PROBLEM  TWO-------------------------------------------
C    Two–dimensional steady-state heat conduction in a hollow cylinder
C-----Implementation of ASTM and comparison with updating method------
C--------------------------------------------------------------------------------------
C************************************************************************
```

ENTRY GRID

LAST=100 ! A relatively large value for non-linear problems

LSOLVE(4)=.TRUE. ! Solve the energy equation

TITLE(4)= ‘ .TEMP. ‘ ! Title for temperature field print out

LPRINT(4)=.TRUE.

TITLE(13)=‘ .COND. ‘ ! Title for variable conductivity print out

LPRINT(13)=.TRUE. ! Regarding Γ as the 13th variable,

```
MODULE START_L
 PARAMETER(NI=100,NJ=200,NIJ=NI,NFMAX=10,NFX4=NFMAX+4)
REAL*8,DIMENSION(NI,NJ,NFX4)::F
```

NF =	1	2	3	4	……	11	12	13	14
Variable	U	V	p_c	T	……	p	ρ	Γ	C_p

MODE=2

R(1)=1.

XL=2.

YL=1.

L1=7

M1=7

Specify lengths and node numbers of domain

CALL UGRID ! Generate interface position of CV

RETURN

```
      ENTRY START
      METHOD=1             !  Boundary temperature updated method;
      DO 100 J=1,M1        While METHOD= 2 is ASTM method
      DO 101 I=1,L1
      T(I,J)=200.          !  Initial values
      IF(I == 1) T(I,J)=100.*(1.+Y(J))    !  Specify left boundary temperature
101 ENDDO
100 ENDDO
      HTC=5.
      Q=50.               Specify boundary condition parameters
      TF=100.
      GAM1=1.         !  Set up conductivity value for main body
      RETURN
*
      ENTRY DENSE
      RETURN           !  Empty, but keep it
```

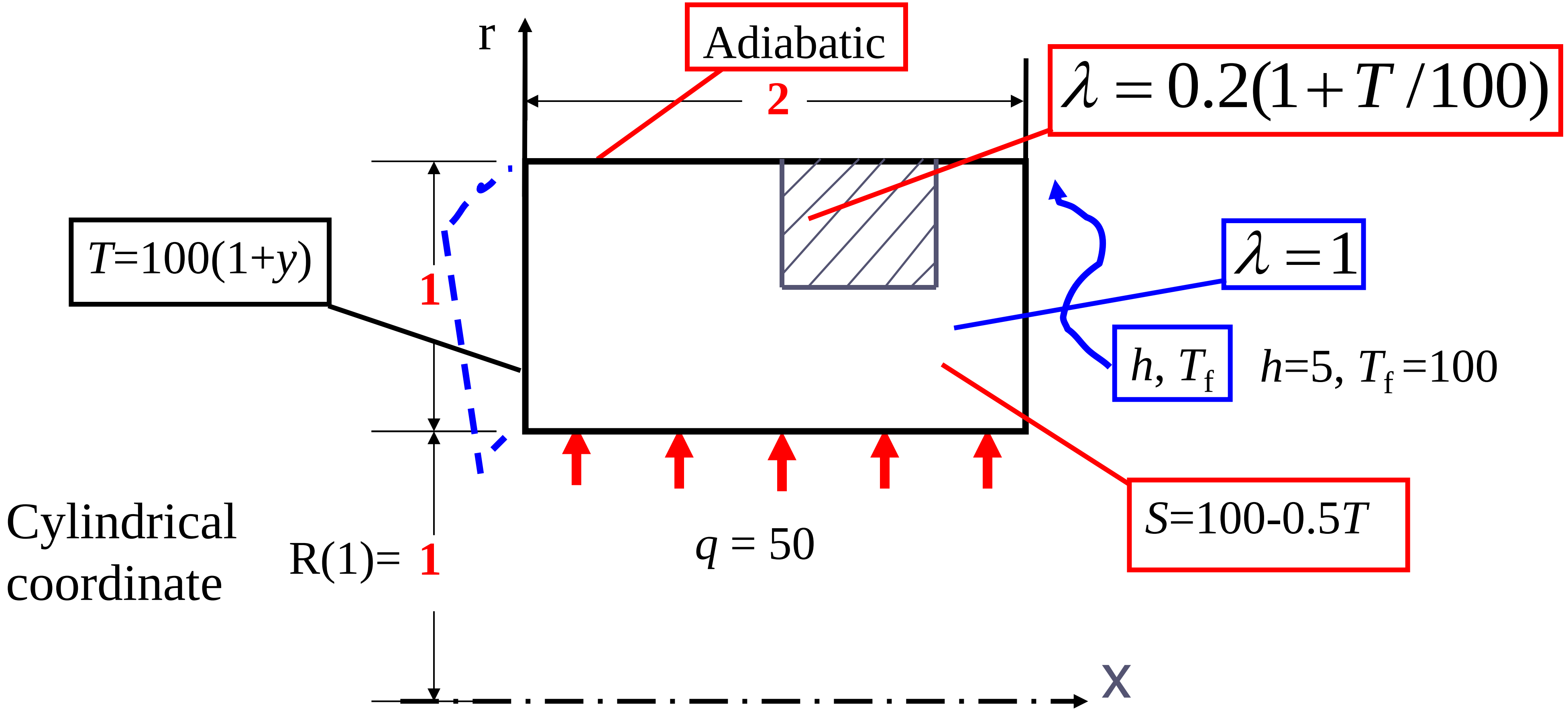


Fig.1 Computational domain

```
      ENTRY BOUND
      DO 300 I=2,L2
      T(I,M1)=T(I,M2)                          ! updated temperature (top boundary)
      T(I,1)=T(I,2)+Q*YDIF(2)/GAM1             ! updated temperature (bottom boundary)
  300 ENDDO
      GY=GAM1/XDIF(L1)                         ! Temporary variable for right boundary
      DO 301 J=2,M2
      T(L1,J)=(HTC*TF+GY*T(L2,J))/(HTC+GY)     ! right boundary, updated temperature
  301 ENDO
      RETURN
```

$$q = \lambda \frac{T(i,1) - T(i,2)}{YDIF(2)}$$

Heat transferring into the region is taken as positive!

$$T(i,1) = T(i,2) + q\frac{YDIF(2)}{\lambda}$$

$$h(T_f - T_{L1}) = \frac{\lambda}{XDIF(L1)}(T_{L1} - T_{L2}) = GY(T_{L1} - T_{L2})$$

$$hT_f + GYT_{L2} = T_{L1}(h + GY)$$

$$T_{L1} = (hT_f + GYT_{L2})/(h + GY)$$

```
      ENTRY OUTPUT
      IF(ITER==0) THEN
      PRINT 403, METHOD
      WRITE(8,403) METHOD
403 FORMAT(1X,'   METHOD =', I1)
      PRINT 401
      WRITE(8,401)
401 FORMAT(1X,'   ITER',11X, 'T(4,5)', 14X, 'T(5,3)')
      ENDIF
      IF (ITER>0) PRINT 402, ITER, T(4,5), T(5.3)
      WRITE(8,402) ITER, T(4,5), T(5,3)
402 FORMAT(1X, I6, 2F20.6)
      IF(ITER==LAST) CALL PRINT
      RETURN
```

METHOD is an indicator for boundary condition treatment for 2nd and 3rd kinds

"I1" shows that the value of METHOD is expressed by an integer with one digit

！ Integer has at most six digits;，2 floating-point data with 6 digits after decimal and total length of 20 places.

```
      ENTRY GAMSOR
      DO 500 J=1,M1
      DO 501 I=1,L1
      GAM(I,J)=GAM1
501 ENDDO
500 ENDDO
      DO 503 J=4,7
      DO 504 I=4,5
      GAM(I,J)=0.2*(1.+T(I,J)/100.)
504 ENDDO
503 ENDDO
      DO 510 J=2,M2
      DO 511 I=2,L2
      CON(I,J)=100.
      AP(I,J) = -0.5
511 ENDDO
510 ENDDO
```

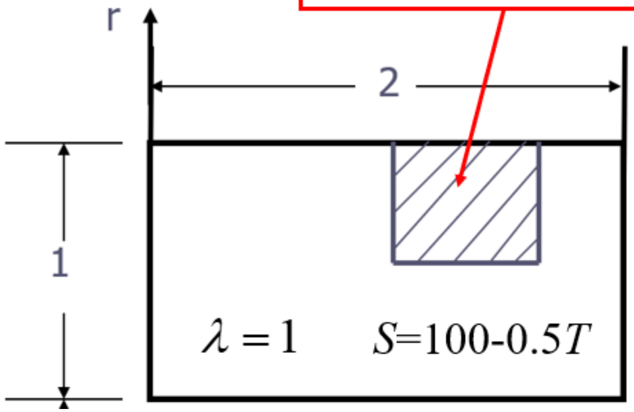
! Specify Γ for whole domain $\Gamma = \lambda = 1$

! Specify variable λ in a small region

! Specify source term S =100-0.5T

CON(I,J) = S_C; AP(I,J) = S_P

$\lambda = 0.2(1+T/100)$



$$-\lambda \frac{\partial T}{\partial y} = 0$$

Top B.: Adiabatic

Bottom B: $q = 50$

Right B: $-\lambda \frac{\partial T}{\partial x} = h\Delta t$

```
      IF(METHOD==1) RETURN   ! Following is for ASTM
      DO 520 I=2,L2
      GAM(I,M1)=0.
      GAM(I,1)=0.  ! Set λ = 0
      CON(I,2)=CON(I,2)+Q*R(1)/ARX(2)
  520 ENDDO                  ! Accumulative
      RES=1./HTC+1./GY
      ARES=1./(RES*XCV(L2))
      DO 521 J=2,M2
      GAM(L1,J)=0.  ! Set λ = 0
      CON(L2,J) = CON(L2,J)+ARES*TF
      AP(L2,J) = AP(L2,J)-ARES
  521 ENDDO !Accumulative
      RETURN
      END
```

Bottom B:

$$S_{c,ad} = \frac{qA}{\Delta V} = \frac{q \cdot XCV(i) \cdot R(1)}{ARX(2) \cdot XCV(i)} = \frac{q \cdot R(1)}{ARX(2)}$$

! CVs next to boundary

$$\frac{A}{\Delta V} = \frac{ARX(j)}{ARX(j) \cdot XCV(i)} = \frac{1}{XCV(i)}$$

$$S_{c,ad} = \frac{A}{\Delta V} \frac{T_f}{\delta x / \Gamma + 1/h} =$$

$$= \underbrace{\frac{1}{XCV(i)} \underbrace{\frac{1}{\delta x / \Gamma + 1/h}}_{RES}}_{ARES} T_f$$

$$S_{P,ad} = -\frac{1}{XCV(i)} \frac{1}{\delta x / \Gamma + 1/h}$$

11-2-3 Results analysis

COMPUTATION FOR AXISYMMETRICAL SIMULATION

****

METHOD =1 ! For updating method

ITER	T(4,5)	T(5,3)
0	200.000000	200.000000
1	196.503891	193.806549
2	194.450150	190.325912
3	192.184113	187.114395
4	189.861618	184.072250
5	187.567535	181.222870
6	185.361771	178.597488
7	183.282364	176.208923
8	181.350449	174.055115
9	179.575180	172.125107
10	177.957458	170.403229

Initial field

11	176.492798	168.871887
12	175.173325	167.513016
13	173.989273	166.309189
14	172.930008	165.243973
15	171.984665	164.302246
16	171.142624	163.470215
17	170.393753	162.735428
18	169.728561	162.086731
19	169.138290	161.514206
20	168.614944	161.008957
21	168.151245	160.563156
22	167.740601	160.169846
23	167.377090	159.822830
24	167.055481	159.516693
25	166.770981	159.246658

26	166.519409	159.008408
27	166.296982	158.798203
28	166.100388	158.612778
29	165.926620	158.449173
30	165.773102	158.304855
31	165.637451	158.177505
32	165.517609	158.065186
33	165.411758	157.966049
34	165.318222	157.878601
35	165.235626	157.801422
36	165.162720	157.733337
37	165.098282	157.673233
38	165.041412	157.620209
39	164.991196	157.573425
40	164.946838	157.532135
41	164.907684	157.495712
42	164.873108	157.463547

43	164.842590	157.435181
44	164.815643	157.410141
45	164.791870	157.388062
46	164.770844	157.368561
47	164.752319	157.351334
48	164.735947	157.336151
49	164.721497	157.322754
50	164.708740	157.310913
51	164.697495	157.300476
52	164.687561	157.291245
53	164.678772	157.283127
54	164.671051	157.275940
55	164.664200	157.269608
56	164.658157	157.264008
57	164.652847	157.259094
58	164.648148	157.254730
59	164.643982	157.250885
60	164.640289	157.247482

61	164.637070	157.244492
62	164.634201	157.241837
63	164.631683	157.239502
64	164.629471	157.237442
65	164.627502	157.235626
66	164.625778	157.234024
67	164.624268	157.232590
68	164.622894	157.231339
69	164.621689	157.230225
70	164.620636	157.229279
71	164.619736	157.228409
72	164.618896	157.227646
73	164.618179	157.226990
74	164.617538	157.226379
75	164.616974	157.225861
76	164.616486	157.225418
77	164.616058	157.225021
78	164.615662	157.224655
79	164.615341	157.224350
80	164.615036	157.224060

81	164.614746	157.223816
82	164.614517	157.223587
83	164.614304	157.223389
84	164.614120	157.223236
85	164.613968	157.223068
86	164.613815	157.222931
87	164.613693	157.222839
88	164.613571	157.222717
89	164.613495	157.222641
90	164.613403	157.222549
91	164.613312	157.222488
92	164.613251	157.222412
93	164.613205	157.222382
94	164.613159	157.222321
95	164.613113	157.222275
96	164.613037	157.222229
97	164.613007	157.222214
98	164.612976	157.222168
99	164.612946	157.222153
100	164.612930	157.222137
(! ITER	T(4,5)	T(5,3))

The 1st three digits after decimal unchanged during 5 iterations!

! LAST = 100

```
LPRINT(4)=.TRUE.
TITLE(4)=' TEMP '
```

```
IF(ITER==LAST) CALL PRINT
RETURN
```

Node numbers: 7 * 7

Temperature field

**************************** TEMP ****************************

I =	1	2	3	4	5	6	7
J							
7	2.00E+02	1.75E+02	1.70E+02	1.64E+02	1.48E+02	1.25E+02	2.00E+02
6	1.90E+02	1.75E+02	1.70E+02	1.64E+02	1.48E+02	1.25E+02	1.12E+02
5	1.70E+02	1.69E+02	1.69E+02	1.65E+02	1.49E+02	1.26E+02	1.13E+02
4	1.50E+02	1.60E+02	1.68E+02	1.66E+02	1.52E+02	1.28E+02	1.14E+02
3	1.30E+02	1.52E+02	1.68E+02	1.70E+02	1.57E+02	1.33E+02	1.16E+02
2	1.10E+02	1.49E+02	1.72E+02	1.75E+02	1.63E+02	1.39E+02	1.19E+02
1	1.00E+02	1.54E+02	1.77E+02	1.80E+02	1.68E+02	1.44E+02	2.00E+02

