Numerical Heat Transfer

(数值传热学)

Chapter 9 Introduction to BFC and Collocated Grid System

Instructor Tao, Wen-Quan

Key Laboratory of Thermo-Fluid Science & Engineering
Int. Joint Research Laboratory of Thermal Science & Engineering
Xi’an Jiaotong University
Innovative Harbor of West China, Xian
2023-Nov-08

9.1 Treatments of Irregular Domain in FDM,FVM

9.2 Introduction to Body-fitted Coordinates

9.3 Boundary Normalization for Generating Body-Fitted Coordinates

(Chapter 10 of Textbook)

9.4 Introduction to Collocated Grid System

(Section 6-10 of Textbook)

9.1 Treatments of Irregular Domain in FDM,FVM

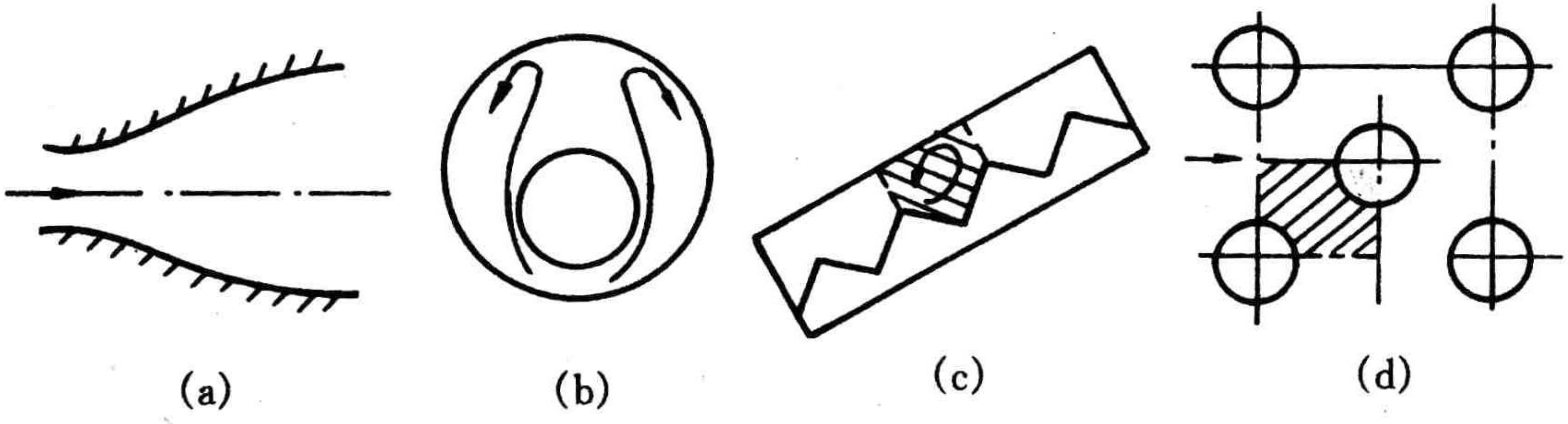
9.1.1 Conventional orthogonal coordinates can not deal with variety of complicated geometries

9.1.2 Methods in FDM,FVM to deal with complicated geometries

1. Structured grid (结构化网格)
   1) **Domain extension method**
   2) **Special orthogonal coordinates**
   3) **Composite grid** (组合网格)
   4) **Body-fitted coordinate** (适体坐标系)
2. Unstructured grid (非结构化)

9.1 Treatments of Irregular Domain in FDM,FVM

9.1.1 Conventional orthogonal (正交)coordinates can not deal with variety of complicated geometries



Plane nozzle

Eccentric annulus (偏心圆环)

Solar collector

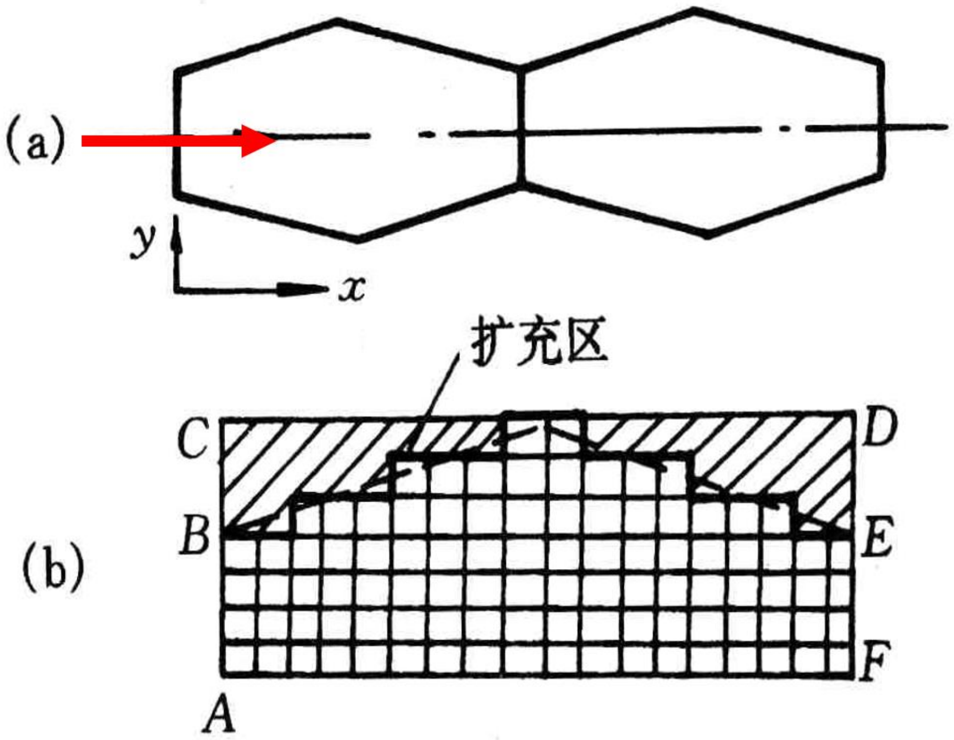
Tube bank

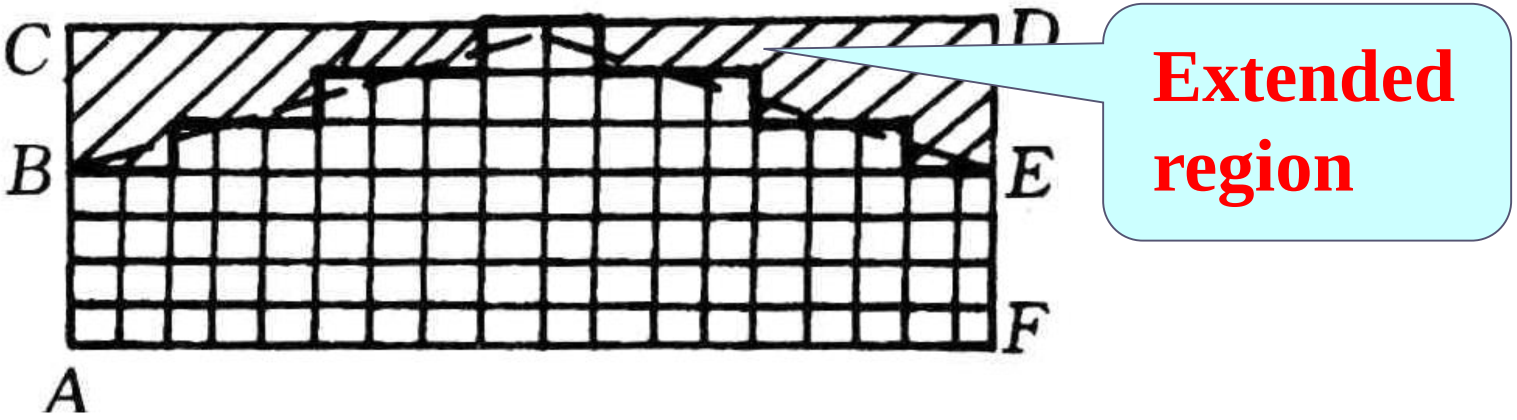
9.1.2 Methods in FDM,FVM to deal with complicated geometries

1. Structured grid (结构化网格)

1) Domain extension method (区域扩充法)

An irregular domain is extended to a regular one, the irregular boundary is replaced by a **step-wise approximation,** and simulation is performed in a conventional coordinate within the extended domain.





