# Numerical Heat Transfer

# (数值传热学)

## Chapter 11 Application Examples of the General Code for 2D Elliptical FF & HT Problems (3)

**Instructor：Fang, Wen-Zhen; Tao, Wen-Quan**

**Email: fangwenzhen@xjtu.edu.cn**

Key Laboratory of Thermo-Fluid Science & Engineering
Xi'an Jiaotong University

2023-Nov-29

11.1 2D steady heat conduction without source term in Cartesian coordinate

11.2 Steady heat conduction in a hollow cylinder

11.3 Fully-developed heat transfer in a square duct

11.4 Fully developed heat transfer in annular space with straight fin at inner wall

11.5 Fluid flow and heat transfer in a 2-D sudden expansion

11.6 Complicated fully developed fluid flow and heat transfer in square duct

---

11.7 Impinging flow on a rotating disc

11.8 Turbulent flow and heat transfer in duct with a central jet

# 11-7 Impinging flow on a rotating disc
## ---Discretization of source term of momentum equation in cylindrical coordinate

### 11-7-1 Physical problem and its math formulation

**Known**: A rotating disc with $\omega$=100 is partially covered by a shell (壳体). Fluid flows into the shell through the central inlet of the shell with inlet velocity $U_{in}=100$; impinges onto the disc and then leaves the disc (盘) through the gap between the shell and the disc. Fluid viscosity $\eta=1$.

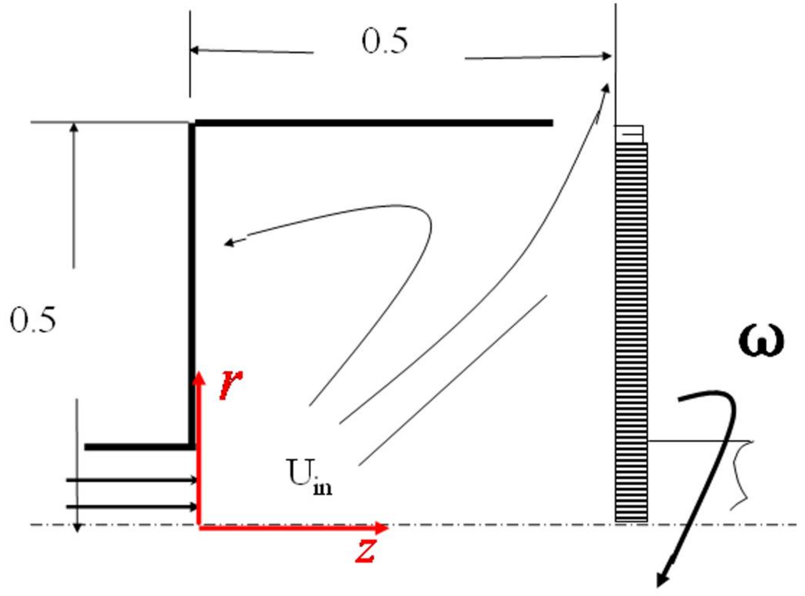
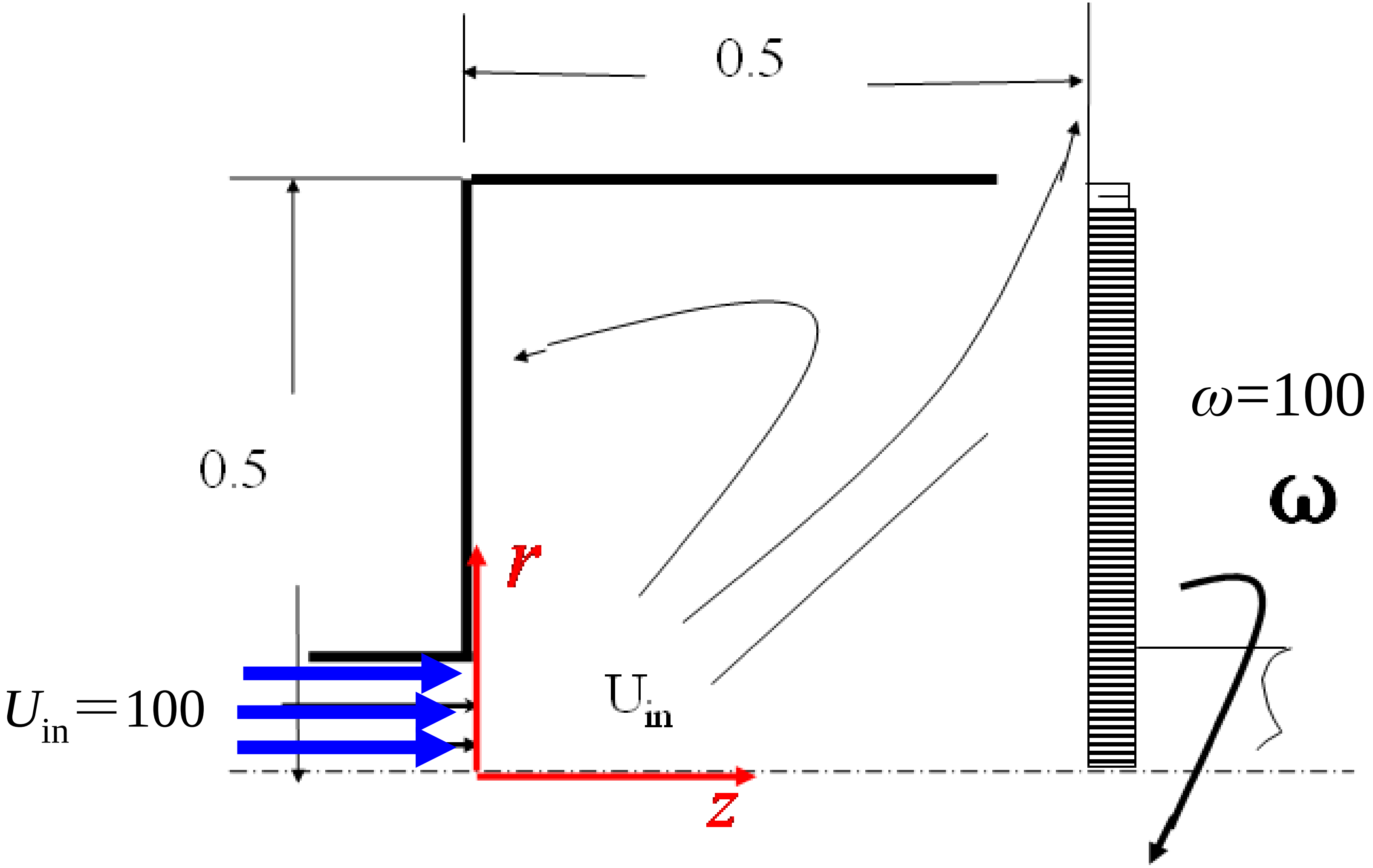
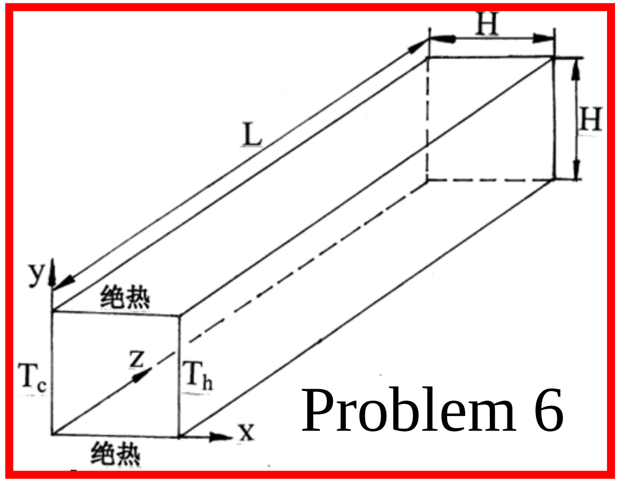
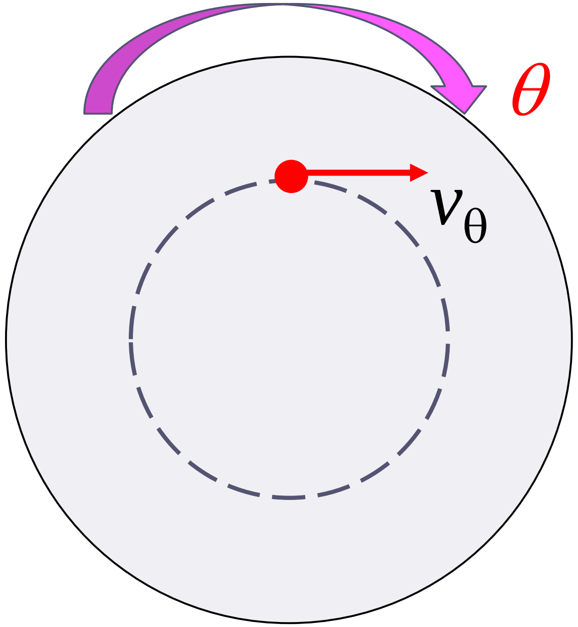


Fig.1 Schematic diagram of problem 7



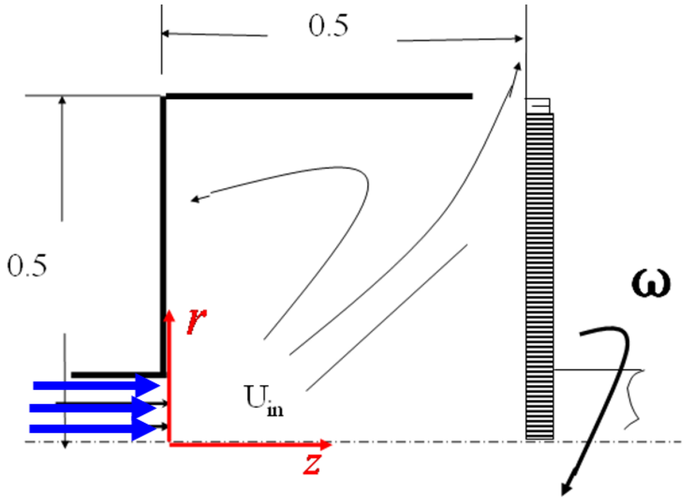




No change along the circumferential ($\theta$) direction（圆周方向）

**Find**： Velocity and pressure distribution in the cavity.

**Solution**： This is a fluid flow problem in three-dimensional cylindrical coordinate. The fluid flow is caused by the impingement of the inlet flow and the rotating effect of the disc. The circumferential velocity, $v_\theta$, is purely caused by the rotating disc. Thus, there exists $v_\theta$, but no circumferential pressure drop. The velocity along the $\theta$ direction is uniform when in steady state.




Circumferential $\theta$

- Original N-S eqs. in cylindrical coordinate are:

$$z\ \text{direction}: \rho\left(v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}+\cancel{\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}}\right)=-\frac{\partial p}{\partial z}+\eta\left(\frac{\partial^2 v_z}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\cancel{\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}}\right)$$

$$r\ \text{direction}: \rho\left(v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}+\cancel{\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}}\right)=-\frac{\partial p}{\partial r}+\eta\left(\frac{\partial^2 v_r}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right)+\cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}}-\cancel{\frac{2}{r}\frac{\partial v_\theta}{\partial \theta}}\right)$$

$$+\rho\frac{v_\theta^2}{r}-\eta\frac{v_r}{r^2}$$

**Source term**

$$\theta\ \text{direction}: \rho\left(v_r\frac{\partial v_\theta}{\partial r}+v_z\frac{\partial v_\theta}{\partial z}+\cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}}\right)=0+\eta\left(\frac{\partial^2 v_\theta}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right)+\cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}}+\cancel{\frac{2}{r}\frac{\partial v_\theta}{\partial \theta}}\right)$$

**Source term**

$$-\rho\frac{v_r v_\theta}{r}-\eta\frac{v_\theta}{r^2}$$

$\boxed{\textbf{Zero pressure gradient!}}$

**There exists $v_\theta$, but $\dfrac{\partial}{\partial \theta}$ should be zero for this problem.**

➢ Thus, governing equations of the three velocities are:

$$z\ \text{direction}: \rho(v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z})=-\frac{\partial p}{\partial z}+\eta(\frac{\partial^2 v_z}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_z}{\partial r}))$$

$$r\ \text{direction}: \rho(v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z})=-\frac{\partial p}{\partial r}+\eta(\frac{\partial^2 v_r}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_r}{\partial r}))$$

$$+\rho\frac{v_\theta^2}{r}-\eta\frac{v_r}{r^2}$$

**Source term**

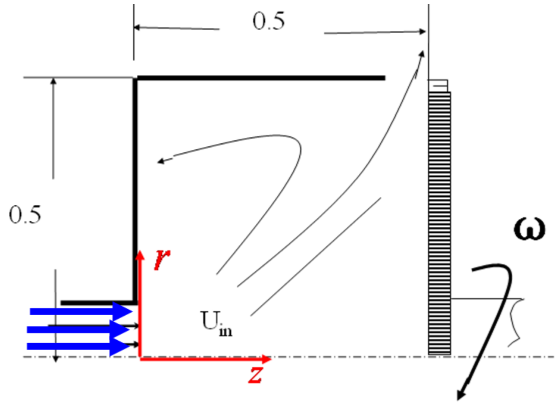
$$\theta\ \text{direction}: \quad \rho(v_r\frac{\partial v_\theta}{\partial r}+v_z\frac{\partial v_\theta}{\partial z})= 0+\eta(\frac{\partial^2 v_\theta}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_\theta}{\partial r}))$$

$$-\rho\frac{v_r v_\theta}{r}-\eta\frac{v_\theta}{r^2}$$

**Source term**

# 11-7-2 Numerical method

**(1)** There are **three** velocity components, but **no terms contain** $\partial/\partial\theta$, such as no terms with $\partial/\partial z$ in Example 6.




$$v_z: \quad \rho\left(v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\eta\left(\frac{\partial^2 v_z}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right)$$

$$v_r: \quad \rho\left(v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\eta\left(\frac{\partial^2 v_r}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right)\right)+\rho\frac{v_\theta^2}{r}-\eta\frac{v_r}{r^2}$$

$$v_\theta: \quad \rho\left(v_r\frac{\partial v_\theta}{\partial r}+v_z\frac{\partial v_\theta}{\partial z}\right)=\eta\left(\frac{\partial^2 v_\theta}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right)\right)-\rho\frac{v_r v_\theta}{r}-\eta\frac{v_\theta}{r^2}$$

**scalar variable**

**(2)** $v_\theta$ is not in convection terms of $v_z, v_r$, but it is included in source term of $v_r$. Thus, $v_\theta$ can be viewed as a **scalar variable** (such as $T$) coupled with $v_r, v_z$; it is a 2-D cylindrical case with MODE＝2.