```
TITLE(13)=' COND '
LPRINT(13)=.TRUE.
```

```
IF(ITER==LAST) CALL PRINT
RETURN
```

Node numbers: 7 * 7

Thermal conductivity

*************************** COND ***************************

I =	1	2	3	4	5	6	7
J							
7	1.00E+00	1.00E+00	1.00E+00	5.28E-01	4.95E-01	1.00E+00	1.00E+00
6	1.00E+00	1.00E+00	1.00E+00	5.28E-01	4.95E-01	1.00E+00	1.00E+00
5	1.00E+00	1.00E+00	1.00E+00	5.29E-01	4.98E-01	1.00E+00	1.00E+00
4	1.00E+00	1.00E+00	1.00E+00	5.33E-01	5.05E-01	1.00E+00	1.00E+00
3	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00
2	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00
1	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00

COMPUTATION FOR AXISYMMETRICAL SIMULATION
****

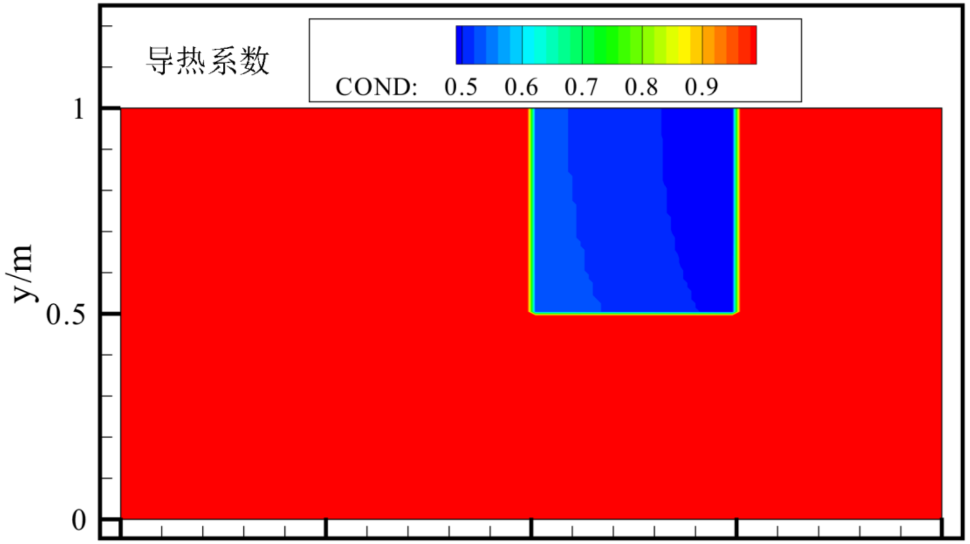
METHOD =2

ITER	T(4,5)	T(5,3)
0	200.000000	200.000000
1	163.633240	156.107574
2	164.603409	157.204285
3	164.612839	157.222092
4	164.612747	157.221954
5	164.612747	157.221954
6	164.612747	157.221954
7	164.612747	157.221970
8	164.612747	157.221954
9	164.612747	157.221970
10	164.612747	157.221954
11	164.612747	157.221970
12	164.612747	157.221954

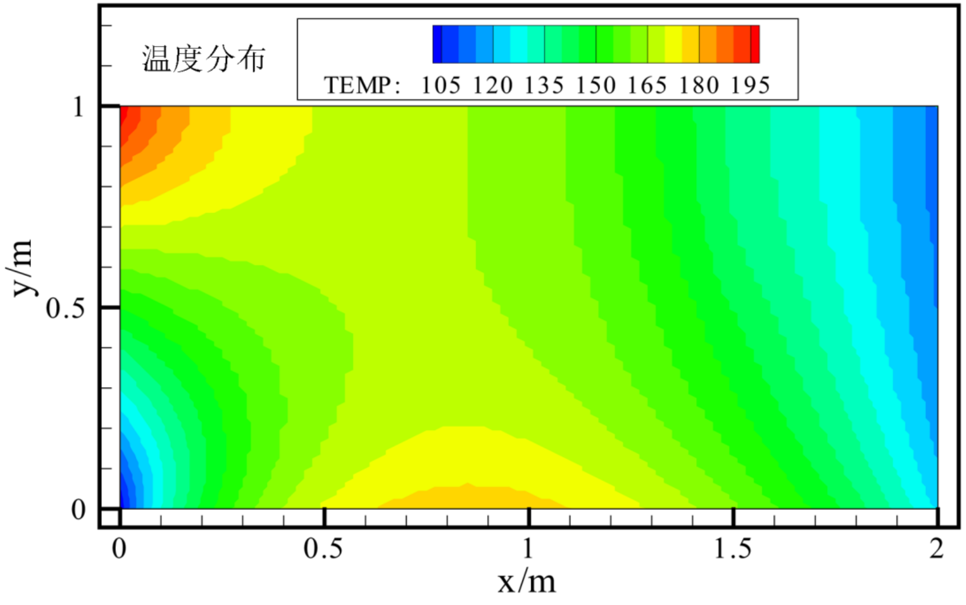
！ For ASTM

In order to keep the 1st three digits after decimal unchanged during 5 iterations, Method 1 needs 90 iterations, while Method 2 only needs 8 iterations! Convergence speed of Method 2 is 10 times of Method 1!

ASTM is recommended.



Thermal conductivity field



Temperature field

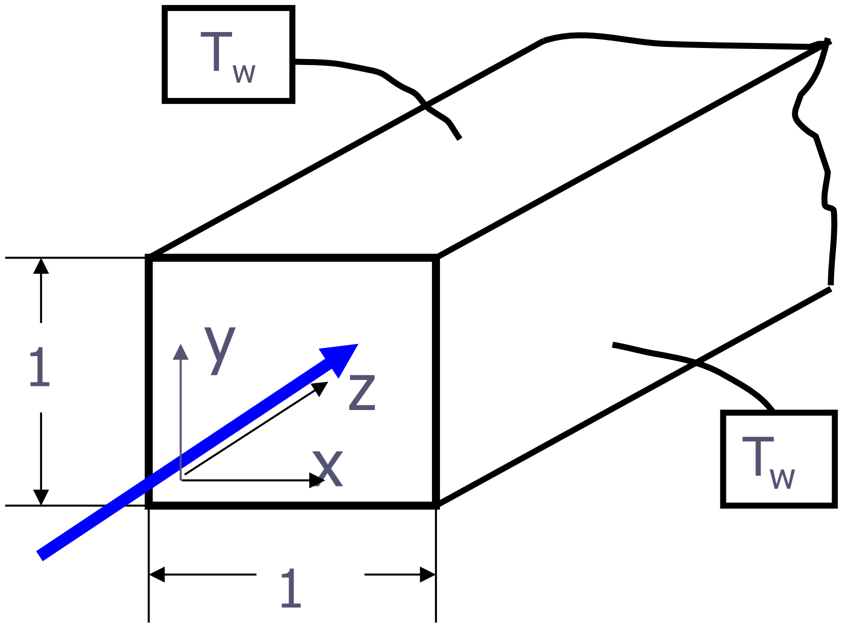
Fig.2 Computational results

11-3 Example 3 Fully-developed heat transfer in a square duct －Numerical techniques for FDHT

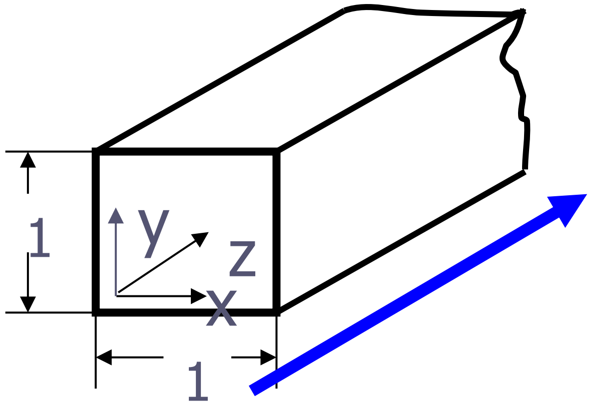
11-3-1 Physical problem and its math formulation

Known： Fully developed laminar heat transfer of fluid with constant properties (Fig. 1) in a square duct. The wall temperatures are uniform.

Find： Velocity and temperature distribution in cross section and fRe and Nu.



Solution: For fully developed laminar flow in a straight duct, $u = 0$, $v = 0$, and $\partial w/\partial z = 0$



$$\rho(\cancelto{0}{u\frac{\partial w}{\partial x}} + \cancelto{0}{v\frac{\partial w}{\partial y}} + w\cancelto{0}{\frac{\partial w}{\partial z}}) = -\frac{\partial p}{\partial z} + \eta(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \cancelto{0}{\frac{\partial^2 w}{\partial z^2}})$$

$$\eta(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}) - \frac{\partial p}{\partial z} = 0 \quad \xrightarrow{\boxed{\text{Neglecting cross section variation of } p}} \quad \eta(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}) - \frac{dp}{dz} = 0$$

GE: $$\eta(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}) - \frac{dp}{dz} = 0$$ Axial velocity w

GGE: $$\frac{\partial(\rho^*\Phi)}{\partial t} + div(\rho^*\vec{u}\Phi) = div(\Gamma_\Phi grad\Phi) + S_\phi^*$$

Compared with standard form, w-eq. is of conduction type.

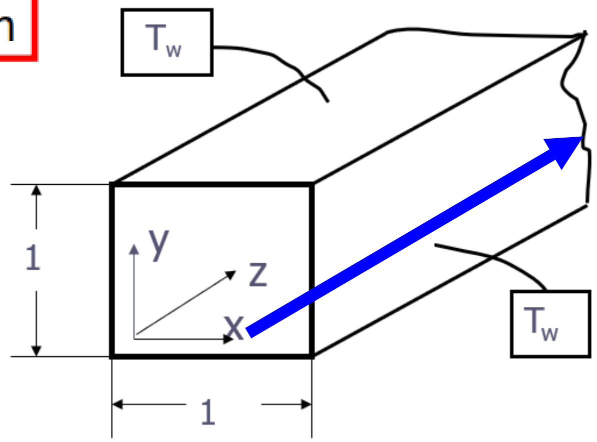
Thus, $\Gamma_\phi = \eta$ $S_C = -dp/dz$

Governing equation for fluid temperature:

$$\rho c_p\left(\cancelto{0}{u\frac{\partial T}{\partial x}}+\cancelto{0}{v\frac{\partial T}{\partial y}}+w\frac{\partial T}{\partial z}\right)=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\cancelto{0}{\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)}$$

Neglecting axial heat conduction

Thus: $$\rho c_p w\frac{\partial T}{\partial z}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)$$



- In summary, the total Governing Eqs.:

$$\eta\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\frac{dp}{dz}=0$$

B.C.: no slip at walls

$$\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)=\rho c_p w\frac{\partial T}{\partial z}$$

B.C.: T_w at walls

11-3-2 Numerical methods

(1) Dimensionless temperature

Define dimensionless temperature $\Theta = \dfrac{T - T_w}{T_b - T_w}$

T_b: average bulk temperature

Then: $T = \Theta(T_b - T_w) + T_w, \quad \dfrac{\partial \Theta}{\partial z} = 0$

$$\Rightarrow \frac{\partial T}{\partial z} = \Theta \frac{dT_b}{dz}$$

Energy eq. is transformed into following conduction equation with source term:

$$\frac{\partial}{\partial x}(\lambda \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(\lambda \frac{\partial T}{\partial y}) - \rho c_p w \frac{\partial T}{\partial z} = 0 \Rightarrow \frac{\partial}{\partial x}(\lambda \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(\lambda \frac{\partial T}{\partial y}) \underline{- \rho c_p w \Theta \frac{dT_b}{dz}} = 0$$

Compared with the standard form: $\Gamma_\phi = \lambda$ $\quad S_C = -\rho c_p w \Theta \dfrac{dT_b}{dz}$

(2) Numerical methods

1.This flow problem is governed by two conduction-type equations with source term;

$$\eta\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\frac{dp}{dz}=0 \qquad \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)-\rho c_p w\Theta\frac{dT_b}{dz}=0$$

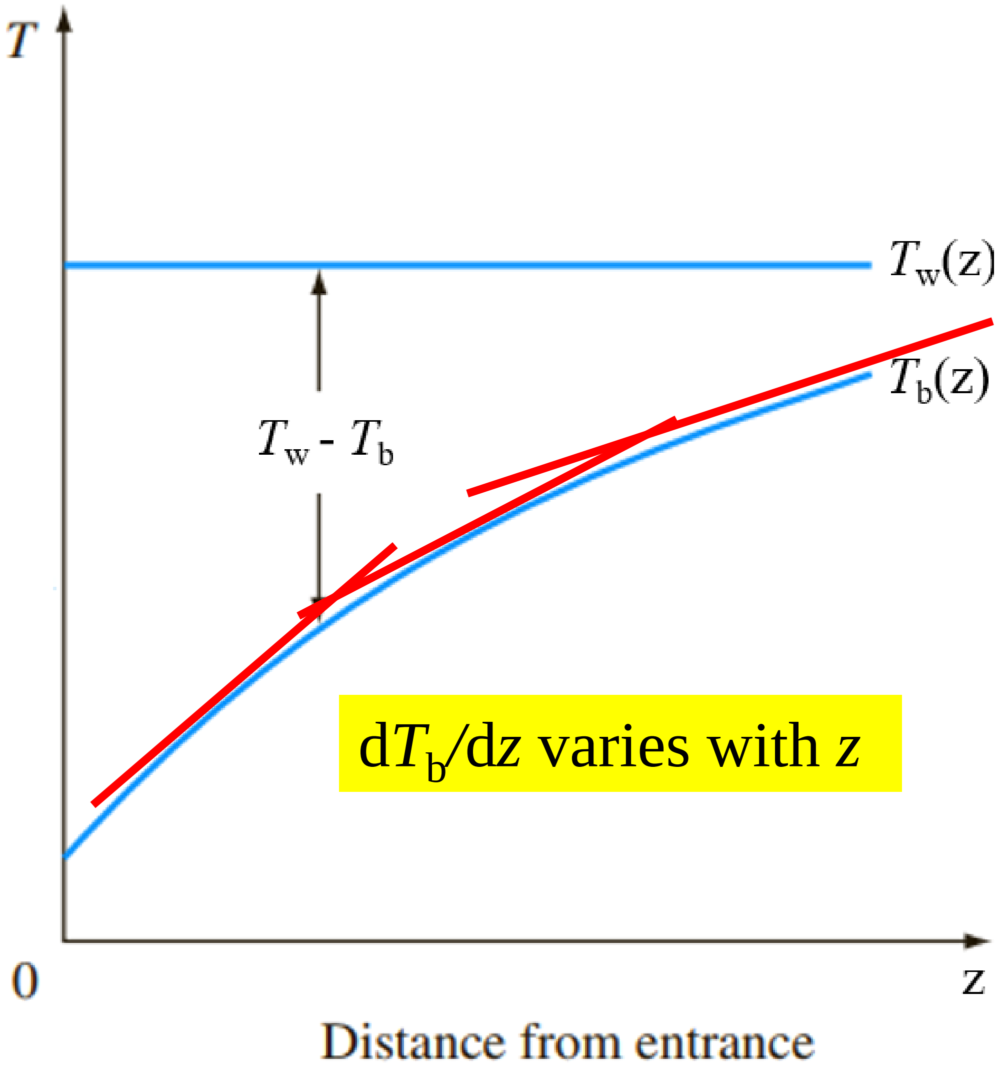
2.The two equations are partially coupled: Velocity *w* is in the source term of temperature. However, temperature is not included in *w*-equation. Thus *w*-eq. should be solved first;

3.For uniform wall temperature, dT_b/dz does not equal constant and an assumed value can be used for simulation. During iteration, the dimensionless temperature Θ (which is included in the source term of temperature) should be updated.

$$S_C=-\rho c_p w\Theta\frac{dT_b}{dz}$$

During simulation, the value of dT_b/dz is assumed and Θ is updated iteratively.

$$S_C = -\rho c_p w \Theta \frac{dT_b}{dz}$$



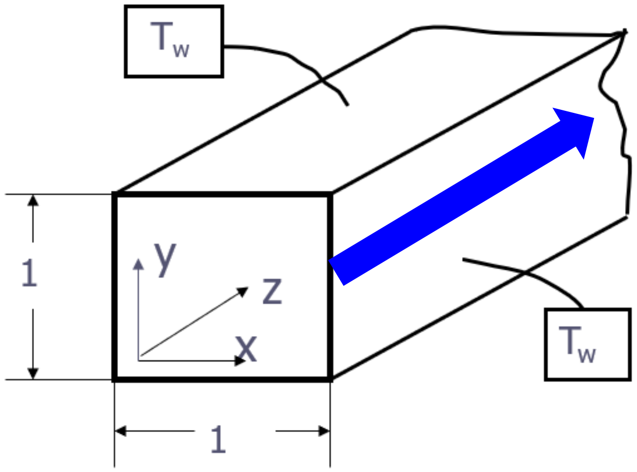


Fig. 2 Streamwise variation of fluid temperature at uniform wall temperature condition

11-3-3 Program reading

MODULE USER_L

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C*****************************************************************
      INTEGER*4 I,J
      REAL*8 AMU, DEN, RHOCP, DPDZ, DTBDZ, ASUM, TSUM, AR,
     1 WR, WBAR, TB, DH, RE, FRE, ANU, TW, QW, THETA, DTDZ
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
     SUBROUTINE USER
C******************************************************************
     USE START_L
     USE USER_L
     IMPLICIT NONE
C*******************************************************************
C------------------------------   PROBLEM THREE----------------------------------------
      Fully developed laminar fluid flow and heat transfer in a square duct
C-------------------------------------------------------------------------------------------------
C*******************************************************************
```

ENTRY GRID

TITLE(4)=‘ .THETA. ' ! Title of dimensionless temperature for output

TITLE(5)='. W/WBAR. ' ! Title of dimensionless velocity for output

LSOLVE(5)=.TRUE. ! *w* (5th variable) solved first, temperature is not solved temporarily

LPRINT(4)=.TRUE.