(1) Flow field simulation

(a) Set zero velocity at the boundaries of extended region at B-C-D-E: $u = v = 0$;

(b) Set a very large viscosity in the extended region

$$\eta = 10^{25} \sim 10^{30};$$

(c) Set interface diffusivity by harmonic mean

(2) Temperature field prediction

(a) First kind boundary condition with uniform temperature: The same as for velocity： in the extended region the thermal conductivity is set to be very large, $\lambda = 10^{25} \sim 10^{30}$， and boundary temperatures are given；

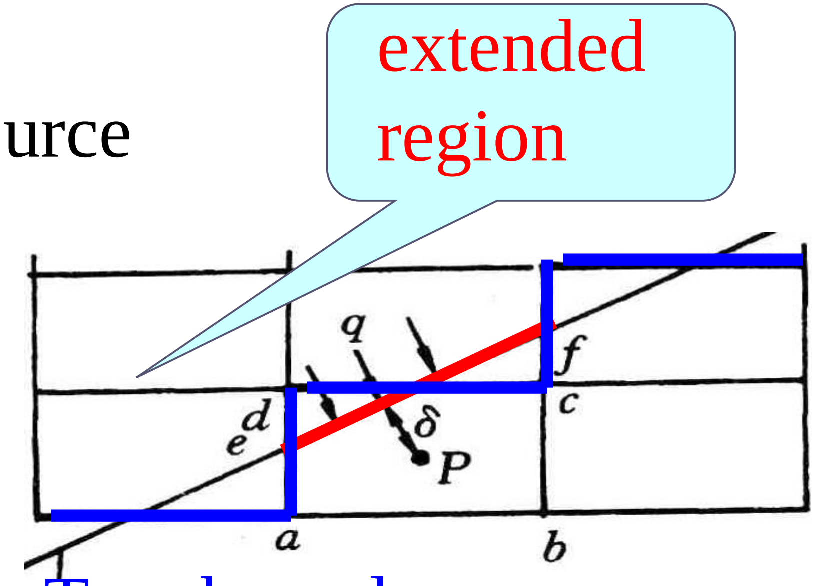
(b) Second kind boundary conditions by ASTM

Specified boundary heat flux distribution (not necessary uniform)

For CV. *P* adding additional source term:

$$S_{c,ad} = \frac{q \bullet \overline{ef}}{\Delta V_P}$$

And setting zero conductivity for the extended region to avoid outward heat transfer.



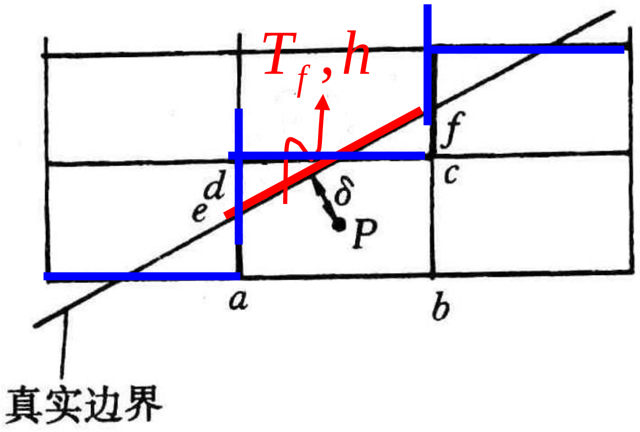
extended region

True boundary

(c) Third kind boundary conditions by ASTM

Specified external convective heat transfer coefficient and fluid temperature, *h* and T_f,

For CV. *P* following source term is added



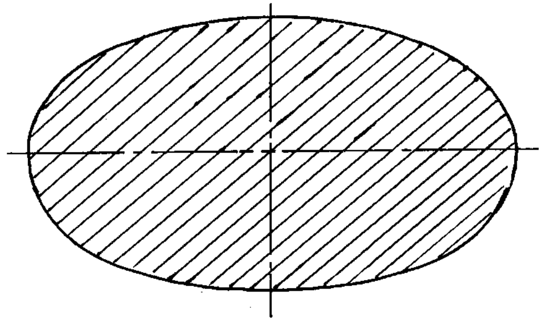
$$S_{C,ad}=\frac{\overline{ef}}{\Delta V_P}\frac{T_f}{1/h+\delta/\lambda};\quad S_{P,ad}=-\frac{\overline{ef}}{\Delta V_P}\frac{1}{1/h+\delta/\lambda};$$

And setting zero conductivity ($\lambda = 0$) for the extended region to avoid outward heat transfer.

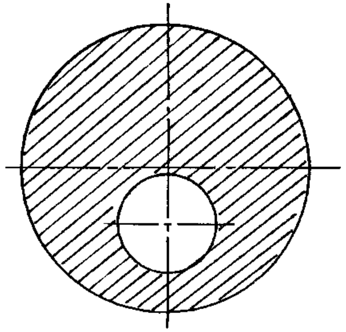
For not very complicated geometries, it is a convenient and efficient method.

2) Adopting special orthog**o**nal (正交的) coordinates

There are 14 orthogonal coordinates, and they can be used to deal with some irregular regions



Elliptical coordinate can be used to simulate flow in elliptic tube

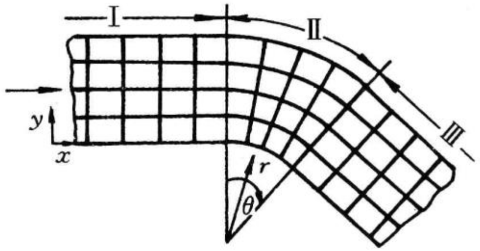


Bi-polar coordinate (双极坐标）can be used for flow in a biased annulus(偏心环)

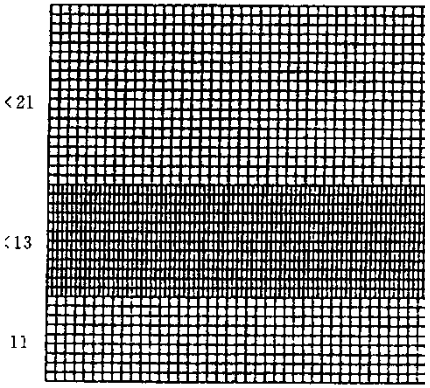
3) Composite coordinate (组合坐标)(block structured)

The entire domain is divided into several blocks, for each block individual coordinate is adopted and solutions are exchanged at the interfaces between different blocks.

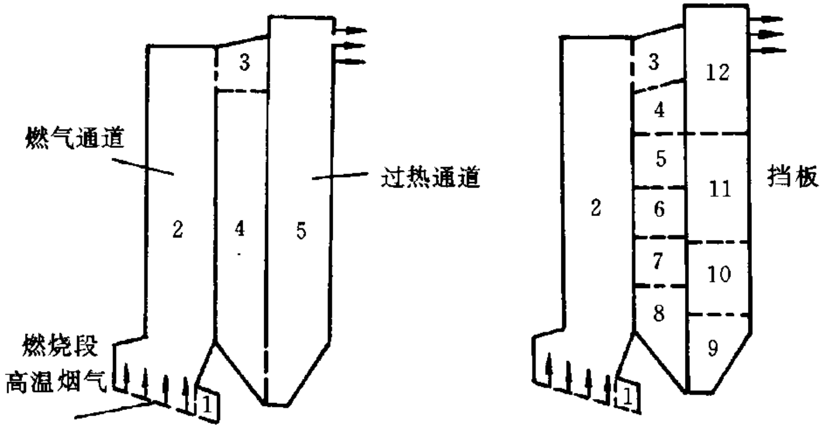
Mathematically it is called domain decomposition method (区域分解法）.



Grid lines are continuous. The entire domain can be solved by ADI.