**(3)** In $v_\theta$ eq., $rv_\theta$ can be taken as the variable to be solved to enhance solution stability.

The original $v_\theta$ momentum equation is:

$$\rho\left(v_r\frac{\partial v_\theta}{\partial r}+v_z\frac{\partial v_\theta}{\partial z}\right)=\eta\left(\frac{\partial^2 v_\theta}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right)\right)-\rho\frac{v_r v_\theta}{r}-\eta\frac{v_\theta}{r^2}$$

**It is transformed to**: $\rho\left(v_r\frac{\partial (rv_\theta)}{\partial r}+v_z\frac{\partial (rv_\theta)}{\partial z}\right)=$

$$\eta\left(\frac{\partial^2 (rv_\theta)}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial (rv_\theta)}{\partial r}\right)\right)-\frac{2\eta}{r}\frac{\partial (rv_\theta)}{\partial r}$$

$rv_\theta$ taken as variable

（4）Numerical treatment of source term in $v_r$ :

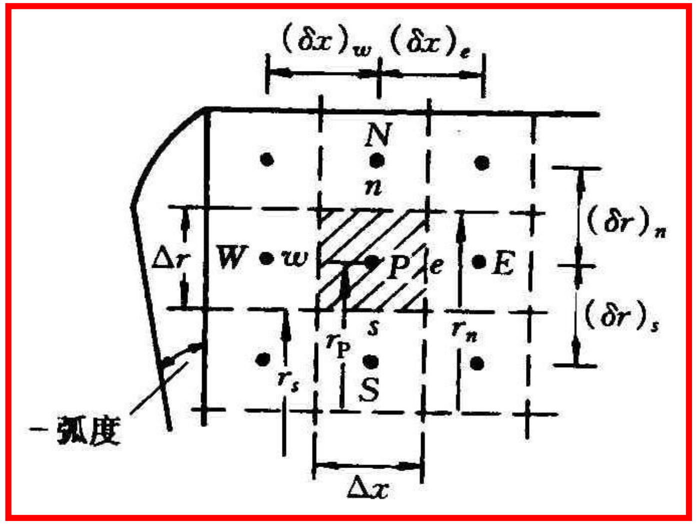
$$\rho\left(v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\eta\left(\frac{\partial^2 v_r}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right)\right)+\rho\frac{v_\theta^2}{r}-\eta\frac{v_r}{r^2}$$

$$S_{v_r}=\rho\frac{v_\theta^2}{r}-\eta\frac{v_r}{r^2}=\rho\frac{(rv_\theta)^2}{r^3}\underbrace{-\eta\frac{1}{r^2}}_{S_P}v_r \qquad S_\phi=S_c+S_p\phi$$

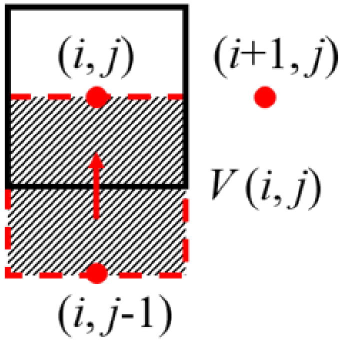
# Numerical treatment of source term of $rv_\theta$

$$\rho\left(v_r \frac{\partial (rv_\theta)}{\partial r} + v_z \frac{\partial (rv_\theta)}{\partial z}\right) = \eta\left(\frac{\partial^2 (rv_\theta)}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial (rv_\theta)}{\partial r}\right)\right) - \frac{2\eta}{r}\frac{\partial (rv_\theta)}{\partial r}$$

$$S_{(rv_\theta)} = -\frac{2\eta}{r}\frac{\partial (rv_\theta)}{\partial r} = -\frac{2\eta}{r_P}\frac{(rv_\theta)_n - (rv_\theta)_s}{YCV(j)}$$




Idea of FUD

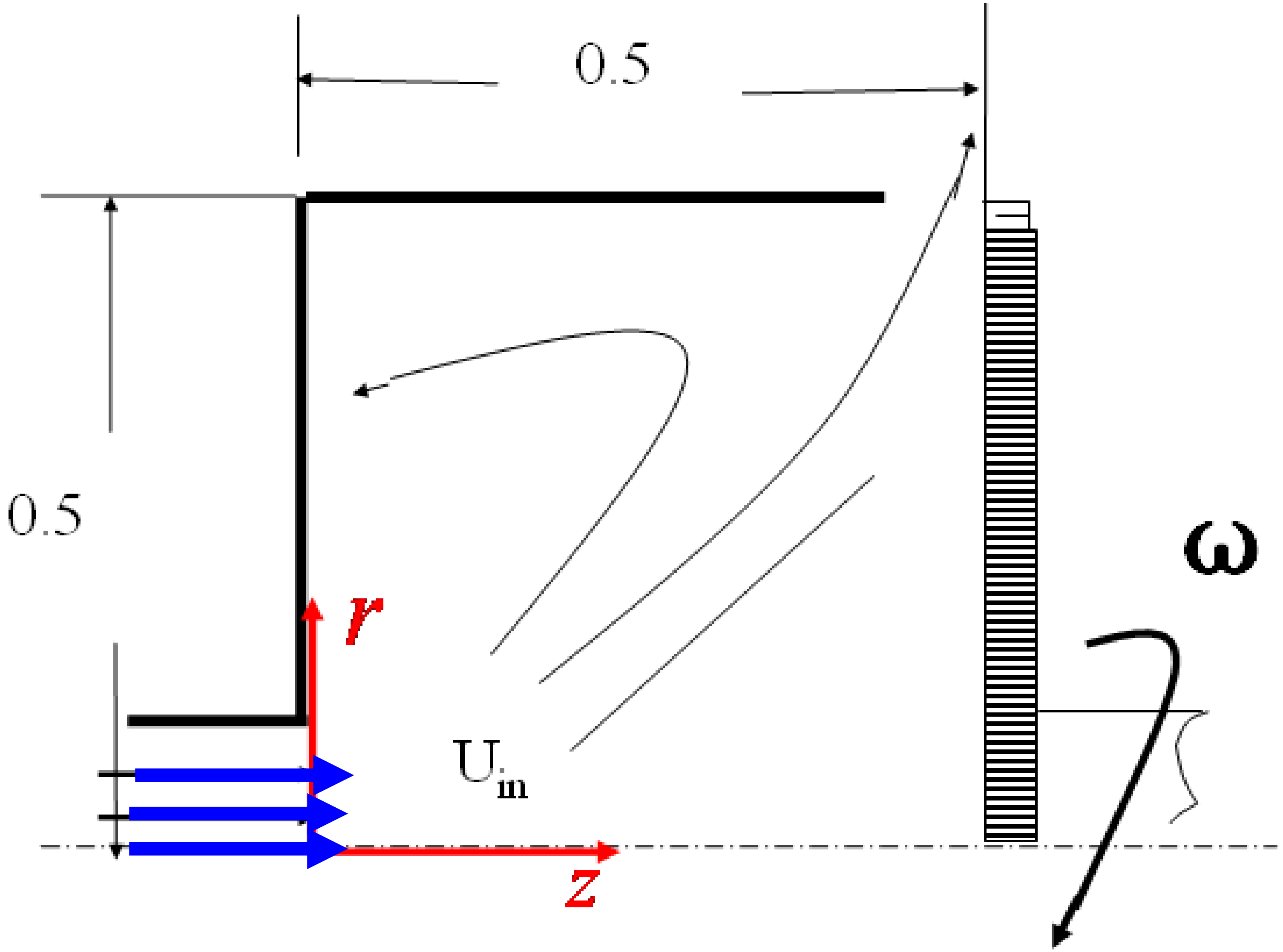



$$-\frac{2}{r_P}\frac{\eta}{YCV(j)}[(rv_\theta)_P - (rv_\theta)_S]$$

$$= \underbrace{\frac{2}{r_P}\frac{\eta (rv_\theta)_S}{YCV(j)}}_{S_C} - \underbrace{\frac{2}{r_P}\frac{\eta}{YCV(j)}}_{S_P}(rv_\theta)_P$$

Introducing a minor value of $S_p$ to enhance solution stability.

# Fig.1 Schematic diagram of Example 7



# 11-7-3 Program reading

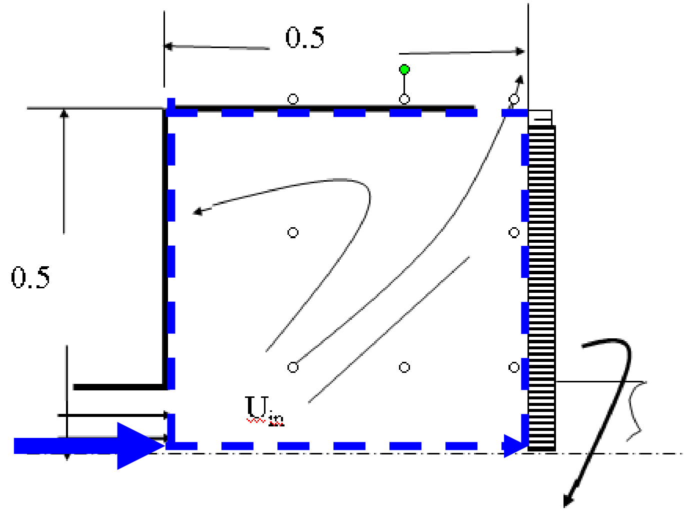
```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C*******************************************************************
       INTEGER*4 I,J
        REAL*8 OMEGA, UIN, AMU, FLOWIN, AR, ADD, FL,
      1 RSWM, RHOM, FLT
        END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
       SUBROUTINE USER
C********************************************************************
       USE START_L
       USE USER_L
       IMPLICIT NONE
C*********************************************************************
C--------------------------------PROBLEM EIGHT-------------------------------------
C                          Laminar impinging flow over a rotating disk
C*********************************************************************
```

**ENTRY GRID**

```
TITLE(1)=' .VEL  U.'
TITLE(2)=' .VEL  V.'
TITLE(3)=' .STR FN.'
TITLE(5)=' .R.VTH.'
TITLE(11)='PRESSURE'
RELAX(1)=0.8
RELAX(2)=0.8
LSOLVE(1)=.TRUE.
LSOLVE(5)=.TRUE.
LPRINT(1)=.TRUE.
LPRINT(2)=.TRUE.
LPRINT(3)=.TRUE.
LPRINT(5)=.TRUE.
LPRINT(11)=.TRUE.
LAST=25
MODE=2
R(1)=0.
XL=0.5
YL=0.5
L1=7
M1=7
CALL UGRID
RETURN
```

Regarding ($rv_\theta$) as 5$^{th}$ variable

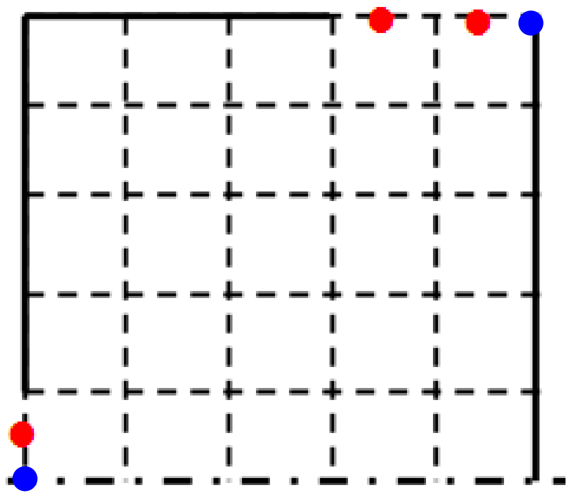
In SIMPLER code, when the 1$^{st}$ variable is set to be solved, the 2$^{nd}$ and 3$^{rd}$ ones are automatically solved.

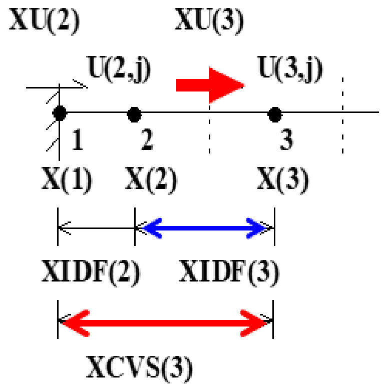



```
      ENTRY START
      OMEGA=100.
      UIN=100.
      DO 100 J=1,M1
      DO 101 I=1,L1
      U(I,J)=0.
      V(I,J)=0.
      F(I,J,5)=0.
      F(L1,J,5)=R(J)**2*OMEGA
101   ENDDO          ! Velocity on disc, causing circumferential flow
100   ENDDO
      U(2,2)=UIN
      AMU=1.
      RETURN
*
      ENTRY DENSE
      RETURN
```

5th variable is R.VTheta

$$\boxed{r \bullet v_{\theta} = r \cdot \omega r = \omega r^{2}}$$





One way for obtaining outlet velocity of open system:

Assuming that the 1st derivatives at outlet =constant

$$\frac{v_{i,M1}-v_{i,M2}}{\Delta y}=k=const \longrightarrow v_{i,M1}=v_{i,M2}+k\Delta y=v_{i,M2}+C$$

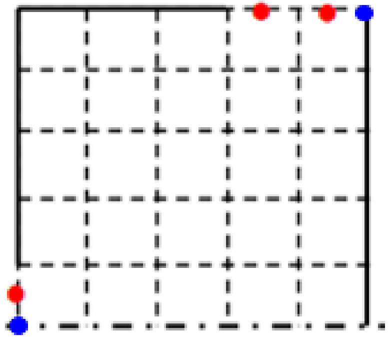
**$C$ is determined according to total mass conservation**

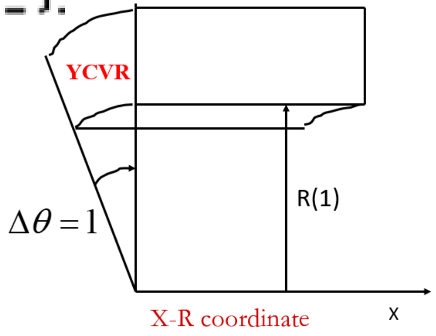
$$\sum_{i=2}^{L2}\rho_{i,M1}(v_{i,M2}+C)\Delta x_i=FLOWIN \longrightarrow$$

$$C=\frac{FLOWIN-\sum\rho_{i,M1}v_{i,M2}\Delta x_i}{\sum\rho_{i,M1}\Delta x_i}$$

$\underline{v_{i,M1}=v^*_{i,M2}+C}$ is taking as boundary condition for next iteration.

**In this example this method is used**






```
ENTRY BOUND
 IF(ITER.NE.0)   FLOWIN=RHO(1,2)*U(2,2)*YCVR(2)
 FL=0.
 AR=0.
 DO 301 I=L3,L2
 FLT=R(M1)*XCV(I)*RHO(I,M1)
 AR=AR+FLT   ! Denominator
 FL=FL+FLT*V(I,M2)
             ! 2nd part of the Numerator
301 ENDDO
 ADD=(FLOWIN-FL)/AR
 DO 302 I=L3,L2
 V(I,M1)=V(I,M2)+ADD
                  ! C---ADD
302 ENDDO
 RETURN
```

! FLOWIN =

$$\sum \rho(I,M1) \bullet XCV(I,M1) \bullet R(M1) \bullet (V(I,M2)+C)$$

$$! \quad C = \frac{\text{FLOWIN-}\sum \rho(i,M1) \bullet XCV(i) \bullet R(M1) \bullet V(i,M2)}{\sum \rho(i,M1) \bullet XCV(i) \bullet R(M1)}$$

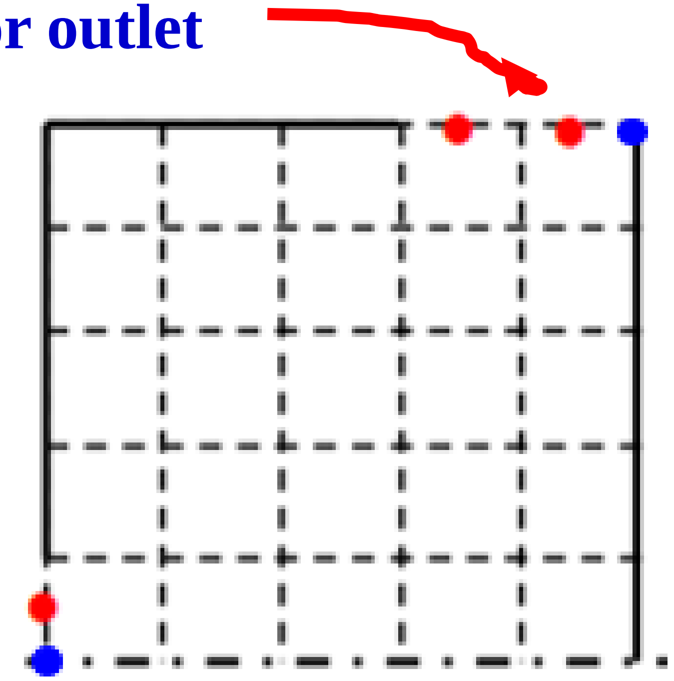
! C-method is adopted to guarantee the total mass conservation condition

```
      ENTRY OUTPUT
       IF(ITER==0) THEN
       PRINT 401
       WRITE(8,401)
 401 FORMAT(1X,'  ITER',7X,'SMAX',11X,'SSUM',10X,'U(4,4)',
      & 9X,'V(4,4)')
        ELSE
        PRINT 403
        WRITE(8,403) ITER,SMAX,SSUM,U(4,4),V(4,4)
 403 FORMAT(1X,I6,1P5E15.4)
        ENDIF
        IF(ITER==LAST) CALL PRINT
        RETURN
```

```
      ENTRY GAMSOR
      IF(ITER== 0) THEN                 ! Constant viscosity, calculation once is
                                          enough
      DO 500 J=1,M1
      DO 501 I=1,L1
      GAM(I,J)=AMU
501 ENDDO
502 ENDDO
      GAM(L3,M1)=0.                     ! Local one-way for outlet
      GAM(L2,M1)=0.
      ENDIF

   ! GAM(1:L1, 1)=0   ??
```

No needed in cylindrical coordinate



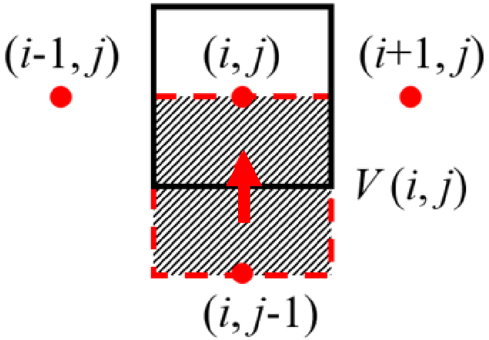
```
IF(NF== 2) THEN
 DO 502 J=3,M2
 DO 503 I=2,L2
 RSWM=FY(J)*F(I,J,5)+FYM(J)*F(I,J-1,5)
 RHOM=FY(J)*RHO(I,J)+FYM(J)*RHO(I,J-1)
 CON(I,J)=RHOM*RSWM**2/RMN(J)**3
 AP(I,J)=-AMU/RMN(J)**2
503 ENDDO
502 ENDDO
 ENDIF
```