LPRINT(5)=.TRUE.

LAST=22

XL=0.5 ! Symmetry, only 1/4 domain needs to be solved

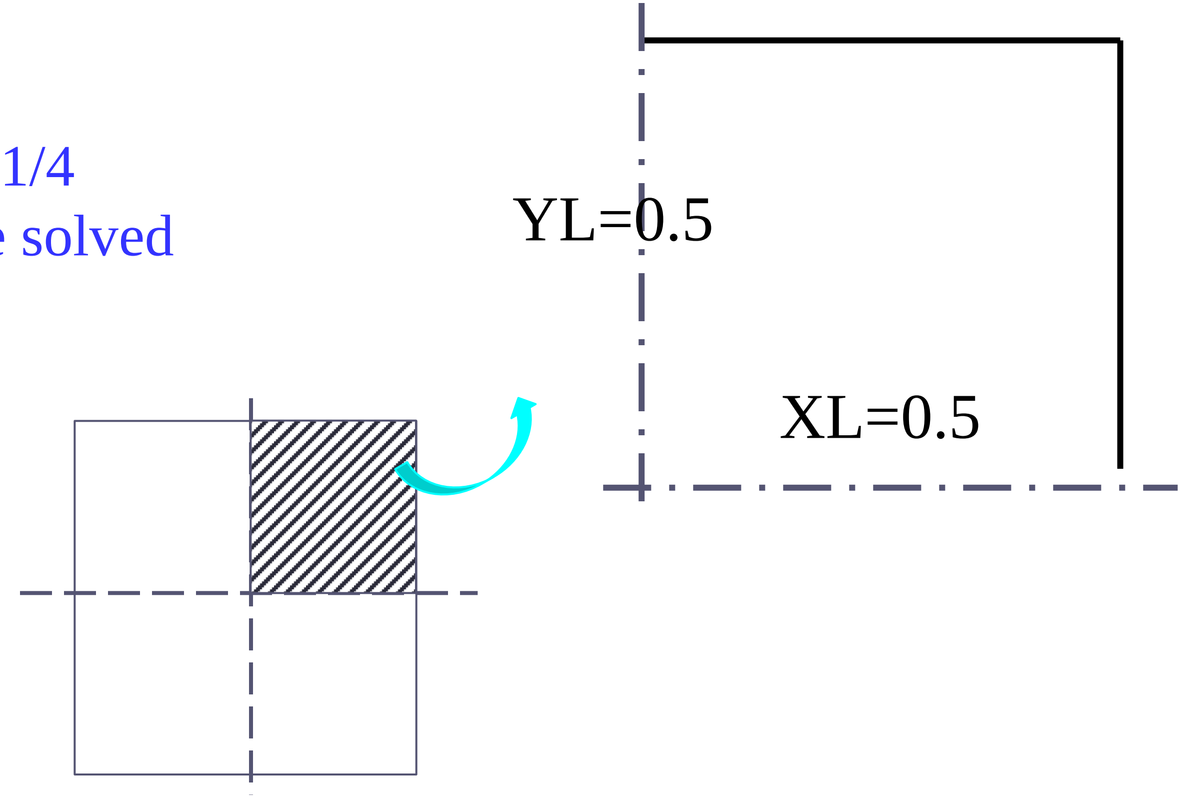
YL=0.5

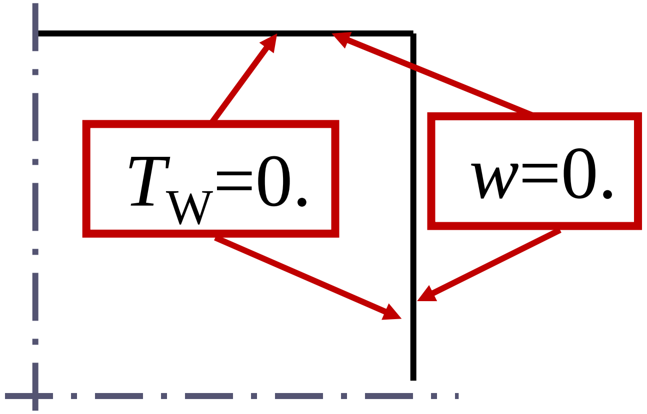
L1=7

M1=7

CALL UGRID

RETURN






```
ENTRY START
  TW=0.                 ! Wall temperature
  DO 100 J=1,M1
  DO 100 I=1,L1
  W(I,J)=0.             ! Set up initial fields, and w=0 at walls
  T(I,J)=1.
  T(I,M1)=TW            ! Set up wall temp. for right and top walls
  T(L1,J)=TW
100 CONTINUE
  AMU=1.
  DEN=1.                ! Set up properties; η = 1 (very large), to
  COND=1.               ensure laminar flow.
  CP=1.                 ! This is not a true flow problem, and there is no convection.
  RHOCP=DEN*CP          RHOCP here is for the source term in conduction equation.
  DPDZ=-100.            ! This value must be less than zero
  DTBDZ=5.              ! Fluid is heated. The value is
  RETURN                arbitrarily assumed
```

$$\frac{\partial(\rho^*\Phi)}{\partial t} + div(\rho^*\vec{u}\Phi) = div(\Gamma_\Phi grad\Phi) + S_\phi^*$$

```
ENTRY DENSE
RETURN
```
! Empty, but keep it

$$T_b = \frac{\iint w(i,j)T(i,j)dA_{i,j}}{\iint w(i,j)dA_{i,j}}$$

```
ENTRY BOUND
ASUM=0.
WSUM=0.
TSUM=0.
```
! Initial values for summation

$$w_m = \frac{\iint w(i,j)dA_{i,j}}{\iint dA_{i,j}}$$

```
DO 300 J=2,M2
DO 301 I=2,L2
AR=XCV(I)*YCV(J)
WR=W(I,J)*AR
WSUM=WSUM+WR
ASUM=ASUM+AR
TSUM=TSUM+WR*T(I,J)
301 ENDDO
300 ENDDO
```

Element area $A_{i,j}$ (AR)

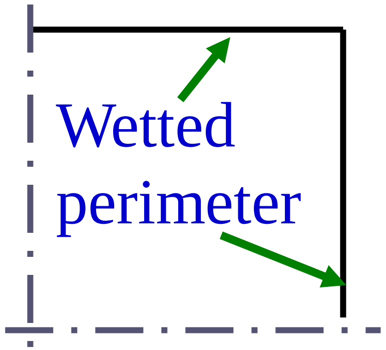
$\iint w(i,j)dA_{i,j}$ (WSUM)

$\iint dA_i$ (ASUM)

$\iint w(i,j)T(i,j)dA_{i,j}$ (TSUM)

$$w_m = \frac{\iint w(i,j)dA_{i,j}}{\iint dA_{i,j}}$$

$$T_b = \frac{\iint w(i,j)T(i,j)dA_{i,j}}{\iint w(i,j)dA_{i,j}}$$




```
WBAR=WSUM/ASUM
TB=TSUM/(WSUM+1.E-30)
DH=4.*XL*YL/(XL+YL)
RE=DEN*WBAR*DH/AMU
FRE=-2.*DPDZ*DH/(DEN*WBAR**2+1.E-30)*RE
QW=DTBDZ*RHOCP*WSUM/(XL+YL)
ANU=QW*DH/(COND*(TW-TB)+1.E-30)
```

! To avoid overflow, a small value is added.

$$f\,\mathrm{Re} = -\frac{(dp/dz)D_h}{\frac{1}{2}\rho w_m^2}\mathrm{Re}$$

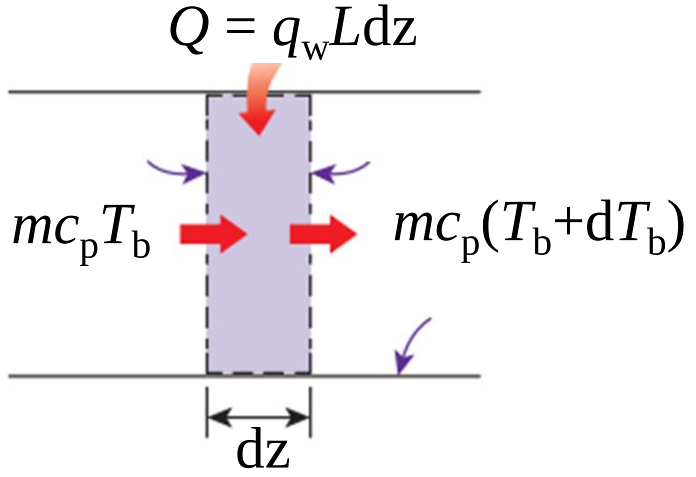
$$q_w = \frac{dT_b}{dz}\rho c_p \sum (w_{i,j}A_{i,j})\frac{1}{XL+YL}$$

$$Nu = \frac{hD_h}{\lambda} = \frac{D_h}{\lambda}\frac{q_w}{(T_w - T_b)}$$

Energy balance:

$$Q = mc_p dT_b/dz = q_w L$$

$$m = \sum (\rho w_{i,j}A_{i,j})$$



```
      IF(ITER>10)   LSOLVE(5)=.FALSE.
      LSOLVE(4)=.TRUE.
      CONTINUE
      RETURN
*
      ENTRY OUTPUT
      IF(ITER==0) THEN
      PRINT 401
      WRITE(8,401)
401 FORMAT(1X,' ITER',12X,'F.RE',17X,'NU')
      ELSE
      PRINT 402, ITER,FRE,ANU
      WRITE(8,402) ITER,FRE,ANU
402 FORMAT(1X,I6,1P2E20.4)
       ENDIF
      IF(ITER/=LAST) RETURN
      DO 410 J=1,M1
      DO 411 I=1,L1
      W(I,J)=W(I,J)/WBAR
      T(I,J)=(T(I,J)-TW)/(TB-TW)
411 ENDDO
410 ENDDO
      CALL PRINT
      RETURN
```

! Switch of solved variable; solve T when ITER >=10

1P2E20.4, Scientific expression of data

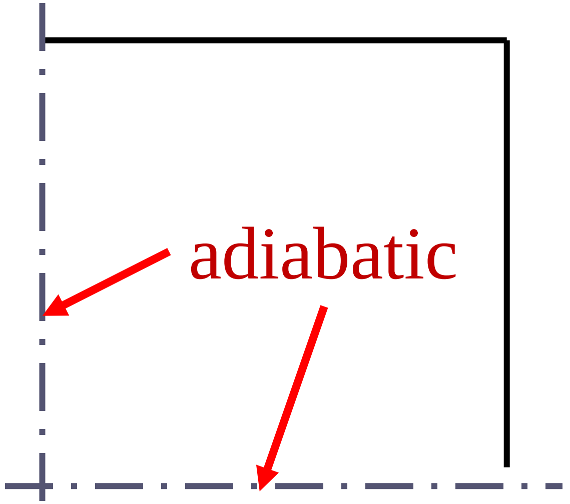
! Dimensionless to make the result more general

$$\bar{w} = \frac{w}{w_m}$$

$$\Theta = \frac{T - T_w}{T_b - T_w}$$

Bottom, Left

```
      ENTRY GAMSOR
      DO 500 I=1,L1
      DO 500 J=1,M1
      GAM(I,J)=AMU                          ! Γ for velocity w
      IF(NF== 4) GAM(I,J)=COND              ! Γ for temperature T
      GAM(I,1)=0.
      GAM(1,J)=0.                           ! Symmetric=adiabatic for both w and T.
500 CONTINUE
      IF(NF== 4) GOTO 511
      DO 510 J=2,M2
      DO 510 I=2,L2
      CONTON(I,J)=-DPDZ                     ! Source term of w
 510 CONTINUE
      RETURN
 511 DO 520 J=2,M2
      DO 520 I=2,L2
      THEAT=(T(I,J)-TW)/(TB-TW+1.E-30)      ! Updating Θ
      DTDZ=THEAT*DTBDZ
520 CON(I,J)=-RHOCP*W(I,J)*DTDZ             ! Source term S_c of temp.
      RETURN
      END
```



$$\boxed{S_C = -\rho c_p w \Theta \frac{dT_b}{dz}}$$

$$\frac{\partial T}{\partial z} = \Theta \frac{dT_b}{dz}$$

11-3-4 Results analysis

COMPUTATION IN CARTESIAN COORDINATES
****

ITER	F.RE	NU
0	0.0000E+00	0.0000E+00
1	6.5168E+01	-3.8363E+00
2	5.6545E+01	-4.4212E+00
3	5.5151E+01	-4.5330E+00
4	5.4891E+01	-4.5545E+00
5	5.4841E+01	-4.5587E+00
6	5.4831E+01	-4.5595E+00
7	5.4829E+01	-4.5596E+00
8	5.4829E+01	-4.5596E+00
9	5.4829E+01	-4.5596E+00
10	5.4829E+01	-4.5596E+00
11	5.4829E+01	4.5875E+00
12	5.4829E+01	3.3408E+00
13	5.4829E+01	3.0894E+00
14	5.4829E+01	3.0361E+00
15	5.4829E+01	3.0257E+00

Energy eq. has not been solved. The values are meaningless

Switch of solved variable

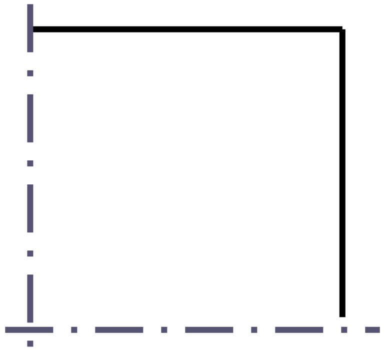
```
IF(ITER>10) LSOLVE(5)=.FALSE.
LSOLVE(4)=.TRUE.
```

16	5.4829E+01	3.0240E+00
17	5.4829E+01	3.0238E+00
18	5.4829E+01	3.0237E+00
19	5.4829E+01	3.0237E+00
20	5.4829E+01	3.0238E+00
21	5.4829E+01	3.0238E+00
22	5.4829E+01	3.0238E+00

Four digits after decimal remain unchanged in successive 6 iterations

*************************** .W/WBAR. ***************************

I =	1	2	3	4	5	6	7
J							
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
6	0.00E+00	4.58E-01	4.34E-01	3.83E-01	2.95E-01	1.44E-01	0.00E+00
5	0.00E+00	1.12E+00	1.06E+00	9.12E-01	6.72E-01	2.95E-01	0.00E+00
4	0.00E+00	1.58E+00	1.48E+00	1.26E+00	9.12E-01	3.83E-01	0.00E+00
3	0.00E+00	1.87E+00	1.74E+00	1.48E+00	1.06E+00	4.34E-01	0.00E+00
2	0.00E+00	2.00E+00	1.87E+00	1.58E+00	1.12E+00	4.58E-01	0.00E+00
1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00



No decoration before output (未作修饰)

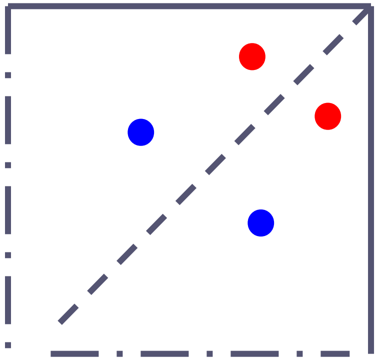
(initial values)

*************************** .THETA. ***************************

I =	1	2	3	4	5	6	7
J							
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
6	-6.63E-01	2.41E-01	2.14E-01	1.65E-01	1.02E-01	3.41E-02	0.00E+00
5	-6.63E-01	7.38E-01	6.53E-01	5.00E-01	3.07E-01	1.02E-01	0.00E+00
4	-6.63E-01	1.22E+00	1.08E+00	8.19E-01	5.00E-01	1.65E-01	0.00E+00
3	-6.63E-01	1.61E+00	1.42E+00	1.08E+00	6.53E-01	2.14E-01	0.00E+00
2	-6.63E-01	1.84E+00	1.61E+00	1.22E+00	7.38E-01	2.41E-01	0.00E+00
1	-6.63E-01	-6.63E-01	-6.63E-01	-6.63E-01	-6.63E-01	-6.63E-01	0.00E+00

No decoration(未作修饰) (initial values)