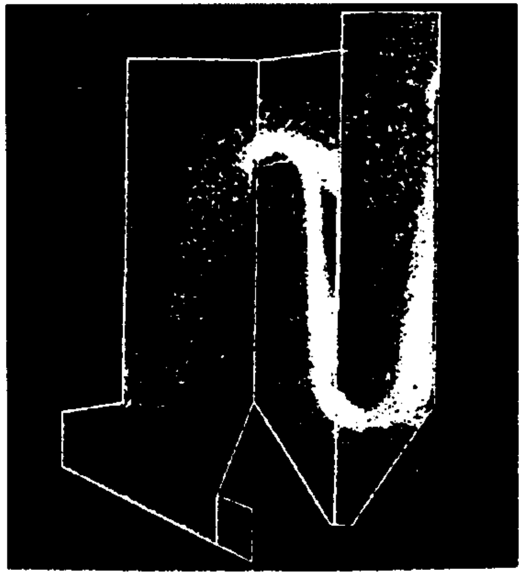
Grid lines are discontinuous



(a) 原设计

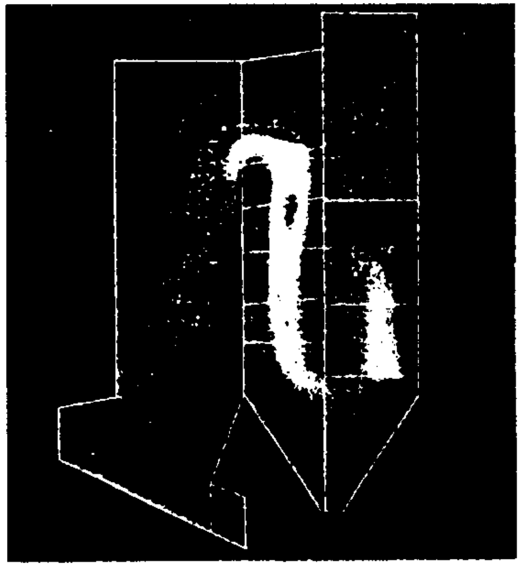
(b) 改型设计

Application example



(a) 原设计

Original design



(b) 改型设计

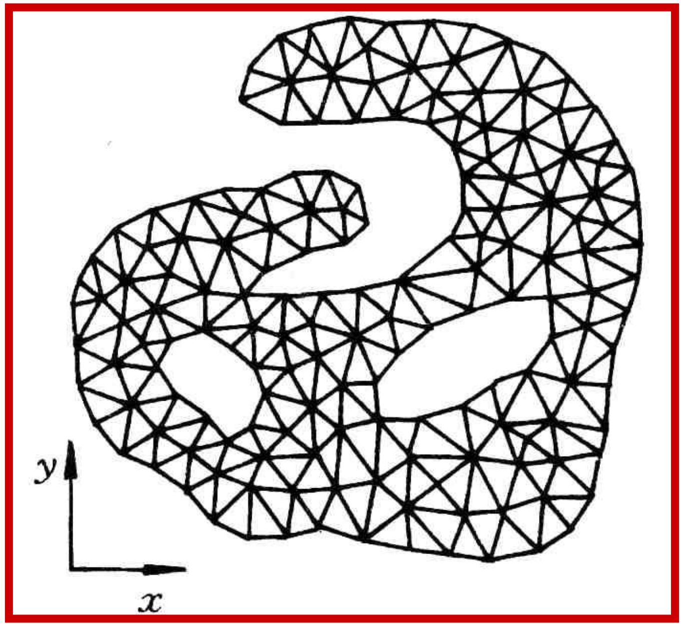
Improved design

4) Body-fitted coordinates (适体坐标)

In such coordinates the coordinates are fitted with(适应) the domain boundaries; The generation of such coordinates by numerical methods is the one major concerns of this chapter. It was proposed by TTM in Colorado University in 1974.

2. Unstructured grid (非结构化网格)

There are no fixed rules for the relationship between different nodes, and such relationship should be specially stored for each node. Computationally expensive. Suitable for very complicated geometries, and widely adopted in engineering computation.



9.2 Introduction to Body-Fitted Coordinates

9.2.1 Basic idea for solving physical problems by BFC

9.2.2 Why domain can be simplified by BFC

9.2.3 Methods for generation of BFC

9.2.4 Requirements for grid system constructed by BFC

9.2.5 Basic solution procedure by BFC

9.2 Introduction to Body-Fitted Coordinates

9.2.1 Basic idea for solving physical problems by body-fitted coordinates (BFC)

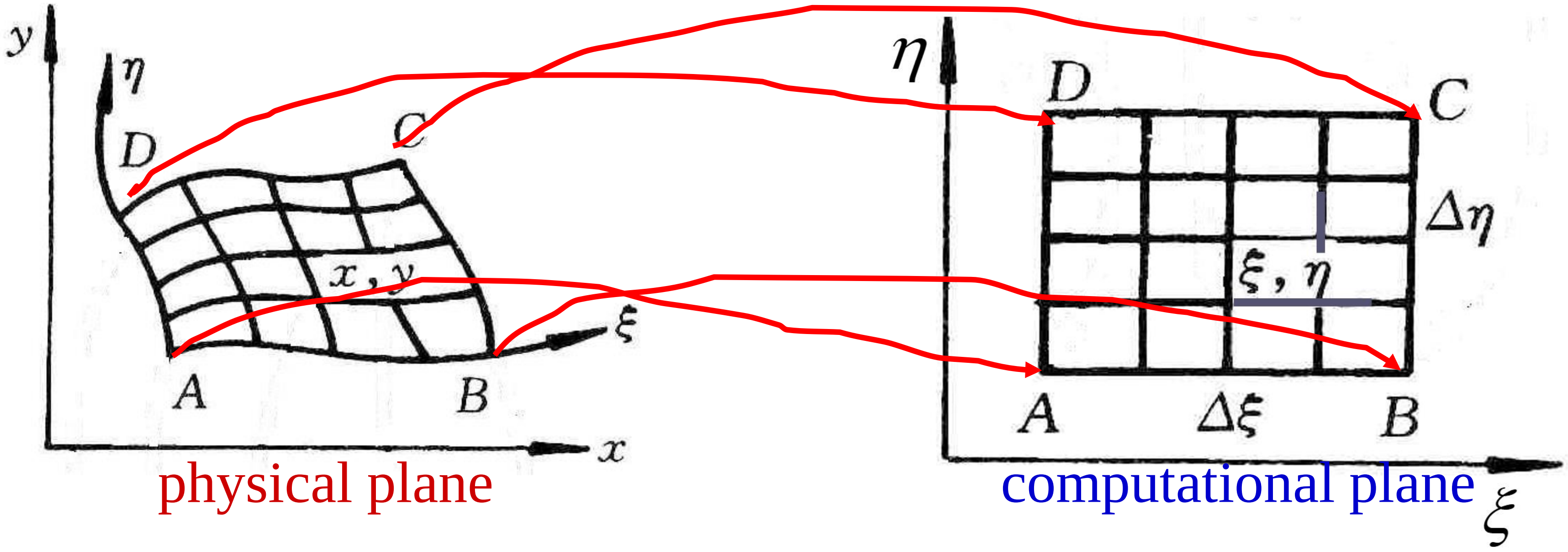
1.In the numerical simulation of physical problems the most ideal coordinate is the one which fits with the boundaries of the studied problem, called body-fitted coordinates(适体坐标系): Cartesian coordinate is the body-fitted one for rectangles, polar coordinate is the one for **a**nnular spaces.

2.The existing orthogonal coordinates can not deal with variety of complicated geometries in different fields ; Thus artif**i**cially (人为地) constructed body-fitted coordinates are necessary to meet the different practical requirements.

9.2.2 Why domain can be simplified by BFC

1.Assuming that a BFC has been constructed in Cartesian coordinate *x*-*y*, denoted by $\xi - \eta$ ；

2.Regarding ξ and η as the two coordinates of a Cartesian coordinate in an imaginary （假想的） computational plane, then the irregular geometry in physical plane transforms to a rectangle in the computational plane.