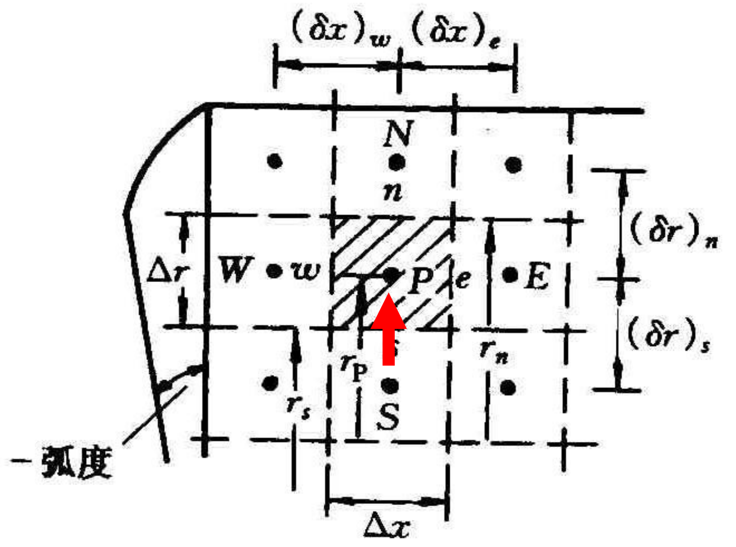
**! Source term of $v_r$ –eq.**

**! $rv_\theta$ Is interpolated from main nodes**

**! Interface density is interpolated from node density for the source term of $v_r$**

$$S_{v_r} = \rho\frac{v_\theta^2}{r} - \eta\frac{v_r}{r^2} = \underline{\rho\frac{(rv_\theta)^2}{r^3}} \underline{-\eta\frac{1}{r^2}} v_r$$



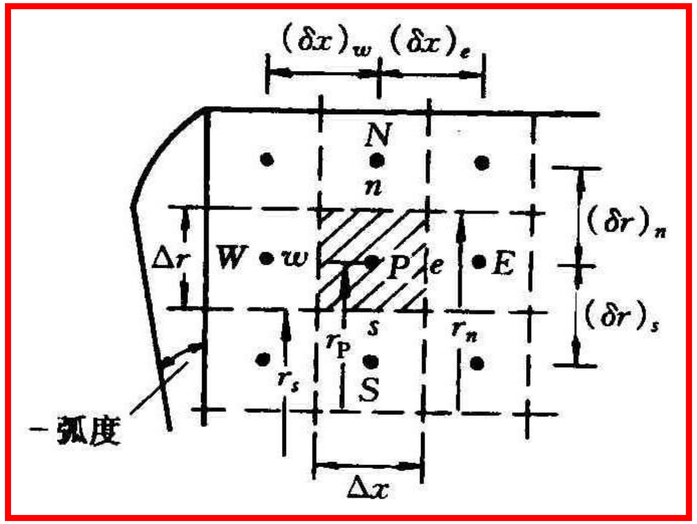
```
510 IF(NF/=5) RETURN
    DO 512 J=2,M2
    DO 513 I=2,L2
    AR=2.*AMU/YCVR(J)
    CON(I,J)=AR*F(I,J-1,5)
    AP(I,J)=-AR
512 ENDDO
513 ENDDO
    RETURN
    END
```

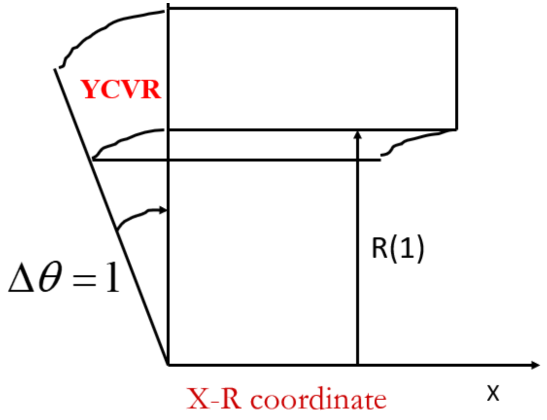
**! Source term of $rv_\theta$ is calculated at main node**

$$S_{(rv_\theta)} = \frac{2}{r_P}\frac{\eta (rv_\theta)_S}{YCV(j)} - \frac{2}{r_P}\frac{\eta}{YCV(j)}(rv_\theta)_P$$

$$= \underline{\frac{2\eta}{YCVR(j)}(rv_\theta)_S} - \underline{\frac{2\eta}{YCVR(j)}}(rv_\theta)_P$$

**CON(I,J)=AR*F(I,J-1,5)** **AP(I,J)=-AR**



# 11-7-4 Results analysis

COMPUTATION FOR AXISYMMETRICAL SITUATION

*********************************************************

| ITER | SMAX | SSUM | U(4,4) | V(4,4) |
|---|---|---|---|---|
| 0 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| 1 | 3.1852E-01 | 0.0000E+00 | 3.3742E+00 | 4.8158E+00 |
| 2 | 3.6224E-01 | 1.1921E-07 | 2.9314E+00 | 7.6065E+00 |
| 3 | 1.1265E-01 | 7.4506E-09 | 1.8755E+00 | 8.5863E+00 |
| 4 | 6.1974E-02 | -3.7253E-08 | 1.5199E+00 | 8.8029E+00 |
| 5 | 3.2279E-02 | -3.1665E-08 | 1.2971E+00 | 8.4019E+00 |
| 6 | 1.7869E-02 | -4.0280E-08 | 1.2738E+00 | 7.6836E+00 |
| 7 | 1.2370E-02 | 5.1223E-09 | 1.3363E+00 | 6.8852E+00 |
| 8 | 1.0312E-02 | -1.1176E-08 | 1.4400E+00 | 6.1421E+00 |
| 9 | 7.9294E-03 | -2.9569E-08 | 1.5480E+00 | 5.5244E+00 |
| 10 | 5.9429E-03 | 4.8894E-08 | 1.6437E+00 | 5.0452E+00 |
| 11 | 4.6140E-03 | -1.6531E-08 | 1.7207E+00 | 4.6926E+00 |
| 12 | 3.3741E-03 | 3.1199E-08 | 1.7787E+00 | 4.4432E+00 |
| 13 | 2.6291E-03 | -5.1106E-08 | 1.8202E+00 | 4.2728E+00 |

| | | | !U(4,4) | !V(4,4) |
|---|---|---|---|---|
| 14 | 1.9695E-03 | -2.6543E-08 | 1.8486E+00 | 4.1597E+00 |
| 15 | 1.4364E-03 | 6.2981E-08 | 1.8674E+00 | 4.0867E+00 |
| 16 | 1.0142E-03 | -4.5111E-08 | 1.8792E+00 | 4.0409E+00 |
| 17 | 6.9815E-04 | 8.9640E-09 | 1.8864E+00 | 4.0129E+00 |
| 18 | 4.6667E-04 | 3.8388E-08 | 1.8906E+00 | 3.9963E+00 |
| 19 | 3.0389E-04 | 3.3469E-09 | 1.8929E+00 | 3.9868E+00 |
| 20 | 1.9290E-04 | -1.1176E-08 | 1.8941E+00 | 3.9816E+00 |
| 21 | 1.1830E-04 | 5.2169E-09 | 1.8946E+00 | 3.9790E+00 |
| 22 | 7.0846E-05 | 4.6941E-08 | 1.8947E+00 | 3.9778E+00 |
| 23 | 4.0823E-05 | 5.4388E-08 | 1.8947E+00 | 3.9773E+00 |
| 24 | 2.2590E-05 | -8.0094E-08 | 1.8945E+00 | 3.9772E+00 |
| 25 | 1.1003E-05 | -3.8743E-08 | 1.8944E+00 | 3.9773E+00 |

****************************** .VEL U. *****************************

| I = | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| J | | | | | | |
| 7 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| 6 | 0.00E+00 | -1.33E+00 | -2.67E+00 | -2.12E+00 | -8.37E-01 | 0.00E+00 |
| 5 | 0.00E+00 | -1.86E+00 | -2.70E+00 | -1.86E+00 | -6.39E-01 | 0.00E+00 |
| 4 | 0.00E+00 | -2.17E-01 | 1.89E+00 | 2.90E+00 | 1.65E+00 | 0.00E+00 |
| 3 | 0.00E+00 | 1.33E+01 | 1.97E+01 | 1.92E+01 | 1.04E+01 | 0.00E+00 |
| 2 | 1.00E+02 | 8.63E+01 | 7.43E+01 | 5.99E+01 | 3.27E+01 | 0.00E+00 |
| 1 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |

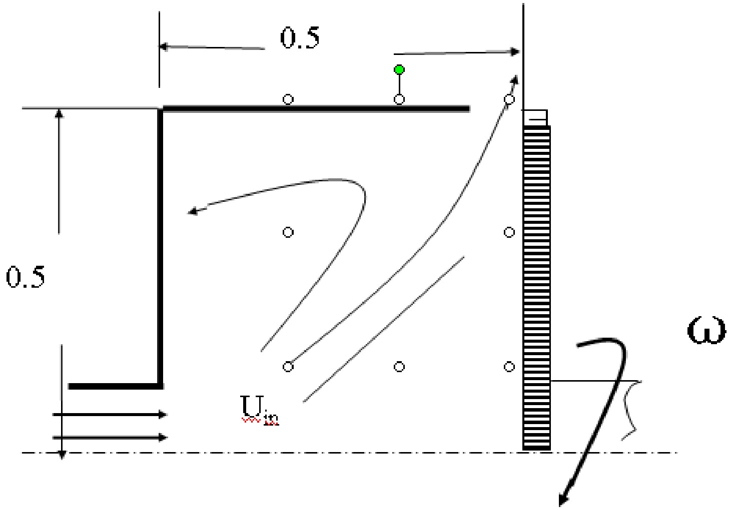
****************************** .VEL V. ******************************

| I = / J | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 7 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 3.99E+00 | 6.01E+00 | 0.00E+00 |
| 6 | 0.00E+00 | -1.50E+00 | -1.50E+00 | 6.18E-01 | 6.44E+00 | 8.45E+00 | 0.00E+00 |
| 5 | 0.00E+00 | -4.17E+00 | -2.98E+00 | 1.81E+00 | 1.00E+01 | 1.20E+01 | 0.00E+00 |
| 4 | 0.00E+00 | -6.53E+00 | -1.84E+00 | 3.98E+00 | 1.34E+01 | 1.60E+01 | 0.00E+00 |
| 3 | 0.00E+00 | 6.87E+00 | 5.96E+00 | 7.21E+00 | 1.36E+01 | 1.63E+01 | 0.00E+00 |
| 2 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |

************************* .STR FN. *************************

| J \ I = | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 7 | 5.00E-01 | 5.00E-01 | 5.00E-01 | 5.00E-01 | 3.00E-01 | 0.00E+00 |
| 6 | 5.00E-01 | 5.60E-01 | 6.20E-01 | 5.95E-01 | 3.38E-01 | 0.00E+00 |
| 5 | 5.00E-01 | 6.25E-01 | 7.15E-01 | 6.60E-01 | 3.60E-01 | 0.00E+00 |
| 4 | 5.00E-01 | 6.31E-01 | 6.67E-01 | 5.88E-01 | 3.19E-01 | 0.00E+00 |
| 3 | 5.00E-01 | 4.31E-01 | 3.72E-01 | 3.00E-01 | 1.63E-01 | 0.00E+00 |
| 2 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |

**At the shell flow rate is constant**

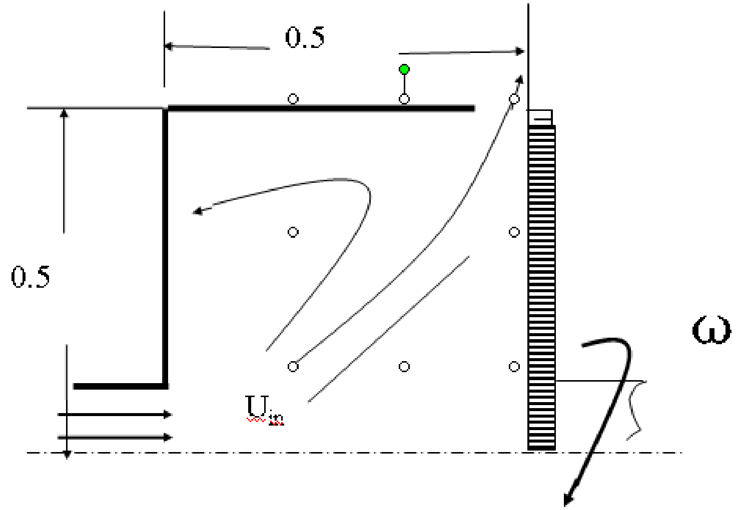



**Zero flow rate on disc**

**************************** R. VTH ****************************

| J \ I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 7 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 2.50E+01 |
| 6 | 0.00E+00 | 1.24E-01 | 5.24E-01 | 1.64E+00 | 5.76E+00 | 1.26E+01 | 2.02E+01 |
| 5 | 0.00E+00 | 2.02E-01 | 7.28E-01 | 1.69E+00 | 3.66E+00 | 7.75E+00 | 1.23E+01 |
| 4 | 0.00E+00 | 1.40E-01 | 4.46E-01 | 8.49E-01 | 1.53E+00 | 3.54E+00 | 6.25E+00 |
| 3 | 0.00E+00 | 5.15E-02 | 1.49E-01 | 2.47E-01 | 3.84E-01 | 1.09E+00 | 2.25E+00 |
| 2 | 0.00E+00 | 4.66E-03 | 1.84E-02 | 3.72E-02 | 5.53E-02 | 1.55E-01 | 2.50E-01 |
| 1 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |

$\varpi * r^2$



**************************** PRESSURE ****************************

| J \ I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 7 | -4.93E+02 | -4.81E+02 | -4.57E+02 | -3.68E+02 | -3.47E+02 | -3.61E+02 | -3.61E+02 |
| 6 | -5.08E+02 | -4.96E+02 | -4.72E+02 | -3.94E+02 | -3.61E+02 | -3.61E+02 | -3.61E+02 |
| 5 | -5.38E+02 | -5.26E+02 | -5.02E+02 | -4.46E+02 | -3.89E+02 | -3.61E+02 | -3.47E+02 |
| 4 | -6.85E+02 | -6.47E+02 | -5.72E+02 | -4.92E+02 | -3.60E+02 | -2.41E+02 | -1.81E+02 |
| 3 | -1.15E+03 | -9.63E+02 | -5.97E+02 | -4.57E+02 | -1.85E+02 | 1.02E+02 | 2.46E+02 |
| 2 | -3.01E+02 | -3.62E+02 | -4.84E+02 | -3.04E+02 | 1.83E+02 | 6.20E+02 | 8.39E+02 |
| 1 | 0.00E+00 | -6.11E+01 | -4.27E+02 | -2.28E+02 | 3.67E+02 | 8.79E+02 | **1.10E+03** |

