# Numerical Heat Transfer

# (数值传热学)

## Chapter 11 Application Examples of the General Code for 2D Elliptical FF & HT Problems (2)

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11.1 2D steady heat conduction without source term in Cartesian coordinate

11.2 Steady heat conduction in a hollow cylinder

11.3 Fully-developed heat transfer in a square duct

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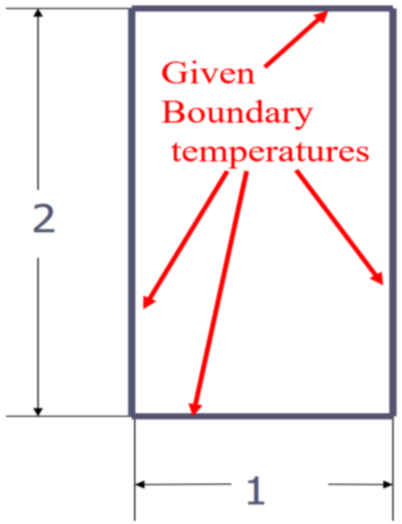
11.4 Fully developed heat transfer in annular space with straight fin at inner wall

11.5 Fluid flow and heat transfer in a 2-D sudden expansion

11.6 Complicated fully developed fluid flow and heat transfer in square duct

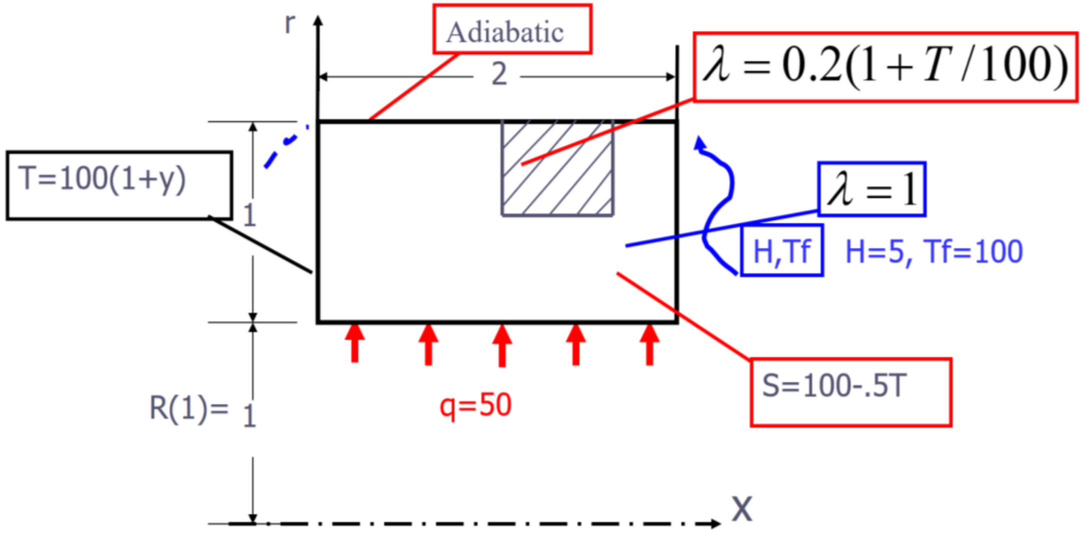
11.7 Impinging flow on a rotating disc

11.8 Turbulent flow and heat transfer in duct with a central jet

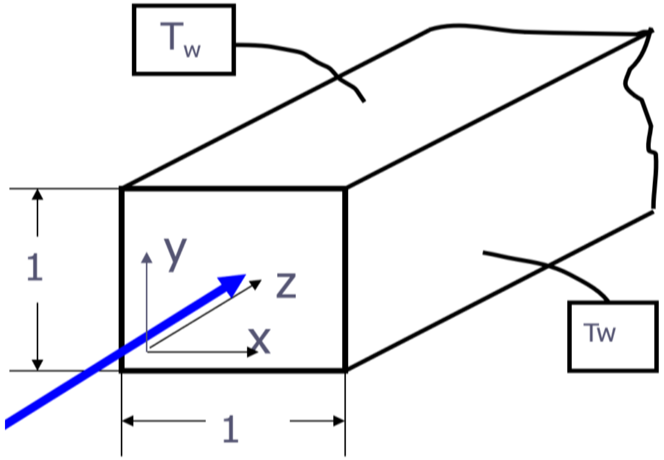



## Knowing USER structure

**USER: Includes modules GRID, START, DENSE, BOUND, OUTPUT, GAMSOR**




Execution of ASTM




Solution of FDHT

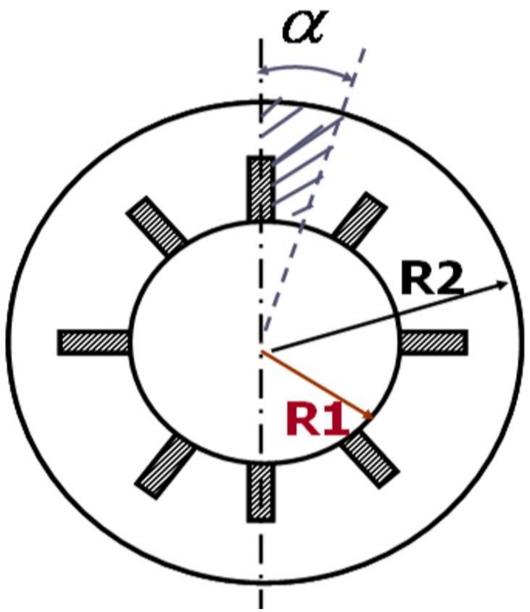
# 11-4 Fully developed heat transfer in annular space with straight fin at inner wall －Numerical methods for conjugated problems

## 11-4-1 Physical Problem and its math formulation

Known：Laminar heat transfer with constant properties in annular space with straight fins at inner wall (Fig. 1).

Its outer wall is adiabatic, while inner wall temperature is circumferentially uniform（周向均匀壁温）.

$R_1$=1, $R_2$=2, the angle between two successive fins equals 30°. Ratio of fin thermal conductivity over fluid one is ten.




$$\frac{\lambda_{fin}}{\lambda_{fluid}} = 10$$

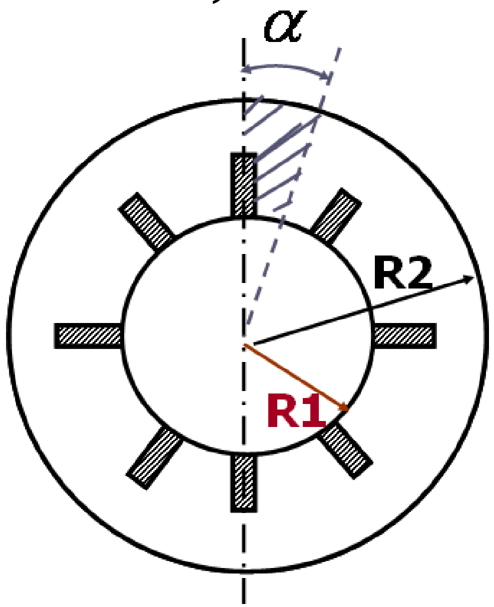
$$\frac{R_2}{R_1} = 2$$

$$\alpha = 15^\circ$$

Fig.1 Cross section view of Problem 4

**Find**: Cross-sectional distributions of velocity and temperature, and *fRe*、*Nu*.

**Solution**: Similar to problem 3, $u$=0, $v$=0, ∂w/∂z = 0, the governing eq. for axial velocity $w$:




$$\frac{1}{r}\frac{\partial}{\partial r}(r\eta\frac{\partial w}{\partial r})+\frac{1}{r}\frac{\partial}{\partial\theta}(\frac{\eta}{r}\frac{\partial w}{\partial\theta})-\frac{dp}{dz}=0$$

$div(\eta grad w)$ — Source term

（Polar coordinate)

The governing eq. of temperature in the fully developed region：

$$div(\lambda grad T)-\rho c_p w\frac{\partial T}{\partial z}=0$$

Source term

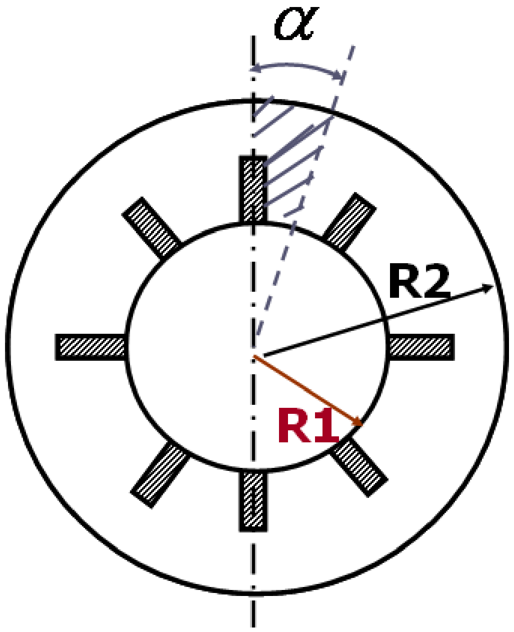
# 11-4-2 Numerical methods

（1）This problem is governed by two conduction-type equations with source term；

$$div(\eta \text{grad} w)-\frac{dp}{dz}=0 \qquad div(\lambda \text{grad} T)-\rho c_p w\frac{\partial T}{\partial z}=0$$

（2）Velocity is not coupled with temperature, and can be solved first;

（3）The fin can be regarded as a special fluid with a very large viscosity; hence the entire flow region can be solved simultaneously---conjugated problem(耦合问题);




（4）The half of the region between two successive fins can be taken as computational domain due to symmetry;

(5) In calculation of cross sectional temperature distribution, it can assume that at the whole section $\partial T/\partial z = C$

$\frac{\partial T}{\partial z} = \Theta \frac{dT_b}{dz}$ (more rigorous for problem 3)

(6) It is assumed that the fin surface coincides with radius.

(7) The fin and fluid temperatures are solved at same time (simultaneously) --- conjugated problem (耦合问题)

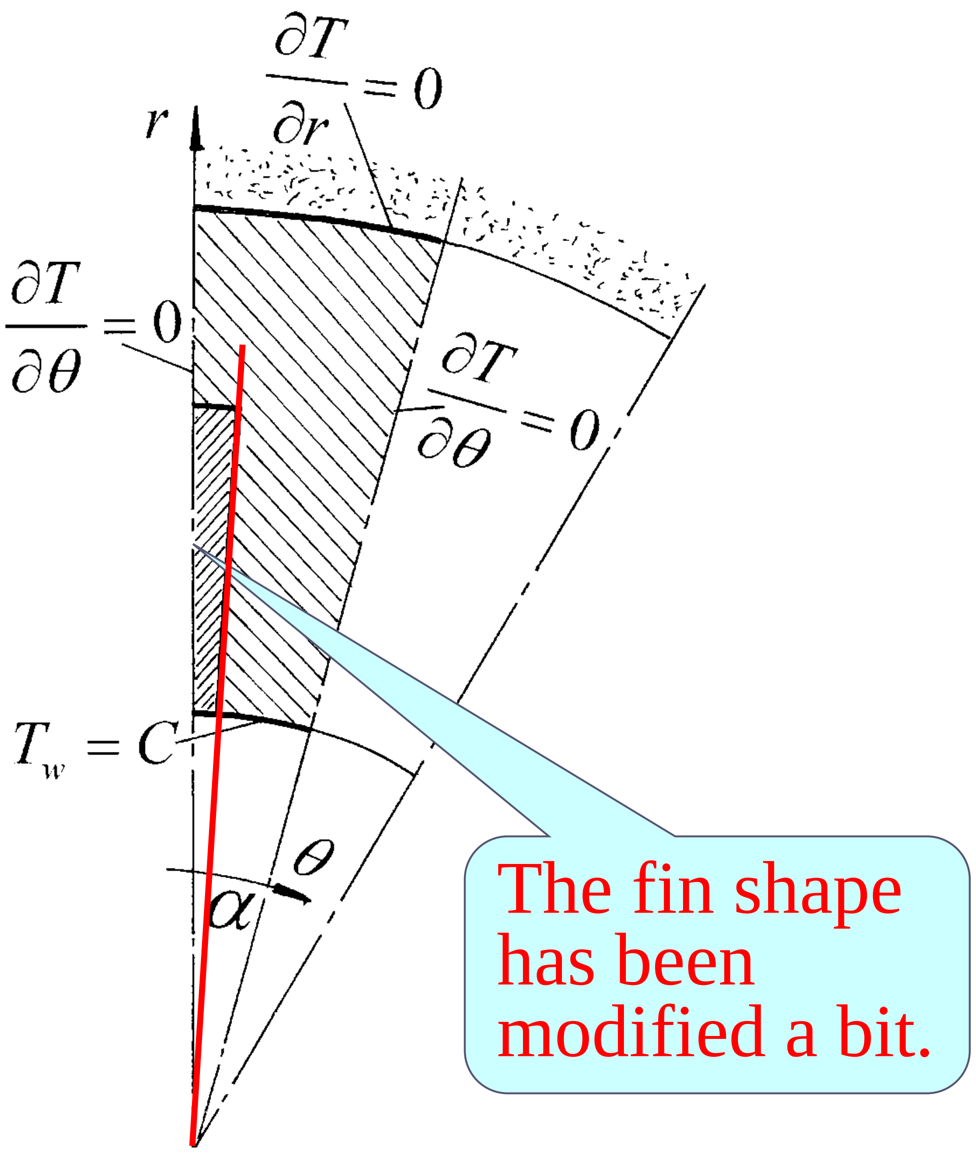
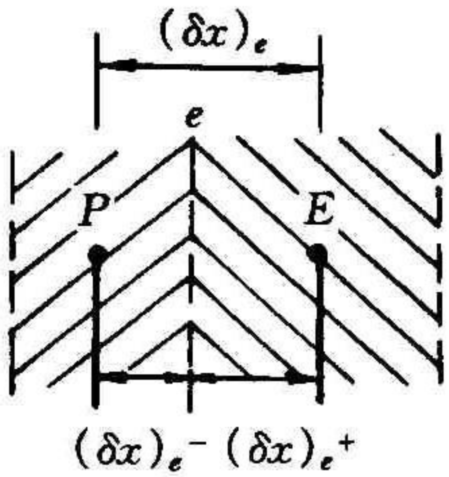



Fig. 2 Computational domain

# Conjugated heat transfer (耦合传热)

If conductivities of P, E are different, the harmonic mean is required to ensure the continuum of heat flux at interface




$$\frac{(\delta x)_e}{\lambda_e} = \frac{(\delta x)_{e^+}}{\lambda_E} + \frac{(\delta x)_{e^-}}{\lambda_P}$$