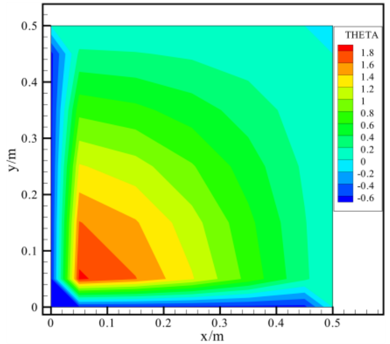
Decoration: before output, set:
THETA (1， j) = THETA(2,j)
THETA (i,1) = THETA(i,2)



Symmetry about diagonal

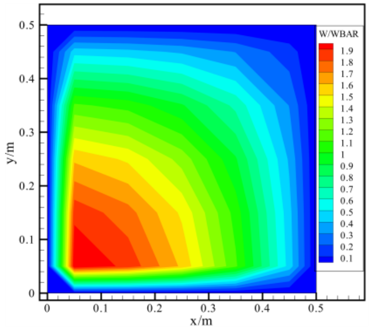
No decoration

Θ

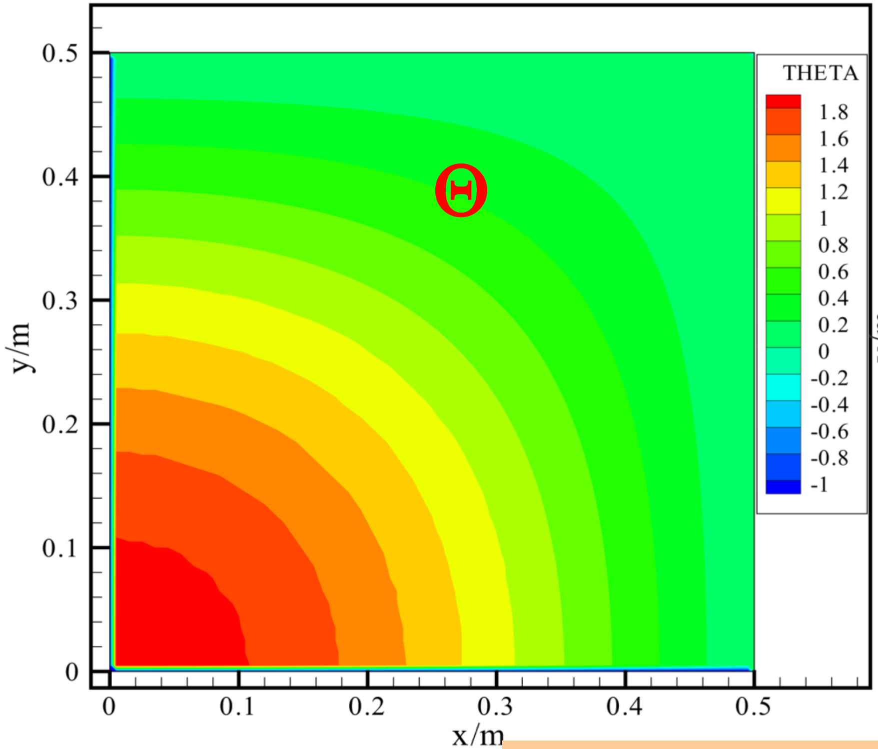


No decoration

W/WBAR



With decoration



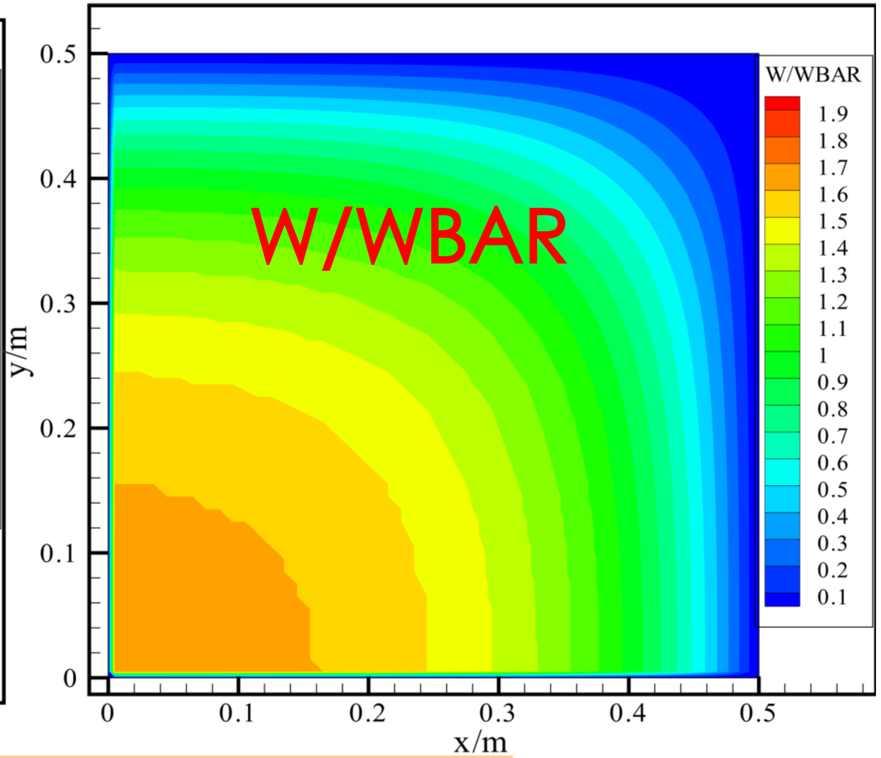


Fig. 3 Results of Problem 3

Do the assumed values of d*p*/dz= -100, dT_b/dz=5 affect *fRe* and *Nu*?

GE: $$\eta\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\frac{dp}{dz}=0 \qquad \lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-\rho c_p w\Theta\frac{dT_b}{dz}=0$$

Introducing characteristic length X_L, characteristic velocity w_m, the above Eqs. can be dimensionless

$$\frac{\partial^2 \overline{w}}{\partial \overline{x}^2}+\frac{\partial^2 \overline{w}}{\partial \overline{y}^2}+\frac{1}{8}\left(1+\frac{X_L}{Y_L}\right)^2 f\,\mathrm{Re}=0 \qquad \frac{\partial^2 \Theta}{\partial x^2}+\frac{\partial^2 \Theta}{\partial y^2}+Nu\cdot\overline{w}\cdot\Theta=0$$

where $\overline{w}=w/w_m$ $\quad \Theta=\dfrac{T-T_w}{T_b-T_w}$

Thus, the assumed values of d*p*/dz, dT_b/dz do not affect the calculated *fRe* and *Nu*

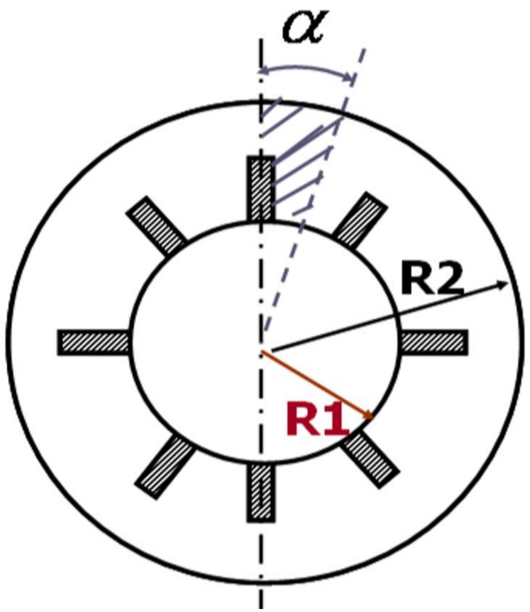
11-4 Fully developed heat transfer in annular space with straight fin at inner wall －Numerical methods for conjugated problems

11-4-1 Physical Problem and its math formulation

Known：Laminar heat transfer with constant properties in annular space with straight fins at inner wall (Fig. 1).

Its outer wall is adiabatic, while inner wall temperature is circumferentially uniform（周向均匀壁温）.

R_1=1, R_2=2, the angle between two successive fins equals 30°. Ratio of fin thermal conductivity over fluid one is ten.



$$\frac{\lambda_{fin}}{\lambda_{fluid}}=10$$

$$\frac{R_2}{R_1}=2$$

$$\alpha=15^{\circ}$$

Fig.1 Cross section view of Problem 4

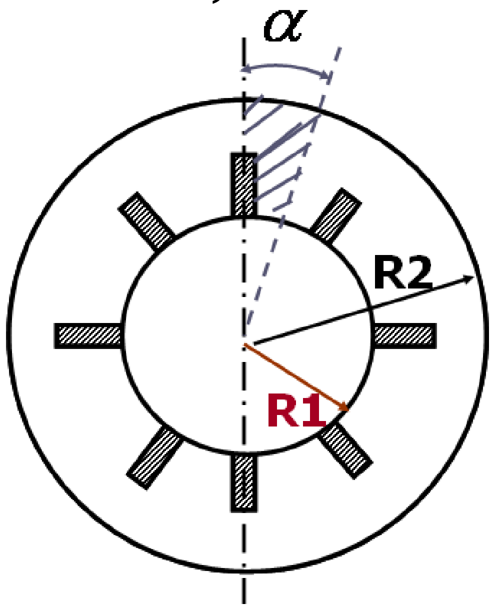
Find： Cross-sectional distributions of velocity and temperature, and *fRe*、*Nu*.

Solution： Similar to problem 3, u=0, v=0, ∂w/∂z = 0, the governing eq. for axial velocity w:

$$\underbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r\eta\frac{\partial w}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\eta}{r}\frac{\partial w}{\partial\theta}\right)}_{div(\eta gradw)}-\frac{dp}{dz}=0$$

Source term

（Polar coordinate)



The governing eq. of temperature in the fully developed region：

$$div(\lambda gradT)-\rho c_p w\frac{\partial T}{\partial z}=0$$

Source term

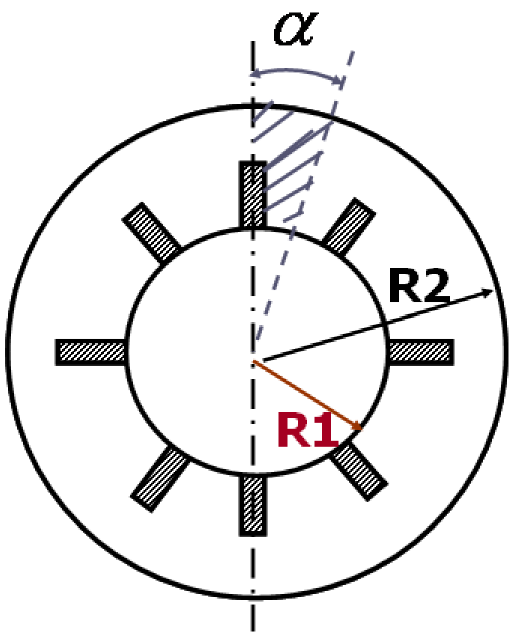
11-4-2 Numerical methods

（1）This problem is governed by two conduction-type equations with source term;

$$div(\eta \mathrm{grad} w) - \frac{dp}{dz} = 0 \qquad div(\lambda \mathrm{grad} T) - \rho c_p w \frac{\partial T}{\partial z} = 0$$

（2）Velocity is not coupled with temperature, and can be solved first;

（3）The fin can be regarded as a special fluid with a very large viscosity; hence the entire flow region can be solved simultaneously---conjugated problem(耦合问题);



（4）The half of the region between two successive fins can be taken as computational domain due to symmetry;

(5) In calculation of cross sectional temperature distribution, it can assume that at the whole section $\partial T/\partial z = C$

(6) It is assumed that the fin surface coincides with radius.

(7) The fin and fluid temperatures are solved at same time (simultaneously) --- conjugated problem (耦合问题)

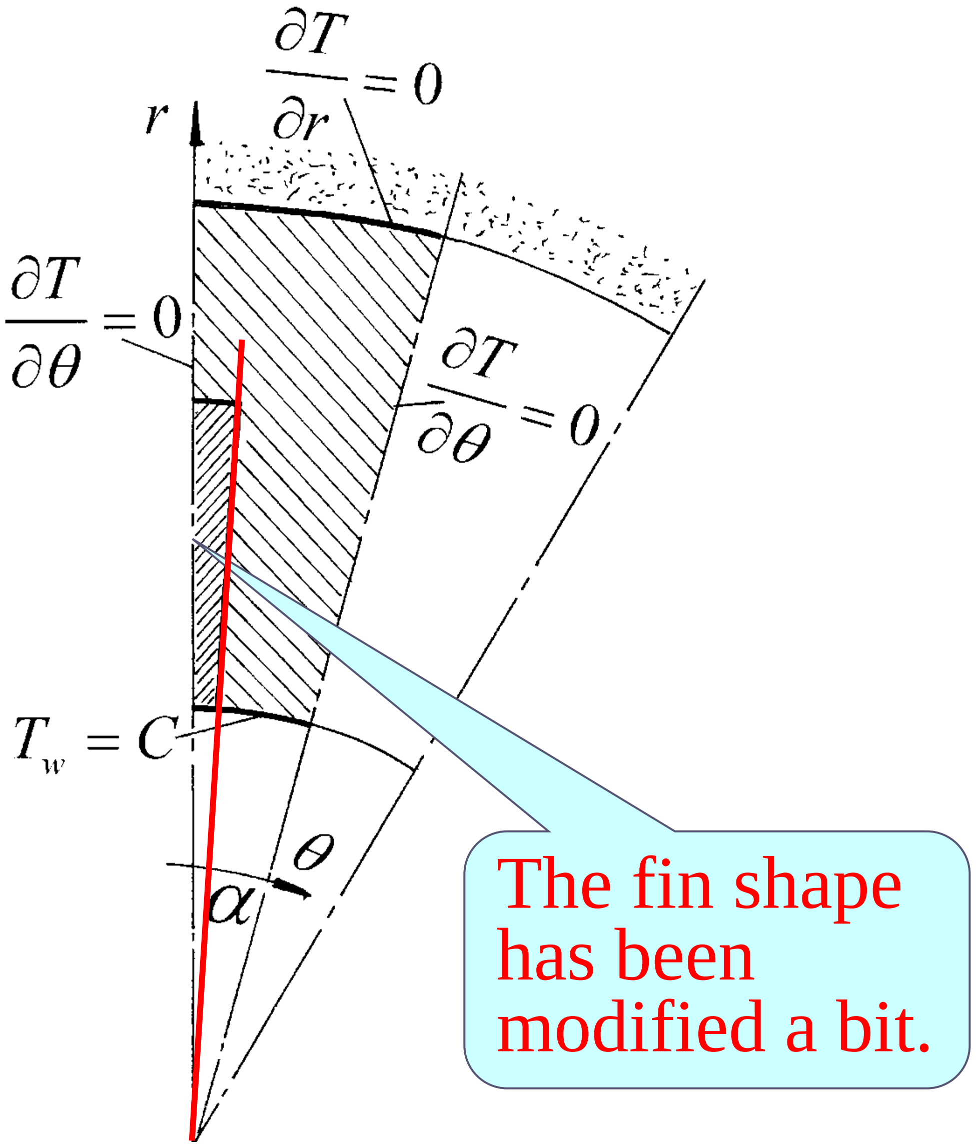
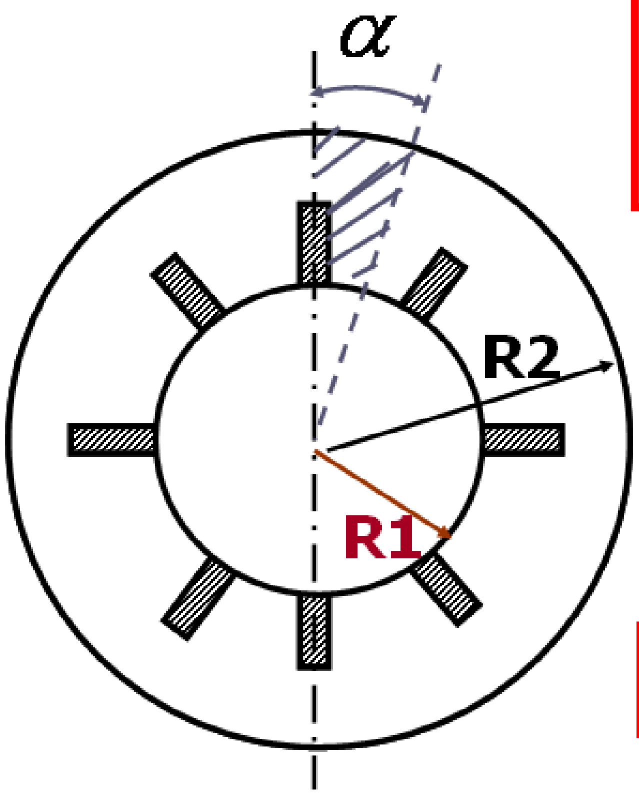