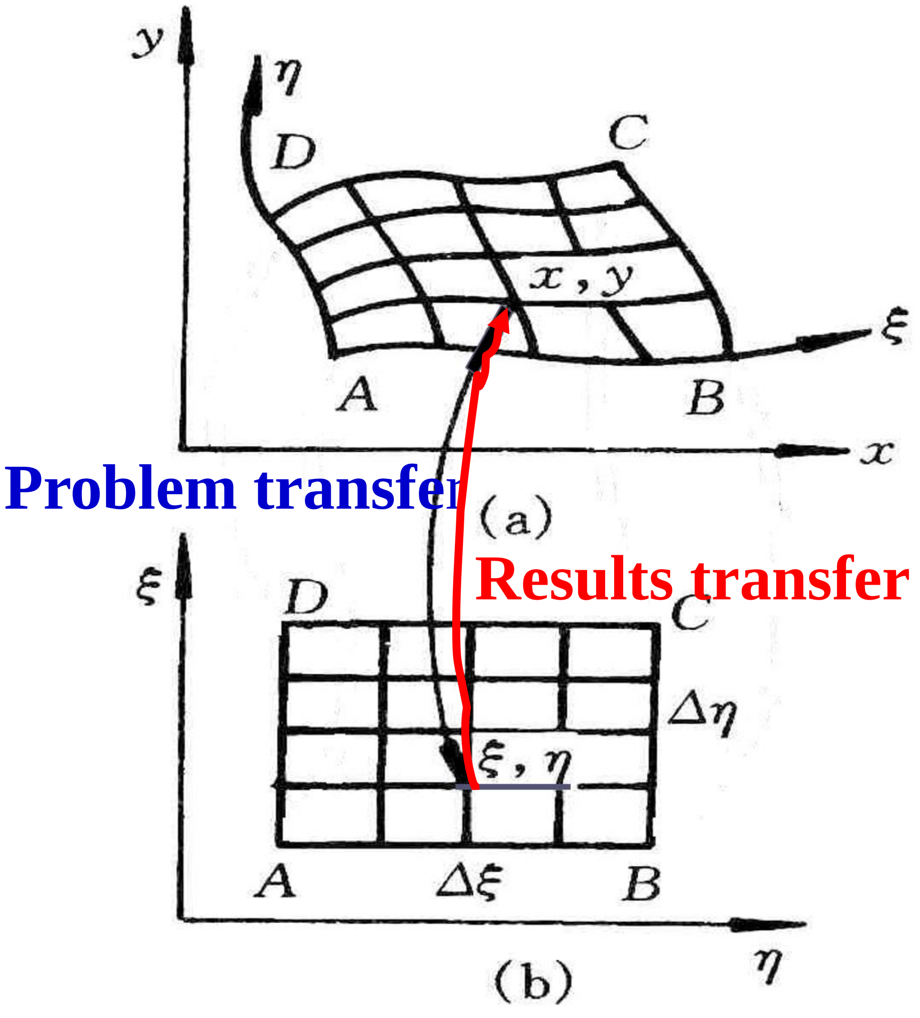
physical plane

computational plane

3.The grids in computational plane are always uniformly distributed, thus once grid number is given, the grid system in computational plane can be constructed with ease.

4.Simulation is first conducted in the computational plane , then the converged solution is transferred from the computational plane to physical plane.

In such a way the simulation domain is greatly simplified.



5.In order to transfer solutions from computational domain to physical domain, it is necessary to obtain the corresponding relations of nodes between the two planes.

The so-called grid generation technique here refers to the methods by which from (ξ, η) in the computational plane the corresponding (x, y) in the physical Cartesian coordinate can be obtained.

9.2.3 Methods for generation of BFC

1. Conforming mapping (保角变换法)

2. Algebraic method (代数法)

The correspondent relations between grids of the two planes are represented by algebraic equations.

3. PDE method(微分方程法)

The relations are obtained through solving a PDE. Three kinds of PDE, hyperbolic, parabolic and elliptic, all can be used to provide such relations.

9.2.4 Requirements for grid system constructed by BFC

1. The nodes in the two planes should be one to one correspondent（一一对应）.

2. Grid lines in **physical plane** should be normal to the boundary .

3. The grid spacing in the **physical plane** can be controlled easily.

9.2.5 Procedure of solving problem by BFC

1. Generating grid： find the one to one correspondence between $(\xi,\eta)\leftrightarrow(x,y)$;

2. Transforming governing eqs. and boundary conditions from physical plane to computational plane；

3. Discretizing gov. eq. and solving the algebraic equations in the computational plane

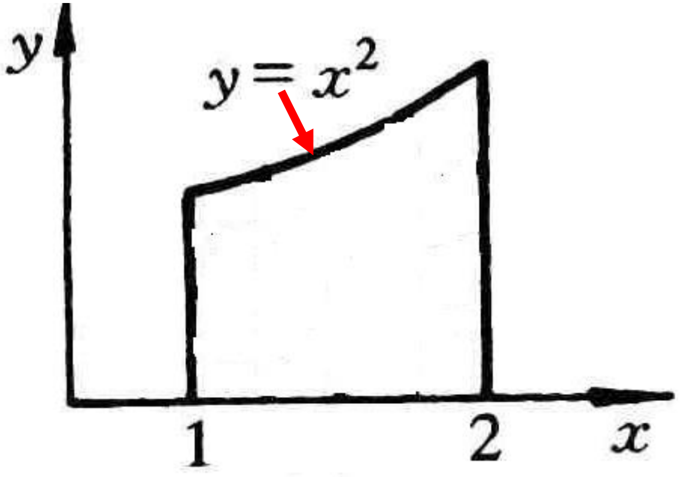
4. Transferring solutions from the computational plane to the physical plane.

9.3 Boundary Normalzation for Generating Body-Fitted Coordinates (simple algebraic method)

For some cases we can obtain body-fitted coordinates just by boundary normalization (边界规范化).

1. 2-D nozzle

A plane nozzle is given by the profile $y = x^2$, its body-fitted coordinates can be constructed as follows:

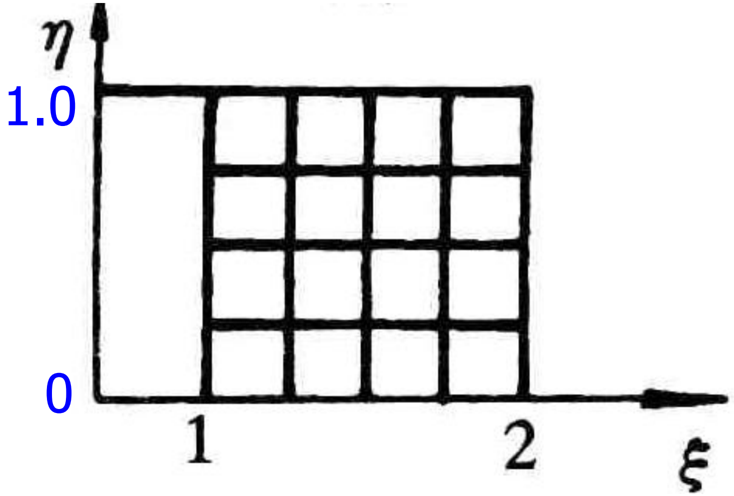


$$\boxed{\xi = x}$$

$$\boxed{\eta = y / y_{\max}}$$

normalization

$$y_{\max} = x^2$$

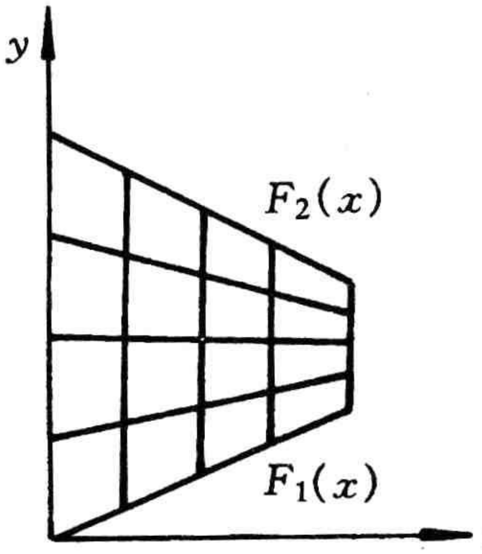


2. Trapezoid (梯形) enclosure

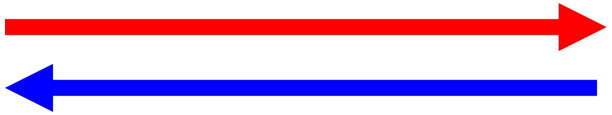
Functions of two tilted **(倾斜的)** boundaries are given by：

$$F_1(x),\ F_2(x)$$

The grid in the trapezoid enclosure is generated.