Harmonic mean

In MAIN program of teaching code, we have

$$\frac{(\delta x)_e}{\Gamma_e} = \frac{(\delta x)_{e^+}}{\Gamma_E} + \frac{(\delta x)_{e^-}}{\Gamma_P}$$

- For the old temperature G.E. in textbook:

$$\frac{\partial(\rho T)}{\partial t} + \frac{\partial(\rho u T)}{\partial x} + \frac{\partial(\rho v T)}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\lambda}{c_p}\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\lambda}{c_p}\frac{\partial T}{\partial y}\right) + \frac{S_T}{c_p}$$

Nominal diffusivity: $\Gamma = \lambda / c_p$

$$\frac{(\delta x)_e}{\Gamma_e} = \frac{(\delta x)_{e^+}}{\Gamma_E} + \frac{(\delta x)_{e^-}}{\Gamma_P}$$

$$\frac{(\delta x)_e}{\lambda / c_{p,e}} = \frac{(\delta x)_{e^+}}{\lambda_E / c_{p,E}} + \frac{(\delta x)_{e^-}}{\lambda_P / c_{p,P}}$$

assuming $c_{p,e} = c_{p,E} = c_{p,P}$

$$\frac{(\delta x)_e}{\lambda_e} = \frac{(\delta x)_{e^+}}{\lambda_E} + \frac{(\delta x)_{e^-}}{\lambda_P}$$

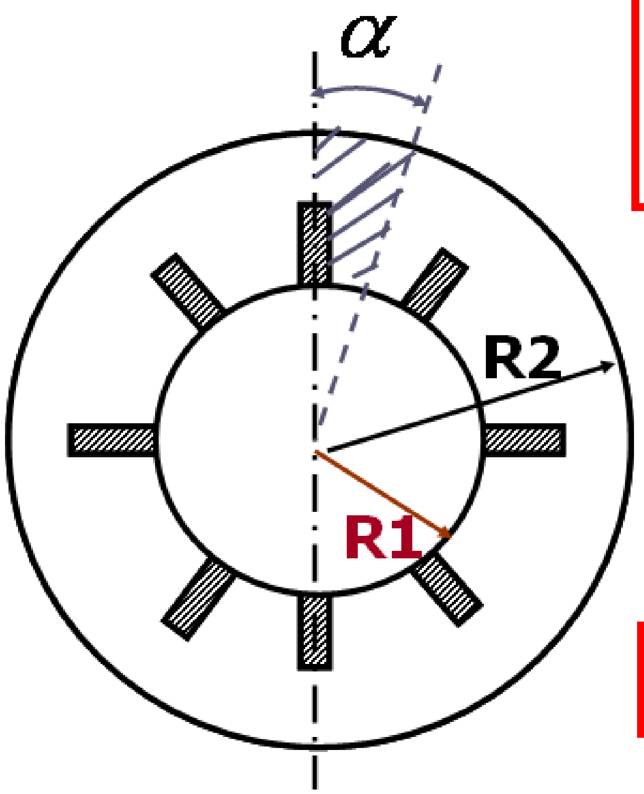
For steady problems, such assumption is OK; while for the transient problem, it does not work.

- For the new temperature GE, we have $\Gamma = \lambda$

$$\frac{\partial(\rho c_p T)}{\partial t} + \frac{\partial(\rho c_p u T)}{\partial x} + \frac{\partial(\rho c_p v T)}{\partial y} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + S_T .$$

We do not need any additional assumption, it automatically satisfies the continuum of heat flux at fluid-solid interface.

# Fully developed heat transfer in annular space with straight fin at inner wall




$$\frac{\lambda_{fin}}{\lambda_{fluid}} = 10$$

$$\frac{R_2}{R_1} = 2$$

$$\alpha = 15^\circ$$

Fig.1 Cross section view of Problem 4

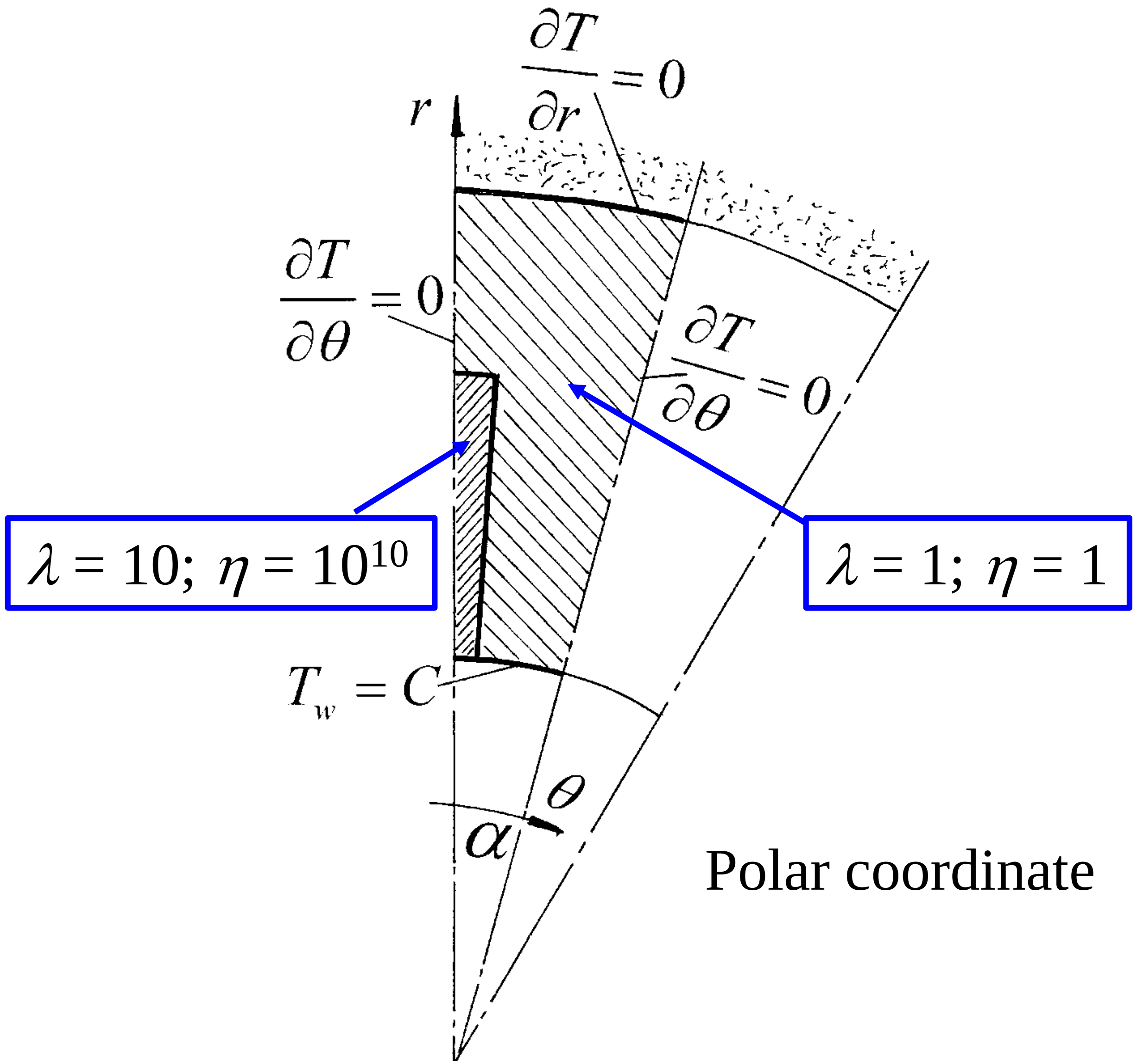



Fig. 2 Computational domain

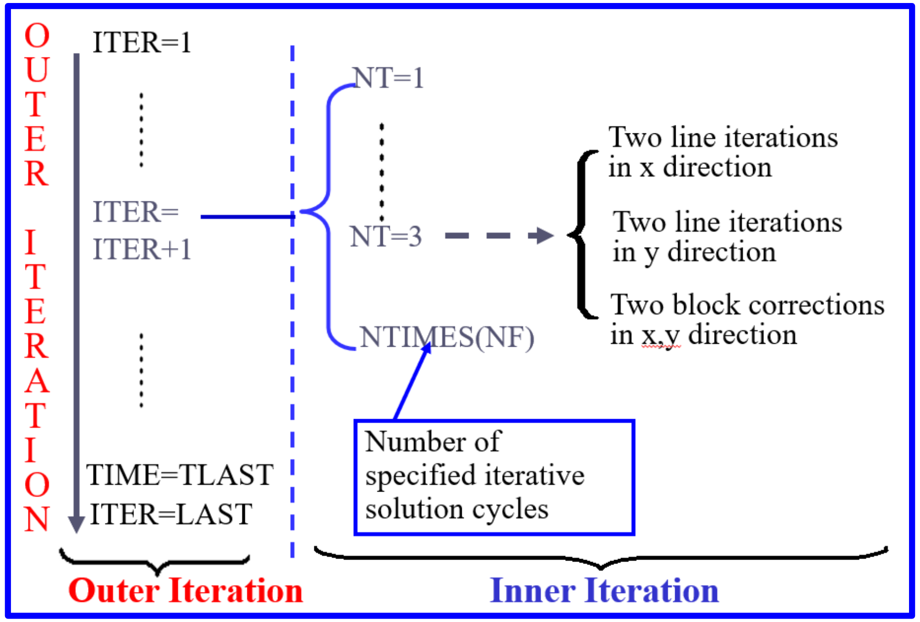
# 11-4-3 Program reading

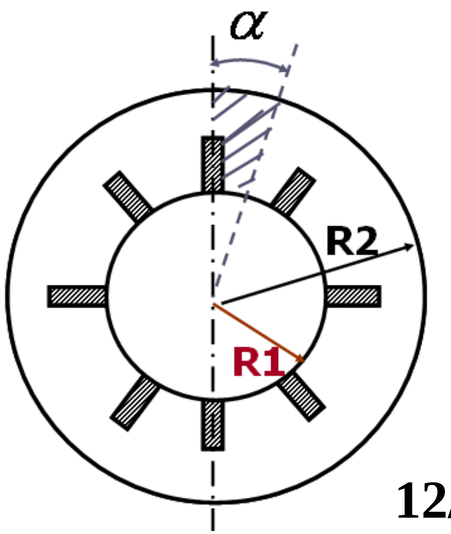
```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C******************************************************************
      INTEGER*4 I, J
      REAL*8 PI, TW, AMU, DPDZ, COND, RHOCP, DTDZ, WSUM, ASUM,
     1 TSUM, AR, WBAR, WP, DH, RE, FRE,TBULK, HTP, HTC, ANU
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE USER
C*******************************************************************
      USE START_L
      USE USER_L
      IMPLICIT NONE
C********************************************************************
C------------------------------------PROBLEM FOUR-----------------------------------------
C     Fully developed laminar fluid flow and heat transfer in annular duct with
C-------------------------------longitudinal fins on inner tube------------------------------
C********************************************************************
```

**ENTRY GRID**

```
TITLE(4)='.THETA.'         ! 4th variable for temperature
TITLE(5)='.W/WBAR.'        ! 5th variable for velocity
LSOLVE(5)=.TRUE.
LPRINT(4)=.TRUE.           ! Velocity solved first, temperature next
LPRINT(5)=.TRUE.
LAST=6
NTIMES(4)=4                ! Both equations are linear, NTIMES may take larger values to decrease outer iteration times.
NTIMES(5)=4
MODE=3                     ! Polar coordinate
R(1)=1.                    ! Specify the bottom radius
PI=3.14159
THL=15.*PI/180.            ! Transform from degree to radian (弧度)
YL=1.                      ! Equivalence (XL, THL)
L1=7
M1=7
CALL UGRID
RETURN
```

```
 EQUIVALENCE(X,TH),(XU,THU),(XDIF,THDIF),(XCV,THCV),
1(XCVS,THCVS),(XL,THL)
```



```
ENTRY START
TW=1.            !  Set up cross sectional wall temperature
DO 100 J=1,M1
DO 101 I=1,L1
F(I,J,4)=TW
F(I,J,5)=0.
101 ENDDO
100 ENDDO
AMU=1.           !  Very large viscosity to ensure laminar flow
COND=1.
RHOCP=1.
DPDZ=-2000.      !  Pressure gradient should be less than zero
DTDZ=100.        !  Set up axial gradient of fluid temperature
RETURN

ENTRY DENSE
RETURN           !  Empty, but keep it.
```

! Initial fields of $w$ (=0) and $T$ (= $T_w$)

! Also specified the unchanged BC for $w$ and $T$. $w$=0 at the wall; $T$= $T_w$ at the bottom wall

$$\frac{\partial(\rho^*\Phi)}{\partial t}+div(\rho^*\vec{u}\Phi)=div(\Gamma_\Phi grad\Phi)+S_\phi^*$$

! This is not a true flow problem, and there is no convection. RHOCP here is for the source term in conduction equation.

$$S_c=-\rho c_p w\frac{\partial T}{\partial z}$$

```
ENTRY BOUND
ASUM=0.
WSUM=0.
TSUM=0.
DO 300 J=2,M2
DO 301 I=2,L2
IF(I>2.OR.I=2 .AND.J>4) THEN
AR=YCVR(J)*THCV(I)
WSUM=WSUM+F(I,J,5)*AR
TSUM=TSUM+AR*F(I,J,4)*F(I,J,5)
ASUM=ASUM+AR
ENDIF
301 ENDDO
300 ENDDO
WBAR=WSUM/ASUM
WP=(R(1)+R(M1))*THL+(1.+THCV(2))*(RMN(5)-R(1))
DH=4.*ASUM/WP
RE=RHOCON*WBAR*DH/AMU
FRE=-2.*DPDZ*DH/(RHOCON*WBAR**2+1.E-30)*RE
```

！ Initial values for summation

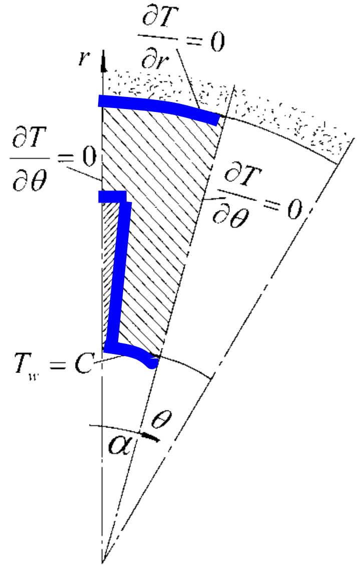
$$AR(\text{面积元}) = YCV(j)*R(j)*XCV(i) = YCV(j)*R(j)*THCV(i) = YCVR(j)*THCV(i)$$