Fig. 2 Computational domain

Fully developed heat transfer in annular space with straight fin at inner wall



$$\frac{\lambda_{fin}}{\lambda_{fluid}} = 10$$

$$\frac{R_2}{R_1} = 2$$

$$\alpha = 15^{\circ}$$

Fig.1 Cross section view of Problem 4

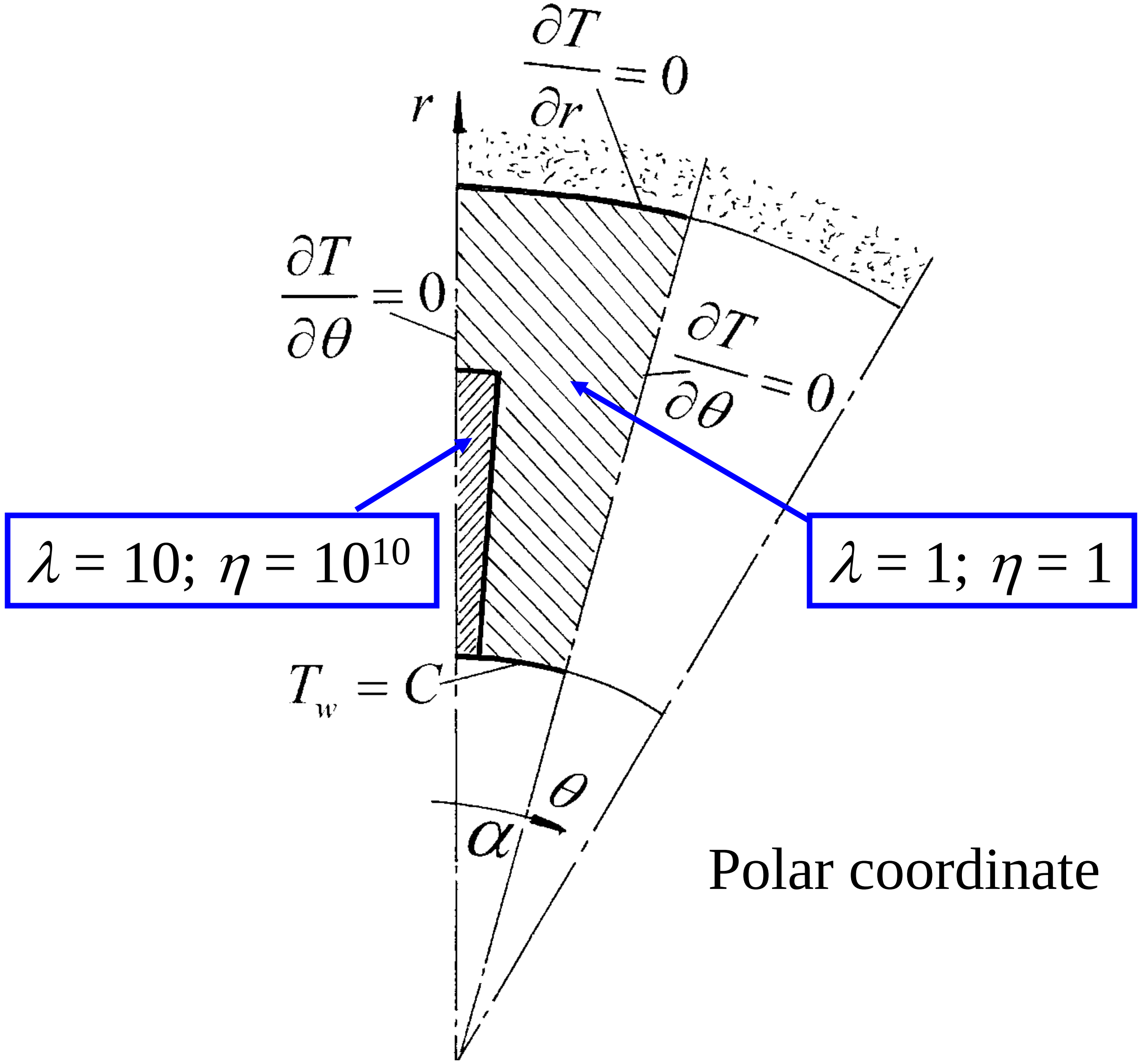


Fig. 2 Computational domain

11-4-3 Program reading

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C*************************************************************
      INTEGER*4 I, J
      REAL*8 PI, TW, AMU, DPDZ, COND, RHOCP, DTDZ, WSUM, ASUM,
    1 TSUM, AR, WBAR, WP, DH, RE, FRE,TBULK, HTP, HTC, ANU
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE USER
C**************************************************************
      USE START_L
      USE USER_L
      IMPLICIT NONE
C***************************************************************
C--------------------------------PROBLEM FOUR------------------------------------
C    Fully developed laminar fluid flow and heat transfer in annular duct with
C---------------------------longitudinal fins on inner tube---------------------------
C***************************************************************
```

ENTRY GRID

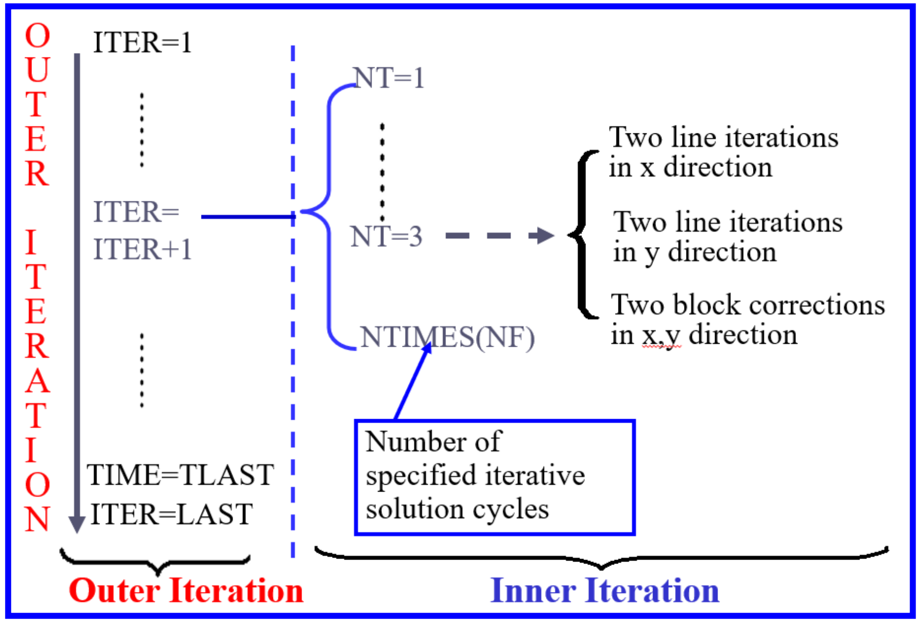
TITLE(4)='.THETA.'　　! 4^{th} variable for temperature

TITLE(5)='.W/WBAR.'　　! 5^{th} variable for velocity

LSOLVE(5)=.TRUE.
LPRINT(4)=.TRUE.
LPRINT(5)=.TRUE.　　! Velocity solved first, temperature next

LAST=6

NTIMES(4)=4
NTIMES(5)=4　　! Both equations are linear, NTIMES may take larger values to decrease outer iteration times.



MODE=3　　! Polar coordinate

R(1)=1.　　! Specify the bottom radius

PI=3.14159

THL=15.*PI/180.　　! Transform from degree to radian (弧度)

YL=1.　　! Equivalence (XL, THL)

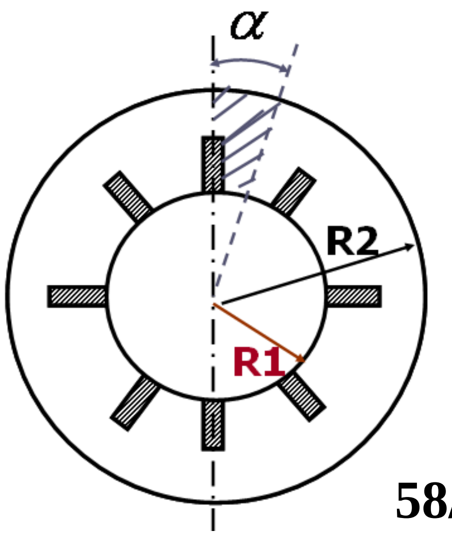
```
 EQUIVALENCE(X,TH),(XU,THU),(XDIF,THDIF),(XCV,THCV),
1(XCVS,THCVS),(XL,THL)
```

L1=7

M1=7

CALL UGRID

RETURN



```
ENTRY START
TW=1.              !  Set up cross sectional wall temperature
DO 100 J=1,M1
DO 101 I=1,L1
F(I,J,4)=TW
F(I,J,5)=0.
101 ENDDO
100 ENDDO
```

! Initial fields of w (=0) and T (= T_w)

! Also specified the unchanged BC for w and T. w=0 at the wall; T= T_w at the bottom wall

```
AMU=1.             !  Very large viscosity to ensure laminar flow
COND=1.
RHOCP=1.
DPDZ=-2000.        !  Pressure gradient should be less than zero
DTDZ=100.          !  Set up axial gradient of fluid temperature
RETURN
```

$$\frac{\partial(\rho^{*}\Phi)}{\partial t}+div(\rho^{*}\vec{u}\Phi)=div(\Gamma_{\Phi}grad\Phi)+S_{\phi}^{*}$$

! This is not a true flow problem, and there is no convection. RHOCP here is for the source term in conduction equation.

$$S_c=-\rho c_p w\frac{\partial T}{\partial z}$$

```
ENTRY DENSE
RETURN
```

! Empty, but keep it.

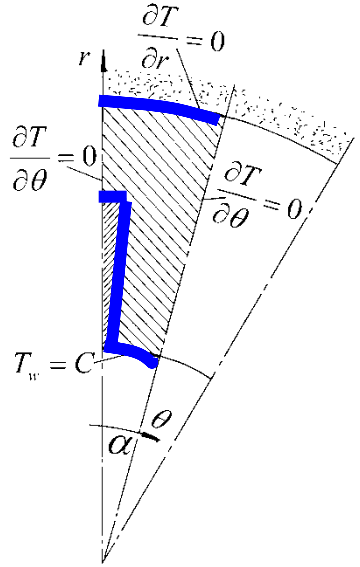
```
ENTRY BOUND
      ASUM=0.
      WSUM=0.      ! Initial values for summation
      TSUM=0.
      DO 300 J=2,M2
      DO 301 I=2,L2
      IF(I>2.OR.I=2 .AND.J>4) THEN
      AR=YCVR(J)*THCV(I)
      WSUM=WSUM+F(I,J,5)*AR
      TSUM=TSUM+AR*F(I,J,4)*F(I,J,5)
      ASUM=ASUM+AR
      ENDIF
301 ENDDO
300 ENDDO
      WBAR=WSUM/ASUM  ! Mean velocity
      WP=(R(1)+R(M1))*THL+(1.+THCV(2))*(RMN(5)-R(1))
      DH=4.*ASUM/WP
      RE=RHOCON*WBAR*DH/AMU
      FRE=-2.*DPDZ*DH/(RHOCON*WBAR**2+1.E-30)*RE
```

$$AR(\text{面积元}) = YCV(j)*R(j)*XCV(i)$$
$$= YCV(j)*R(j)*THCV(i)$$
$$= YCVR(j)*THCV(i)$$

! Exclude(排除)solid region for flow area

$$\sum w(i,j)dA_{i,j}$$
$$\sum w(i,j)T(i,j)dA_{i,j}$$

! Flow area



! Length of wetted perimeter(润湿边界的周长)

$$f\,\mathrm{Re} = \frac{-(dp/dx)D_h}{(1/2)\rho w_m^2}\mathrm{Re}$$

```
TBULK=TSUM/(WSUM+1.E-30)     ! Mean temperature
HTP=WP-R(M1)*THL             ! Length of perimeter for heat transfer
HTC=RHOCP*WSUM*DTDZ/((TW-TBULK+1.E-30)*HTP)
ANU=HTC*DH/COND              ! Nu = hD_e / λ
IF(ITER<3) RETURN
LSOLVE(4)=.TRUE.
LSOLVE(5)=.FALSE.
RETURN
```

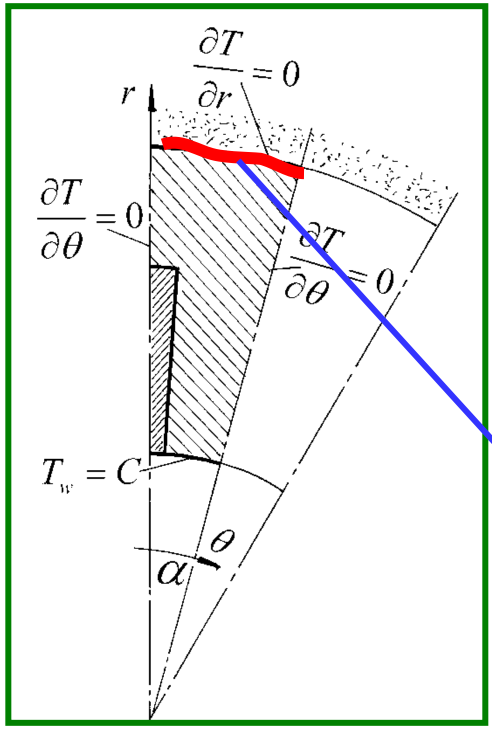
$T_b = \iint w(i,j)T(i,j)dA_{i,j} \Big/ \iint w(i,j)dA_{i,j}$

$Nu = hD_e / \lambda$

Switch solution variable

$$q = \rho c_p (W_m A \frac{\partial T}{\partial z}) \bullet 1/(HTP \bullet 1)$$

$$h = q/(T_w - T_b)$$



! This length is adiabatic, hence should be excluded in HTP.