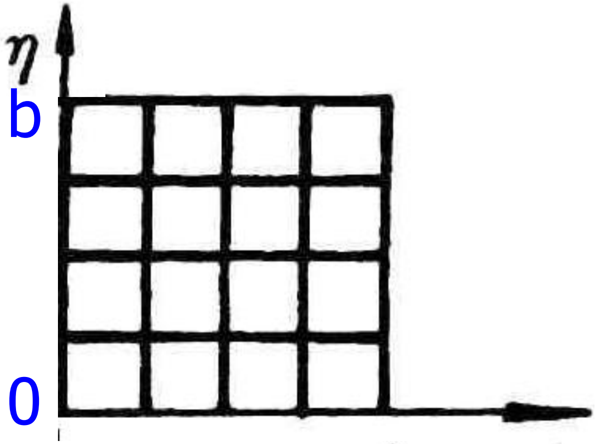


$$\boxed{\xi = ax}$$

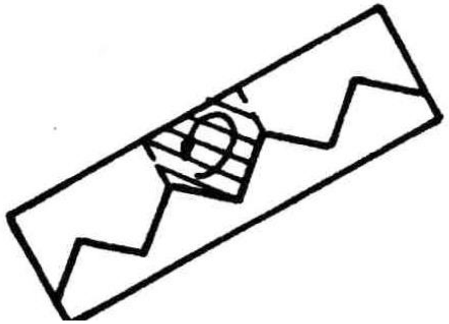


$$\boxed{\eta = b\frac{y - F_1(x)}{F_2(x) - F_1(x)}}$$

normalization



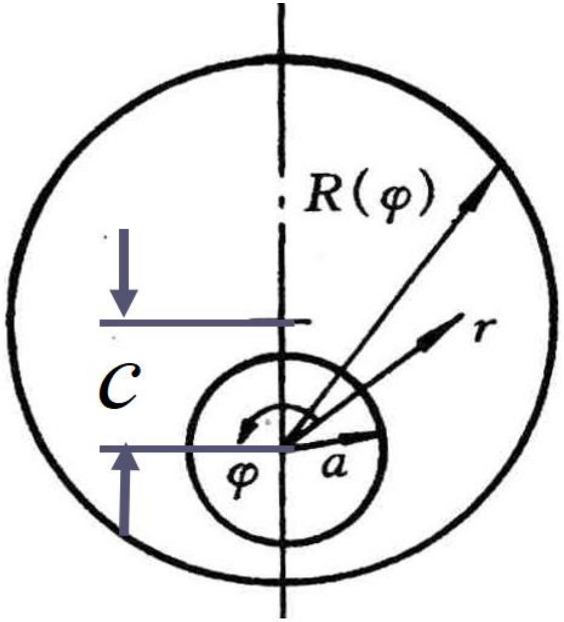
Normalized by the distance between top and bottom，a and b are coefficients for enlargement or reduction.



Solar collector

3. Eccentric annular space

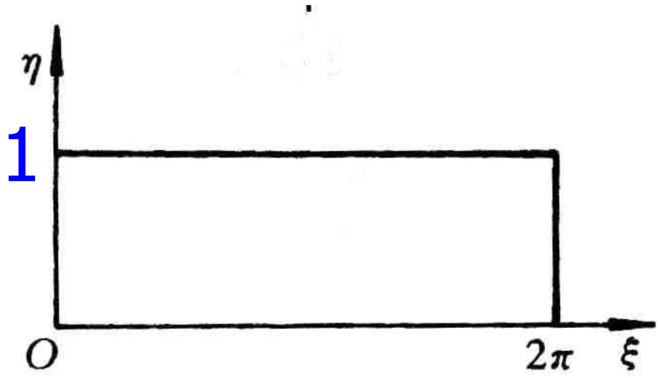
Given two radiuses (R,*a*) and the eccentric (偏心的)distance *c*



$$\boxed{\xi = \varphi}$$

$$\boxed{\eta = \frac{r - a}{R(\varphi) - a}}$$

normalization

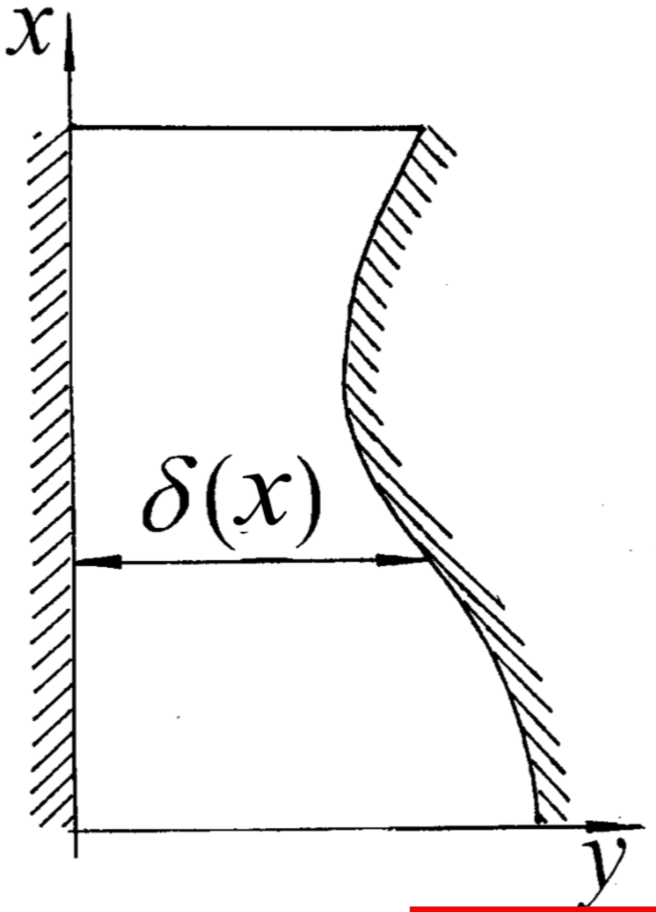


Normalized by the distance between outer and inner circles.

Prusa,Yao, ASME J H T, 1983， 105:105-116

4. Plane duct with one irregular boundary

Given the profile of the irregular boundary $y=\delta(x)$

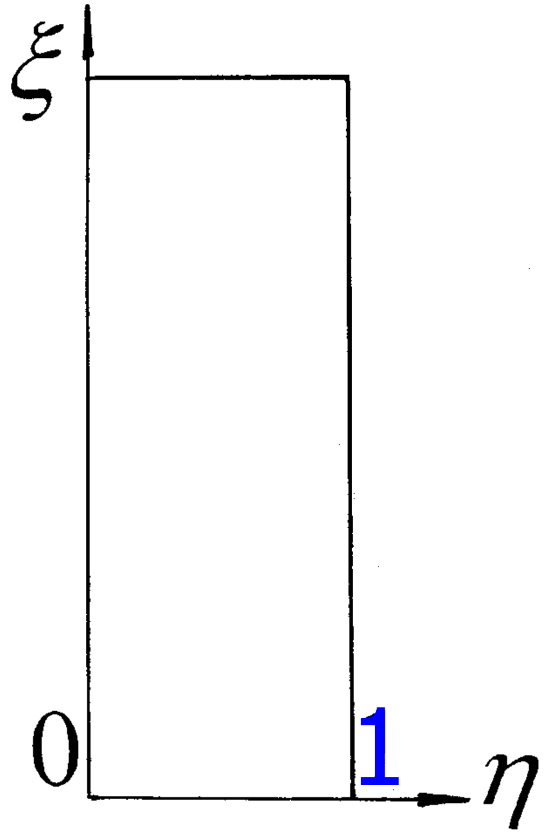


$$\boxed{\xi = x}$$

$$\boxed{\eta = \frac{y}{\delta(x)}}$$

normalization

Normalized by the distance between left and right boundaries



Sparrow-Faghri-Asako, p.479 of Textbook

9.4 Introduction to Collocated Grid System

9.4.1 Historical Development of Grid System

1965

Harlow-Welch proposed the staggered grid system to overcome the checkboard pressure difficulty.

1974

Thompson-Thomas-Mastin proposed the body-fitted coordinate method to meet the requirement of irregular domain of FV.

1980s

At beginning of 1980s in university of MN and Chicago researchers began to study non-staggered grid system.

1987

In 1987 Peric et al. published their paper in C&F introducing this new grid system to CFD/NHT communities。

1990s

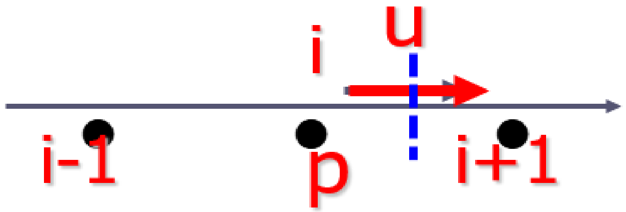
In 1990s, the unstructured grids were gradually adopted in FV computations because of its good adaptation of irregular region .

2000s

Unstructured and collocated grid system is widely used in many soft-wares because of its wide adaptability.