！ Exclude(排除)solid region for flow area

$$\sum w(i,j)dA_{i,j}$$

$$\sum w(i,j)T(i,j)dA_{i,j}$$

！ Flow area




！ Mean velocity

！ Length of wetted perimeter(润湿边界的周长)

$$f\,\mathrm{Re} = \frac{-(dp/dx)D_h}{(1/2)\rho w_m^2}\mathrm{Re}$$

```
TBULK=TSUM/(WSUM+1.E-30)          ! Mean temperature
HTP=WP-R(M1)*THL                  ! Length of perimeter for heat transfer
HTC=RHOCP*WSUM*DTDZ/((TW-TBULK+1.E-30)*HTP)
ANU=HTC*DH/COND                   ! Nu = hD_e / λ
IF(ITER<3) RETURN
LSOLVE(4)=.TRUE.
LSOLVE(5)=.FALSE.
RETURN
```

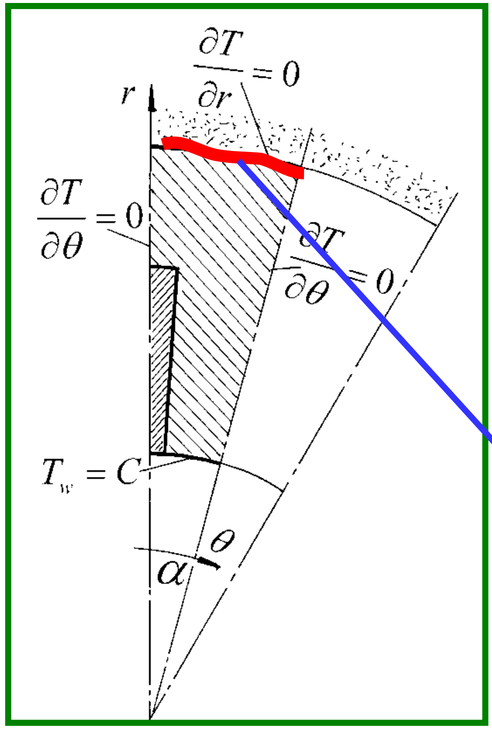
$$T_b = \iint w(i,j)T(i,j)dA_{i,j} \Big/ \iint w(i,j)dA_{i,j}$$

$$Nu = hD_e / \lambda$$

Switch solution variable

$$q = \rho c_p (W_m A \frac{\partial T}{\partial z}) \bullet 1/(HTP \bullet 1)$$

$$h = q/(T_w - T_b)$$




! This length is adiabatic, hence should be excluded in HTP.

```
      ENTRY OUTPUT
      IF(ITER==0) THEN                    ! The head of output
      PRINT 401
      WRITE(8,401)
401 FORMAT(1X,' ITER',12X,'F.RE',17X,'NU')
      ELSE
      PRINT 402, ITER, FRE, ANU
      WRITE(8,402) ITER,FRE,ANU
402 FORMAT(1X,I6,1P2E20.4)
      ENDIF
      IF(ITER/=LAST) RETURN
      DO 410 J=1,M1
      DO 411 I=1,L1                        ! Output of
      F(I,J,5)=F(I,J,5)/WBAR                 dimensionless
      F(I,J,4)=(F(I,J,4)-TW)/(TBULK-TW+1.E-30)  results
411 ENDDO
410 ENDDO
      CALL PRINT
      RETURN
```

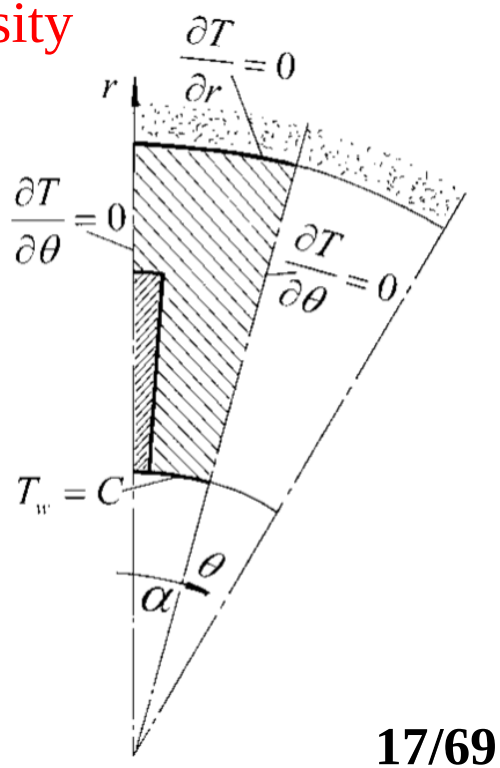
$$\Theta = \frac{T - T_w}{T_b - T_w};\ \Theta_w = \frac{T_w - T_w}{T_b - T_w} = 0$$

```
ENTRY GAMSOR
 DO 500 I=1,L1
 DO 501 J=1,M1
 GAM(I,J)=AMU                       ! Γ for velocity(specified first)
 IF(NF==4) GAM(I,J)=COND            ! Γ for temperature
 GAM(1,J)=0.      }  ! Symmetry=adiabatic (for both w and T)
 GAM(L1,J)=0.     }
 IF(NF==4) GAM(I,M1)=0.   ! North BC: adiabatic for T; w = 0 specified in START.
 IF(J<=4) GAM(2,J)=1.E10  ! Fin is regarded as fluid with large viscosity
 IF(NF==4.AND.J<=4) GAM(2,J)=10.*COND   ! Fin conductivity
501 ENDDO
500 ENDDO
 DO 510 J=2,M2
 DO 511 I=2,L2
 CON(I,J)=-DPDZ     ! Source term of w-eq., should be less than zero
 IF(NF==4) CON(I,J)=-DTDZ*F(I,J,4)*RHOCP
511 ENDDO
510 ENDDO
 RETURN
 END
```

(left、right BCs)

! Source of Temperature eq.

$$-\rho c_p w\frac{dT}{dz}$$



# 11-4-4 Results analysis

COMPUTATION IN POLAR COORDINATES
*******************************************

| ITER | F.RE | NU |
|---|---|---|
| 0 | 0.0000E+00 | 0.0000E+00 |
| 1 | 6.5484E+01 | 1.9787E+10 |
| 2 | 6.5484E+01 | 2.3588E+33 |
| 3 | 6.5484E+01 | 2.3588E+33 |
| 4 | 6.5484E+01 | 1.5098E+00 |
| 5 | 6.5484E+01 | 1.5098E+00 |
| 6 | 6.5484E+01 | 1.5098E+00 |

Solving flow only

! NTIMES=4, only one outer iteration solution is converged

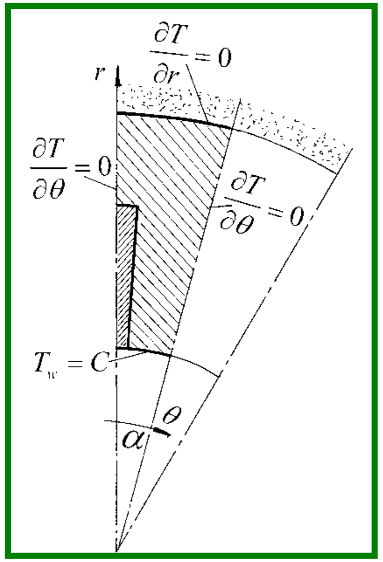
! NTIMES=4, only one outer iteration solution is converged

**************************** .W/WBAR. ****************************

| J \ I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 7 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| 6 | 0.00E+00 | 8.18E-01 | 8.50E-01 | 8.91E-01 | 9.25E-01 | 9.43E-01 | 0.00E+00 |
| 5 | 0.00E+00 | 1.10E+00 | 1.30E+00 | 1.50E+00 | 1.64E+00 | 1.72E+00 | 0.00E+00 |
| 4 | 0.00E+00 | 4.37E-09 | 4 .57E-01 | 1.05E+00 | 1.41E+00 | 1.58E+00 | 0.00E+00 |
| 3 | 0.00E+00 | 3.34E-09 | 3.01E-01 | 7.45E-01 | 1.03E+00 | 1.18E+00 | 0.00E+00 |
| 2 | 0.00E+00 | 1.43E-09 | 1.63E-01 | 3.91E-01 | 5.36E-01 | 6.06E-01 | 0.00E+00 |
| 1 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |

$w$=0 of fin region

Symmetric line, not decorated (initial values).




Symmetric line, not decorated (initial values)

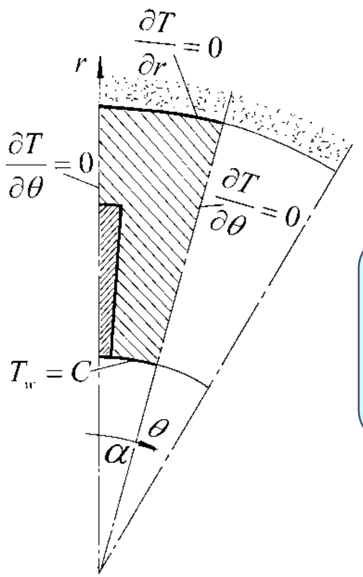
*************************** .THETA. ***************************

Adiabatic, not decorated

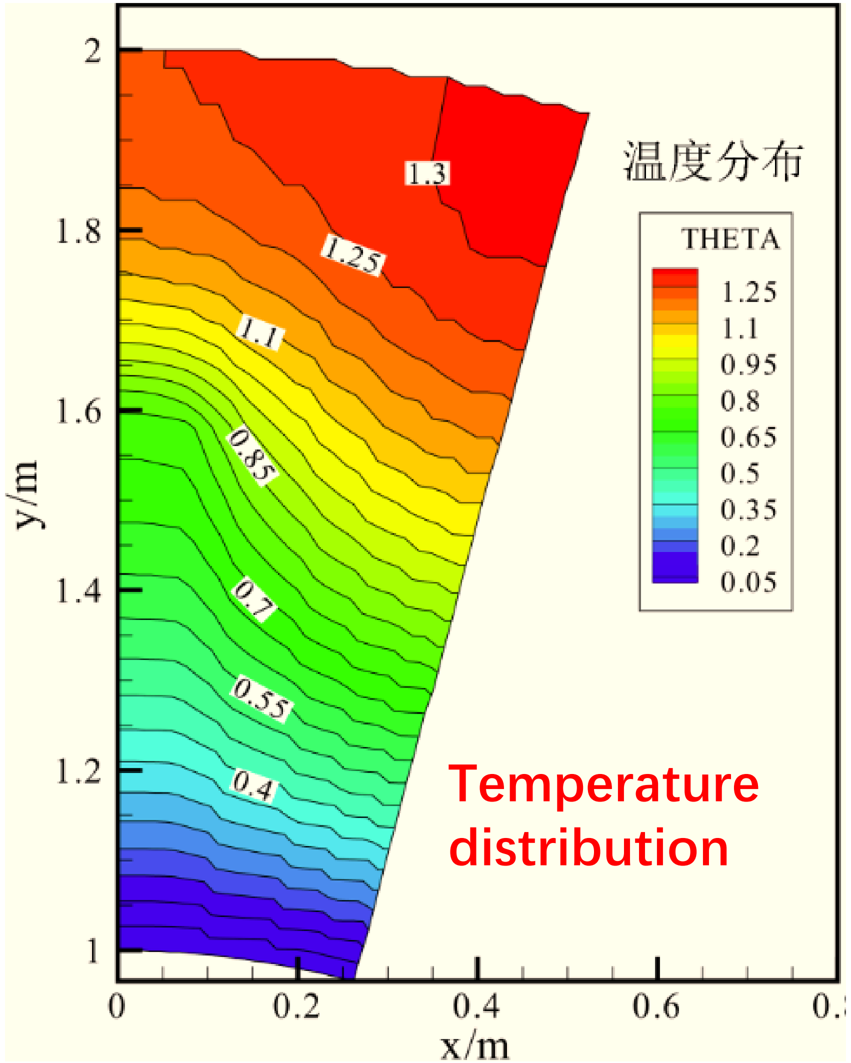
| I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| J | | | | | | | |
| 7 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| 6 | 0.00E+00 | 1.24E+00 | 1.26E+00 | 1.28E+00 | 1.30E+00 | 1.31E+00 | 0.00E+00 |
| 5 | 0.00E+00 | 1.03E+00 | 1.09E+00 | 1.15E+00 | 1.19E+00 | 1.21E+00 | 0.00E+00 |
| 4 | 0.00E+00 | 6.34E-01 | 7.15E-01 | 8.24E-01 | 8.96E-01 | 9.32E-01 | 0.00E+00 |
| 3 | 0.00E+00 | 4.48E-01 | 4.80E-01 | 5.36E-01 | 5.78E-01 | 6.00E-01 | 0.00E+00 |
| 2 | 0.00E+00 | 1.76E-01 | 1.86E-01 | 2.04E-01 | 2 .18E-01 | 2.26E-01 | 0.00E+00 |
| 1 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |

$$\Theta_w = \frac{T_w - T_w}{T_b - T_w} = 0$$

Symmetric line, not decorated.