$p_{min}$

$p_{max}$

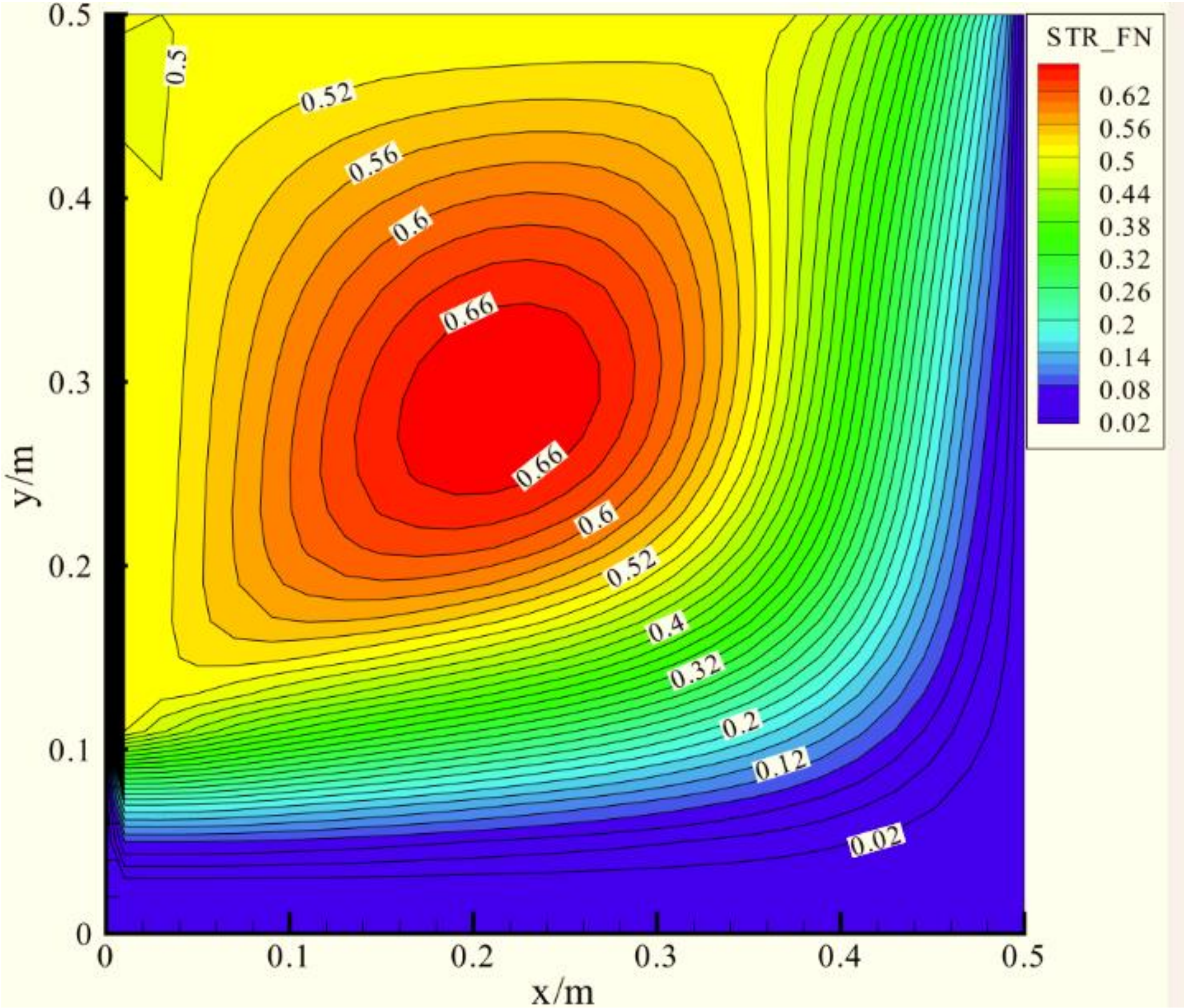


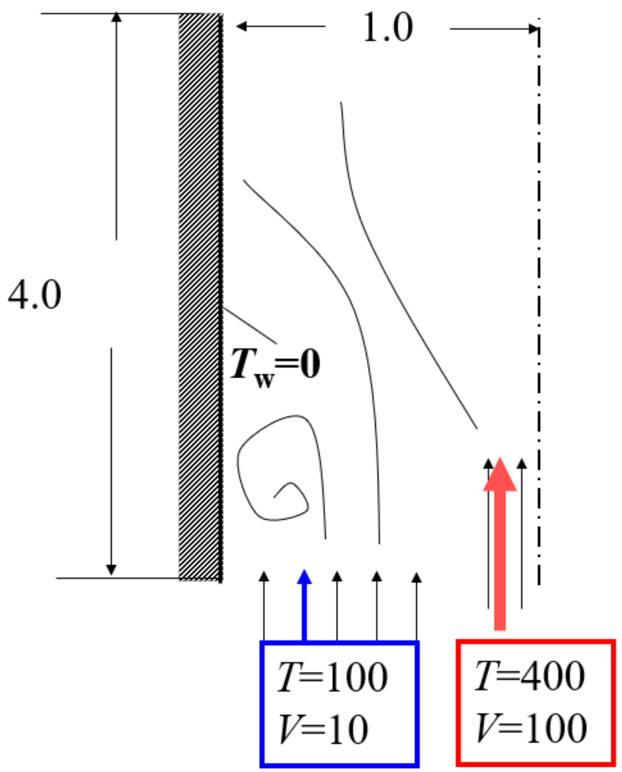

Fig.2 Schematic diagram of Section 7

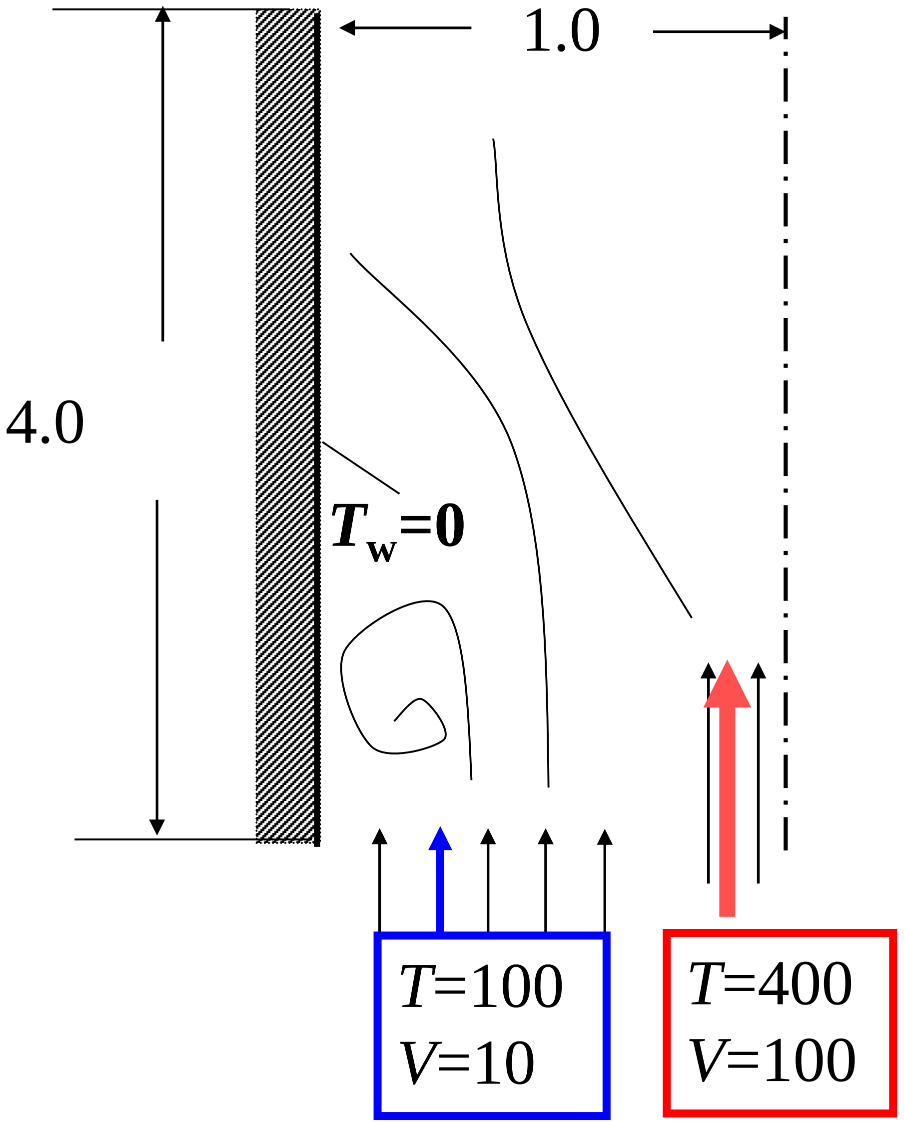
# 11-8 Turbulent flow and heat transfer in duct with a stepwise inlet velocity distribution ---k-epsilon turbulence model with WFM

## 11-8-1 Physical problem and its math formulation

**Known:** A stream with a central jet goes into a parallel channel; Flow is in turbulent state, $\eta = 10^{-6}$ and $Pr = 0.7$.

**Find:** Adopt the standard $k$-$\varepsilon$ model and the wall function method to determine velocity and temperature fields in the channel.






Flow is in turbulent state, $\eta = 10^{-6}$ and $Pr$ = 0.7

**Fig. 1 of Example 8**

# Governing equations

$$\begin{cases} \dfrac{\partial u_k}{\partial x_k} = 0 \\ \dfrac{\partial u_i}{\partial t} + \dfrac{\partial(\rho u_k u_i)}{\partial x_k} = -\dfrac{\partial p_{eff}}{\partial x_i} + \dfrac{\partial}{\partial x_k}[\overbrace{(\eta_l + \eta_t)}^{\eta_{\text{eff}}}\dfrac{\partial u_i}{\partial x_k}] + S_i \quad ; p_{eff} = p + p_t \\ \dfrac{\partial(\rho^* \phi)}{\partial t} + \dfrac{\partial(\rho^* u_k \phi)}{\partial x_k} = \dfrac{\partial}{\partial x_k}[\underbrace{(\Gamma_l + \Gamma_t)}_{\Gamma_{eff}}\dfrac{\partial \phi}{\partial x_k}] + S_\phi \end{cases}$$

$$u:\ S = \frac{\partial}{\partial x}\left(\eta_{\text{eff}}\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\eta_{\text{eff}}\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial z}\left(\eta_{\text{eff}}\frac{\partial w}{\partial x}\right)$$

$$v:\ S = \frac{\partial}{\partial x}\left(\eta_{\text{eff}}\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial y}\left(\eta_{\text{eff}}\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left(\eta_{\text{eff}}\frac{\partial w}{\partial y}\right)$$

$$w:\ S = \frac{\partial}{\partial x}\left(\eta_{\text{eff}}\frac{\partial u}{\partial z}\right) + \frac{\partial}{\partial y}\left(\eta_{\text{eff}}\frac{\partial v}{\partial z}\right) + \frac{\partial}{\partial z}\left(\eta_{\text{eff}}\frac{\partial w}{\partial z}\right)$$

Additional Eqs. are needed to determine turbulent viscosity $\eta_t$, so as to close model

# Using $k$-$\varepsilon$ model to determine $\eta_t$

## $k$ equation

$$\boxed{G = \frac{\eta_t}{\rho}\frac{\partial u_i}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}$$

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_j k)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\eta_l + \frac{\eta_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + \underbrace{\overbrace{\eta_t \frac{\partial u_j}{\partial x_i}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)}^{\boxed{\rho G}} - \rho\varepsilon}_{\boxed{\text{Source term}}}$$

## $\varepsilon$ equation:

$$\underbrace{\frac{\partial(\rho\varepsilon)}{\partial t}}_{\boxed{\text{transient}}} + \underbrace{\frac{\partial(\rho u_j \varepsilon)}{\partial x_j}}_{\boxed{\text{convection}}} = \underbrace{\frac{\partial}{\partial x_j}\left[\left(\eta_l + \frac{\eta_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]}_{\boxed{\text{diffusion}}} + \underbrace{C_1 \frac{\varepsilon}{k}\eta_t \frac{\partial u_j}{\partial x_i}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) - C_2 \rho \frac{\varepsilon^2}{k}}_{\boxed{\text{source}}}$$

$\sigma_\varepsilon$ Prandtl number of $\varepsilon$; $C_1, C_2$ are empirical coefficients

**turbulent viscosity:**

$$\eta_t = C'_\mu \rho k^{1/2} l = C'_\mu C_D \rho k^{1/2+3/2} \frac{l}{C_D k^{3/2}} = C_\mu \rho k^2 / \varepsilon$$

$$C'_\mu C_D \to C_\mu \qquad \varepsilon = C_D \frac{k^{3/2}}{l}$$

**Governing equation is:**

$$div(\rho\vec{u}\phi) = div(\Gamma_{\phi} grad\phi) + S_{\phi}$$

where $\phi = u, v, T, k, \varepsilon, p, p'$

➢ The diffusion coefficients are:

| NF= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 11 |
|---|---|---|---|---|---|---|---|---|---|
| Variable | $U$ | $V$ | $P_C$ | $T$ | $k$ | $\varepsilon$ | $\eta_t$ | $G$ | $P$ |
| $\Gamma_{\phi}$ | $\eta_t$ | $\eta_t$ | / | $\frac{\eta_t c_p}{\mathrm{Pr}_t}$ | $\frac{\eta_t}{\sigma_k}$ | $\frac{\eta_t}{\sigma_{\varepsilon}}$ | | | |
| $\alpha$ | 0.8 | 0.8 | | 1.0 | 0.6 | 0.6 | 0.6 | | |

$$\eta_{\mathrm{eff}} = \eta + \eta_t \approx \eta_t$$

For our new temperature G.E.: $\Gamma_t = \lambda_t = \eta_t c_p / \mathrm{Pr}_t$

- The source terms are:

$$u: \quad S_u = \frac{\partial}{\partial x}(\eta_{\mathrm{eff}}\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(\eta_{\mathrm{eff}}\frac{\partial v}{\partial x})$$

$$v: \quad S_v = \frac{\partial}{\partial x}(\eta_{\mathrm{eff}}\frac{\partial u}{\partial y}) + \frac{\partial}{\partial y}(\eta_{\mathrm{eff}}\frac{\partial v}{\partial y})$$

$$k: \quad S_k = \eta_t G - \rho\varepsilon$$

$$\varepsilon: \quad S_\varepsilon = \frac{c_1\varepsilon\eta_t G}{k} - \frac{c_2\rho\varepsilon^2}{k}$$

$$G = \frac{\eta_t}{\rho}\left\{2\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right\}$$

- Boundary conditions are：

**（1） Inlet：**

Fluctuation kinetic energy $k$ – taking 1% of kinetic energy of oncoming flow；

Dissipation rate $\varepsilon$ – determined by following eq.

$$\varepsilon = \frac{c_\mu \rho k^2}{\eta_t}$$

where $\eta_t$ is determined by $\mathrm{Re}_t = \frac{\rho V(2L_{in})}{\eta_t} = 100$

（2） **Wall**: adopting Wall Function Method；

（3） **Outlet**: taking local one–way;

(4) **At symmetric line**: normal velocity component ($u$) = 0, all others have their first order normal derivatives equal to zero!

## 11-8-2 Numerical method

### (1) Source term treatment for $k-\varepsilon$

$$S_\phi = S_c + S_p\phi$$

$$S_k = \eta_t G - \rho\varepsilon = \underbrace{\eta_t G}_{S_C} \underbrace{-\left(\frac{\rho\varepsilon}{k^*}\right)}_{S_P} k$$

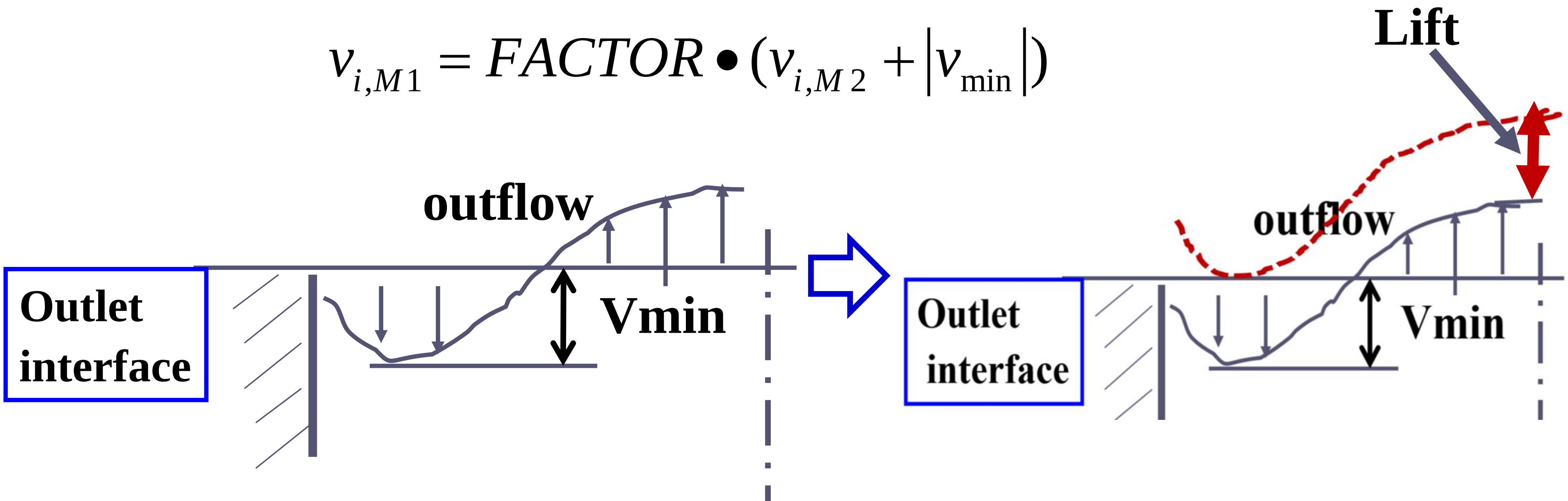
$$S_\varepsilon = \frac{c_1\varepsilon\eta_t G}{k} - \frac{c_2\rho\varepsilon^2}{k} = \underbrace{\frac{c_1\varepsilon\eta_t G}{k}}_{S_C} \underbrace{-\left(\frac{c_2\rho\varepsilon^*}{k}\right)}_{S_P}\varepsilon$$