```
      ENTRY OUTPUT
      IF(ITER==0) THEN                    ! The head of output
      PRINT 401
      WRITE(8,401)
401 FORMAT(1X,' ITER',12X,'F.RE',17X,'NU')
      ELSE
      PRINT 402, ITER, FRE, ANU
      WRITE(8,402) ITER,FRE,ANU
402 FORMAT(1X,I6,1P2E20.4)
      ENDIF
      IF(ITER/=LAST) RETURN
      DO 410 J=1,M1                       ! Output of
      DO 411 I=1,L1                         dimensionless
      F(I,J,5)=F(I,J,5)/WBAR                results
      F(I,J,4)=(F(I,J,4)-TW)/(TBULK-TW+1.E-30)
411 ENDDO
410 ENDDO
      CALL PRINT
      RETURN
```

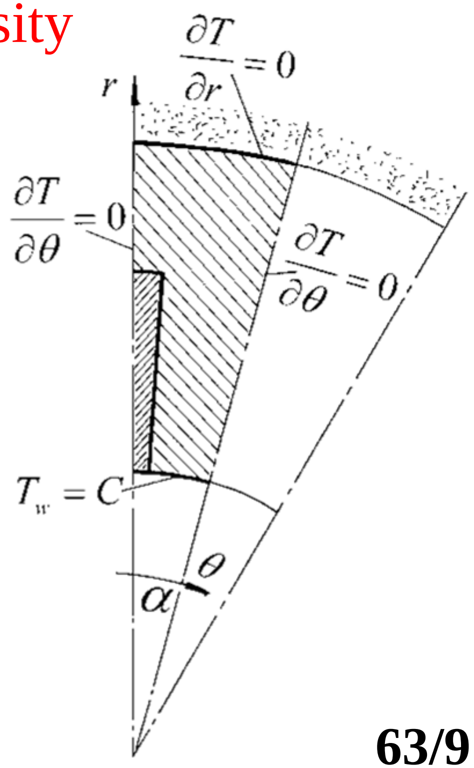
$$\boxed{\Theta = \frac{T - T_w}{T_b - T_w};\ \Theta_w = \frac{T_w - T_w}{T_b - T_w} = 0}$$

```
ENTRY GAMSOR
  DO 500 I=1,L1
  DO 501 J=1,M1
  GAM(I,J)=AMU                       ! Γ for velocity(specified first)
  IF(NF==4) GAM(I,J)=COND            ! Γ for temperature
  GAM(1,J)=0.
  GAM(L1,J)=0.                       ! Symmetry=adiabatic (for both w and T)
  IF(NF==4) GAM(I,M1)=0.    ! North BC: adiabatic for T; w = 0 specified in START.
  IF(J<=4) GAM(2,J)=1.E10   ! Fin is regarded as fluid with large viscosity
  IF(NF==4.AND.J<=4) GAM(2,J)=10.*COND  ! Fin conductivity
501 ENDDO
500 ENDDO
  DO 510 J=2,M2
  DO 511 I=2,L2
  CON(I,J)=-DPDZ      ! Source term of w-eq., should be less than zero
  IF(NF==4) CON(I,J)=-DTDZ*F(I,J,4)*RHOCP
511 ENDDO
510 ENDDO
  RETURN
  END
```

(left、right BCs)

! Source of Temperature eq.

$$-\rho c_p w \frac{dT}{dz}$$



11-4-4 Results analysis

COMPUTATION IN POLAR COORDINATES
****

ITER	F.RE	NU
0	0.0000E+00	0.0000E+00
1	6.5484E+01	1.9787E+10
2	6.5484E+01	2.3588E+33
3	6.5484E+01	2.3588E+33
4	6.5484E+01	1.5098E+00
5	6.5484E+01	1.5098E+00
6	6.5484E+01	1.5098E+00

Solving flow only

! NTIMES=4, only one outer iteration solution is converged

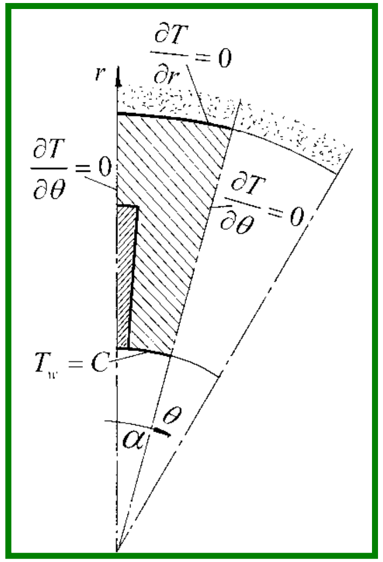
! NTIMES=4, only one outer iteration solution is converged

*************************** .W/WBAR. ***************************

I =	1	2	3	4	5	6	7
J							
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
6	0.00E+00	8.18E-01	8.50E-01	8.91E-01	9.25E-01	9.43E-01	0.00E+00
5	0.00E+00	1.10E+00	1.30E+00	1.50E+00	1.64E+00	1.72E+00	0.00E+00
4	0.00E+00	4.37E-09	4 .57E-01	1.05E+00	1.41E+00	1.58E+00	0.00E+00
3	0.00E+00	3.34E-09	3.01E-01	7.45E-01	1.03E+00	1.18E+00	0.00E+00
2	0.00E+00	1.43E-09	1.63E-01	3.91E-01	5.36E-01	6.06E-01	0.00E+00
1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

w=0 of fin region

Symmetric line, not decorated (initial values).



Symmetric line, not decorated (initial values)

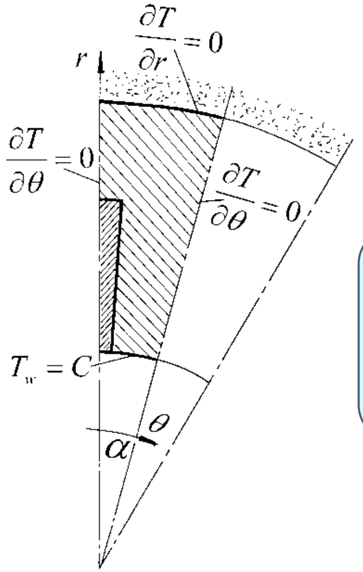
************************** .THETA. **************************

Adiabatic, not decorated

I =	1	2	3	4	5	6	7
J							
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
6	0.00E+00	1.24E+00	1.26E+00	1.28E+00	1.30E+00	1.31E+00	0.00E+00
5	0.00E+00	1.03E+00	1.09E+00	1.15E+00	1.19E+00	1.21E+00	0.00E+00
4	0.00E+00	6.34E-01	7.15E-01	8.24E-01	8.96E-01	9.32E-01	0.00E+00
3	0.00E+00	4.48E-01	4.80E-01	5.36E-01	5.78E-01	6.00E-01	0.00E+00
2	0.00E+00	1.76E-01	1.86E-01	2.04E-01	2 .18E-01	2.26E-01	0.00E+00
1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

$$\Theta_w = \frac{T_w - T_w}{T_b - T_w} = 0$$

Symmetric line, not decorated.



Symmetric line, not decorated

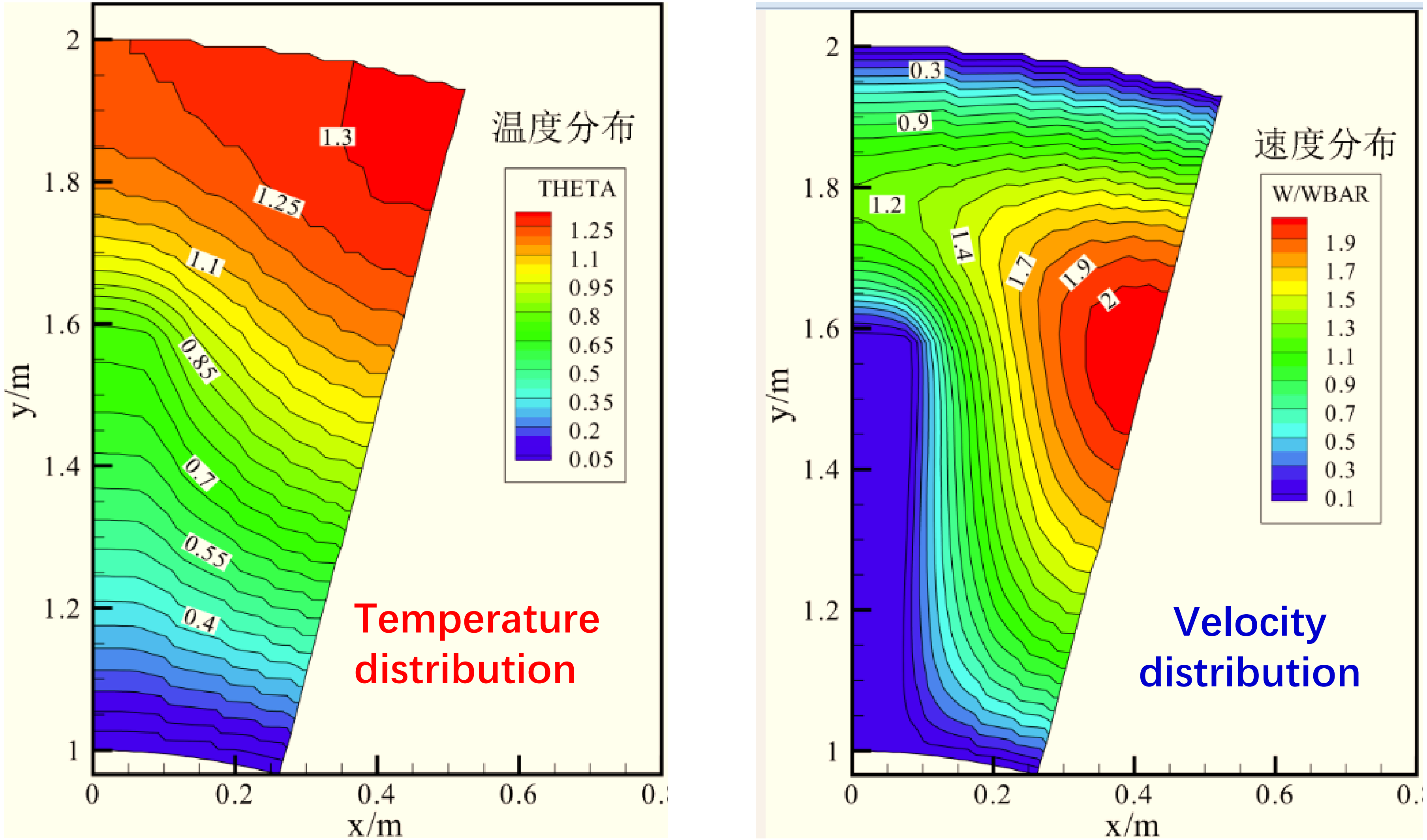


Fig.3 Result of Problem 4

11-5 Fluid flow and heat transfer in a 2-D sudden expansion---Solution of Navier Stokes equation

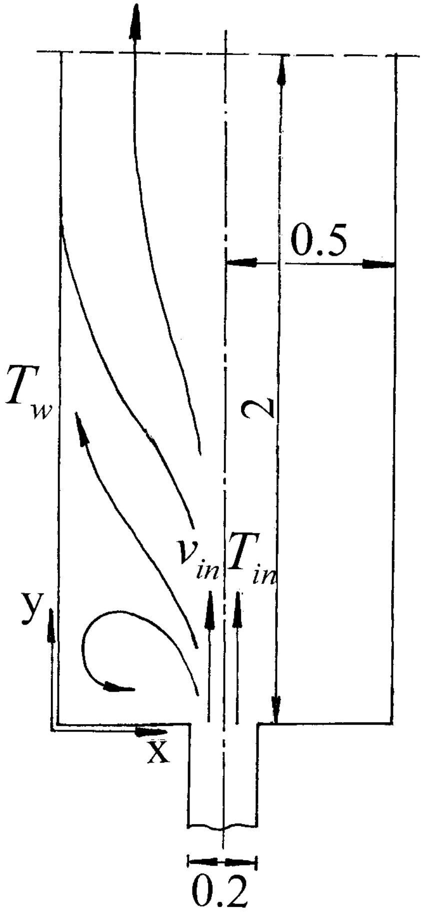
11-5-1 Physical problem and its math formulation

Known： Laminar flow and heat transfer in a parallel duct shown in Fig. 1: Uniform inlet velocity, $\boldsymbol{V_{in}=100}$, and uniform inlet temperature, $\boldsymbol{T_{in}= 500}$; Duct wall are at uniform temperature, $\boldsymbol{T_w= 300}$; Fluid **Pr = 0.7**, molecular dynamic viscosity $\eta = 1$; density varies according to：

$$\rho = \rho_{ref} \frac{T_{ref}}{T}$$

where referenced density $\rho_{ref} = 1$, and $T_{ref} = 300$.

Note that: compressible fluids can undergo incompressible flow, as long as $\nabla \cdot \mathbf{u} = 0$.



Laminar flow and heat transfer in a parallel duct

Find： Distributions of velocity, temperature, density and fluid pressure in the duct.

Solution： Solve the Navier Stokes equation and temperature governing equation

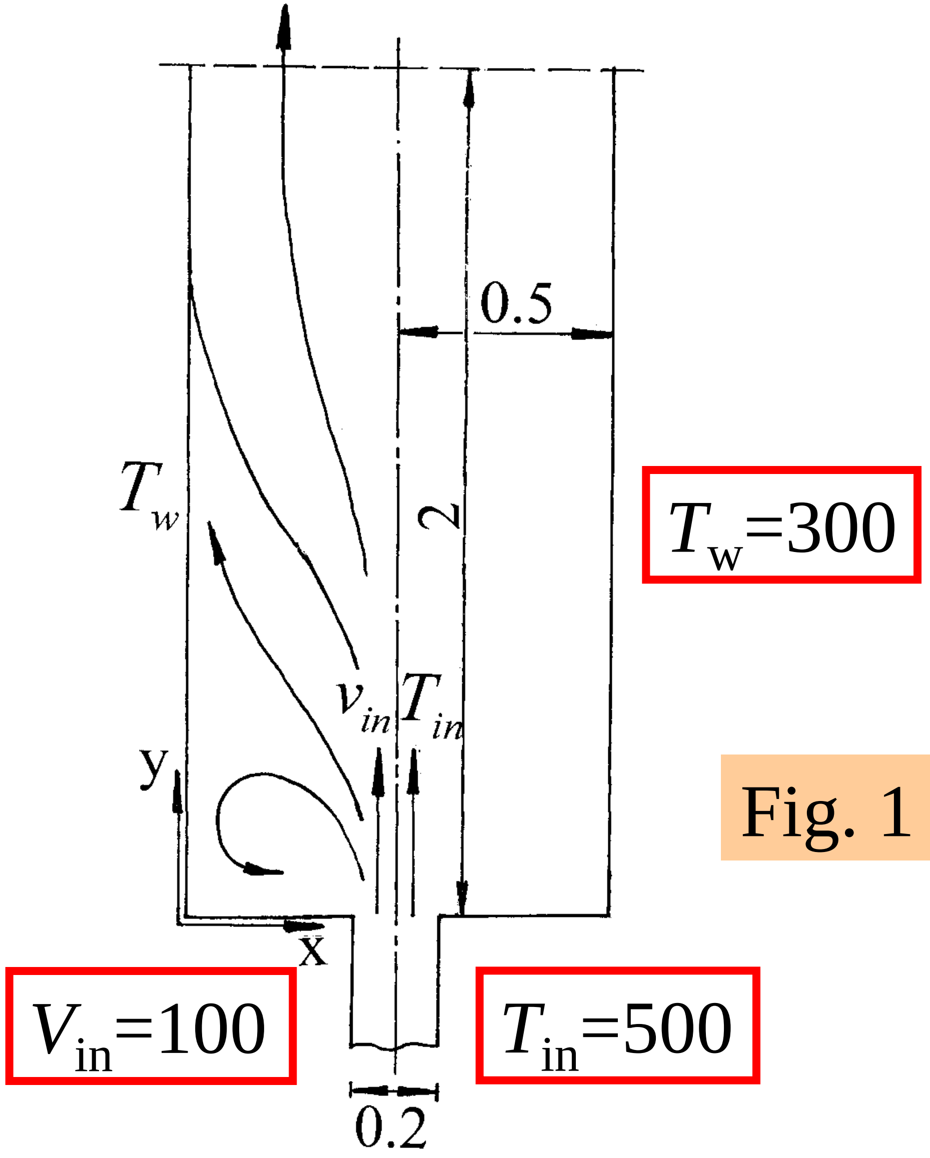


Fig. 1

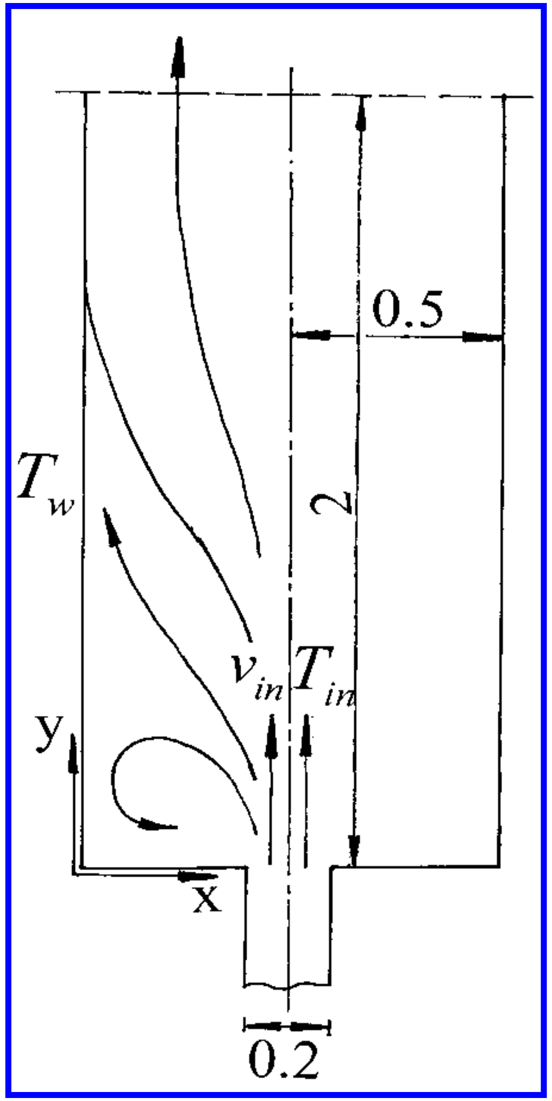
- The governing equations of velocity and temperature:

$$u: \quad div(\rho \vec{u} u) = -\frac{\partial p}{\partial x} + div(\eta\, grad u) + 0$$

$$v: \quad div(\rho \vec{u} v) = -\frac{\partial p}{\partial y} + div(\eta\, grad v) + 0$$

$$T: \quad div(\rho c_p \vec{u} T) = div(\lambda\, grad T) + 0$$

$\frac{\partial p}{\partial x}$ $\frac{\partial p}{\partial y}$ have been treated as source term in **MAIN** program



- Boundary conditions:

At symmetric line: $u = 0; \ \frac{\partial v}{\partial x} = 0; \ \frac{\partial T}{\partial x} = 0$

At inlet: u, v, T are specified;

At solid wall: $u = v = 0; \ T = T_w$

11-5-2 Numerical methods

（1）This is an open-flow system. Determination of normal velocity at the outlet boundary for open flow is important. We set outlet boundary in region without recirculation, and adopt local one-way method with total mass conservation；

（2）Convergence condition for flow field iteration: SSUM and SMAX less than pre-specified values or 4 to 5 digits of printed values remain unchanged during 5 to 10 successive iterations；

（3）Variation of density with temperature is specified in ENTRY DENSE. Momentum equations are coupled with temperature equation.

11-5-3 Program reading

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C*************************************************************
      INTEGER*4 I,J
      REAL*8 TIN, TW, VIN, VOUT, PR, AMU, COND, TREF, RHOREF,
   1   RHOT, FLOWIN, FL, FACTOR
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
     SUBROUTINE USER
C*************************************************************
     USE START_L      !Difference in section number and problem number:
     USE USER_L                 !Section No is five;
     IMPLICIT NONE              !Prob. No. 6 of the original code
C*************************************************************
C------------------------- ----- -PROBLEM SIX---------------------------- -----------
C    Laminar fluid flow and heat transfer in a two-dimensional sudden expansion
C--------------------------------------------------------------------------------------
```

```
ENTRY GRID
TITLE(1)=' .VEL  U.'        '   VEL_U'
TITLE(2)=' .VEL  V.'        '   VEL_V'
TITLE(3)=' .STR FN.'        '  STR_FN'
TITLE(4)=' . TEMP .'        '   TEMP. '
TITLE(11)='PRESSURE'        'PRESSURE'
TITLE(12)=' DENSITY'        ' DENSITY'
LPRINT(1)=.TRUE.
LPRINT(2)=.TRUE.
LPRINT(3)=.TRUE.
LPRINT(4)=.TRUE.
LPRINT(11)=.TRUE.
LPRINT(12)=.TRUE.
LAST= 60
LSOLVE(1)=.TRUE.
LSOLVE(4)=.TRUE.
RELAX(1)=0.8
RELAX(2)=0.8
XL= 0.5
YL= 2.
L1=7
M1=12
CALL UGRID
RETURN
```