Symmetric line, not decorated



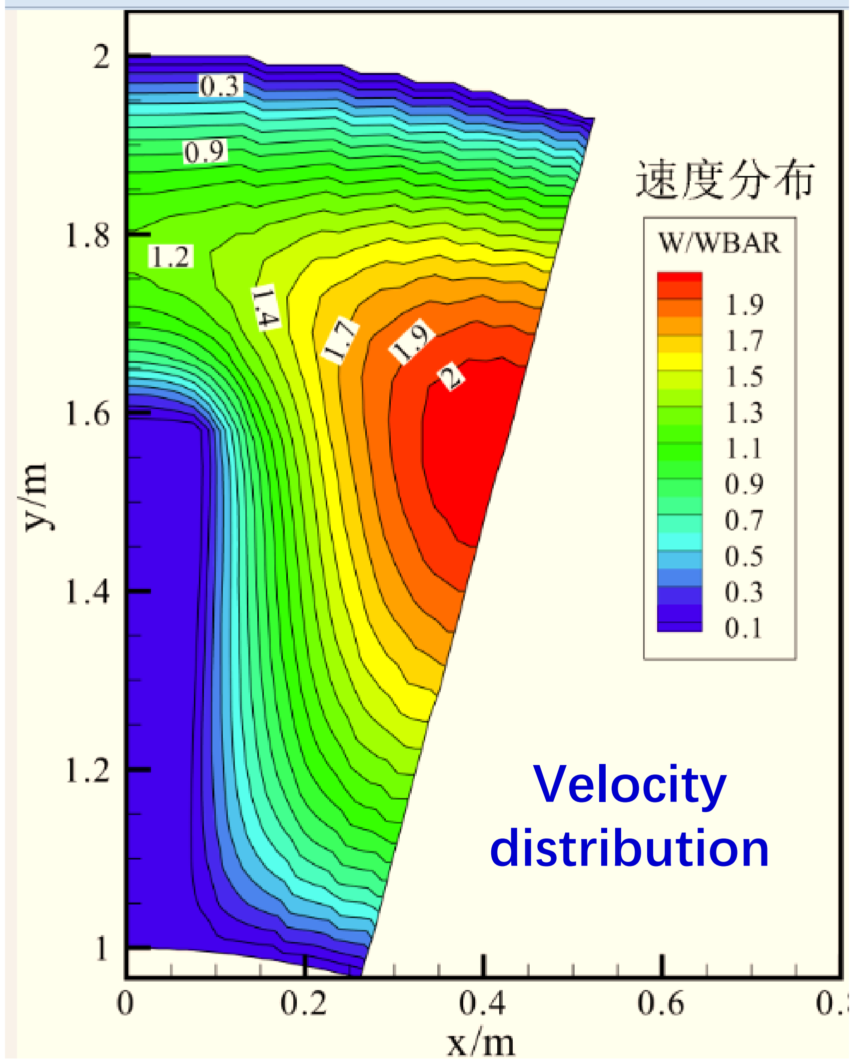




Fig.3 Result of Problem 4

# 11-5 Fluid flow and heat transfer in a 2-D sudden expansion---Solution of Navier Stokes equation

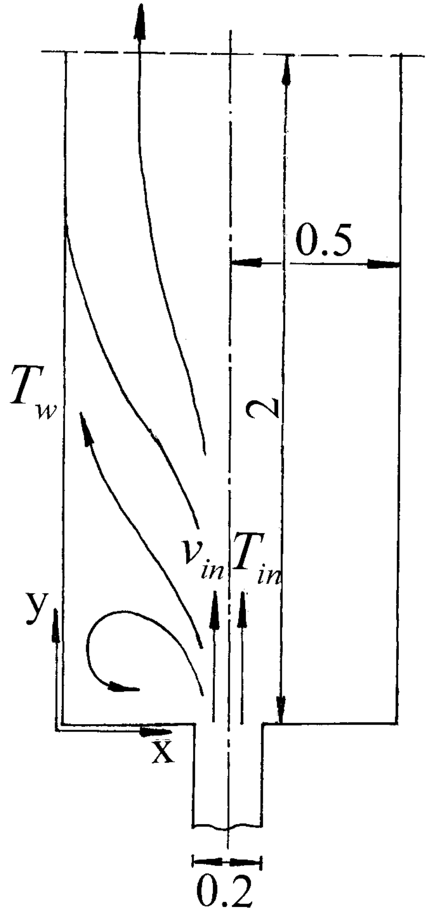
## 11-5-1 Physical problem and its math formulation

Known: Laminar flow and heat transfer in a parallel duct shown in Fig. 1: Uniform inlet velocity, $\boldsymbol{V_{in}}$**=100,** and uniform inlet temperature, $\boldsymbol{T_{in}}$**= 500**; Duct wall are at uniform temperature, $\boldsymbol{T_w}$**= 300**; Fluid **Pr = 0.7**, molecular dynamic viscosity $\eta = 1$; density varies according to:

$$\rho = \rho_{ref} \frac{T_{ref}}{T}$$

where referenced density $\rho_{ref} = 1$, and $T_{ref} = 300$.

Note that: compressible fluids can undergo incompressible flow, as long as $\nabla \cdot \mathbf{u} = 0$.



# Laminar flow and heat transfer in a parallel duct

**Find:** Distributions of velocity, temperature, density and fluid pressure in the duct.

**Solution:** Solve the Navier Stokes equation and temperature governing equation

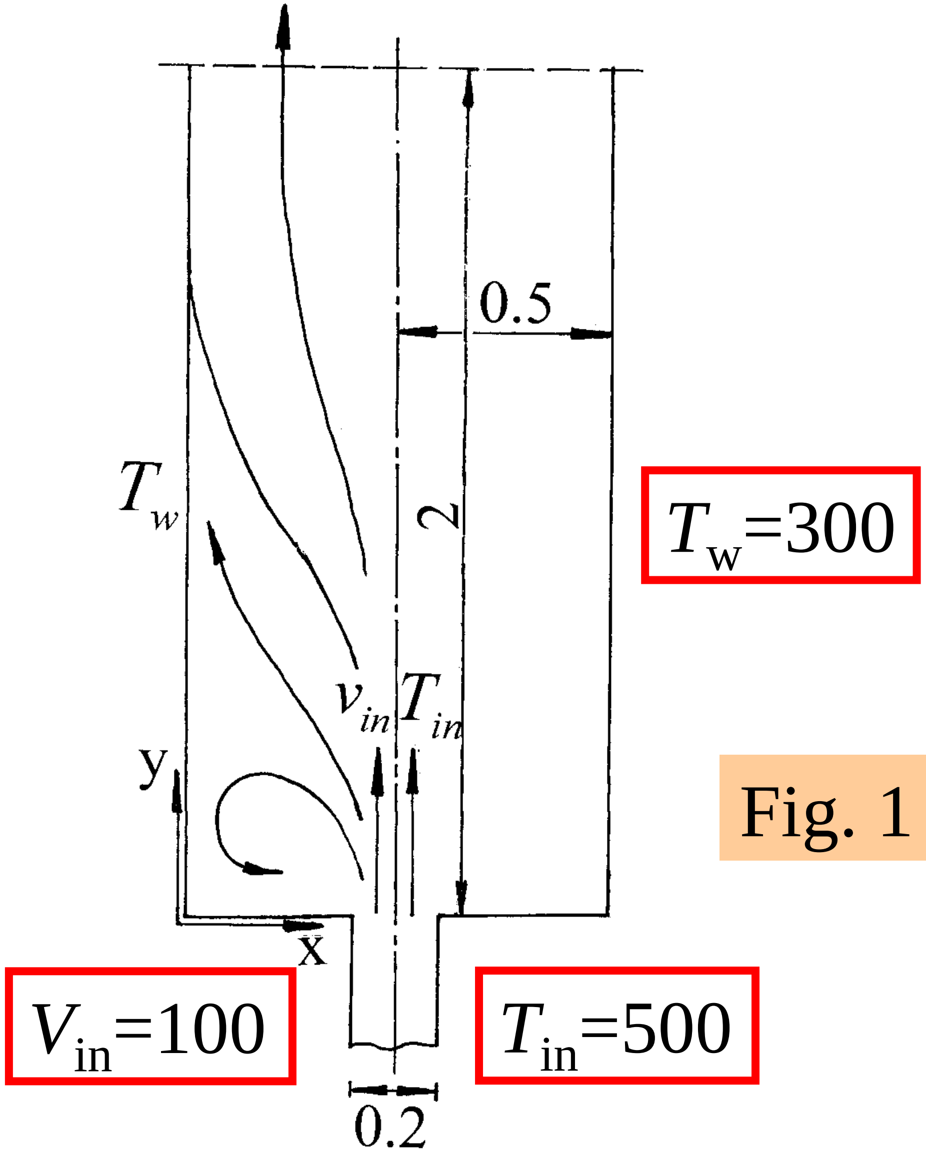



Fig. 1

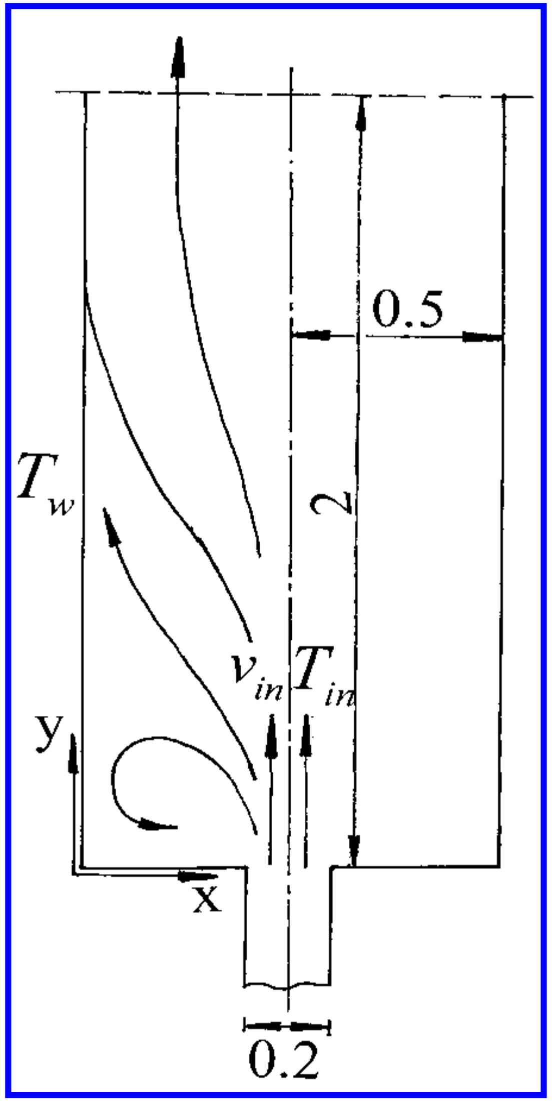
- The governing equations of velocity and temperature:

$$u: \quad div(\rho \vec{u} u) = -\frac{\partial p}{\partial x} + div(\eta \, grad u) + 0$$

$$v: \quad div(\rho \vec{u} v) = -\frac{\partial p}{\partial y} + div(\eta \, grad v) + 0$$

$$T: \quad div(\rho c_p \vec{u} T) = div(\lambda \, grad T) + 0$$

$\frac{\partial p}{\partial x}$ $\frac{\partial p}{\partial y}$ have been treated as source term in **MAIN** program




- Boundary conditions:

At symmetric line: $u = 0; \ \frac{\partial v}{\partial x} = 0; \ \frac{\partial T}{\partial x} = 0$

At inlet: $u, v, T$ are specified;

At solid wall: $u = v = 0; \ T = T_w$

# 11-5-2 Numerical methods

（1）This is an open-flow system. Determination of normal velocity at the outlet boundary for open flow is important. We set outlet boundary in region without recirculation, and adopt local one-way method with total mass conservation；

（2）Convergence condition for flow field iteration: SSUM and SMAX less than pre-specified values or 4 to 5 digits of printed values remain unchanged during 5 to 10 successive iterations；

（3）Variation of density with temperature is specified in ENTRY DENSE. Momentum equations are coupled with temperature equation.

# 11-5-3 Program reading

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C***************************************************************
      INTEGER*4 I,J
      REAL*8 TIN, TW, VIN, VOUT, PR, AMU, COND, TREF, RHOREF,
     1   RHOT, FLOWIN, FL, FACTOR
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE USER
C****************************************************************
      USE START_L        !Difference in section number and problem number:
      USE USER_L                    !Section No is five;
      IMPLICIT NONE                 !Prob. No. 6 of the original code
C*****************************************************************
C-------------------------- ----- -PROBLEM SIX----------------------------- -----------
C    Laminar fluid flow and heat transfer in a two-dimensional sudden expansion
C----------------------------------------------------------------------------------
```

**ENTRY GRID**

TITLE(1)=‘ .VEL U.' ' VEL_U'
TITLE(2)=‘ .VEL V.' ‘ VEL_V’
TITLE(3)=‘ .STR FN.' ‘ STR_FN’
TITLE(4)=‘ . TEMP .' ‘ TEMP. '
TITLE(11)=‘PRESSURE' ‘PRESSURE’
TITLE(12)=‘ DENSITY' ‘ DENSITY'

Titles for print out

LPRINT(1)=.TRUE.
LPRINT(2)=.TRUE.
LPRINT(3)=.TRUE.
LPRINT(4)=.TRUE.
LPRINT(11)=.TRUE.
LPRINT(12)=.TRUE.
LAST= 60
LSOLVE(1)=.TRUE.
LSOLVE(4)=.TRUE.
RELAX(1)=0.8
RELAX(2)=0.8
XL= 0.5
YL= 2.
L1=7
M1=12
CALL UGRID
**RETURN**