## （2）Lift (提升）of outlet velocity

In order to avoid negative outlet velocity during iteration, we may adopt method for lifting temporary (暂时的) outlet velocity:

$$FACTOR = \frac{FLOWIN}{\sum_{i=2}^{L2}[(V_{i,M2} + |V_{\min}|) * RHO_{i,M1} * XCV(i)]}$$

$$v_{i,M1} = FACTOR \bullet (v_{i,M2} + |v_{\min}|)$$



$$S_u = \frac{\partial}{\partial x}(\eta_{\text{eff}}\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(\eta_{\text{eff}}\frac{\partial v}{\partial x})$$

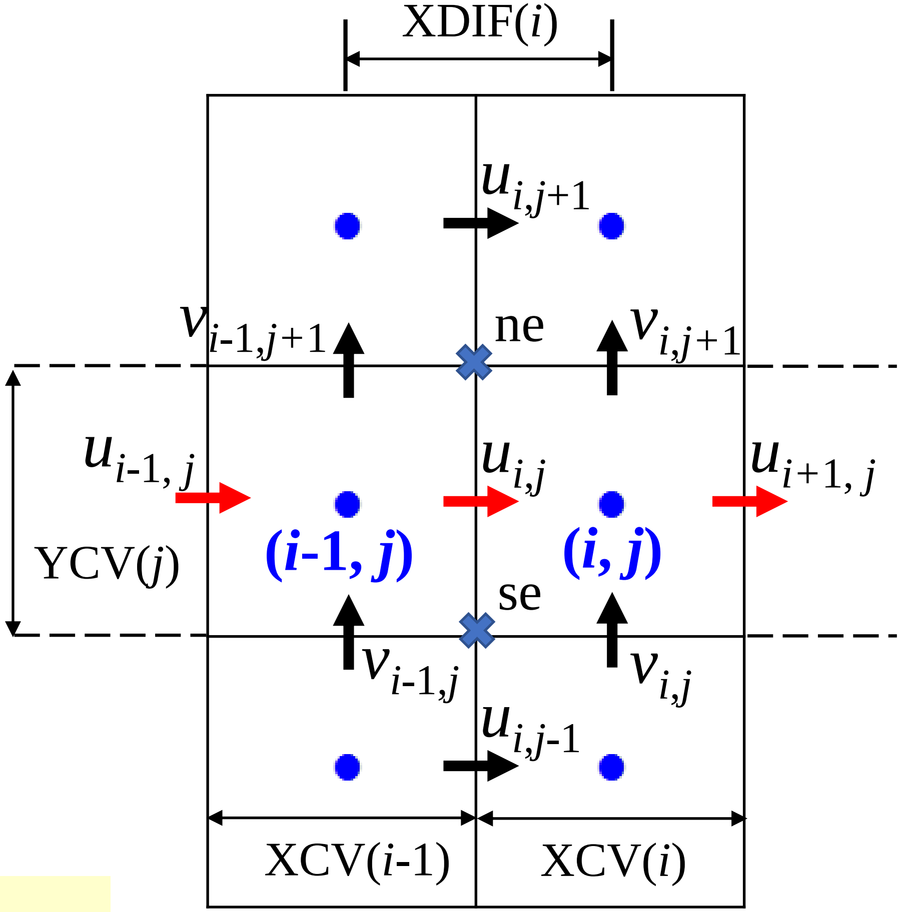
## (3) Treatment of source term in *u*-momentum equation

$$S_u = \frac{\partial}{\partial x}(\eta_t \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(\eta_t \frac{\partial v}{\partial x}) \qquad \eta_{\text{eff}} = \eta + \eta_t \approx \eta_t$$

$$\frac{\partial}{\partial x}(\eta_t \frac{\partial u}{\partial x}) = \frac{1}{XDIF(i)}$$

$$\{GAM(i,j)\frac{u(i+1,j)-u(i,j)}{xcv(i)} -$$

$$GAM(i-1,j)\frac{u(i,j)-u(i-1,j)}{xcv(i-1)}\}$$




The above term is taken as $S_c$ of $u$-equation!

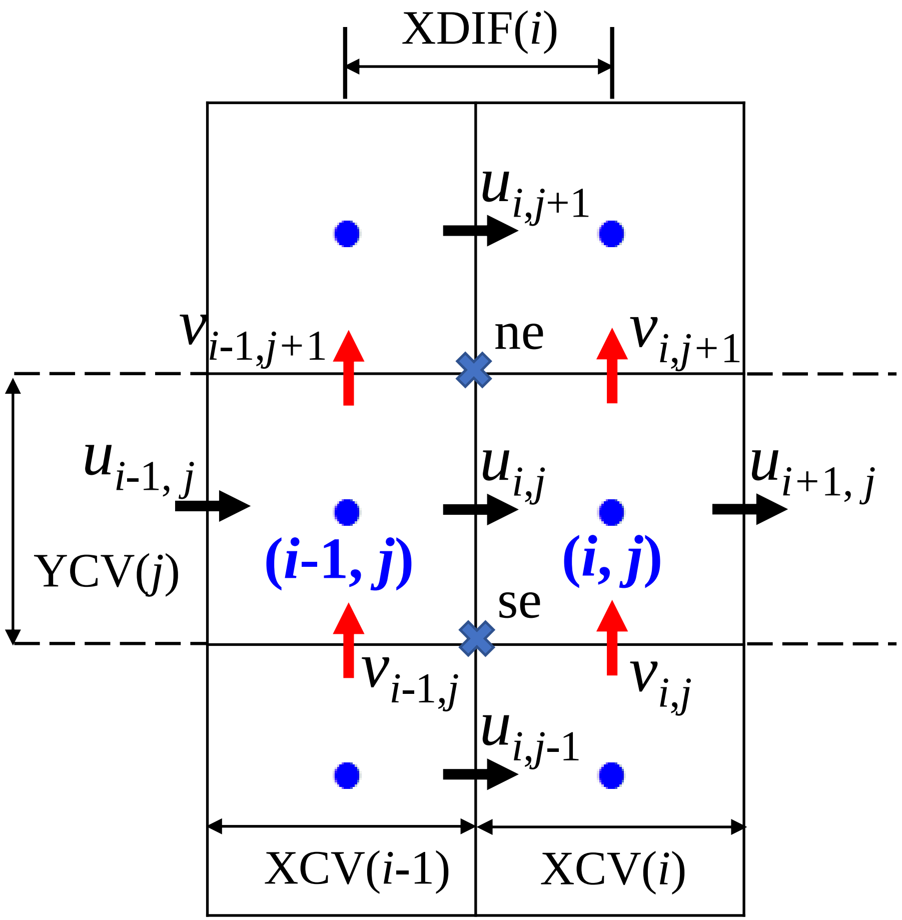
## ➢ source term in *u*-equation

$$\frac{\partial}{\partial y}(\eta_t \frac{\partial v}{\partial x}) = \frac{1}{YCV(j)}$$

$$\{\eta_{t,ne} \frac{v(i,j+1) - v(i-1,j+1)}{XDIF(i)} -$$

$$\eta_{t,se} \frac{v(i,j) - v(i-1,j)}{XDIF(i)}\}$$

Also, taken as $S_c$ of *u*-equation!




## (4) Flow field and temperature are solved separately

Because velocities are not coupled with temperature, the turbulent flow field can be solved first, then the fluid temperature.

# 11-8-3 Program reading

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C*********************************************************************
      INTEGER*4 I,J
      REAL*8 CMU, C1, C2, PRT, PRK, PRD, PRPRT, PFN, CMU4,
     1 AFL, VMIN, REL, AMT, ALOG, GAP, GAMM, DUDX, DUDY, DVDX,
     1 DVDY, DISS, AMU, PR, FLOWIN， FL, FACTOR
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE USER
C**********************************************************************
      USE START_L
      USE USER_L
      IMPLICIT NONE
C**********************************************************************
C-----------------------------PROBLEM TEN--------------------------------
C      Turbulent fluid flow and heat transfer in a parallel duct with stepwise
C                                   inlet velocity distribution
C**********************************************************************
```

*

**ENTRY GRID**

**TITLE(1)=‘ .VEL  U.'**
**TITLE(2)=‘ .VEL  V.'**
**TITLE(3)=‘ .STR  FN.'**
**TITLE(4)=‘ . TEMP .'**
**TITLE(5)='KIN  ENE'**
**TITLE(6)=‘ .DISIPA.’**
**TITLE(7)='TURB  VI'**
**TITLE(11)='PRESSURE'**
**TITLE(12)=‘ DENSITY'**

**!All are titles for printing**

```
RELAX(1)=0.8
RELAX(2)=0.8
RELAX(5)=0.6
RELAX(6)=0.6
RELAX(7)=0.6
LSOLVE(1)=.TRUE.
LSOLVE(5)=.TRUE.
LSOLVE(6)=.TRUE.
LPRINT(1)=.TRUE.
LPRINT(2)=.TRUE.
LPRINT(3)=.TRUE.
LPRINT(4)=.TRUE.
LPRINT(5)=.TRUE.
LPRINT(6)=.TRUE.
LPRINT(7)=.TRUE.
LPRINT(11)=.TRUE.
LAST=100
XL=1.
YL=4.
L1=7
M1=9
CPCON=1000.
CALL UGRID
RETURN
```

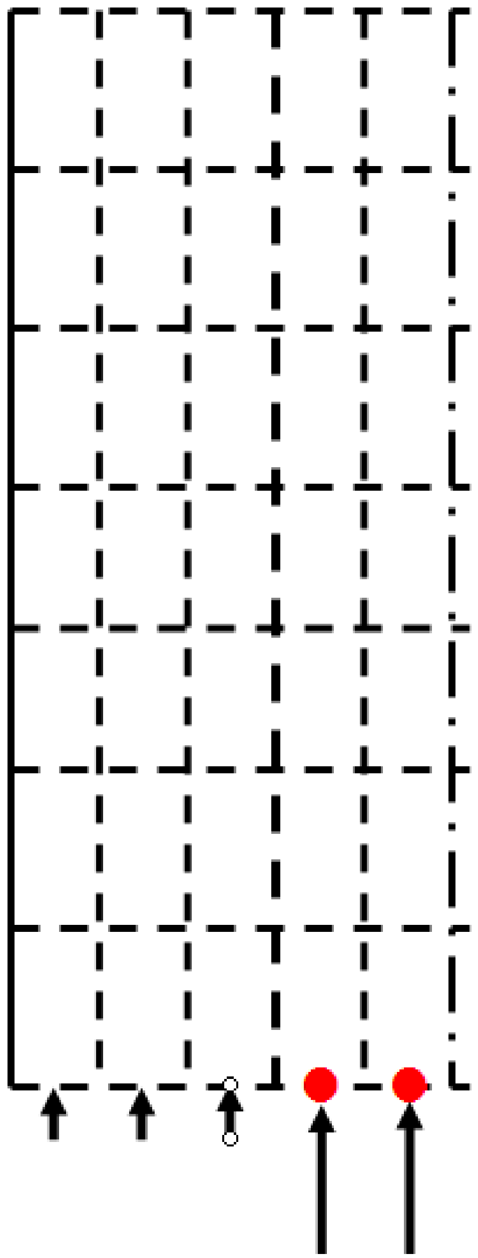
**! NF=7 for turbulent viscosity $\eta_t$**

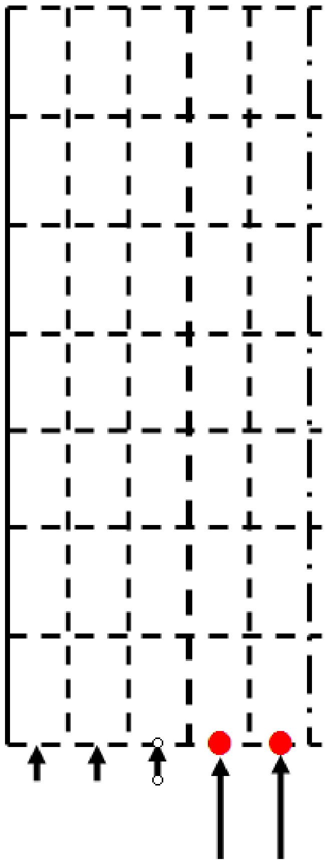
**!All logical values for solving and printing**

**Regarding $\eta_t$ as the 7th element of F( i, j, NF)**

**! $C_p$ in the $\Gamma$ expression for temperature**

$$\Gamma_t = \lambda_t = \eta_t c_p / \Pr_t$$





```
ENTRY START
DO 100 J=1,M1
DO 101 I=1,L1
U(I,J)=0.
V(I,J)=10.
V(1,J)=0.
V(I,2)=10.
IF(I.GT.4) V(I,2)=100.
T(I,J)=100.
T(1,J)=0.
IF(I.GT.4) T(I,1)=400.
AKE(I,J)=0.005*V(I,2)**2
DIS(I,J)=0.1*AKE(I,J)**2
101 ENDDO
100 ENDDO
```

**1% of inlet kinetic energy, initial value, also B.C. for inlet**

$\eta_t$ : determied from

$$\mathrm{Re}_t = \frac{\rho V(2L_{in})}{\eta_t} = 100$$

$$100 = \frac{1\times 100\times 1.0}{\eta_t}, \eta_t = 1.0$$

$$\boxed{\varepsilon = C_\mu \rho k^2 / \eta_t = 0.09\times 1\times k^2 \approx 0.1k^2}$$

**Initial value, also B.C. for inlet !**

```
AMU=1.E-6   ! Attention, very small value, turbulent flow
CMU=0.09
C1=1.44
C2=1.92
PRT=0.9
PRK=1.0
PRD=1.3
PR=0.7
PRPRT=PR/PRT
PFN=9.*(PRPRT-1.)/PRPRT**.25   ! P function of WFM for T
CMU4=CMU**.25
RETURN

ENTRY DENSE
RETURN
```

! Constants of Standard $k$-$\varepsilon$

Most widely accepted values of model constants

| $C_1$ | $C_2$ | $C_\mu$ | $\sigma_k$ | $\sigma_\varepsilon$ | $\sigma_T$ |
|---|---|---|---|---|---|
| 1.44 | 1.92 | 0.09 | 1.0 | 1.3 | 0.9-1.0 |

$$T^+ = \frac{\sigma_t}{\kappa}\ln(Ey^+) + P\sigma_t$$

$$\boxed{P = 8.96\left(\frac{\sigma_l}{\sigma_t} - 1\right)\left(\frac{\sigma_l}{\sigma_t}\right)^{-1/4}}$$