9.4.2 Key point of staggered grid system---1-δ pressure difference

In the staggered grid system by moving velocity position to the interface of two grids We realize two requirements:



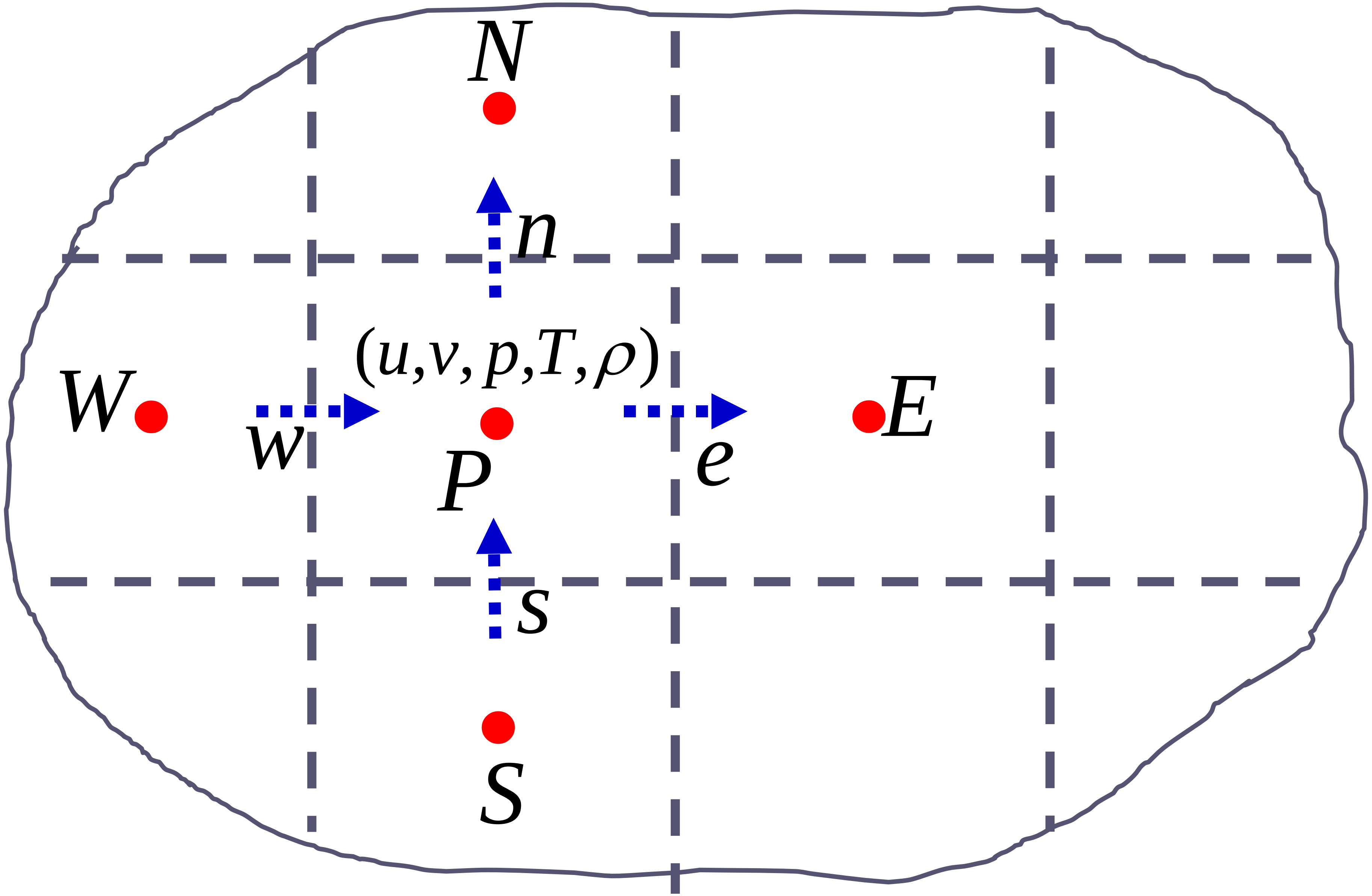
Staggered

(1) 1-δ pressure difference appears in the discretized equations,

(2) This 1-δ pressure difference can form central difference for pressure gradient in the momentum equation.

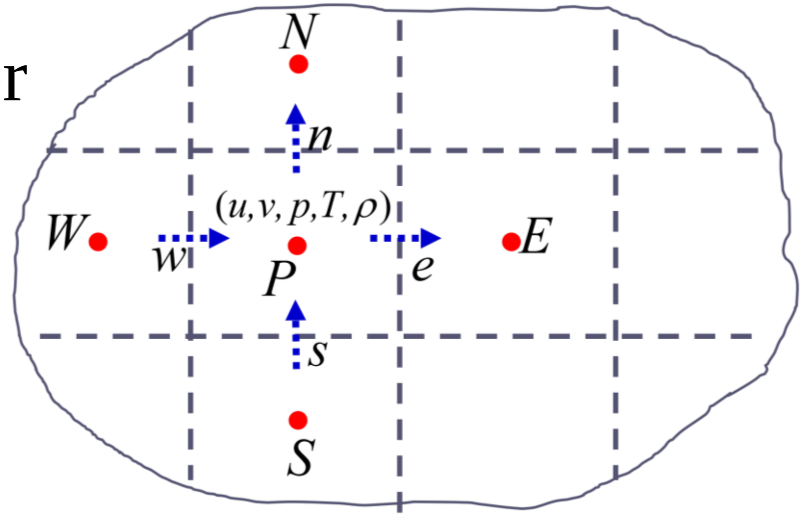
In order that only one grid system is used for all variables successfuly these two conditions must be satisfied.

9.4.2 Momentum interpolation---key point of collocated grid system



Non-staggered grid system

1)Descretization of momentum equation



By integrating the u-momentum equation over control volume *P*:

$$a_P u_P = \sum a_{nb} u_{nb} + b + A_P (p_w - p_e)$$

$$u_P = [\sum a_{nb} u_{nb} + b + A_P (p_w - p_e)] / a_P$$

$$u_P = [\underline{\frac{\sum a_{nb} u_{nb} + b}{a_P}}] + \frac{A_P}{a_P}(p_w - p_e) \qquad u_P = u_P + \frac{A_P}{a_P}(p_w - p_e)$$

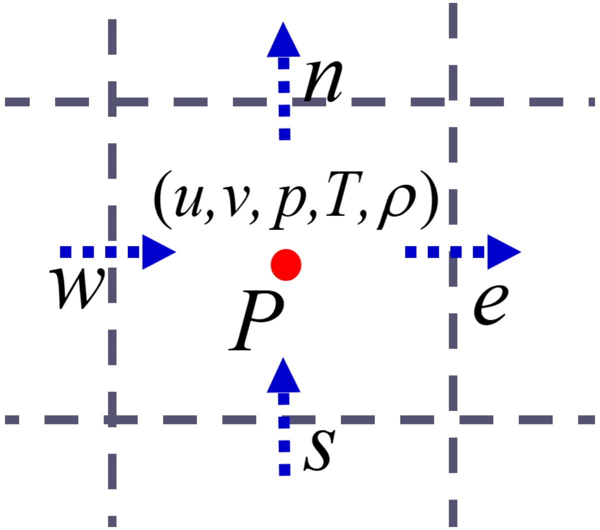
Similarly, $u_E = u_E + \frac{A_P}{a_P})_{\mathrm{E}}(p_w - p_e)_{\mathrm{E}}$

For the interface pressure linear interpolation between neighbor grids may be used.

2)Descretization of mass condervation

By integrating the mass conservation equation over control volume P:

$$(\rho uA)_e-(\rho uA)_w+(\rho vA)_n-(\rho vA)_s=0$$



We don’t have velocity at the four interfaces. How to determine them? It is here that we may introduce the method which can meet the two requirements!

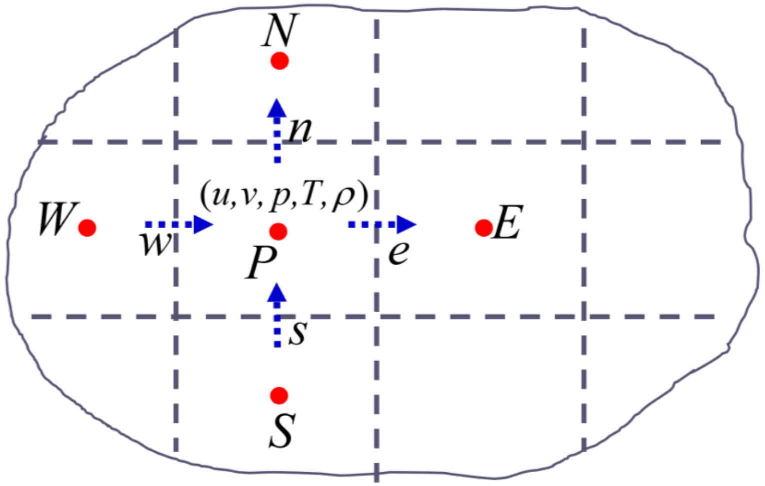
3) Momentum interpolation for interface velocity

Rhie-Chow proposed: the interface velocity should use the form of momentum equation at grids:

$$u_P=u_P+\frac{A_P}{a_P}(p_w-p_e)$$

$$u_e = u_e + \frac{A_P}{a_P})_e(p_P - p_E)$$

$$= u_e - \frac{A_P}{a_P})_e(p_E - p_P)$$



Based on such interface velocity equation we can derive the pressure correction equation which contains the two requirements for guarantee overcoming the checkboard pressure distribution.