Titles for print out

```
MODULE START_L
 PARAMETER(NI=100,NJ=200,NIJ=NI,NFMAX=10,NFX4=NFMAX+4)
REAL*8,DIMENSION(NI,NJ,NFX4)::F
```

NF =	1	2	3	4	……	11	12	13	14
Variable	U	V	p_c	T	……	p	ρ	Γ	C_p

！ In SIMPLER code when the 1st variable is set to be solved, the 2nd , 3rd and 11th ones (v, p_c, p) are automatically to be solved.

！ Underrelaxation of velocity is organized in the solution process.

$$\left(\frac{a_P}{\alpha}\right)\phi_P = \sum a_{nb}\phi_{nb} + b + (1-\alpha)\frac{a_P}{\alpha}\phi_P^0$$

！ half of computation domain

```
      ENTRY START
      TIN=500
      TW=300.
      VIN=100.
      VOUT=VIN*XCV(L2)/X(L1)*TW/TIN        ! Estimation of outlet normal velocity
      DO 100 J=1,M1
      DO 101 I=1,L1
      U(I,J)=0
      V(I,J)=VOUT
      V(I,2)=0
      V(1,J)=0.
      T(I,J)=TW
 101  ENDDO
 100  ENDDO
      V(L2,2)=VIN
      T(L2,1)=TIN                          ! Different j for V and T
       PR= .7
      AMU=1.
      AMUP=AMU*CPCON/PR
      TREF=300.
      RHOREF=1.
      RHOT=RHOREF*TREF
      RETURN
```

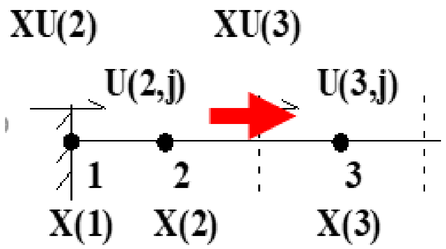
! Initial values and some BCs (U(I,J)=0 … V(1,J)=0.)

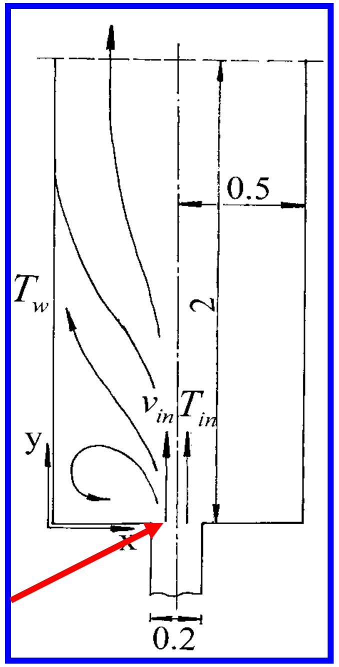
! Estimation of outlet normal velocity

! Different j for V and T

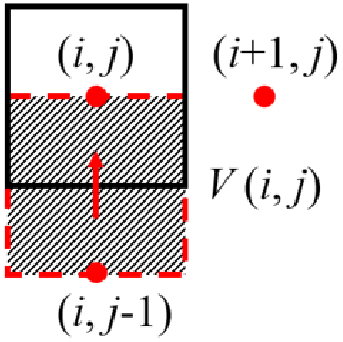
Variable	IST-1	JST-1
ϕ, p, p'	1	1
u	2	1
v	1	2

$$\Pr = \eta c_p / \lambda,$$
$$\lambda = \eta c_p / \Pr$$





(L2,1)

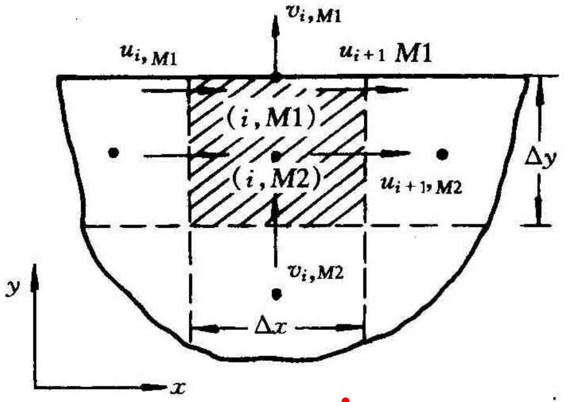


Total mass conservation for case of outlet without recirculation

（1）Assuming that relative changes of outlet normal velocity =constant

$$\frac{v_{i,M1}-v_{i,M2}}{v_{i,M2}}=k=const$$

$$v_{i,M1}=v_{i,M2}(1+k)=f\,v_{i,M2}$$



f is determined according to total mass conservation :

$$\sum_{i=2}^{L2}\rho_{i,M1}v_{i,M1}\Delta x_i=\sum_{i=2}^{L2}\rho_{i,M1}f\,v_{i,M2}\Delta x_i=FLOWIN$$

$$f=\frac{FLOWIN}{\sum_{2}^{L2}\rho_{i,M1}v_{i,M2}\Delta x_i}$$

$$\boxed{v_{i,M1}=f\bullet v_{i,M2}^{*}}$$

FACTOR method

It is regarded as the boundary condition for next iteration.

(2) Assuming that the 1st derivatives at outlet =constant

$$\frac{v_{i,M1}-v_{i,M2}}{\Delta y}=k=const \longrightarrow v_{i,M1}=v_{i,M2}+k\Delta y=v_{i,M2}+C$$

C is determined according to total mass conservation

$$\sum_{i=2}^{L2}\rho_{i,M1}(v_{i,M2}+C)\Delta x_i=FLOWIN \longrightarrow$$

$$C=\frac{FLOWIN-\sum\rho_{i,M1}v_{i,M2}\Delta x_i}{\sum\rho_{i,M1}\Delta x_i}$$

$v_{i,M1}=v^*_{i,M2}+C$ is taking as boundary condition for next iteration.

When fully developed at outlet, $f = 1$, C = 0;
Otherwise, there is some differences between the two treatments. In this example, FACTOR method will be used

```
      ENTRY DENSE
      DO 200 J=1,M1
      DO 201 I=1,L1
      RHO(I,J)=RHOT/T(I,J)
201 ENDDO
200 ENDDO
      RETURN
*
      ENTRY BOUND
      IF(ITER==0)  FLOWIN=RHO(L2,1)*V(L2,2)*XCV(L2)
      FL=0.
      DO 301 I=2,L2
      FL=FL+RHO(I,M1)*V(I,M2)*XCV(I)
301 ENDDO
      FACTOR=FLOWIN/FL
      DO 302 I=2,L2
      V(I,M1)=V(I,M2)*FACTOR
      T(I,M1)=T(I,M2）
302 ENDDO
      RETURN
```

! Variable density

! RHOT=RHOREF*TREF

!Inlet flow rate calculation

!Outlet flow rate calculation

$$\text{Factor} = \frac{\text{FLOWIN}}{\sum_{\text{i=2}}^{\text{L2}} \rho_{\text{i,M1}} * V_{i,M2} * XCV(i)}$$

$$v_{i,M1} = f \bullet v^*_{i,M2}$$

Only for print out purpose—decoration! It can be executed after getting converged solution.

```
      ENTRY OUTPUT
      IF(ITER==0)  THEN
      WRITE(8,401)
401 FORMAT(1X,'   ITER',7X,'SMAX',11X,'SSUM',10X,'V(4,7)',
     1  9X,'T(4,7)')
      ELSE
      PRINT 403, ITER, SMAX,  SSUM, V(4,7), T(4,7)
      WRITE(8,403) ITER, SMAX, SSUM, V(4,7), T(4,7)
403 FORMAT(1X,I6,1P4E15.3)
      ENDIF
      IF (ITER==LAST) CALL PRINT
      RETURN
*
```

Print out SMAX,SSUM for observing the convergence of the iteration

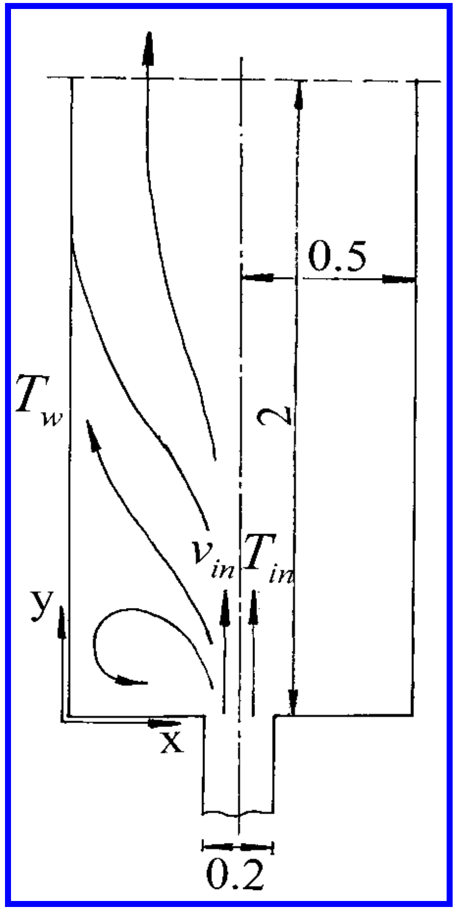
! Residual of mass conservation:

! $b = [(\rho u^*)_w - (\rho u^*)_s]A_e + [(\rho v^*)_s - (\rho v^*)_n]A_n$

! $\mathrm{SSUM} = \sum b(i,j)$

! SMAX = maxval (b)

```
      ENTRY GAMSOR
      DO 500 J=1,M1
      DO 501 I=1,L1
      GAM(I,J)=AMU          ! For solving fluid flow
      IF(NF==4) GAM(I,J)=AMUP     ! For solving temperature
      IF(NF/=1) GAM(L1,J)=0.  ! Except u, others---symmetry
      GAM(I,M1)=0.    ! Local one way for both u and T,
501 ENDDO               identical to adiabatic.
500 ENDDO
      RETURN
      END
```



11-5-4 Results analysis

COMPUTATION IN CARTISIAN COORDINATES

****

ITER	SMAX	SSUM	V(4,7)	T(4,7)
0	0.000E+00	0.000E+00	1.200E+01	3.000E+02
1	2.366E+00	5.960E-08	1.269E+01	[illegible]
2	1.068E+00	3.576E-07	1.526E+01	3.574E+02
3	1.059E+00	-2.980E-07	1.600E+01	3.609E+02
4	6.520E-01	-8.941E-08	1.609E+01	3.630E+02
5	1.605E-01	4.433E-07	1.618E+01	3.645E+02
6	1.039E-01	-8.754E-08	1.606E+01	3.655E+02
7	5.972E-02	-8.196E-08	1.594E+01	3.663E+02
8	3.817E-02	-3.101E-07	1.576E+01	3.668E+02
9	2.447E-02	-5.243E-07	1.559E+01	3.672E+02
10	1.535E-02	2.674E-07	1.543E+01	3.675E+02
11	9.663E-03	-8.473E-07	1.529E+01	3.677E+02
12	5.899E-03	4.657E-10	1.516E+01	3.678E+02

Total mass conservation is artificially made!

			!V(4,7)	!T(4,7)
13	4.332E-03	-2.432E-07	1.506E+01	3.678E+02
14	3.456E-03	2.751E-07	1.498E+01	3.678E+02
15	2.698E-03	7.753E-08	1.491E+01	3.678E+02
16	2.052E-03	1.475E-07	1.486E+01	3.678E+02
17	1.539E-03	-5.428E-07	1.481E+01	3.678E+02
18	1.133E-03	2.519E-07	1.478E+01	3.677E+02
19	8.994E-04	2.108E-07	1.476E+01	3.677E+02
20	7.056E-04	5.479E-07	1.474E+01	3.677E+02
21	5.436E-04	2.256E-07	1.473E+01	3.677E+02
22	4.111E-04	9.380E-08	1.472E+01	3.676E+02
23	3.100E-04	1.485E-07	1.471E+01	3.676E+02
24	2.303E-04	2.160E-07	1.470E+01	3.676E+02
25	1.793E-04	4.192E-07	1.470E+01	3.676E+02
26	1.447E-04	-1.086E-08	1.470E+01	3.676E+02
27	1.149E-04	-9.684E-08	1.469E+01	3.676E+02
28	8.990E-05	1.732E-09	1.469E+01	3.676E+02
29	6.926E-05	-5.815E-07	1.469E+01	3.676E+02
30	5.170E-05	-3.065E-07	1.469E+01	3.676E+02
31	3.837E-05	-5.491E-07	1.469E+01	3.676E+02
32	3.084E-05	2.732E-07	1.469E+01	3.676E+02

33	2.032E-05	-9.269E-07	1.469E+01	3.676E+02
34	2.015E-05	3.659E-08	1.469E+01	3.676E+02
35	1.213E-05	4.555E-07	1.469E+01	3.676E+02
36	9.591E-06	-1.184E-07	1.469E+01	3.676E+02
37	6.249E-06	4.063E-07	1.469E+01	3.676E+02
38	4.888E-06	-2.038E-08	1.469E+01	3.676E+02
39	3.099E-06	1.491E-07	1.469E+01	3.676E+02
40	3.695E-06	4.564E-07	1.469E+01	3.676E+02
41	2.980E-06	-3.393E-07	1.469E+01	3.676E+02
42	2.923E-06	1.307E-06	1.469E+01	3.676E+02
43	3.150E-06	-3.455E-07	1.469E+01	3.676E+02
44	2.787E-06	5.100E-07	1.469E+01	3.676E+02
45	3.219E-06	-2.657E-07	1.469E+01	3.676E+02
46	2.980E-06	-8.977E-07	1.469E+01	3.676E+02
47	2.503E-06	-2.419E-07	1.469E+01	3.676E+02
48	2.205E-06	5.658E-08	1.469E+01	3.676E+02
49	3.517E-06	-9.167E-07	1.469E+01	3.676E+02
50	3.576E-06	-1.444E-07	1.469E+01	3.676E+02
51	3.278E-06	2.954E-07	1.469E+01	3.676E+02

ITER	SMAX	SSUM	V(4,7)	T(4,7)
52	2.772E-06	1.221E-08	1.469E+01	3.676E+02
53	2.146E-06	5.844E-07	1.469E+01	3.676E+02
54	2.104E-06	5.236E-07	1.469E+01	3.676E+02
55	2.921E-06	3.407E-07	1.469E+01	3.676E+02
56	2.712E-06	1.156E-07	1.469E+01	3.676E+02
57	2.801E-06	2.216E-07	1.469E+01	3.676E+02
58	3.005E-06	8.967E-08	1.469E+01	3.676E+02
59	2.886E-06	4.362E-07	1.469E+01	3.676E+02
60	2.623E-06	5.034E-07	1.469E+01	3.676E+02

That SMAX reduces to a certain value can be regarded as an indicator of convergence

In the iteration process, SSUM takes a very small value from beginning to the end. This can not be regarded as an indicator of convergence. Because it is due to our treatment of outflow boundary condition!