```
      ENTRY BOUND
      IF(ITER == 0) THEN
      FLOWIN=0.
      DO 310 I=2,L2
      FLOWIN=FLOWIN+RHO(I,1)*V(I,2)*XCV(I)      ! Flow rate at inlet
310 ENDDO
      ELSE
      FL=0.
      AFL=0.
      VMIN=0.
      ENDIF
      DO 301 I=2,L2
      IF(V(I,M2)< 0.) VMIN=DMAX1(VMIN,-V(I,M2))      ! Search for Vmin
      AFL=AFL+RHO(I,M1)*XCV(I)
      FL=FL+RHO(I,M1)*V(I,M2)*XCV(I)
      FACTOR=FLOWIN/(FL+AFL*VMIN)
301 ENDDO
      DO 302 I=2,L2
      V(I,M1)=(V(I,M2)+VMIN)*FACTOR
302 ENDDO
      DO 303 J=2,M2
      AKE(L1,J)=AKE(L2,J)      ! symmetry; decoration for print out
      DIS(L1,J)=DIS(L2,J)
303 ENDDO
      RETURN
```

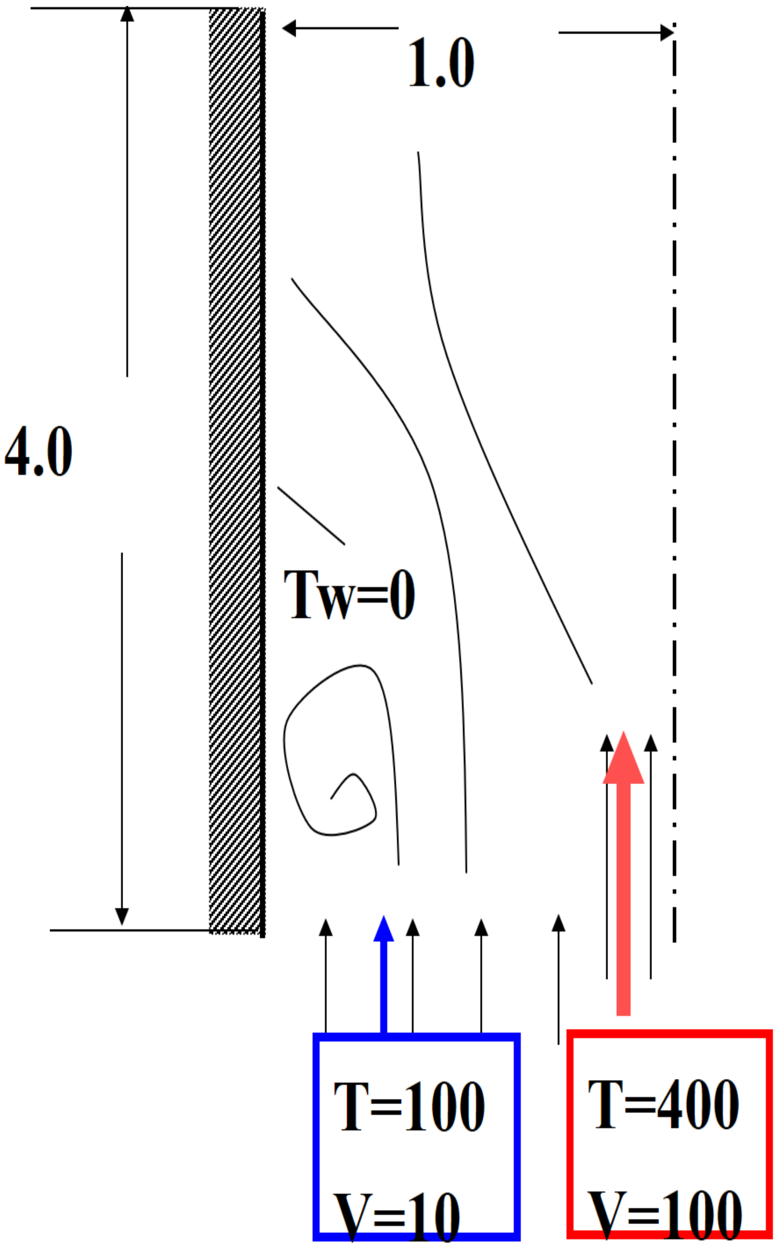
$$FACTOR = \frac{FLOWIN}{\sum_{i=2}^{L2}[(V_{i,M2} + |V_{\min}|) * RHO_{i,M1} * XCV(i)]}$$

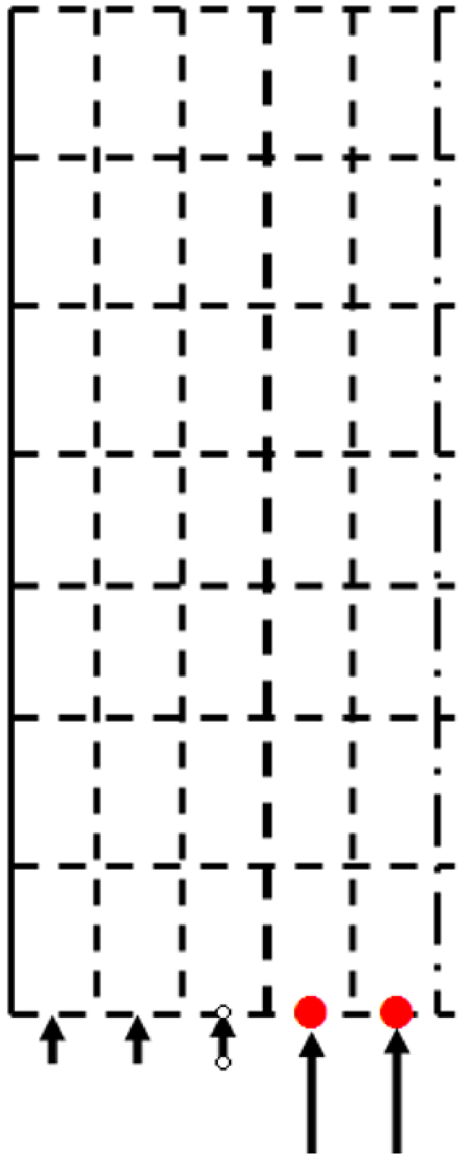
! Search for $V_{\min}$

$$!v_{i,M1} = FACTOR \bullet (v_{i,M2} + |v_{\min}|)$$

```
      ENTRY OUTPUT
      IF(ITER==0) THEN
      PRINT 401
      WRITE(8,401)
 401 FORMAT(1X,'   ITER',6X,'SMAX',6X,'SSUM',5X,'V(6,6)',
   1 4X,'T(5,6)',4X,'KE(5,6)')
      ELSE
      PRINT 403, ITER, SMAX, SSUM, V(6,6),T(5,6), AKE(5,6)
      WRITE(8,403) ITER,SMAX,SSUM,V(6,6),T(5,6),AKE(5,6)
403 FORMAT(1X,I6,1P5E11.3)
      ENDIF
      IF(ITER>=55) THEN
      LSOLVE(4)=.TRUE.
      LSOLVE(1)=.FALSE.
      LSOLVE(5)=.FALSE.
      LSOLVE(6)=.FALSE.
      ENDIF
      IF（ITER==LAST) CALL PRINT
      RETURN
```

**! Switch off the solution variables: Flow is not coupled with temperature！ After obtaining converged flow field, temperature is solved**


1.0
4.0
Tw=0
T=100
V=10
T=400
V=100



```
      ENTRY GAMSOR
      IF(NF== 3) RETURN
      IF(NF== 1) THEN
      REL=1.-RELAX(7)          ! NF=7 for turbulent viscosity
      DO 500 J=1,M1
      DO 501 I=1,L1
      AMT=CMU*RHO(I,J)*AKE(I,J)**2/(DIS(I,J)+1.E-30)
      IF(ITER==0) AMUT(I,J)=AMT   ! Initial values
      AMUT(I,J)=RELAX(7)*AMT+REL*AMUT(I,J)
501   ENDDO                    ! Underrelaxation for turbulent viscosity
500   ENDDO
      FACTOR=1.
      ELSE
      IF(NF== 4) FACTOR=CPCON/PRT
      IF(NF== 5) FACTOR=1./PRK
      IF(NF== 6) FACTOR=1./PRD
      DO 520 J=1,M1
      DO 521 I=1,L1             ! Laminar part is omitted.
      GAM(I,J)=AMUT(I,J)*FACTOR
      IF(NF/= 1) GAM(L1,J)=0. ! Symmetric line, u=0
      GAM(I,M1)=0. ! Local one way for outlet
521   ENDDO
520   ENDDO
```

$$\eta_t = \frac{c_\mu \rho k^2}{\varepsilon}$$

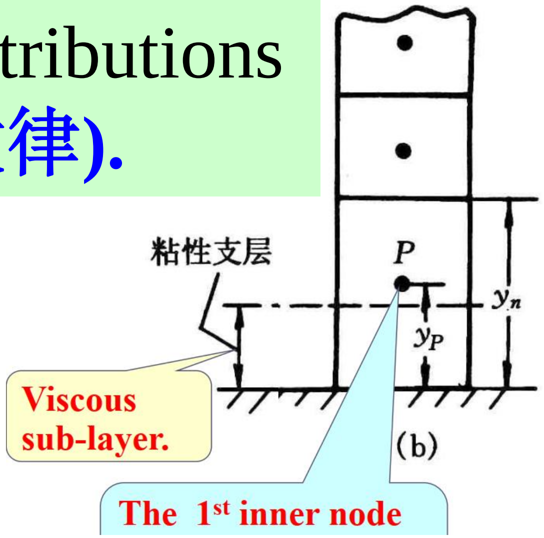
$$\boxed{\mathrm{Pr_t} = \eta_t c_p / \lambda_t, \quad \lambda_t = \eta_t c_p / \mathrm{Pr_t}}$$

$$\boxed{(\eta_l + \frac{\eta_t}{\sigma_k}) - \text{for } k; \quad (\eta_l + \frac{\eta_t}{\sigma_\varepsilon}) - \text{for } \varepsilon}$$

# Fundamentals of Wall Function Method

1) Assuming that the dimensionless velocity and temp. distributions outside the viscous sub-layer are of **logarithmic law(对数律).**

For $x_P^+ > 11.0$
$$\begin{cases} u^+ = \dfrac{1}{\kappa}\ln(E x_P^+), & \dfrac{1}{\kappa}\ln(E) = 5.0 \sim 5.5 \\ T^+ = \dfrac{\sigma_t}{\kappa}\ln(E x_P^+) + P\sigma_t; & P = 8.96\left(\dfrac{\sigma_l}{\sigma_t} - 1\right)\left(\dfrac{\sigma_l}{\sigma_t}\right)^{-1/4} \end{cases}$$




2) Placing the **1st inner node P** outside the viscous sub-layer, where logarithmic law is valid ( $x_P^+ > 11$ ).

3) The **effective turbulent viscosity and thermal conductivity** between the 1st inner node and wall determined by :

$$\eta_B = \left(\frac{x_P^+}{u_P^+}\right)\eta_l \qquad \lambda_B = \left(\frac{x_P^+}{T_P^+}\right)\mathrm{Pr}_l\,\lambda_l \qquad x_P^+ = \frac{\rho x (C_\mu^{1/4} k^{1/2})}{\eta_l}$$

# Fundamentals of Wall Function Method

4) The boundary condition of $k$ equation $\partial k/\partial n = 0$

5) The $\boldsymbol{\varepsilon}$ **at 1st inner node** is determined by: $\varepsilon = C_\mu^{3/4} k_P^{3/2} / (\kappa y_P)$

- For Solid wall: adopting wall function method

(1)Velocity — normal to wall $\frac{\partial \phi}{\partial n})_w = 0$;

Velocity — parallel to wall $\phi_w = 0$ , and $\eta_B$ determined by WFM

$$\eta_B = (\frac{x_P^+}{u_P^+})\eta_l \qquad x_P^+ = \frac{\rho x (C_\mu^{1/4} k^{1/2})}{\eta_l}$$

(2) Temperature

$\lambda_B$ determined by WFM $\lambda_B = (\frac{x_P^+}{T_P^+}) \Pr_l \lambda_l$

# WFM implementation!

```
DO 530 J=2,M2
SELECT CASE (NF)        ! For u, p', k, ε
CASE (1,3,5,6)
GAM(1,J)=0.
CASE (2)                ! For velocity v, WFM should be used!
GAM(1,J)=AMU            ! First, laminar viscosity is given for the left wall
XPLUS(J)=RHO(2,J)*SQRT(AKE(2,J))*CMU4*XDIF(2)/AMU
IF(XPLUS(J)>11.5)   GAM(1,J)=AMU*XPLUS(J)/
1 (ALOG(9.*XPLUS(J))*2.5)   ! Turbulence viscosity
CASE (4)  !  For temperature, WFM for temperature
GAM(1,J)=AMU*CPCON/PR!  First, laminar thermal conductivity
IF(XPLUS(J)>11.5) GAM(1,J)=AMU*CPCON/PRT*XPLUS(J)
1 /(2.5*ALOG(9.*XPLUS(J))+PFN)  ! Turbulence thermal conductivity
ENDSELECT
530 ENDDO
```

For $u$, $p$'and $k$--adiabatic; For $\varepsilon$, set up its value of 1st inner node, **then cut the connection to its boundary. All lead to $\Gamma$=0**

$$\eta_B = \left(\frac{x_P^+}{u_P^+}\right)\eta_l$$

$$\boxed{x^+ = \frac{\rho x (C_\mu^{1/4} k^{1/2})}{\eta_l}}$$

$$\lambda_B = \left(\frac{x_P^+}{T_P^+}\right)\mathrm{Pr}_l\,\lambda_l$$

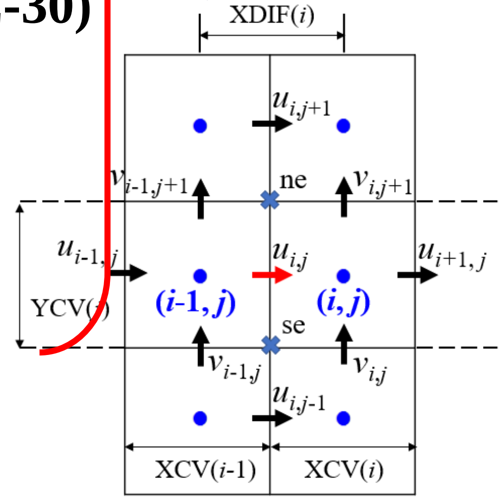
W F M Implementation

$$S_u = \frac{\partial}{\partial x}\left(\eta_t \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\eta_t \frac{\partial v}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\eta_t \frac{\partial u}{\partial x}\right) = \frac{1}{XDIF(i)}\left\{GAM(i,j)\frac{u(i+1,j)-u(i,j)}{xcv(i)} - GAM(i-1,j)\frac{u(i,j)-u(i-1,j)}{xcv(i-1)}\right\}$$

```
IF(NF==1) THEN
DO 590 J=2,M2
DO 591 I=3,L2
CON(I,J)=(GAM(I,J)*(U(I+1,J)-U(I,J))/XCV(I)
1 -GAM(I-1,J)*(U(I,J)-U(I-1,J))/XCV(I-1))/XDIF(I)
GAMP=GAM(I,J+1)*GAM(I-1,J+1)/(GAM(I,J+1)+GAM(I-1,J+1)+1.E-30)
GAMP=GAMP+GAM(I,J)*GAM(I-1,J)/(GAM(I,J)+GAM(I-1,J)+1.E-30)
GAMM=GAM(I,J-1)*GAM(I-1,J-1)/(GAM(I,J-1)+GAM(I-1,J-1)+1.E-30)
GAMM=GAMM+GAM(I,J)*GAM(I-1,J)/(GAM(I,J)+GAM(I-1,J)+1.E-30)
CON(I,J)=CON(I,J)+(GAMP*(V(I,J+1)-V(I-1,J+1))
1 -GAMM*(V(I,J)-V(I-1,J)))/(YCV(J)*XDIF(I))
AP(I,J)=0.
591 ENDDO
590 ENDDO
RETURN
```

**Source term for u-eq.**




$$\eta_{t,ne} = \frac{\eta_t(i-1,j)\eta_t(i,j)}{\eta_t(i-1,j)+\eta_t(i,j)} + \frac{\eta_t(i-1,j+1)\eta_t(i,j+1)}{\eta_t(i-1,j+1)+\eta_t(i,j+1)}$$

Refer to textbook page 358

```
509 IF(NF==2) THEN
      DO 594 J=3,M2
      DO 595 I=2,L2
      CON(I,J)=(GAM(I,J)*(V(I,J+1)-V(I,J))/YCV(J)-
     1  GAM(I,J-1)*(V(I,J)-V(I,J-1))/YCV(J-1))/(YDIF(J))
      GAMP=GAM(I+1,J)*GAM(I+1,J-1)/(GAM(I+1,J)+GAM(I+1,J-1)+1.E-30)
      GAMP=GAMP+GAM(I,J)*GAM(I,J-1)/(GAM(I,J)+GAM(I,J-1)+1.E-30)
      GAMM=GAM(I-1,J)*GAM(I-1,J-1)/(GAM(I-1,J)+GAM(I-1,J-1)+1.E-30)
      GAMM=GAMM+GAM(I,J)*GAM(I,J-1)/(GAM(I,J)+GAM(I,J-1)+1.E-30)
      CON(I,J)=CON(I,J)+(GAMP*(U(I+1,J)-U(I+1,J-1))
     1  -GAMM*(U(I,J)-U(I,J-1)))/(XCV(I)*YDIF(J))
      AP(I,J)=0.
595 ENDDO
594 ENDDO
      RETURN
      ENDIF
```

Source term calculation for *v*- eq.