Non-staggered grid with the momentum interpolation is called the collocated grid system.

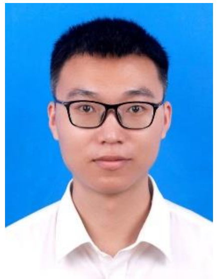
The derivation of pressure correction equation and solution procedure of SIMPLE algorithm at the collocated grids are very similar to those of staggered grids, and can be found in the Textbook（pp. 247-249).

Contents of Numerical Heat Transfer

Part I： Fundamentals of NHT （9 chapters）

Part II： Teaching Code (2 chapters）

Part III of NHT：Study of FLUENT



方文振(Wen-Zhen Fang）

冀文涛（Wen-Tao Ji)

Chapter 12 Basic contents (6 hours)

Applications (6 hours)

任秦龙(Qing-Long Ren)

陈黎（Li Chen）

C 13a Fundamental Applications

C 13b Intermediate Applications

Computer-Aided Project of 2023 Numerical Heat Transfe
Xi’an Jiaotong University

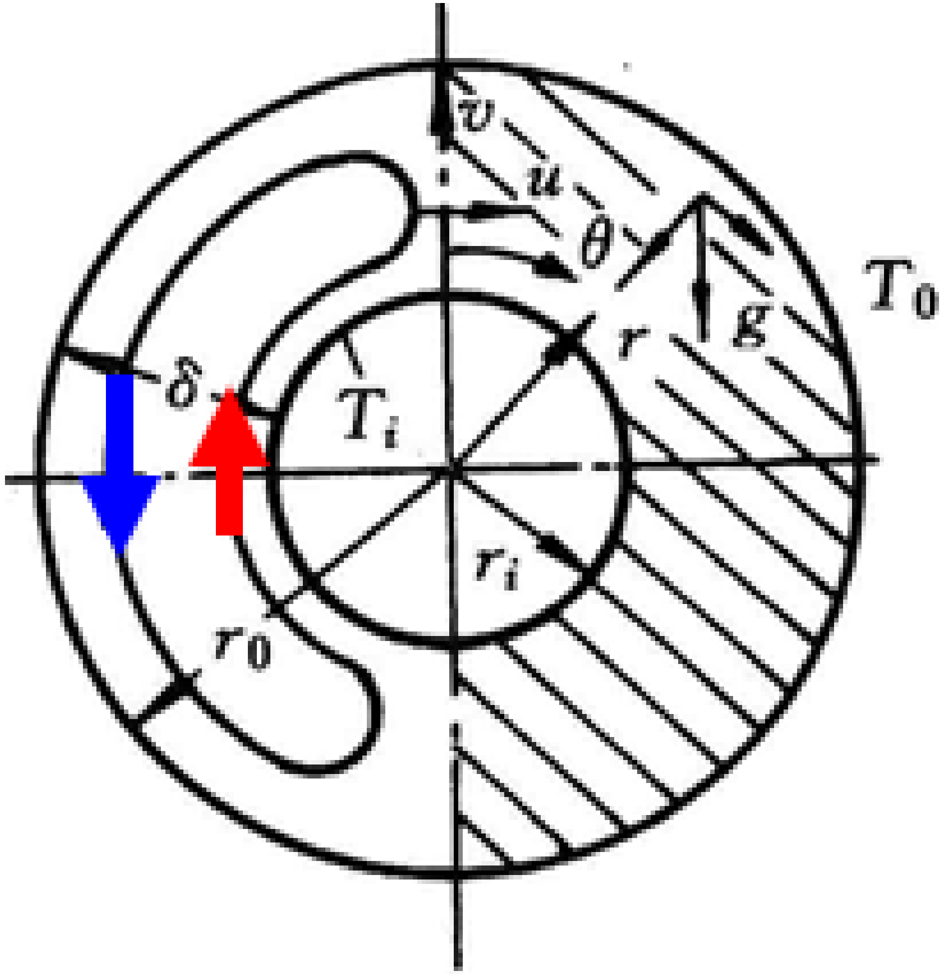
We present three computer-aided projects: one is to be solved by our teaching code (Project 1）, the 2nd and 3rd ones are to be solved by FLUENT (Fundamental , Project 2, Intermediate Project 3) . Every student can choose one project according to your interest and condition.

For the first project the self-developed computer code (USER) should attached in your final report. **Students are encouraged to take Project 1.**

For the second and third project Class F and Class I will have different projects. The instructors will assign the project at the end of the lecture.

Computer-Aided Project (1) of NHT-2023, Xi’an Jiaotong University

（Laminar natural convection in annular space）



$(T_i > T_0)$

1. Project formulation

For air natural convection within an annular space as shown in Fig. 1 , following conditions are given: $\delta / r_o = 0.4$, flow is laminar and the average air temperature is $50^\circ C$ For $\mathrm{Ra} = g\beta\Delta T\delta^3 \nu / a^2 = 10^2, 10^3, 10^4, 10^5$, determine the relative thermal conductivity: $\lambda_{\mathrm{eq}} / \lambda_{air}$ .The temperature difference between inner wall and outer wall is not large, so the Boussinesq assumption can be adopted. By using Tecplot or other software, display the isotherms and streamlines and the variation of $\lambda_{\mathrm{eq}} / \lambda_{air}$ vs. Ra. Natural convection heat transfer rate between the inner and outer surface is expressed by an effective thermal conductivity λ_{eq} as follows:

$$\phi = \frac{2\pi L \lambda_{eq} \Delta T}{\ln(d_2 / d_1)}$$

λ_{eq} is the equivalent thermal conductivity of the entire annular space.

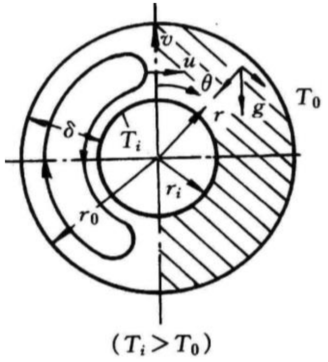
2. Suggestions and Requirements

1) Considering the symmetry of the geometry, only half of the structure should be simulated.
2) The solution should be grid-independent.
3) The project report should be written in the format of the Journal of Xi’an Jiaotong University. Both Chinese and English can be accepted.
4) Examples in teaching codes may be consulted.
5) Please submit in the USER part developed by yourself for solving the problem.

The project report should be due in before April 30, 2024. Teaching assistant group will inform you where to submit your results.