```
MODULE START_L
 PARAMETER(NI=100,NJ=200,NIJ=NI,NFMAX=10,NFX4=NFMAX+4)
REAL*8,DIMENSION(NI,NJ,NFX4)::F
```

| NF = | 1 | 2 | 3 | 4 | …… | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|
| Variable | $U$ | $V$ | $p_c$ | $T$ | …… | $p$ | $\rho$ | $\Gamma$ | $C_p$ |

! In SIMPLER code when the 1st variable is set to be solved, the 2nd , 3rd and 11th ones ($v$, $p_c$, $p$) are automatically to be solved.

! Underrelaxation of velocity is organized in the solution process.

! half of computation domain

$$(\frac{a_P}{\alpha})\phi_P = \sum a_{nb}\phi_{nb} + b + (1-\alpha)\frac{a_P}{\alpha}\phi_P^0$$

$$u = u^* + \alpha_u u'$$ No！！！

Reason： The velocity correction is obtained through mass conservation requirement. Its underrelaxation will violate (破坏) mass conservation condition.

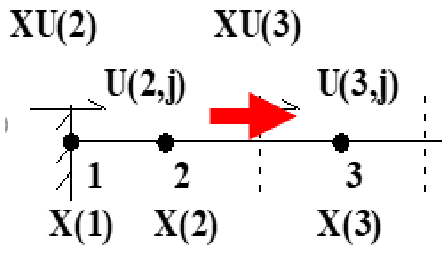
```
ENTRY START
      TIN=500
      TW=300.
      VIN=100.
      VOUT=VIN*XCV(L2)/X(L1)*TW/TIN      ! Estimation of outlet normal velocity
      DO 100 J=1,M1
      DO 101 I=1,L1
      U(I,J)=0
      V(I,J)=VOUT
      V(I,2)=0
      V(1,J)=0.
      T(I,J)=TW
101 ENDDO
100 ENDDO
      V(L2,2)=VIN
      T(L2,1)=TIN
       PR= .7
      AMU=1.
      AMUP=AMU*CPCON/PR
      TREF=300.
      RHOREF=1.
      RHOT=RHOREF*TREF
RETURN
```

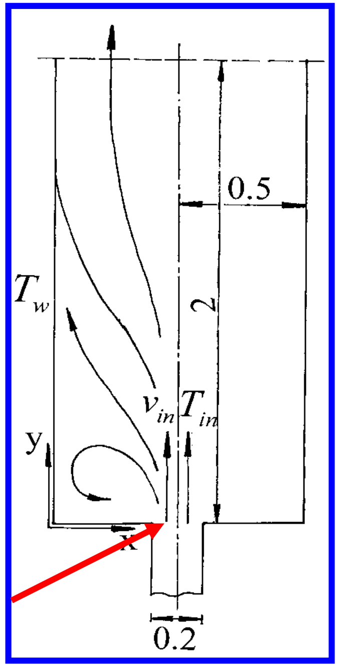
! Initial values and some BCs (U(I,J)=0 to V(1,J)=0.)

| Variable | IST-1 | JST-1 |
|---|---|---|
| $\phi, p, p'$ | 1 | 1 |
| $u$ | 2 | 1 |
| $v$ | 1 | 2 |

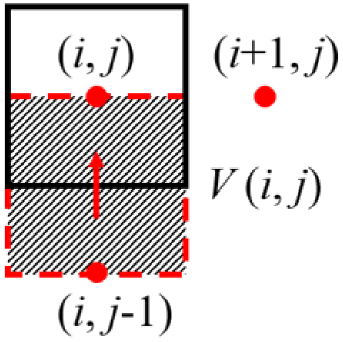
! Different $j$ for $V$ and $T$

$$Pr = \eta c_p / \lambda,$$
$$\lambda = \eta c_p / Pr$$




(L2,1)

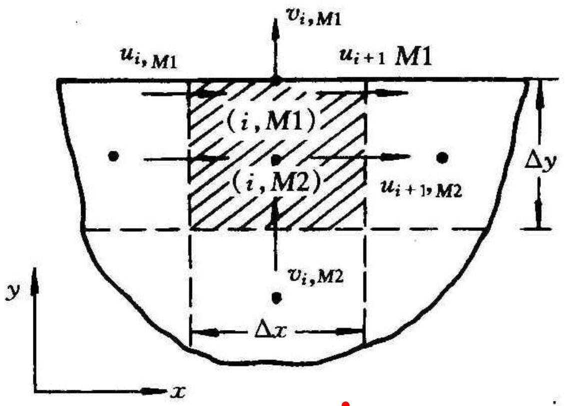


# Total mass conservation for case of outlet without recirculation

（1）Assuming that relative changes of outlet normal velocity =constant

$$\frac{v_{i,M1}-v_{i,M2}}{v_{i,M2}}=k=const$$

$$v_{i,M1}=v_{i,M2}(1+k)=f\,v_{i,M2}$$




*f* is determined according to total mass conservation :

$$\sum_{i=2}^{L2}\rho_{i,M1}v_{i,M1}\Delta x_i=\sum_{i=2}^{L2}\rho_{i,M1}f\,v_{i,M2}\Delta x_i=FLOWIN$$

$$f=\frac{FLOWIN}{\sum_{2}^{L2}\rho_{i,M1}v_{i,M2}\Delta x_i}$$

$$v_{i,M1}=f\bullet v^{*}_{i,M2}$$ FACTOR method

It is regarded as the boundary condition for next iteration.

## (2) Assuming that the 1st derivatives at outlet =constant

$$\frac{v_{i,M1}-v_{i,M2}}{\Delta y}=k=const \rightarrow v_{i,M1}=v_{i,M2}+k\Delta y=v_{i,M2}+C$$

C is determined according to total mass conservation

$$\sum_{i=2}^{L2}\rho_{i,M1}(v_{i,M2}+C)\Delta x_i=FLOWIN \rightarrow$$

$$C=\frac{FLOWIN-\sum\rho_{i,M1}v_{i,M2}\Delta x_i}{\sum\rho_{i,M1}\Delta x_i}$$

$v_{i,M1}=v^*_{i,M2}+C$ is taking as boundary condition for next iteration.

When fully developed at outlet, $f$ = 1, C = 0;
Otherwise, there is some differences between the two treatments. In this example, FACTOR method will be used

```
      ENTRY DENSE
       DO 200 J=1,M1
       DO 201 I=1,L1
       RHO(I,J)=RHOT/T(I,J)
201 ENDDO
200 ENDDO
       RETURN
*
       ENTRY BOUND
       IF(ITER==0)  FLOWIN=RHO(L2,1)*V(L2,2)*XCV(L2)
       FL=0.
       DO 301 I=2,L2
       FL=FL+RHO(I,M1)*V(I,M2)*XCV(I)
301 ENDDO
       FACTOR=FLOWIN/FL
       DO 302 I=2,L2
       V(I,M1)=V(I,M2)*FACTOR
       T(I,M1)=T(I,M2)
302 ENDDO
       RETURN
```

! Variable density

! RHOT=RHOREF*TREF

!Inlet flow rate calculation

!Outlet flow rate calculation

$$\text{Factor} = \frac{\text{FLOWIN}}{\sum_{i=2}^{L2} \rho_{i,M1} * V_{i,M2} * XCV(i)}$$

$$v_{i,M1} = f \bullet v^{*}_{i,M2}$$

Only for print out purpose—decoration! It can be executed after getting converged solution.

```
      ENTRY OUTPUT
      IF(ITER==0)  THEN
      WRITE(8,401)
401 FORMAT(1X,'   ITER',7X,'SMAX',11X,'SSUM',10X,'V(4,7)',
    1  9X,'T(4,7)')
      ELSE
      PRINT 403, ITER, SMAX,  SSUM, V(4,7), T(4,7)
      WRITE(8,403) ITER, SMAX, SSUM, V(4,7), T(4,7)
403 FORMAT(1X,I6,1P4E15.3)
      ENDIF
      IF (ITER==LAST) CALL PRINT
      RETURN
*
```

Print out SMAX,SSUM for observing the convergence of the iteration

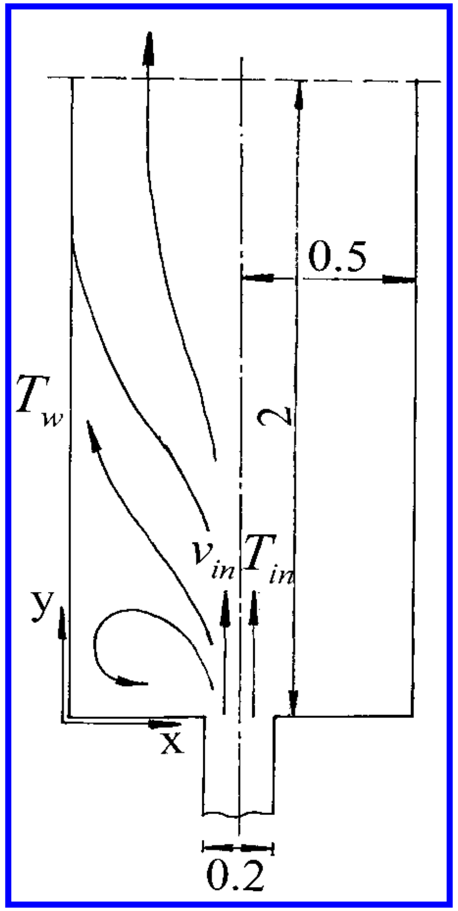
! Residual of mass conservation:

! $b=[(\rho u^*)_w-(\rho u^*)_s]A_e+[(\rho v^*)_s-(\rho v^*)_n]A_n$

! $SSUM=\sum b(i,j)$

! SMAX = maxval (b)