************************** .VEL U. **************************

No decoration

J \ I =	2	3	4	5	6	7
12	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
11	0.00E+00	1.41E-02	3.39E-02	4.04E-02	2.71E-02	0.00E+00
10	0.00E+00	-6.73E-02	-1.96E-01	-2.78E-01	-2.11E-01	0.00E+00
9	0.00E+00	-1.55E-01	-4.33E-01	-5.97E-01	-4.48E-01	0.00E+00
8	0.00E+00	-3.26E-01	-8.75E-01	-1.19E+00	-8.95E-01	0.00E+00
7	0.00E+00	-6.17E-01	-1.61E+00	-2.16E+00	-1.65E+00	0.00E+00
6	0.00E+00	-1.03E+00	-2.62E+00	-3.53E+00	-2.75E+00	0.00E+00
5	0.00E+00	-1.42E+00	-3.67E+00	-5.06E+00	- 4.10E+00	0.00E+00
4	0.00E+00	-1.35E+00	-3.91E+00	.02E+00	-5.42E+00	0.00E+00
3	0.00E+00	1.37E-01	-1.24E+00	69E+00	-6.33E+00	0.00E+00
2	0.00E+0	.64E+00	6.16E+0	03E+00	-7.70E+00	0.00E+00
1	0.00E+	.00E+00	0.00E+0	0E+00	0.00E+00	0.00E+00

u larger than 0

u less than 0

************************** .VEL V. **************************

I =	1	2	3	4	5	6	7
J							
12	0.00E+00	3.73E+00	9.97E+00	1.50E+01	1.87E+01	2.07E+01	1.20E+01
11	0.00E+00	3.76E+00	1.01E+01	1.52E+01	1.89E+01	2.09E+01	1.20E+01
10	0.00E+00	3.65E+00	9.94E+00	1.53E+01	1.95E+01	2.19E+01	1.20E+01
9	0.00E+00	3.37E+00	9.57E+00	1.54E+01	2.04E+01	2.35E+01	1.20E+01
8	0.00E+00	2.76E+00	8.70E+00	1.52E+01	2.17E+01	2.61E+01	1.20E+01
7	0.00E+00	1.59E+00	7.02E+00	1.47E+01	2.35E+01	3.03E+01	1.20E+01
6	0.00E+00	-3.65E-01	4.21E+00	1.36E+01	2.60E+01	3.70E+01	1.20E+01
5	0.00E+00	-3.06E+00	1.81E-01	1.15E+01	2.89E+01	4.66E+01	1.20E+01
4	0.00E+00	-5.60E+00	-4.41E+0[illegible]	8.01E+00	3.09E+01	5.93E+01	1.20E+01
3	0.00E+00	-5.24E+00	-6.77E+[illegible]	1.43E+00	2.77E+01	7.51E+01	1.20E+01
2	0.00E+00	[illegible].00E+00	0.00E[illegible]	0.00E+00	0.00E+00	1.00E+02	0.00E+00

v less than 0

v larger than 0

Inlet *v*

No decoration

************************* .STR FN. ***************************

J \ I =	2	3	4	5	6	7
12	0.00E+00	-3.63E-01	-1.29E+00	-2.63E+00	-4.24E+00	-6.00E+00
11	0.00E+00	-3.66E-01	-1.29E+00	-2.63E+00	-4.25E+00	-6.00E+00
10	0.00E+00	-3.53E-01	-1.26E+00	-2.58E+00	-4.21E+00	-6.00E+00
9	0.00E+00	-3.24E-01	-1.18E+00	-2.48E+00	-4.14E+00	-6.00E+00
8	0.00E+00	-2.64E-01	-1.03E+00	-2.29E+00	-4.00E+00	-6.00E+00
7	0.00E+00	-1.51E-01	-7.61E-01	-1.95E+00	-3.74E+00	-6.00E+00
6	0.00E+00	3.46E-02	-3.26E-01	-1.40E+00	-3.34E+00	-6.00E+00
5	0.00E+00	2.89E-01	2.74E-01	-6.28E-01	-2.74E+00	-6.00E+00
4	0.00E+00	5.31E-01	9.10E-01	2.79E-01	-1.97E+00	-6.00E+00
3	0.00E+00	5.06E-01	1.12E+00	9.96E-01	-1.09E+00	-6.00E+00
2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	-6.00E+00

Stream function =0 at the wall

Total flow rate

********************** . TEMP . **********************

Decoration has been made：
T(I,M1)=T(I,M2)

I =	1	2	3	4	5	6	7
J							
12	3.00E+02	3.08E+02	3.23E+02	3.37E+02	3.48E+02	3.53E+02	3.00E+02
11	3.00E+02	3.08E+02	3.23E+02	3.37E+02	3.48E+02	3.53E+02	3.00E+02
10	3.00E+02	3.09E+02	3.27E+02	3.43E+02	3.55E+02	3.62E+02	3.00E+02
9	3.00E+02	3.11E+02	3.31E+02	3.50E+02	3.65E+02	3.73E+02	3.00E+02
8	3.00E+02	3.12E+02	3.37E+02	3.59E+02	3.75E+02	3.84E+02	3.00E+02
7	3.00E+02	3.14E+02	3.43E+02	3.68E+02	3.87E+02	3.97E+02	3.00E+02
6	3.00E+02	3.16E+02	3.48E+02	3.76E+02	3.98E+02	4.10E+02	3.00E+02
5	3.00E+02	3.18E+02	3.53E+02	3.83E+02	4.07E+02	4.23E+02	3.00E+02
4	3.00E+02	3.18E+02	3.53E+02	3.85E+02	4.12E+02	4.35E+02	3.00E+02
3	3.00E+02	3.15E+02	3.45E+02	3.76E+02	4.10E+02	4.49E+02	3.00E+02
2	3.00E+02	3.06E+02	3.21E+02	3.42E+02	3.88E+02	4.69E+02	3.00E+02
1	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	5.00E+02	3.00E+02

Given wall temperature

Inlet temp.

No decoration

************************** PRESSURE **************************

J \ I =	1	2	3	4	5	6	7
12	8.40E+02	8.40E+02	8.39E+02	8.38E+02	8.34E+02	8.31E+02	8.30E+02
11	8.52E+02	8.52E+02	8.52E+02	8.50E+02	8.48E+02	8.45E+02	8.44E+02
10	8.77E+02	8.77E+02	8.76E+02	8.76E+02	8.75E+02	8.74E+02	8.73E+02
9	8.99E+02	8.98E+02	8.97E+02	8.95E+02	8.94E+02	8.92E+02	8.91E+02
8	**9.12E+02**	9.10E+02	9.08E+02	9.06E+02	9.05E+02	9.02E+02	9.00E+02
7	9.06E+02	9.04E+02	9.01E+02	8.99E+02	8.99E+02	8.96E+02	8.94E+02
6	8.63E+02	8.61E+02	8.56E+02	8.56E+02	8.62E+02	8.59E+02	8.58E+02
5	7.55E+02	7.52E+02	7.46E+02	7.50E+02	7.66E+02	7.69E+02	7.70E+02
4	5.57E+02	5.53E+02	5.45E+02	5.50E+02	5.85E+02	6.02E+02	6.11E+02
3	2.91E+02	2.84E+02	2.72E+02	2.55E+02	3.32E+02	3.56E+02	3.68E+02
2	9.85E+01	8.74E+01	6.54E+01	-3.27E+01	-2.08E+02	9.08E+01	2.40E+02
1	0.00E+00	-1.10E+01	-3.79E+01	-1.77E+02	-4.78E+02	-4.18E+01	1.07E+02

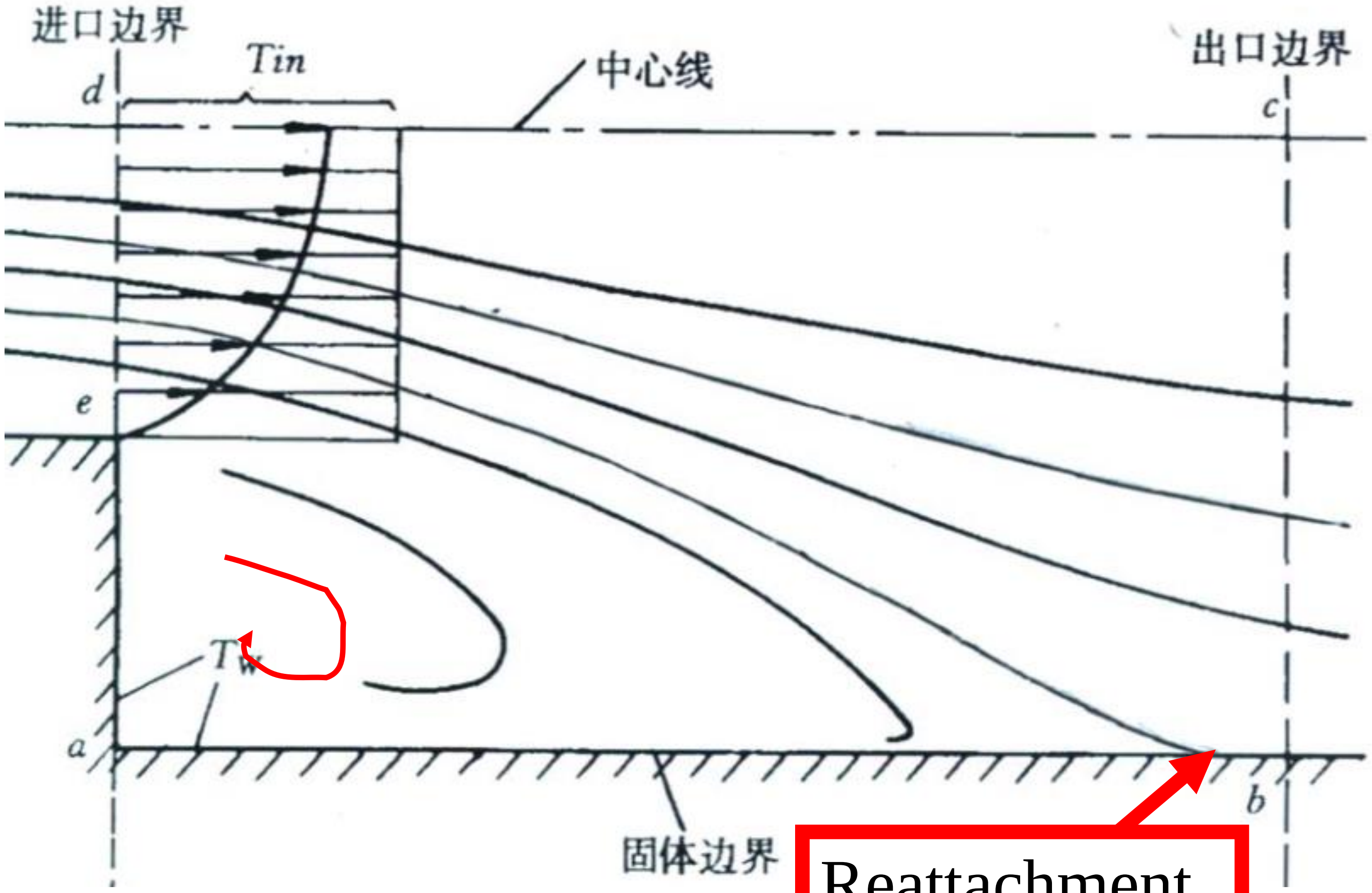
Maximum pressure caused by reattachment of flow

Reference point

$$p(i, j) = p(i, j) - p(IPREF, JPREF)$$

Low pressure region caused by high inlet velocity

From interpolation



Reattachment Point，$p=p_{max}$

Fig.2 of Problem 6

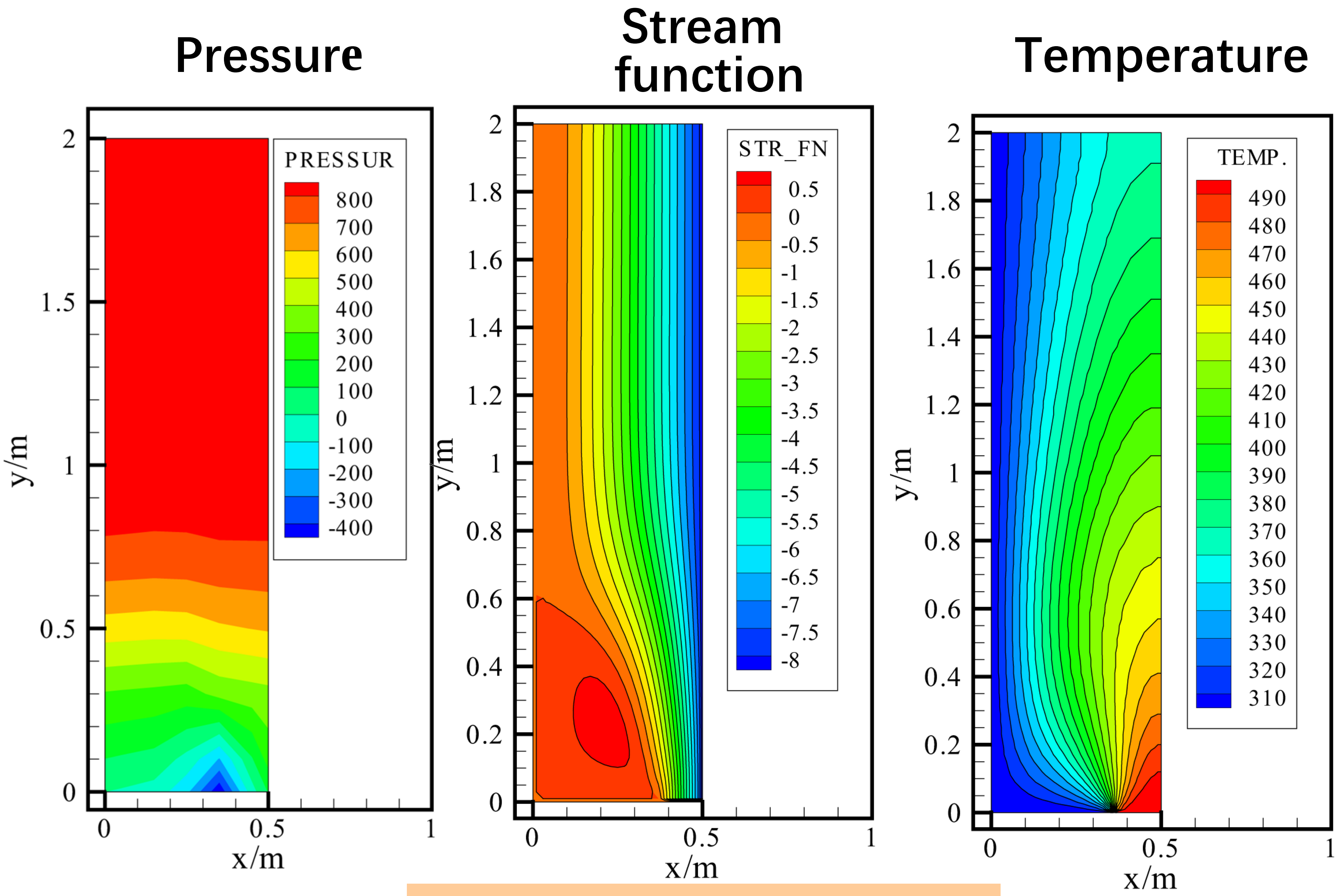


Fig. 3 Results of Problem 6

本组网页地址：http://nht.xjtu.edu.cn 欢迎访问！

Teaching PPT will be loaded on our website

同舟共济

渡彼岸！

People in the same boat help each other to cross to the other bank, where....