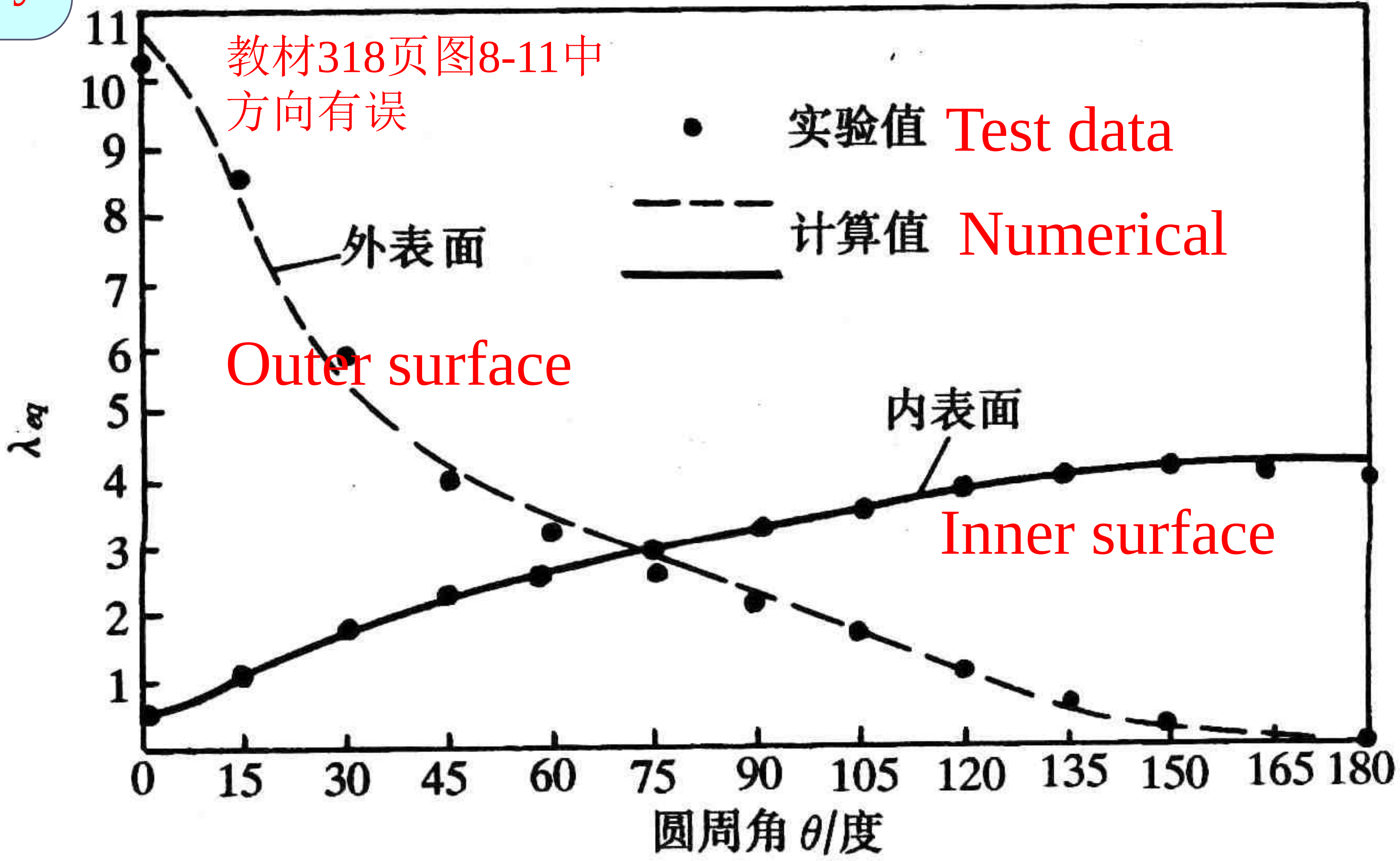
Local dimensionless thermal onductivity

Kuehn T H, Goldstein R J. An experimental and theoretical study of natural convection in the annulus between horizontal concentric cylinders. J Fluid Mech, 1971, 74:605-719



Local relative thermal conductivity

$$\overline{\lambda}_{eq}=\left(\frac{q_{convection}}{q_{conduction}}\right)_{\theta}$$



教材318页图8-11中方向有误

Test data

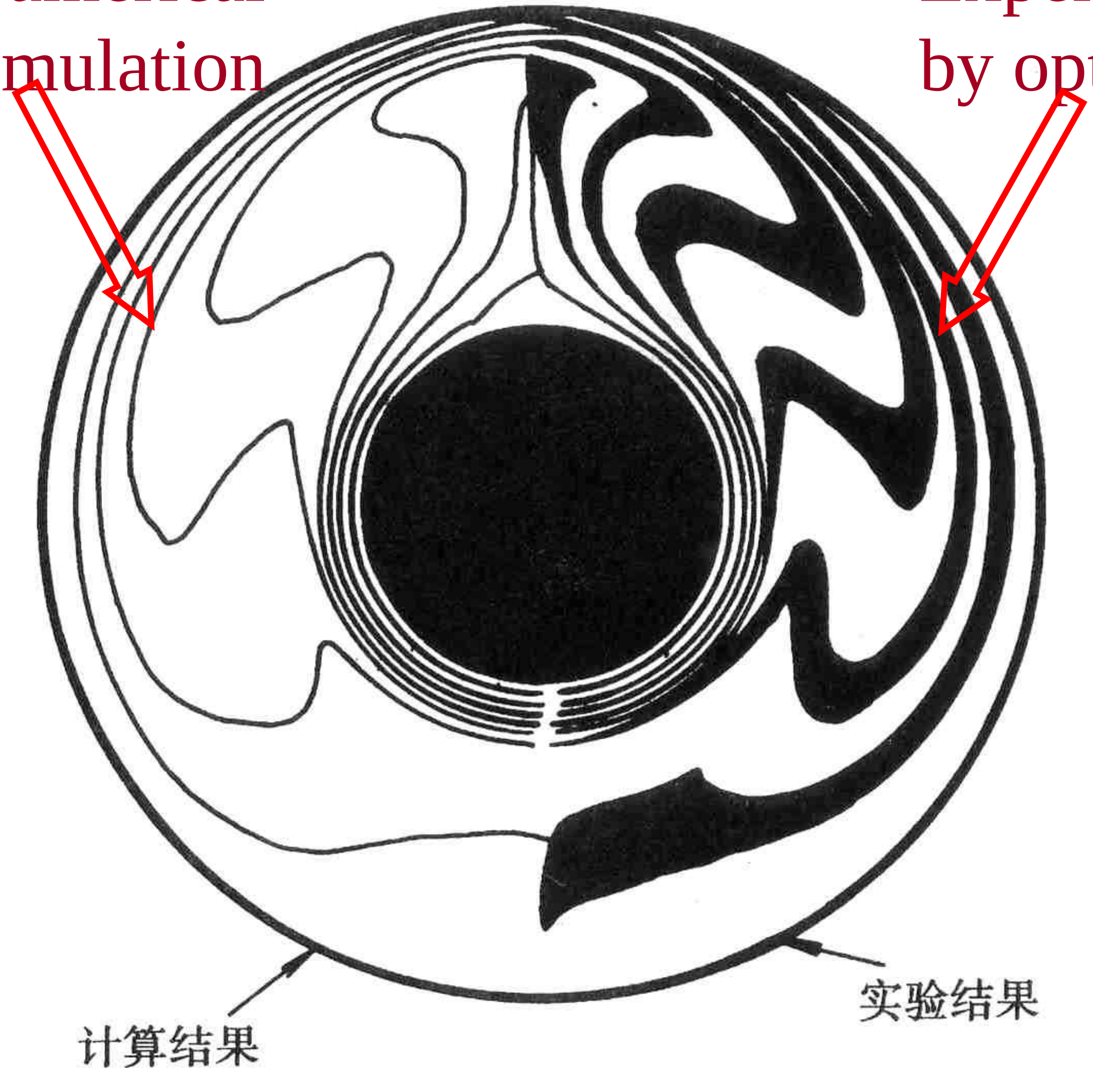
Numerical

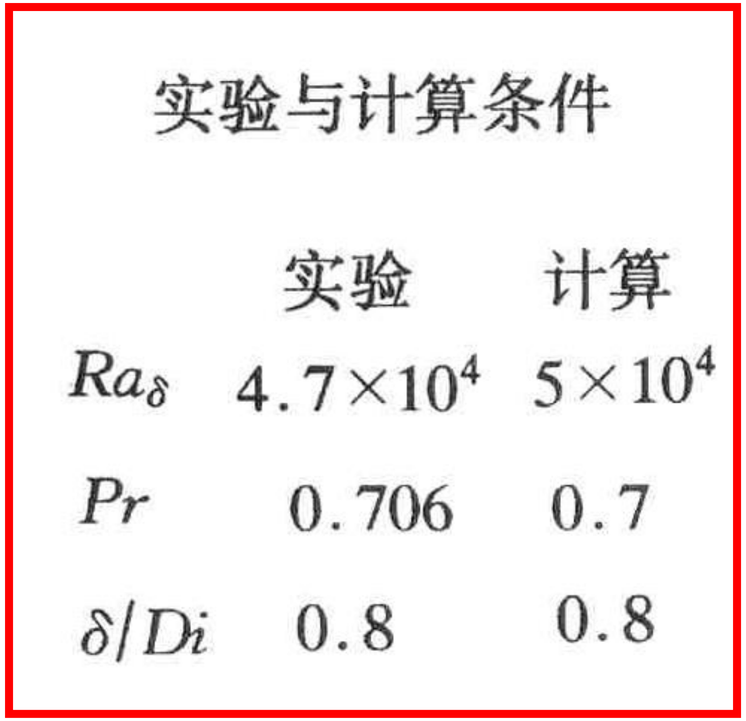
Outer surface

Inner surface

Numerical simulation

Experiment by optical method





实验与计算条件

	实验	计算
Ra_δ	4.7×10^4	5×10^4
Pr	0.706	0.7
δ/Di	0.8	0.8

Comparison of isotherms (等温线)

本组网页地址：http://nht.xjtu.edu.cn　欢迎访问！

Teaching PPT will be loaded on ou website

同舟共济

渡彼岸！

People in the same boat help each other to cross to the other bank, where….