```
      ENTRY GAMSOR
      DO 500 J=1,M1
      DO 501 I=1,L1
      GAM(I,J)=AMU              ! For solving fluid flow
      IF(NF==4) GAM(I,J)=AMUP   ! For solving temperature
      IF(NF/=1) GAM(L1,J)=0.    ! Except u, others---symmetry
      GAM(I,M1)=0.              ! Local one way for both u and T,
 501  ENDDO                       identical to adiabatic.
 500  ENDDO
      RETURN
      END
```



# 11-5-4 Results analysis

COMPUTATION IN CARTISIAN COORDINATES

**************************************************

| ITER | SMAX | SSUM | V(4,7) | T(4,7) |
|---|---|---|---|---|
| 0 | 0.000E+00 | 0.000E+00 | 1.200E+01 | 3.000E+02 |
| 1 | 2.366E+00 | 5.960E-08 | 1.269E+01 | [illegible] |
| 2 | 1.068E+00 | 3.576E-07 | 1.526E+01 | 3.574E+02 |
| 3 | 1.059E+00 | -2.980E-07 | 1.600E+01 | 3.609E+02 |
| 4 | 6.520E-01 | -8.941E-08 | 1.609E+01 | 3.630E+02 |
| 5 | 1.605E-01 | 4.433E-07 | 1.618E+01 | 3.645E+02 |
| 6 | 1.039E-01 | -8.754E-08 | 1.606E+01 | 3.655E+02 |
| 7 | 5.972E-02 | -8.196E-08 | 1.594E+01 | 3.663E+02 |
| 8 | 3.817E-02 | -3.101E-07 | 1.576E+01 | 3.668E+02 |
| 9 | 2.447E-02 | -5.243E-07 | 1.559E+01 | 3.672E+02 |
| 10 | 1.535E-02 | 2.674E-07 | 1.543E+01 | 3.675E+02 |
| 11 | 9.663E-03 | -8.473E-07 | 1.529E+01 | 3.677E+02 |
| 12 | 5.899E-03 | 4.657E-10 | 1.516E+01 | 3.678E+02 |

Total mass conservation is artificially made!

| | | | !V(4,7) | !T(4,7) |
|---|---|---|---|---|
| 13 | 4.332E-03 | -2.432E-07 | 1.506E+01 | 3.678E+02 |
| 14 | 3.456E-03 | 2.751E-07 | 1.498E+01 | 3.678E+02 |
| 15 | 2.698E-03 | 7.753E-08 | 1.491E+01 | 3.678E+02 |
| 16 | 2.052E-03 | 1.475E-07 | 1.486E+01 | 3.678E+02 |
| 17 | 1.539E-03 | -5.428E-07 | 1.481E+01 | 3.678E+02 |
| 18 | 1.133E-03 | 2.519E-07 | 1.478E+01 | 3.677E+02 |
| 19 | 8.994E-04 | 2.108E-07 | 1.476E+01 | 3.677E+02 |
| 20 | 7.056E-04 | 5.479E-07 | 1.474E+01 | 3.677E+02 |
| 21 | 5.436E-04 | 2.256E-07 | 1.473E+01 | 3.677E+02 |
| 22 | 4.111E-04 | 9.380E-08 | 1.472E+01 | 3.676E+02 |
| 23 | 3.100E-04 | 1.485E-07 | 1.471E+01 | 3.676E+02 |
| 24 | 2.303E-04 | 2.160E-07 | 1.470E+01 | 3.676E+02 |
| 25 | 1.793E-04 | 4.192E-07 | 1.470E+01 | 3.676E+02 |
| 26 | 1.447E-04 | -1.086E-08 | 1.470E+01 | 3.676E+02 |
| 27 | 1.149E-04 | -9.684E-08 | 1.469E+01 | 3.676E+02 |
| 28 | 8.990E-05 | 1.732E-09 | 1.469E+01 | 3.676E+02 |
| 29 | 6.926E-05 | -5.815E-07 | 1.469E+01 | 3.676E+02 |
| 30 | 5.170E-05 | -3.065E-07 | 1.469E+01 | 3.676E+02 |
| 31 | 3.837E-05 | -5.491E-07 | 1.469E+01 | 3.676E+02 |
| 32 | 3.084E-05 | 2.732E-07 | 1.469E+01 | 3.676E+02 |

| | | | | |
|---|---|---|---|---|
| 33 | 2.032E-05 | -9.269E-07 | 1.469E+01 | 3.676E+02 |
| 34 | 2.015E-05 | 3.659E-08 | 1.469E+01 | 3.676E+02 |
| 35 | 1.213E-05 | 4.555E-07 | 1.469E+01 | 3.676E+02 |
| 36 | 9.591E-06 | -1.184E-07 | 1.469E+01 | 3.676E+02 |
| 37 | 6.249E-06 | 4.063E-07 | 1.469E+01 | 3.676E+02 |
| 38 | 4.888E-06 | -2.038E-08 | 1.469E+01 | 3.676E+02 |
| 39 | 3.099E-06 | 1.491E-07 | 1.469E+01 | 3.676E+02 |
| 40 | 3.695E-06 | 4.564E-07 | 1.469E+01 | 3.676E+02 |
| 41 | 2.980E-06 | -3.393E-07 | 1.469E+01 | 3.676E+02 |
| 42 | 2.923E-06 | 1.307E-06 | 1.469E+01 | 3.676E+02 |
| 43 | 3.150E-06 | -3.455E-07 | 1.469E+01 | 3.676E+02 |
| 44 | 2.787E-06 | 5.100E-07 | 1.469E+01 | 3.676E+02 |
| 45 | 3.219E-06 | -2.657E-07 | 1.469E+01 | 3.676E+02 |
| 46 | 2.980E-06 | -8.977E-07 | 1.469E+01 | 3.676E+02 |
| 47 | 2.503E-06 | -2.419E-07 | 1.469E+01 | 3.676E+02 |
| 48 | 2.205E-06 | 5.658E-08 | 1.469E+01 | 3.676E+02 |
| 49 | 3.517E-06 | -9.167E-07 | 1.469E+01 | 3.676E+02 |
| 50 | 3.576E-06 | -1.444E-07 | 1.469E+01 | 3.676E+02 |
| 51 | 3.278E-06 | 2.954E-07 | 1.469E+01 | 3.676E+02 |

| ITER | SMAX | SSUM | V(4,7) | T(4,7) |
|---|---|---|---|---|
| 52 | 2.772E-06 | 1.221E-08 | 1.469E+01 | 3.676E+02 |
| 53 | 2.146E-06 | 5.844E-07 | 1.469E+01 | 3.676E+02 |
| 54 | 2.104E-06 | 5.236E-07 | 1.469E+01 | 3.676E+02 |
| 55 | 2.921E-06 | 3.407E-07 | 1.469E+01 | 3.676E+02 |
| 56 | 2.712E-06 | 1.156E-07 | 1.469E+01 | 3.676E+02 |
| 57 | 2.801E-06 | 2.216E-07 | 1.469E+01 | 3.676E+02 |
| 58 | 3.005E-06 | 8.967E-08 | 1.469E+01 | 3.676E+02 |
| 59 | 2.886E-06 | 4.362E-07 | 1.469E+01 | 3.676E+02 |
| 60 | 2.623E-06 | 5.034E-07 | 1.469E+01 | 3.676E+02 |

That SMAX reduces to a certain value can be regarded as an indicator of convergence

In the iteration process, SSUM takes a very small value from beginning to the end. This can not be regarded as an indicator of convergence. Because it is due to our treatment of outflow boundary condition!

************************** .VEL U. **************************

| I = | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| J | | | | | | |
| 12 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| 11 | 0.00E+00 | 1.41E-02 | 3.39E-02 | 4.04E-02 | 2.71E-02 | 0.00E+00 |
| 10 | 0.00E+00 | -6.73E-02 | -1.96E-01 | -2.78E-01 | -2.11E-01 | 0.00E+00 |
| 9 | 0.00E+00 | -1.55E-01 | -4.33E-01 | -5.97E-01 | -4.48E-01 | 0.00E+00 |
| 8 | 0.00E+00 | -3.26E-01 | -8.75E-01 | -1.19E+00 | -8.95E-01 | 0.00E+00 |
| 7 | 0.00E+00 | -6.17E-01 | -1.61E+00 | -2.16E+00 | -1.65E+00 | 0.00E+00 |
| 6 | 0.00E+00 | -1.03E+00 | -2.62E+00 | -3.53E+00 | -2.75E+00 | 0.00E+00 |
| 5 | 0.00E+00 | -1.42E+00 | -3.67E+00 | -5.06E+00 | - 4.10E+00 | 0.00E+00 |
| 4 | 0.00E+00 | -1.35E+00 | -3.91E+00 | .02E+00 | -5.42E+00 | 0.00E+00 |
| 3 | 0.00E+00 | 1.37E-01 | -1.24E+00 | 69E+00 | -6.33E+00 | 0.00E+00 |
| 2 | 0.00E+0 | 2.64E+00 | 6.16E+0 | 03E+00 | -7.70E+00 | 0.00E+00 |
| 1 | 0.00E+ | .00E+00 | 0.00E+0 | 0E+00 | 0.00E+00 | 0.00E+00 |

No decoration

*u* larger than 0

*u* less than 0

*************************** .VEL V. ***************************

| I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| J | | | | | | | |
| 12 | 0.00E+00 | 3.73E+00 | 9.97E+00 | 1.50E+01 | 1.87E+01 | 2.07E+01 | 1.20E+01 |
| 11 | 0.00E+00 | 3.76E+00 | 1.01E+01 | 1.52E+01 | 1.89E+01 | 2.09E+01 | 1.20E+01 |
| 10 | 0.00E+00 | 3.65E+00 | 9.94E+00 | 1.53E+01 | 1.95E+01 | 2.19E+01 | 1.20E+01 |
| 9 | 0.00E+00 | 3.37E+00 | 9.57E+00 | 1.54E+01 | 2.04E+01 | 2.35E+01 | 1.20E+01 |
| 8 | 0.00E+00 | 2.76E+00 | 8.70E+00 | 1.52E+01 | 2.17E+01 | 2.61E+01 | 1.20E+01 |
| 7 | 0.00E+00 | 1.59E+00 | 7.02E+00 | 1.47E+01 | 2.35E+01 | 3.03E+01 | 1.20E+01 |
| 6 | 0.00E+00 | -3.65E-01 | 4.21E+00 | 1.36E+01 | 2.60E+01 | 3.70E+01 | 1.20E+01 |
| 5 | 0.00E+00 | -3.06E+00 | 1.81E-01 | 1.15E+01 | 2.89E+01 | 4.66E+01 | 1.20E+01 |
| 4 | 0.00E+00 | -5.60E+00 | -4.41E+0[illegible] | 8.01E+00 | 3.09E+01 | 5.93E+01 | 1.20E+01 |
| 3 | 0.00E+00 | -5.24E+00 | -6.77E+[illegible] | 1.43E+00 | 2.77E+01 | 7.51E+01 | 1.20E+01 |
| 2 | 0.00E+00 | [illegible].00E+00 | 0.00E[illegible] | 0.00E+00 | 0.00E+00 | 1.00E+02 | 0.00E+00 |

*v* less than 0

*v* larger than 0

Inlet *v*

No decoration

************************ .STR FN. **************************

| J \ I = | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 12 | 0.00E+00 | -3.63E-01 | -1.29E+00 | -2.63E+00 | -4.24E+00 | -6.00E+00 |
| 11 | 0.00E+00 | -3.66E-01 | -1.29E+00 | -2.63E+00 | -4.25E+00 | -6.00E+00 |
| 10 | 0.00E+00 | -3.53E-01 | -1.26E+00 | -2.58E+00 | -4.21E+00 | -6.00E+00 |
| 9 | 0.00E+00 | -3.24E-01 | -1.18E+00 | -2.48E+00 | -4.14E+00 | -6.00E+00 |
| 8 | 0.00E+00 | -2.64E-01 | -1.03E+00 | -2.29E+00 | -4.00E+00 | -6.00E+00 |
| 7 | 0.00E+00 | -1.51E-01 | -7.61E-01 | -1.95E+00 | -3.74E+00 | -6.00E+00 |
| 6 | 0.00E+00 | 3.46E-02 | -3.26E-01 | -1.40E+00 | -3.34E+00 | -6.00E+00 |
| 5 | 0.00E+00 | 2.89E-01 | 2.74E-01 | -6.28E-01 | -2.74E+00 | -6.00E+00 |
| 4 | 0.00E+00 | 5.31E-01 | 9.10E-01 | 2.79E-01 | -1.97E+00 | -6.00E+00 |
| 3 | 0.00E+00 | 5.06E-01 | 1.12E+00 | 9.96E-01 | -1.09E+00 | -6.00E+00 |
| 2 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | -6.00E+00 |

Stream function =0 at the wall

Total flow rate

********************** . TEMP . ************************

Decoration has been made：T(I,M1)=T(I,M2)

| J \ I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 12 | 3.00E+02 | 3.08E+02 | 3.23E+02 | 3.37E+02 | 3.48E+02 | 3.53E+02 | 3.00E+02 |
| 11 | 3.00E+02 | 3.08E+02 | 3.23E+02 | 3.37E+02 | 3.48E+02 | 3.53E+02 | 3.00E+02 |
| 10 | 3.00E+02 | 3.09E+02 | 3.27E+02 | 3.43E+02 | 3.55E+02 | 3.62E+02 | 3.00E+02 |
| 9 | 3.00E+02 | 3.11E+02 | 3.31E+02 | 3.50E+02 | 3.65E+02 | 3.73E+02 | 3.00E+02 |
| 8 | 3.00E+02 | 3.12E+02 | 3.37E+02 | 3.59E+02 | 3.75E+02 | 3.84E+02 | 3.00E+02 |
| 7 | 3.00E+02 | 3.14E+02 | 3.43E+02 | 3.68E+02 | 3.87E+02 | 3.97E+02 | 3.00E+02 |
| 6 | 3.00E+02 | 3.16E+02 | 3.48E+02 | 3.76E+02 | 3.98E+02 | 4.10E+02 | 3.00E+02 |
| 5 | 3.00E+02 | 3.18E+02 | 3.53E+02 | 3.83E+02 | 4.07E+02 | 4.23E+02 | 3.00E+02 |
| 4 | 3.00E+02 | 3.18E+02 | 3.53E+02 | 3.85E+02 | 4.12E+02 | 4.35E+02 | 3.00E+02 |