```
IF(NF==4) THEN
     DO 596 J=2,M2
     DO 597 I=2,L2
     CON(I,J)=0.
     AP(I,J)=0.
597 ENDDO
586 ENDDO
     RETURN
```

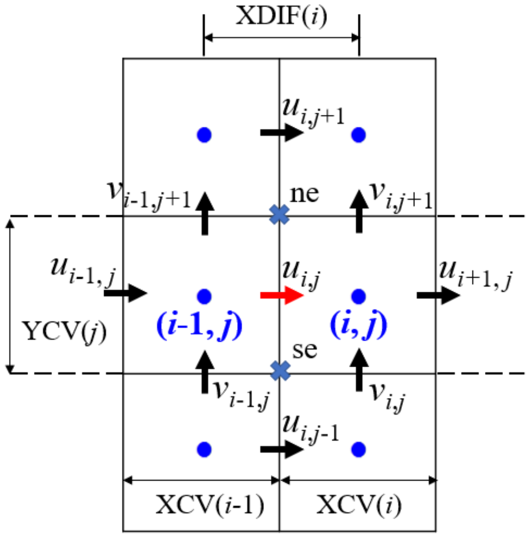
Source term calculation for *T*- eq.

**! Following part is for the source term of *k*- eq.:**

$$S_k = \eta_t G - \rho\varepsilon = \eta_t G - \left(\frac{\rho\varepsilon}{k^*}\right)k \qquad G = \frac{\partial u_i}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

**! Most part of the code is for calculation of GEN term**

```
   ELSE IF(NF==5) THEN
   DO 598 J=2,M2
   DO 599 I=2,L2
   DUDX=(U(I+1,J)-U(I,J))/XCV(I)
   DVDY=(V(I,J+1)-V(I,J))/YCV(J)
   IF(J==2) DUDY=(0.5*(U(I,J+1)-U(I,J))+0.5*(U(I+1,J+1)-
C U(I+1,J)))/YDIF(J+1)
```



```
 IF(J==M2) DUDY=(0.5*(U(I,J)-U(I,J-1))+0.5*(U(I+1,J)-U(I+1,J-1))) /YDIF(J)
 IF(J/=2.AND.J/=M2) DUDY=(0.5*(U(I,J+1)-U(I,J-1))+0.5*(U(I+1,J+1)-
1 U(I+1,J-1)))/(YDIF(J)+YDIF(J+1))
 IF(I==2) DVDX=(0.5*(V(I+1,J)-V(I-1,J))+0.5*(V(I+1,J+1)
1 -V(I-1,J+1)))/(XDIF(I)+XDIF(I+1))
 IF(I==L2)  DVDX=(0.5*(V(I,J)-V(I-1,J))+0.5*(V(I,J+1)
1 -V(I-1,J+1)))/XDIF(I)
 IF(I/=2.AND.I/=L2) DVDX=(0.5*(V(I+1,J)-V(I,J))+0.5*(V(I+1,J+1)
1 -V(I,J+1)))/XDIF(I+1))
  GEN(I,J)=2.*(DUDX**2+DVDY**2)+(DUDY+DVDX)**2      !
  CON(I,J)=GEN(I,J)*AMUT(I,J)
  AP(I,J)=-RHO(I,J)*DIS(I,J)/(AKE(I,J)+1.E-30)
598 ENDDO
599 ENDDO
    RETURN
    ENDIF
```

$$G=\frac{\partial u_i}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$

**Sp of *k*-eq.**

$$S_k=\eta_t G-\rho\varepsilon=\eta_t G-\left(\frac{\rho\varepsilon}{k^*}\right)k$$

$$S_\varepsilon = \frac{c_1 \varepsilon \eta_t G}{k} - \frac{c_2 \rho \varepsilon^2}{k} = \frac{c_1 \varepsilon \eta_t G}{k} - \left(\frac{c_2 \rho \varepsilon^*}{k}\right)\varepsilon$$

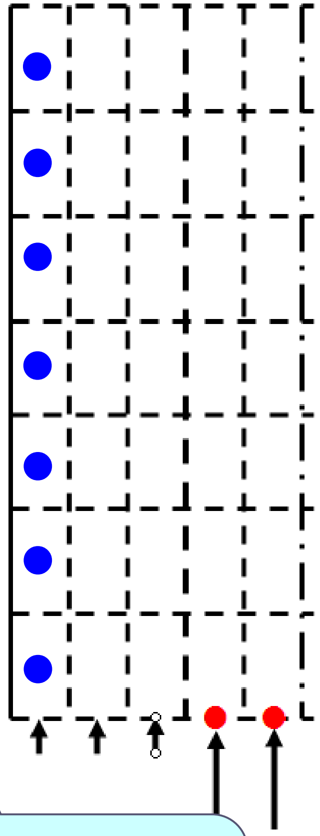
$$S_c = A\phi_{given}, S_P = -A,$$
$$A = 10^{20} \sim 10^{30}$$

**Large source term method**