| 3 | 3.00E+02 | 3.15E+02 | 3.45E+02 | 3.76E+02 | 4.10E+02 | 4.49E+02 | 3.00E+02 |
| 2 | 3.00E+02 | 3.06E+02 | 3.21E+02 | 3.42E+02 | 3.88E+02 | 4.69E+02 | 3.00E+02 |
| 1 | 3.00E+02 | 3.00E+02 | 3.00E+02 | 3.00E+02 | 3.00E+02 | 5.00E+02 | 3.00E+02 |

Given wall temperature

Inlet temp.

No decoration

*************************** PRESSURE ***************************

| J \ I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 12 | 8.40E+02 | 8.40E+02 | 8.39E+02 | 8.38E+02 | 8.34E+02 | 8.31E+02 | 8.30E+02 |
| 11 | 8.52E+02 | 8.52E+02 | 8.52E+02 | 8.50E+02 | 8.48E+02 | 8.45E+02 | 8.44E+02 |
| 10 | 8.77E+02 | 8.77E+02 | 8.76E+02 | 8.76E+02 | 8.75E+02 | 8.74E+02 | 8.73E+02 |
| 9 | 8.99E+02 | 8.98E+02 | 8.97E+02 | 8.95E+02 | 8.94E+02 | 8.92E+02 | 8.91E+02 |
| 8 | **9.12E+02** | 9.10E+02 | 9.08E+02 | 9.06E+02 | 9.05E+02 | 9.02E+02 | 9.00E+02 |
| 7 | 9.06E+02 | 9.04E+02 | 9.01E+02 | 8.99E+02 | 8.99E+02 | 8.96E+02 | 8.94E+02 |
| 6 | 8.63E+02 | 8.61E+02 | 8.56E+02 | 8.56E+02 | 8.62E+02 | 8.59E+02 | 8.58E+02 |
| 5 | 7.55E+02 | 7.52E+02 | 7.46E+02 | 7.50E+02 | 7.66E+02 | 7.69E+02 | 7.70E+02 |
| 4 | 5.57E+02 | 5.53E+02 | 5.45E+02 | 5.50E+02 | 5.85E+02 | 6.02E+02 | 6.11E+02 |
| 3 | 2.91E+02 | 2.84E+02 | 2.72E+02 | 2.55E+02 | 3.32E+02 | 3.56E+02 | 3.68E+02 |
| 2 | 9.85E+01 | 8.74E+01 | 6.54E+01 | -3.27E+01 | -2.08E+02 | 9.08E+01 | 2.40E+02 |
| 1 | 0.00E+00 | -1.10E+01 | -3.79E+01 | -1.77E+02 | -4.78E+02 | -4.18E+01 | 1.07E+02 |

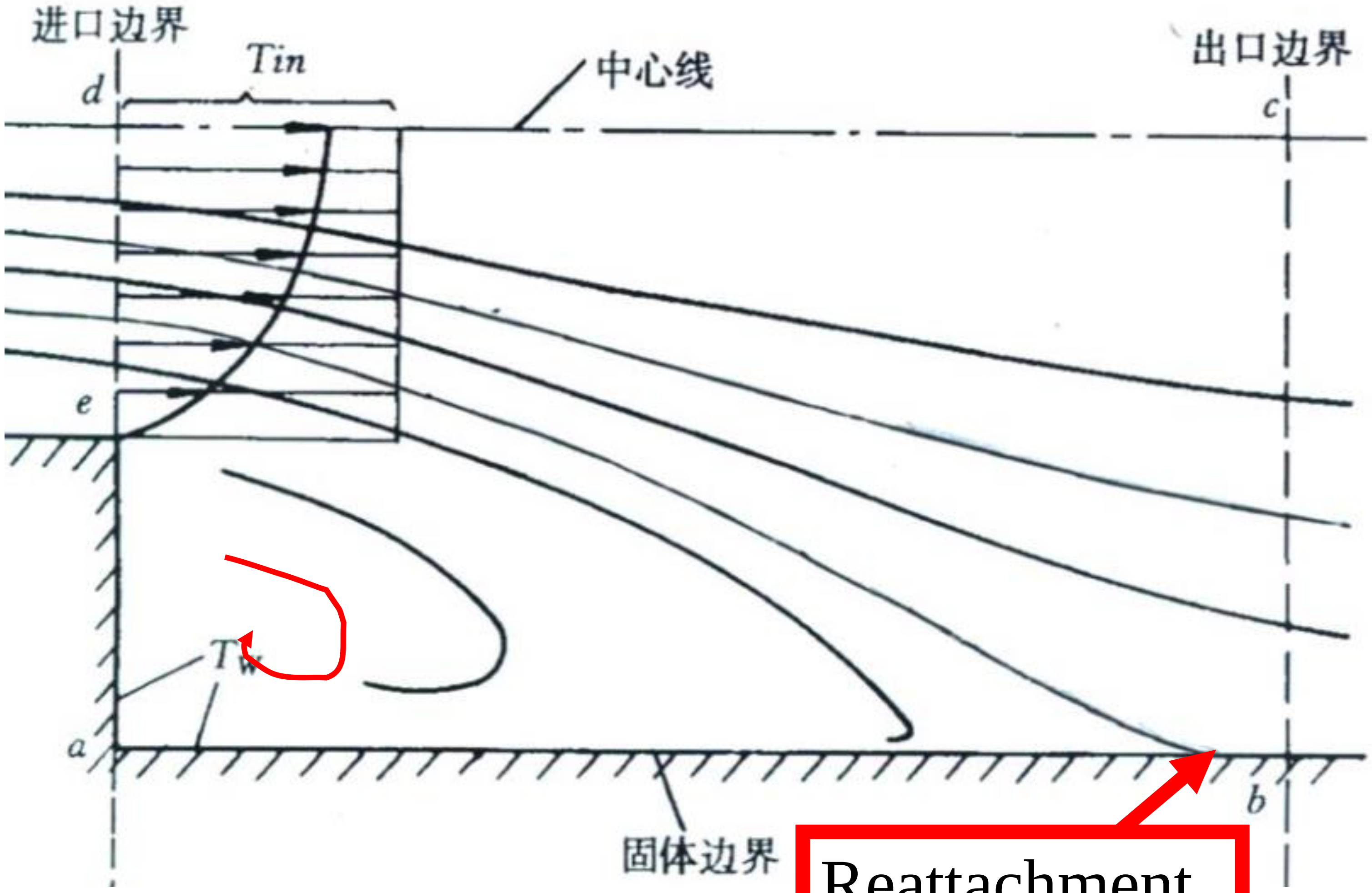
Maximum pressure caused by reattachment of flow

Reference point

$$p(i, j) = p(i, j) - p(IPREF, JPREF)$$

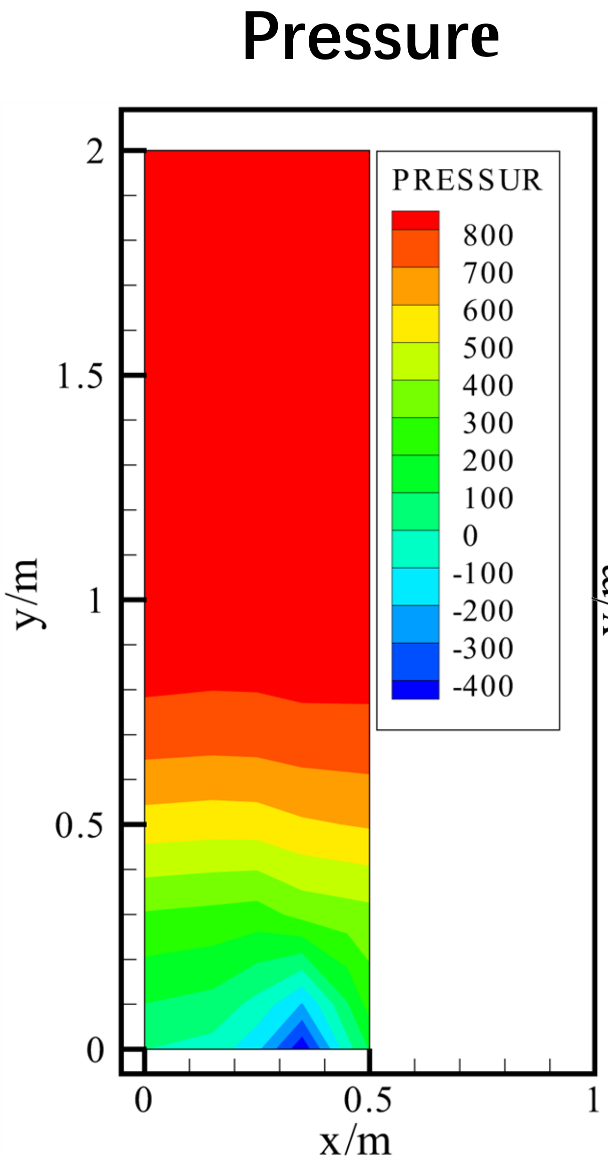
Low pressure region caused by high inlet velocity

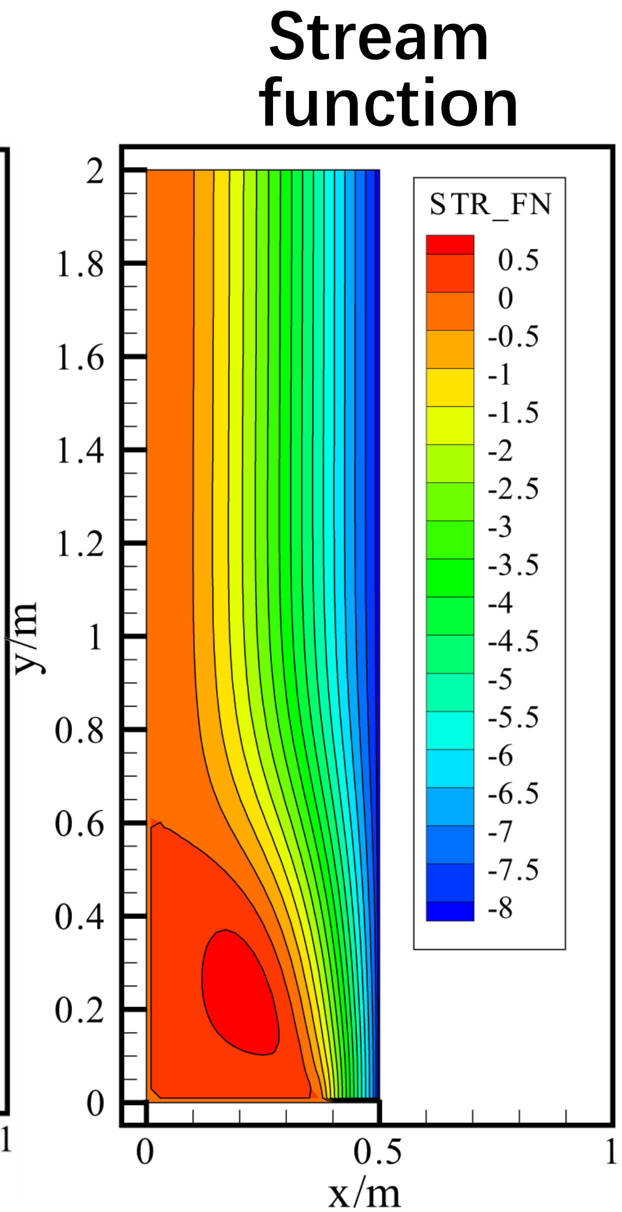
From interpolation




Reattachment Point， $p=p_{max}$

Fig.2 of  Problem 6



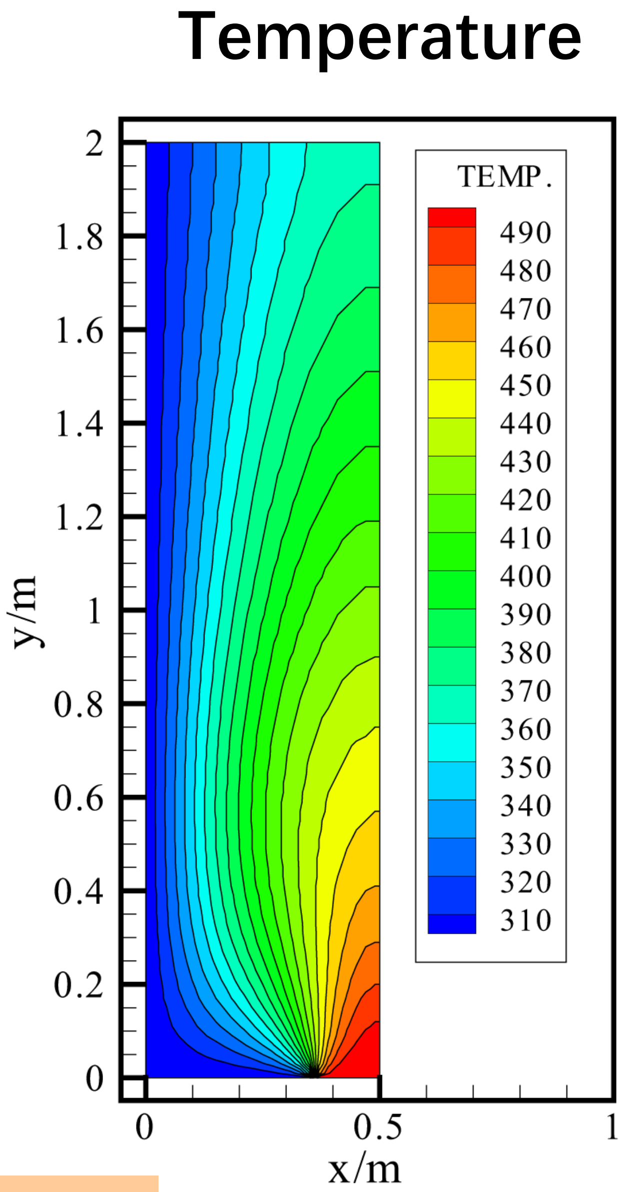



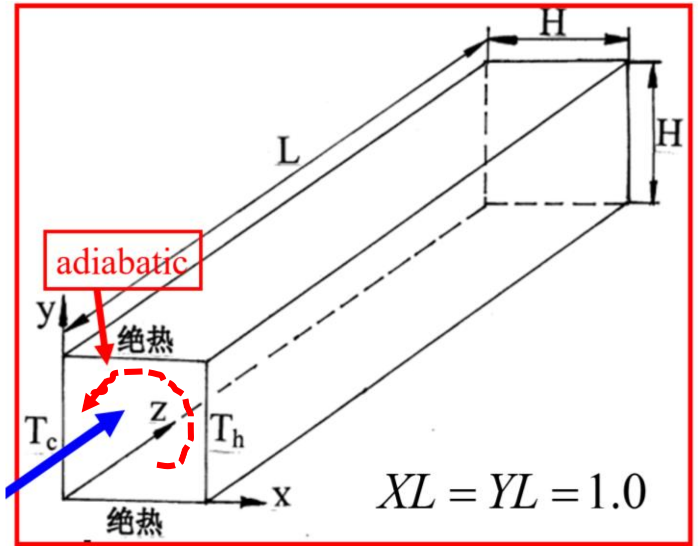

Fig. 3 Results of Problem 6

# 11-6 Complicated fully developed fluid flow and heat transfer in a square duct ---Velocity is regarded as a $\phi$ variable

## 11-6-1 Physical problem and its math formulation

Known: Fully developed heat transfer in a square duct shown in Fig. 1. The effect of gravitation is taken into account by Boussinesq assumption. Duct top and bottom walls are adiabatic, while left and right walls are kept at constant and uniform temperatures: $T_c$=0, $T_h$=1; $Pr$=0.7, $\eta$=1.0, $dp/dz$=-3000, and $\rho g \beta = 10^4$ .

Find: Cross sectional distributions of $u$, $v$, and $w$, temperature distribution and $fRe$.




**Natural convection due to buoyancy**

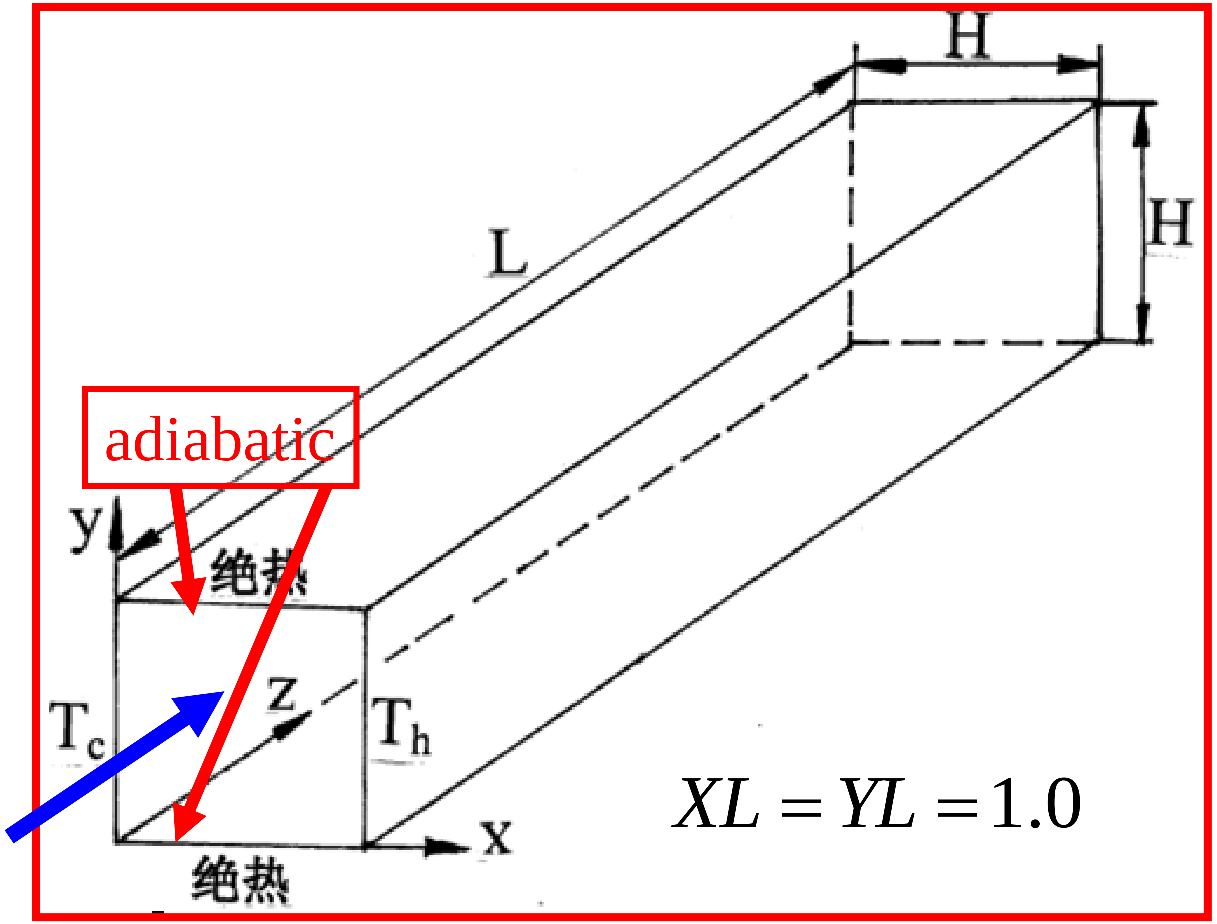



Fig. 1 Physical model of Problem 6

Main feature: When heat transfer goes into the fully developed region, the heat leaves the hot wall goes into the cold wall, *i.e.*, the heat transfer rate is determined by the flow at the cross-section, and the axial flow does not make any contribution to this heat transfer.

# Analysis of the governing eq.:

- According to the fully developed condition

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=-\frac{\partial p}{\partial x}+\eta(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2})$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z})=-\frac{\partial p}{\partial y}+\eta(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2})-\rho g$$

$$\rho(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z})=-\frac{\partial p}{\partial z}+\eta(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2})$$

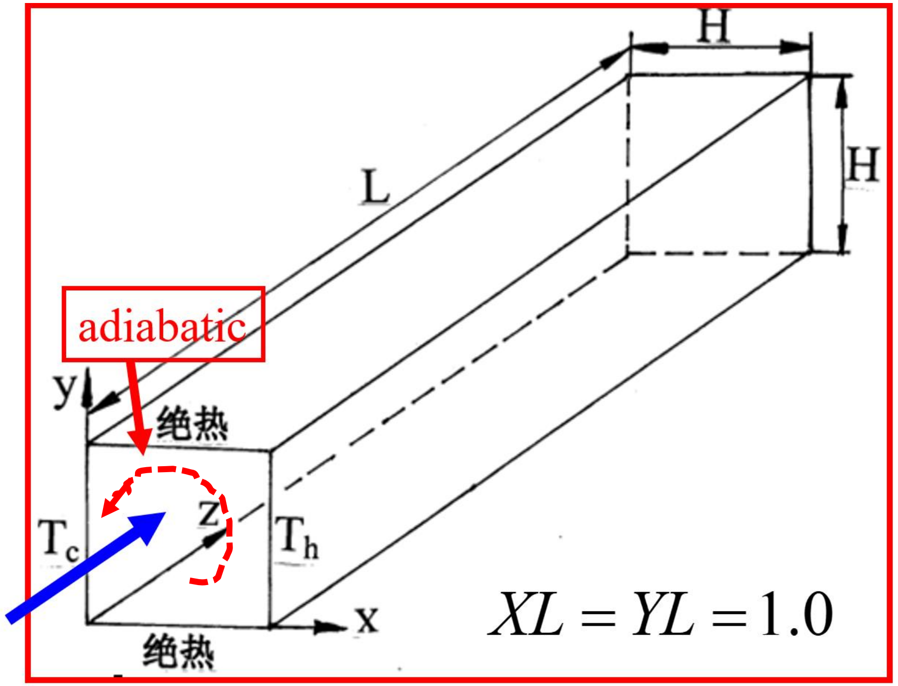
- The axial flow does not make contributions to heat transfer:

$$\rho c_p(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z})=\lambda(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2})$$

# Analysis for the computational domain：

This problem looks like Problem 3 where we take 1/4 of the cross section as the computational domain. Can we still take such practice for this case?

No! Because of the cross sectional natural convection, the entire region must be taken as the computational domain.




## Boundary conditions：

At $x$=0, $T=T_c$: $x=XL$, $T=T_h$

At $y$=0 and $y=YL$: adiabatic

At four walls: $u=v=w=0$.

# Major features of the problem

（1）There are three velocity components： *u, v, w* ; However *u, v* are not coupled with *w* ；

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+\eta(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}-\rho g+\eta(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})$$

$$\rho(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y})=-\frac{\partial p}{\partial z}+\eta(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2})$$

(2) *T* is coupled with velocity *u, v*. The variation of $\rho g$ term with *T* causes the buoyancy, driving the natural convection in cross section.

$$\rho c_p(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})=\lambda(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})$$

# 11-6-2 Numerical methods

(1) Boussinesq assumption is adopted for the density in the source term of *v*-equation: $\rho = \rho_{ref}[1 - \beta(T - T_{ref})]$

Treatment of pressure gradient and gravitation term for *v*-equation

$$\rho(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}) = -\frac{\partial p}{\partial y} - \rho g + \eta(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2})$$

$$-\frac{\partial p}{\partial y} - \rho g = -\frac{\partial p}{\partial y} - \rho_{ref}[1 - \beta(T - T_{ref})]g = -\frac{\partial p}{\partial y} - \rho_{ref}(1 + \beta T_{ref})g + g\rho_{ref}\beta T$$

$$= -\frac{\partial}{\partial y}[\underline{p + \rho_{ref}(1 + \beta T_{ref})gy}] + g\rho_{ref}\beta T = -\frac{\partial p_{eff}}{\partial y} + g\rho_{ref}\beta T$$

$$p_{eff} \approx p$$

The source term in *v*-equation is a function of temperature

## (2) How to use 2-D code for solving three velocity components?

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+\eta(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}+\rho g\beta T+\eta(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})$$

$$\rho c_p(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})=\lambda(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})$$

$$\rho(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y})=-\frac{\partial p}{\partial z}+\eta(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2})$$

- *u*, *v*, *T* are coupled and should be solved simultaneously;
- *u*, *v*, *T* are not coupled with *w*, while *w* is coupled with *u* and *v*; *u*, *v*, *T* are solved first, then *w* is solved. Thus *w* is regarded as a scalar variable.

## (3) The problem studied can be separated into two sub-problems:

(a) Natural convection in a 2-D square cavity: *u*, *v*, *T* are solved;

(b) Fully developed axial flow for solving *w*, with a pre-specified source term of $-dp/dz$.

# Governing equations of the problem studied:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p_{eff}}{\partial x}+\eta\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p_{eff}}{\partial y}+\eta\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\rho g\beta T$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

Natural convection

Solved first to get $u$, $v$ and $T$

$$\rho\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}\right)=-\frac{dp}{dz}+\eta\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)$$

Solved 2nd with known $u$, $v$ and specified pressure gradient!

$dp/dz$ (<0) can be assumed and is specified as -3000.

# 11-6-3 Program reading

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C***************************************************************
      INTEGER*4 I,J
      REAL*8  GBR, DPDZ, PR, AMU, FRE, WBAR, TM
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE USER
C****************************************************************
      USE START_L
      USE USER_L
      IMPLICIT NONE
C*****************************************************************
C------------------------------------PROBLEM SEVEN------------------------------ -----
C           Complex fully developed laminar fluid flow and heat transfer in a
C                                   horizontal square duct
C****************************************************************
```

```
ENTRY GRID
TITLE(1)=' .VEL  U.'
TITLE(2)=' .VEL  V.'
TITLE(3)=' .STR FN.'
TITLE(4)=' . TEMP .'
TITLE(5)=' .W/WBAR.'
TITLE(11)='PRESSURE'
RELAX(1)=0.8
RELAX(2)=0.8
LSOLVE(1)=.TRUE.
LSOLVE(4)=.TRUE.
LPRINT(1)=.TRUE.
LPRINT(2)=.TRUE.
LPRINT(3)=.TRUE.
LPRINT(4)=.TRUE.
LPRINT(5)=.TRUE.
LPRINT(11)=.TRUE.
LAST=25
XL=1.
YL=1.
L1=7
M1=7
CALL UGRID
RETURN
```