```
      DO 600 J=2,M2
      DO 601 I=2,L2
      CON(I,J)=C1*GEN(I,J)*CMU*RHO(I,J)*AKE(I,J)
      AP(I,J)=-C2*RHO(I,J)*DIS(I,J)/(AKE(I,J)+1.E-30)
601 ENDDO
600 ENDDO
      DO 602 J=2,M2
      DISS=CMU*AKE(2,J)**1.5/(0.4*CMU4*XDIF(2))
      CON(2,J)=1.E30*DISS
      AP(2,J)=-1.E30
602 ENDDO
      RETURN
      END
```

$$\varepsilon = \frac{C_\mu^{3/4} k_P^{3/2}}{\kappa y_P}$$

**Adopt large source term method for 1st inner node where i=2;**



**Source term calculation for Epsilon eq.**

# 9.8.4 Results analysis

COMPUTATION IN CARTESIAN COORDINATES

************************************************

| ITER | SMAX | SSUM | V(6,6) | T(5,6) | KE(5,6) |
|---|---|---|---|---|---|
| 1 | 8.411E+00 | 1.421E-14 | 4.326E+01 | 1.000E+02 | 9.108E+00 |
| 2 | 2.675E+00 | 8.882E-15 | 4.354E+01 | 1.000E+02 | 2.939E+01 |
| 3 | 9.943E-01 | -4.441E-15 | 4.409E+01 | 1.000E+02 | 5.808E+01 |
| 4 | 1.321E+00 | 6.661E-16 | 4.538E+01 | 1.000E+02 | 9.042E+01 |
| 5 | 1.147E+00 | -1.998E-15 | 4.668E+01 | 1.000E+02 | 1.233E+02 |
| 6 | 7.209E-01 | 3.331E-16 | 4.747E+01 | 1.000E+02 | 1.550E+02 |
| 7 | 5.410E-01 | 2.109E-15 | 4.762E+01 | 1.000E+02 | 1.848E+02 |
| 8 | 4.211E-01 | 8.882E-16 | 4.725E+01 | 1.000E+02 | 2.119E+02 |
| 9 | 3.760E-01 | 3.886E-15 | 4.642E+01 | 1.000E+02 | 2.363E+02 |
| 10 | 3.451E-01 | -2.776E-15 | 4.521E+01 | 1.000E+02 | 2.577E+02 |
| 11 | 3.723E-01 | -5.773E-15 | 4.376E+01 | 1.000E+02 | 2.760E+02 |
| 12 | 3.797E-01 | -4.441E-16 | 4.217E+01 | 1.000E+02 | 2.912E+02 |
| 13 | 3.811E-01 | 1.044E-14 | 4.054E+01 | 1.000E+02 | 3.031E+02 |
| 14 | 3.785E-01 | -8.216E-15 | 3.899E+01 | 1.000E+02 | 3.120E+02 |
| 15 | 3.723E-01 | -9.437E-15 | 3.757E+01 | 1.000E+02 | 3.183E+02 |
| 16 | 3.714E-01 | -1.332E-15 | 3.633E+01 | 1.000E+02 | 3.226E+02 |
| 17 | 3.640E-01 | -4.441E-16 | 3.529E+01 | 1.000E+02 | 3.254E+02 |
| 18 | 3.615E-01 | 1.776E-15 | 3.446E+01 | 1.000E+02 | 3.273E+02 |

| | | | | | |
|---|---|---|---|---|---|
| 19 | 3.499E-01 | 5.773E-15 | 3.380E+01 | 1.000E+02 | 3.285E+02 |
| 20 | 1.993E-01 | 0.000E+00 | 3.331E+01 | 1.000E+02 | 3.293E+02 |
| 21 | 1.916E-01 | 7.327E-15 | 3.294E+01 | 1.000E+02 | 3.298E+02 |
| 22 | 1.632E-01 | -3.275E-15 | 3.267E+01 | 1.000E+02 | 3.299E+02 |
| 23 | 1.494E-01 | -5.773E-15 | 3.248E+01 | 1.000E+02 | 3.299E+02 |
| 24 | 1.283E-01 | -3.220E-15 | 3.234E+01 | 1.000E+02 | 3.295E+02 |
| 25 | 1.071E-01 | -8.327E-16 | 3.224E+01 | 1.000E+02 | 3.290E+02 |
| 26 | 8.615E-02 | -1.024E-14 | 3.218E+01 | 1.000E+02 | 3.282E+02 |
| 27 | 7.442E-02 | 5.301E-15 | 3.213E+01 | 1.000E+02 | 3.273E+02 |
| 28 | 7.219E-02 | -3.969E-15 | 3.210E+01 | 1.000E+02 | 3.261E+02 |
| 29 | 6.907E-02 | -1.638E-15 | 3.207E+01 | 1.000E+02 | 3.248E+02 |
| 30 | 6.246E-02 | -5.704E-15 | 3.205E+01 | 1.000E+02 | 3.234E+02 |
| 31 | 5.292E-02 | -6.689E-15 | 3.202E+01 | 1.000E+02 | 3.218E+02 |
| 32 | 4.163E-02 | -3.039E-15 | 3.199E+01 | 1.000E+02 | 3.201E+02 |
| 33 | 3.782E-02 | 6.467E-15 | 3.196E+01 | 1.000E+02 | 3.183E+02 |
| 34 | 3.624E-02 | 1.332E-15 | 3.193E+01 | 1.000E+02 | 3.165E+02 |
| 35 | 3.316E-02 | -7.938E-15 | 3.189E+01 | 1.000E+02 | 3.145E+02 |
| 36 | 2.901E-02 | 1.693E-15 | 3.185E+01 | 1.000E+02 | 3.126E+02 |
| 37 | 2.497E-02 | -1.303E-14 | 3.181E+01 | 1.000E+02 | 3.105E+02 |
| 38 | 2.160E-02 | -1.010E-14 | 3.177E+01 | 1.000E+02 | 3.085E+02 |
| 39 | 1.930E-02 | 1.041E-16 | 3.173E+01 | 1.000E+02 | 3.064E+02 |
| 40 | 1.730E-02 | 1.774E-14 | 3.168E+01 | 1.000E+02 | 3.043E+02 |
| 41 | 1.535E-02 | -9.714E-16 | 3.164E+01 | 1.000E+02 | 3.022E+02 |
| 42 | 2.275E-02 | 5.967E-16 | 3.160E+01 | 1.000E+02 | 3.002E+02 |

| | | | V | T | KE |
|---|---|---|---|---|---|
| 43 | 4.093E-02 | -4.635E-15 | 3.156E+01 | 1.000E+02 | 2.981E+02 |
| 44 | 4.235E-02 | -1.457E-15 | 3.152E+01 | 1.000E+02 | 2.961E+02 |
| 45 | 3.395E-02 | 8.327E-16 | 3.148E+01 | 1.000E+02 | 2.941E+02 |
| 46 | 2.645E-02 | 1.388E-16 | 3.144E+01 | 1.000E+02 | 2.921E+02 |
| 47 | 2.060E-02 | 8.188E-16 | 3.140E+01 | 1.000E+02 | 2.901E+02 |
| 48 | 1.581E-02 | 4.718E-15 | 3.136E+01 | 1.000E+02 | 2.882E+02 |
| 49 | 1.193E-02 | -6.939E-16 | 3.133E+01 | 1.000E+02 | 2.863E+02 |
| 50 | 8.833E-03 | -2.772E-15 | 3.130E+01 | 1.000E+02 | 2.845E+02 |
| 51 | 6.423E-03 | 7.556E-15 | 3.127E+01 | 1.000E+02 | 2.827E+02 |
| 52 | 6.119E-03 | -2.288E-15 | 3.124E+01 | 1.000E+02 | 2.810E+02 |
| 53 | 6.003E-03 | -3.456E-15 | 3.121E+01 | 1.000E+02 | 2.793E+02 |
| 54 | 5.891E-03 | -5.551E-15 | 3.118E+01 | 1.000E+02 | 2.776E+02 |
| 55 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 1.000E+02 | 2.760E+02 |
| 56 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.126E+02 | 2.760E+02 |
| 57 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.170E+02 | 2.760E+02 |
| 58 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 59 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 60 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 61 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 62 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 63 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 64 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 65 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 66 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |

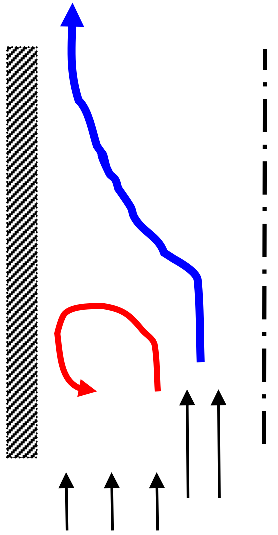
Changing solution variables

Seven iterations of *T* reach converged solution

| | | | | | |
|---|---|---|---|---|---|
| 67 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 68 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 69 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 70 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 71 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 72 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 73 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 74 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 75 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 76 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 77 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 78 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 79 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 80 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 81 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 82 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 83 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 84 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 85 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 86 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 87 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 88 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 89 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |
| 90 | 5.779E-03 | -7.527E-15 | 3.116E+01 | 2.174E+02 | 2.760E+02 |

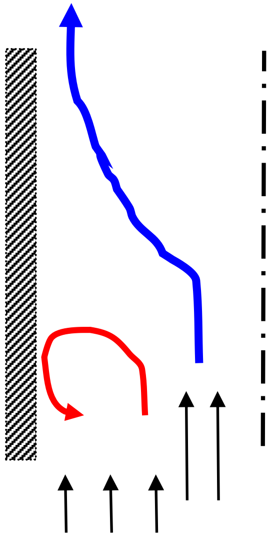
***************************** .VEL U. *****************************

| J \ I = | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 9 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| 8 | 0.00E+00 | 1.56E-02 | 3.75E-02 | 3.84E-02 | 2.04E-02 | 0.00E+00 |
| 7 | 0.00E+00 | -1.65E+00 | -2.68E+00 | -2.78E+00 | -1.33E+00 | 0.00E+00 |
| 6 | 0.00E+00 | -2.37E+00 | -3.56E+00 | -3.57E+00 | -1.63E+00 | 0.00E+00 |
| 5 | 0.00E+00 | -2.38E+00 | -3.88E+00 | -3.98E+00 | -1.66E+00 | 0.00E+00 |
| 4 | 0.00E+00 | -1.39E+00 | -3.33E+00 | -3.86E+00 | -1.45E+00 | 0.00E+00 |
| 3 | 0.00E+00 | 3.74E+00 | -3.47E-01 | -2.75E+00 | -8.62E-01 | 0.00E+00 |
| 2 | 0.00E+00 | 4.44E+00 | 6.55E+00 | -2.87E+00 | -6.77E-01 | 0.00E+00 |
| 1 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |



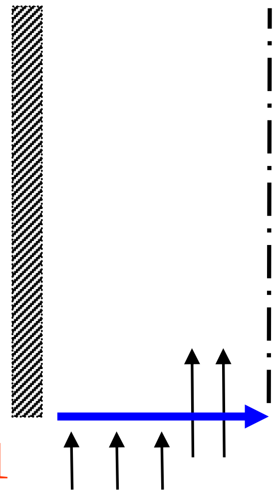
****************************** .VEL V. ******************************

| I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| J | | | | | | | |
| 9 | 0.00E+00 | 8.87E+00 | 3.18E+01 | 4.59E+01 | 6.52E+01 | 7.82E+01 | 1.00E+01 |
| 8 | 0.00E+00 | 8.87E+00 | 3.18E+01 | 4.59E+01 | 6.52E+01 | 7.82E+01 | 1.00E+01 |
| 7 | 0.00E+00 | 4.16E+00 | 2.89E+01 | 4.56E+01 | 6.93E+01 | 8.20E+01 | 1.00E+01 |
| 6 | 0.00E+00 | -2.61E+00 | 2.55E+01 | 4.56E+01 | 7.48E+01 | 8.67E+01 | 1.00E+01 |
| 5 | 0.00E+00 | -9.41E+00 | 2.12E+01 | 4.53E+01 | 8.15E+01 | 9.14E+01 | 1.00E+01 |
| 4 | 0.00E+00 | -1.34E+01 | 1.56E+01 | 4.38E+01 | 8.83E+01 | 9.56E+01 | 1.00E+01 |
| 3 | 0.00E+00 | -2.70E+00 | 3.98E+00 | 3.69E+01 | 9.37E+01 | 9.81E+01 | 1.00E+01 |
| 2 | 1.00E+01 | 1.00E+01 | 1.00E+01 | 1.00E+01 | 1.00E+02 | 1.00E+02 | 1.00E+02 |



****************************  **.STR FN** ****************************

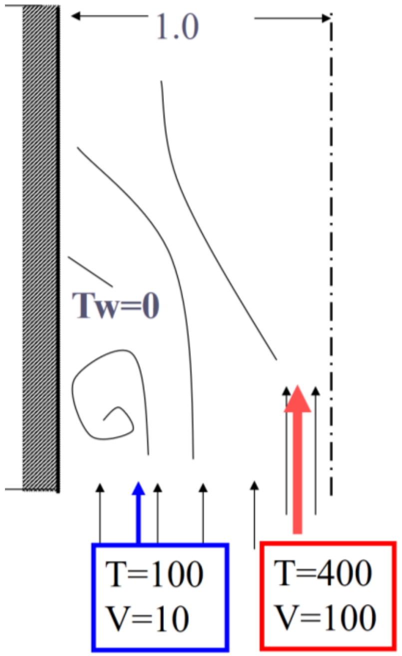
| I = | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| J | | | | | | |
| **9** | **0.00E+00** | **-1.77E+00** | **-8.12E+00** | **-1.73E+01** | **-3.03E+01** | -4.60E+01 |
| 8 | 0.00E+00 | -1.77E+00 | -8.14E+00 | -1.73E+01 | -3.04E+01 | -4.60E+01 |
| 7 | 0.00E+00 | -8.31E-01 | -6.61E+00 | -1.57E+01 | -2.96E+01 | -4.60E+01 |
| 6 | 0.00E+00 | 5.21E-01 | -4.58E+00 | -1.37E+01 | -2.87E+01 | -4.60E+01 |
| 5 | 0.00E+00 | 1.88E+00 | -2.36E+0 0 | -1.14E+01 | -2.77E+01 | -4.60E+01 |
| 4 | 0.00E+00 | 2.68E+00 | -4.55E-01 | -9.22E+00 | -2.69E+01 | -4.60E+01 |
| 3 | 0.00E+00 | 5.39E-01 | -2.57E-01 | -7.64E+00 | -2.64E+01 | -4.60E+01 |
| **2** | **0.00E+00** | **-2.00E+00** | **-4.00E+00** | **-6.00E+00** | **-2.60E+01** | -4.60E+01 |



**Stream function increase along this direction**

**************************** . TEMP . ****************************

| I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| J | | | | | | | |
| 9 | 0.00E+00 | 1.00E+02 | 1.00E+02 | 1.00E+02 | 1.00E+02 | 1.00E+02 | 1.00E+02 |
| 8 | 0.00E+00 | 3.01E+02 | 3.26E+02 | 3.39E+02 | 3.60E+02 | 3.80E+02 | 1.00E+02 |
| 7 | 0.00E+00 | 3.00E+02 | 3.21E+02 | 3.35E+02 | 3.63E+02 | 3.85E+02 | 1.00E+02 |
| 6 | 0.00E+00 | 2.93E+02 | 3.10E+02 | 3.26E+02 | 3.64E+02 | 3.89E+02 | 1.00E+02 |
| 5 | 0.00E+00 | 2.80E+02 | 2.93E+02 | 3.11E+02 | 3.67E+02 | 3.92E+02 | 1.00E+02 |
| 4 | 0.00E+00 | 2.65E+02 | 2.69E+02 | 2.88E+02 | 3.72E+02 | 3.95E+02 | 1.00E+02 |
| 3 | 0.00E+00 | 2.52E+02 | 2.36E+02 | 2.53E+02 | 3.79E+02 | 3.97E+02 | 1.00E+02 |
| 2 | 0.00E+00 | 1.29E+02 | 1.16E+02 | 2.01E+02 | 3.90E+02 | 3.99E+02 | 1.00E+02 |
| 1 | 0.00E+00 | 1.00E+02 | 1.00E+02 | 1.00E+02 | 4.00E+02 | 4.00E+02 | 4.00E+02 |




Given wall temp

**Given inlet temp.**

****************************  KIN ENE  ****************************

Initial values, No decoration!

| J \ I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 9 | 5.00E-01 | 5.00E-01 | 5.00E-01 | 5.00E-01 | 5.00E+01 | 5.00E+01 | 5.00E+01 |
| 8 | 5.00E-01 | 1.59E+02 | 4.93E+02 | 4.65E+02 | 3.53E+02 | 2.15E+02 | 2.15E+02 |
| 7 | 5.00E-01 | 1.90E+02 | 5.34E+02 | 4.85E+02 | 3.35E+02 | 1.74E+02 | 1.74E+02 |
| 6 | 5.00E-01 | 2.20E+02 | 5.83E+02 | 5.22E+02 | 3.20E+02 | 1.37E+02 | 1.37E+02 |
| 5 | 5.00E-01 | 2.39E+02 | 6.06E+02 | 5.46E+02 | 2.94E+02 | 1.06E+02 | 1.06E+02 |
| 4 | 5.00E-01 | 2.15E+02 | 5.40E+02 | 5.31E+02 | 2.54E+02 | 8.23E+01 | 8.23E+01 |
| 3 | 5.00E-01 | 1.15E+02 | 3.30E+02 | 4.69E+02 | 2.06E+02 | 6.62E+01 | 6.62E+01 |
| 2 | 5.00E-01 | 1.88E+01 | 1.03E+01 | 3.22E+02 | 1.46E+02 | 5.55E+01 | 5.55E+01 |
| 1 | 5.00E-01 | 5.00E-01 | 5.00E-01 | 5.00E-01 | 5.00E+01 | 5.00E+01 | 5.00E+01 |

Initial values,
No decoration!

*************************** .DISIPA. ***************************

| J \ I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 9 | 2.50E-02 | 2.50E-02 | 2.50E-02 | 2.50E-02 | 2.50E+02 | 2.50E+02 | 2.50E+02 |
| 8 | 2.50E-02 | 8.18E+03 | 1.25E+04 | 1.13E+04 | 7.78E+03 | 3.60E+03 | 3.60E+03 |
| 7 | 2.50E-02 | 1.07E+04 | 1.44E+04 | 1.28E+04 | 7.79E+03 | 2.82E+03 | 2.82E+03 |
| 6 | 2.50E-02 | 1.34E+04 | 1.71E+04 | 1.53E+04 | 7.94E+03 | 2.12E+03 | 2.12E+03 |
| 5 | 2.50E-02 | 1.51E+04 | 1.93E+04 | 1.80E+04 | 7.66E+03 | 1.50E+03 | 1.50E+03 |
| 4 | 2.50E-02 | 1.29E+04 | 1.79E+04 | 1.98E+04 | 6.81E+03 | 1.01E+03 | 1.01E+03 |
| 3 | 2.50E-02 | 5.08E+03 | 1.04E+04 | 1.99E+04 | 5.46E+03 | 6.63E+02 | 6.63E+02 |
| 2 | 2.50E-02 | 3.34E+02 | 1.53E+02 | 1.52E+04 | 3.43E+03 | 4.02E+02 | 4.02E+02 |
| 1 | 2.50E-02 | 2.50E-02 | 2.50E-02 | 2.50E-02 | 2.50E+02 | 2.50E+02 | 2.50E+02 |

**Initial values, No decoration!**

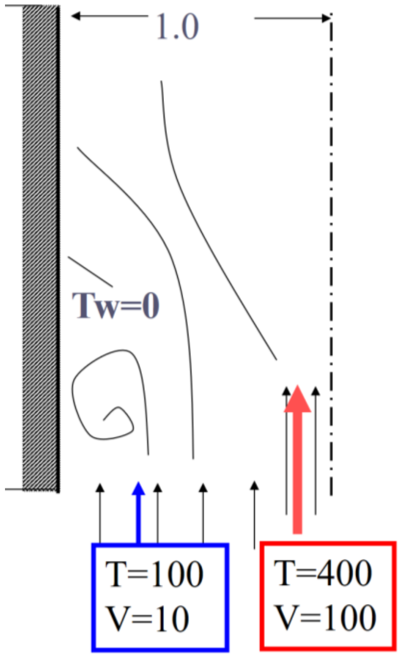
**Initial values,**
**No decoration!**

$$\varepsilon = \frac{C_\mu^{3/4} k_P^{3/2}}{\kappa y_P}$$

**************************** TURB VI ****************************

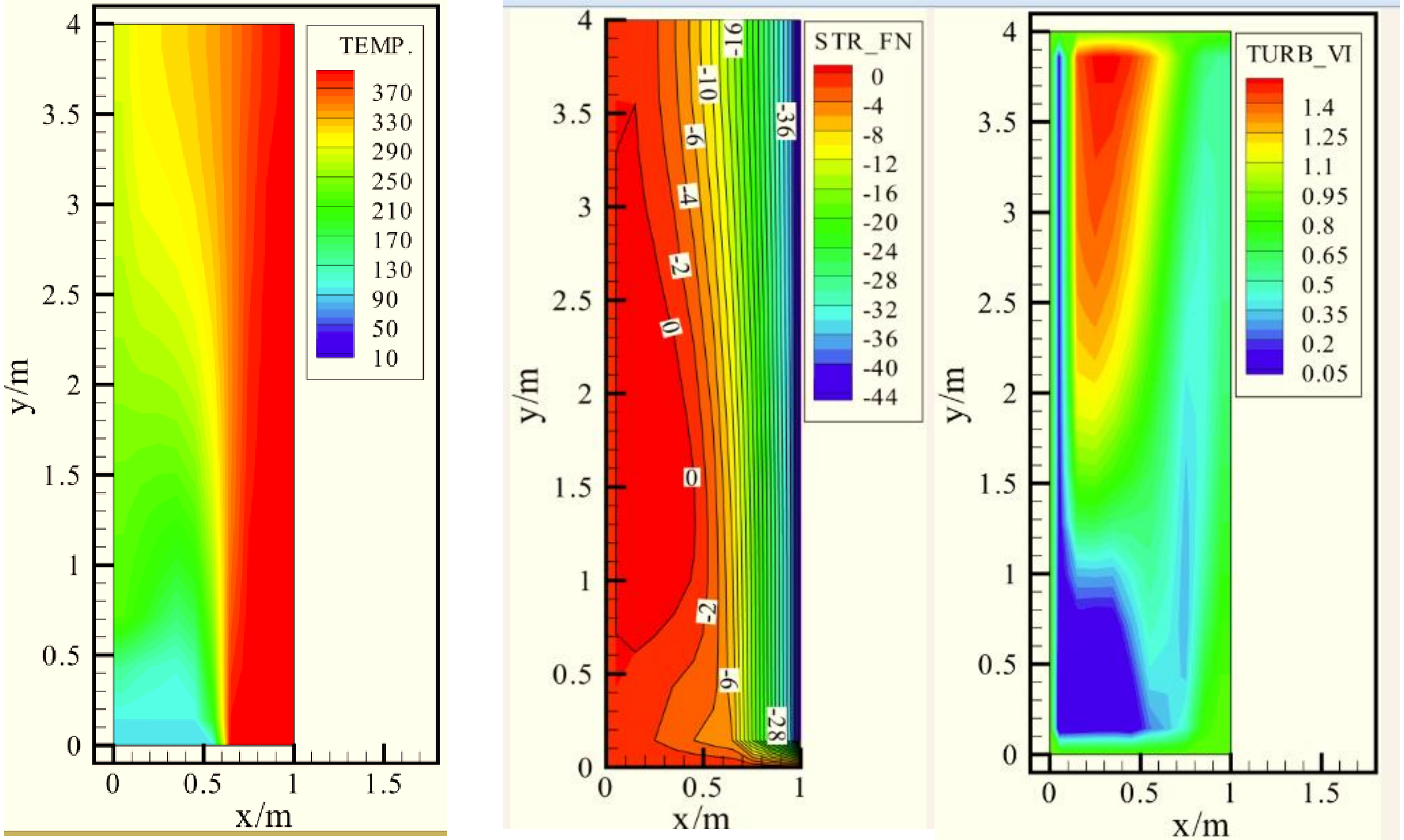
| J \ I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 9 | 9.00E-01 | 9.00E-01 | 9.00E-01 | 9.00E-01 | 9.00E-01 | 9.00E-01 | 9.00E-01 |
| 8 | 9.00E-01 | 2.78E-01 | 1.72E+00 | 1.70E+00 | 1.42E+00 | 1.14E+00 | 1.14E+00 |
| 7 | 9.00E-01 | 3.04E-01 | 1.76E+00 | 1.65E+00 | 1.29E+00 | 9.59E-01 | 9.59E-01 |
| 6 | 9.00E-01 | 3.27E-01 | 1.77E+00 | 1.59E+00 | 1.16E+00 | 7.99E-01 | 7.99E-01 |
| 5 | 9.00E-01 | 3.40E-01 | 1.71E+00 | 1.48E+00 | 1.01E+00 | 6.75E-01 | 6.75E-01 |
| 4 | 9.00E-01 | 3.22E-01 | 1.46E+00 | 1.28E+00 | 8.54E-01 | 6.02E-01 | 6.02E-01 |
| 3 | 9.00E-01 | 2.36E-01 | 9.39E-01 | 9.99E-01 | 7.00E-01 | 5.94E-01 | 5.94E-01 |
| 2 | 9.00E-01 | 9.50E-02 | 6.24E-02 | 6.19E-01 | 5.58E-01 | 6.88E-01 | 6.88E-01 |
| 1 | 9.00E-01 | 9.00E-01 | 9.00E-01 | 9.00E-01 | 9.00E-01 | 9.00E-01 | 9.00E-01 |

**Molecular viscosity** $\eta_l \approx 10^{-6}$



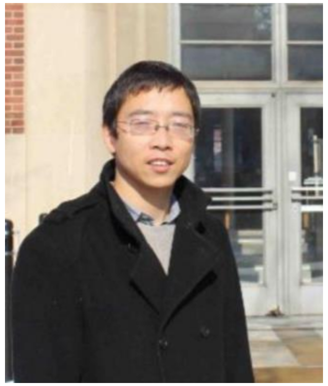

******************************  PRESSURE  ******************************

| I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| J | | | | | | | |
| 9 | 1.44E+03 | 1.43E+03 | 1.41E+03 | 1.33E+03 | 1.21E+03 | 1.14E+03 | 1.12E+03 |
| 8 | 1.36E+03 | 1.35E+03 | 1.33E+03 | 1.28E+03 | 1.20E+03 | 1.15E+03 | 1.13E+03 |
| 7 | 1.20E+03 | 1.19E+03 | 1.17E+03 | 1.17E+03 | 1.17E+03 | 1.16E+03 | 1.16E+03 |
| 6 | 9.40E+02 | 9.31E+02 | 9.11E+02 | 9.19E+02 | 9.28E+02 | 9.26E+02 | 9.25E+02 |
| 5 | 6.02E+02 | 5.92E+02 | 5.72E+02 | 5.96E+02 | 6.22E+02 | 6.25E+02 | 6.27E+02 |
| 4 | 2.24E+02 | 2.16E+02 | 1.99E+02 | 2.54E+02 | 3.08E+02 | 3.24E+02 | 3.32E+02 |
| 3 | 4.20E+01 | 3.16E+01 | 1.09E+01 | 1.03E+02 | 1.39E+02 | 1.44E+02 | 1.46E+02 |
| 2 | 1.31E+01 | 5.48E+00 | -9.74E+00 | -6.55E+01 | 2.53E+01 | 4.85E+01 | 6.02E+01 |
| 1 | 0.00E+00 | -7.61E+00 | -2.01E+01 | -1.50E+02 | -3.17E+01 | 1.07E+00 | 1.27E+01 |


TEMP.
370
330
290
250
210
170
130
90
50
10
y/m
x/m
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
STR_FN
0
-4
-8
-12
-16
-20
-24
-28
-32
-36
-40
-44
-16
-10
-6
-4
-2
0
0
-2
-6
-28
-36
TURB_VI
1.4
1.25
1.1
0.95
0.8
0.65
0.5
0.35
0.2
0.05

# Part I： Fundamentals of NHT and Teaching Code （11 chapters）

## Part II of NHT：Study of FLUENT



冀文涛
（Wen-Tao Ji)

C 12 Basic contents
(6 hours)

任秦龙
(Qing-Long Ren)

Applications
(6 hours)

C 13a Fundamental Applications

C 13b Intermediate Applications

陈黎
（Li Chen）

# Announcement

**Next week, Prof. Wen-Tao Ji is going to attend a oversea meeting. Therefore, the lessons in the next week (Dec. 5 & Dec. 6) will be postponed (延期). Next class is on Dec. 12, 2023.**

下周的课（12月5日、12月6日）取消，课程往后顺延一周。

本组网页地址：http://nht.xjtu.edu.cn 欢迎访问！

*Teaching PPT will be loaded on our website*


同舟共济

渡彼岸！

**People in the same boat help each other to cross to the other bank, where….**