! *w* is treated as fifth variable!

! Not for *w*; With known *u*, *v*, the *w* eq is linear.

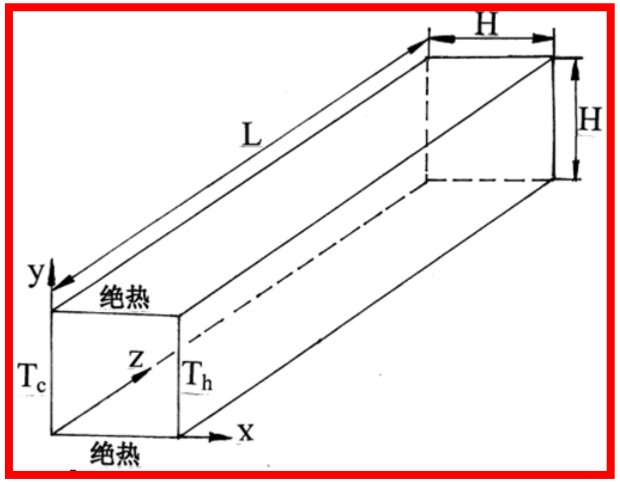
! *u*, *v*, *p*, *T* are solved first

! In SIMPLER code, when the 1st variable is set to be solved, the 2nd and 3rd ones ($v$ and $p_c$) are automatically regarded as variables to be solved.

! Computation for the entire region

```fortran
ENTRY START
      GBR=1.E4            ! ρgβ
      DPDZ=-3000.
      DO 100 J=1,M1
      DO 101 I=1,L1
      U(I,J)=0.
      V(I,J)=0.      !Initial temperature and some boundary conditions
      T(I,J)=0.
      T(L1,J)=1.
      F(I,J,5)=100.    ! Initial field for axial velocity w
      IF (I==1.OR.I==L1)  F(I,J,5)=0.
      IF (J==1.OR.J==M1)  F(I,J,5)=0.   !Boundary cond. of w at four walls w=0
101 ENDDO
100 ENDDO
      PR=0.7
      AMU=1.
      AMUP=AMU*CPCON/PR
      RETURN

ENTRY DENSE
RETURN
```




! $\rho g \beta$

!Initial temperature and some boundary conditions

! Initial field for axial velocity $w$

!Boundary cond. of $w$ at four walls $w$=0

! CPCON=1, default value

$$!\Pr = \mu c_p / \lambda; \quad \lambda = \mu c_p / \Pr$$

! $\lambda$ for temperature

Bossinesq assumption: It ignores density differences except where they appear in terms multiplied by $g$

```
ENTRY BOUND
FRE=0.
IF(ITER<20) RETURN        ! w is not solved when ITER<20
IF(.NOT.LSOLVE(5)) THEN
LSOLVE(1)=.FALSE.
LSOLVE(5)=.TRUE.
ENDIF
WBAR=0.
DO 302 J=2,M2
DO 303 I=2,L2
WBAR=WBAR+F(I,J,5)*XCV(I)*YCV(J)   ! For computing average velocity
303 ENDDO
302 ENDDO
FRE=-DPDZ*2.*4.*(XL*YL)**3/(XL+YL)**2/(WBAR*AMU)
RETURN
```

! Switch of the solved variables, only executed once. The default value of LSOLVE is FALSE. When ITER=20, .NOT.LSOLVE(5) is TRUE; When ITER>=21, it is .FALSE.

! Once 5th variable is solved, these two statements are not needed to executed any more.

! Computing ($fRe$) according to definition; Shown in the next page.

! WBAR=WBAR/(XL*YL)   Because both XL and YL =1, this calculation is ignored!

FRE=-DPDZ*2.*4.*(XL*YL)**3/(XL+YL)**2/(WBAR*AMU)

$$f\,\mathrm{Re} = -[(dp/dz)D_h / \frac{1}{2}\rho w_m^2]\frac{\rho w_m D_h}{\eta}$$

$$f\,\mathrm{Re} = -2[(dp/dz)D_h^2 / w_m \eta] = \frac{-2dp/dz}{\eta(\sum w_{i,j}\Delta A_{i,j}/A)} \bullet (\frac{4A}{P})^2$$

$$= \frac{-2dp/dz}{\eta\sum w_{i,j}\Delta A_{i,j}} \bullet (\frac{4A}{P})^2 A$$

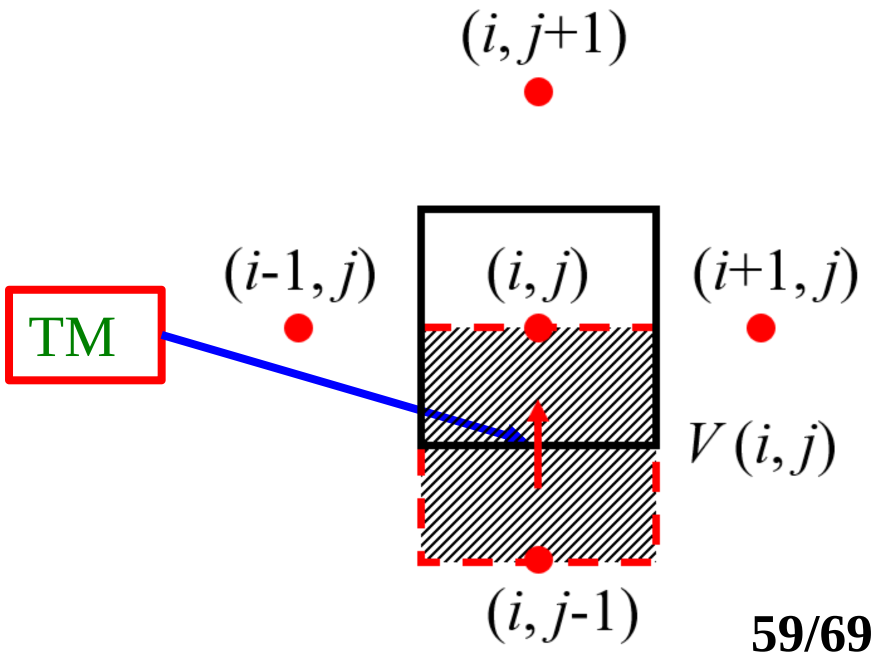
$$= \frac{-2dp/dz}{\eta\sum w_{i,j}\Delta A_{i,j}} \bullet (\frac{4XL*YL}{2(XL+YL)})^2 \bullet XL*YL$$

$$= \frac{-2dp/dz}{\eta\sum w_{i,j}\Delta A_{i,j}} \bullet \frac{4(XL*YL)^3}{(XL+YL)^2}$$

```
      ENTRY OUTPUT
      IF(ITER==0) THEN
      PRINT401
      WRITE(8,401)
401 FORMAT(1X,'  ITER',6X,'SMAX',8X,'SSUM',7X,'V(6,4)',
     &  6X,'T(2,6)',6X,'F.RE')
       ELSE
       PRINT 403, ITER, SMAX, SSUM, V(6,4), T(2,6), FRE
       WRITE(8,403) ITER,SMAX,SSUM,V(6,4),T(2,6),FRE
403 FORMAT(1X,I6,1P5E12.3)
       ENDIF
      IF(ITER/=LAST) RETURN
      DO 410 J=1,M1
      DO 411 I=1,L1
      F(I,J,5)=F(I,J,5)/WBAR     !Dimensionless output for w
411 ENDDO
410 ENDDO
      CALL PRINT
      RETURN
```

```
      ENTRY GAMSOR
      DO 500 J=1,M1
      DO 501 I=1,L1
      GAM(I,J)=AMU           ! Γ for velocity
      IF(NF== 4) THEN
      GAM(I,J)=COND          ! Γ for temp.
      GAM(I,1)=0.
      GAM(I,M1)=0.           ! Adiabatic for south and north boundaries
      ENDIF
501ENDDO
500ENDDO
      DO 510 J=2,M2
      DO 511 I=2,L2
      IF(NF==2) THEN
      IF(J/=2) THEN               ! JST = 3 for v-eq
      TM=(T(I,J)+T(I,J-1))*0.5    ! Source term of v-eq.
      CON(I,J)=TM*GBR
      ENDIF
      ENDIF
      IF(NF==5)  CON(I,J)=-DPDZ
511 ENDDO
510 ENDDO                   ! Source term of w-eq.
      RETURN
      END
```

$GBR = g \rho_{ref} \beta T$



# 11-6-4 Results analysis

COMPUTATION IN CARTESIAN COORDINATES

****************************************************

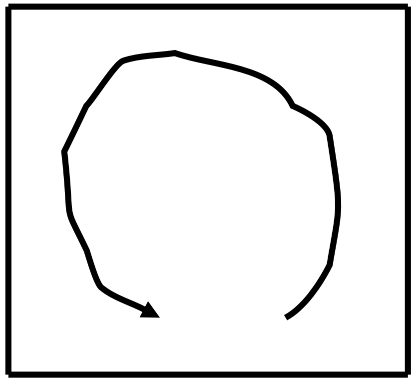
| ITER | SMAX | SSUM | V(6,4) | T(2,6) | F.RE |
|---|---|---|---|---|---|
| 0 | 0.000E+00 | 0.000E+00 | 0.000E+00 | 0.000E+00 | 0.000E+00 |
| 1 | 0.000E+00 | 0.000E+00 | 0.000E+00 | 1.000E-01 | 0.000E+00 |
| 2 | 1.273E+01 | -1.907E-06 | 1.016E+01 | 2.848E-01 | 0.000E+00 |
| 3 | 6.308E+00 | 1.073E-06 | 1.926E+01 | 3.445E-01 | 0.000E+00 |
| 4 | 2.978E+00 | 7.153E-07 | 2.076E+01 | 3.826E-01 | 0.000E+00 |
| 5 | 1.237E+00 | -5.960E-07 | 2.284E+01 | 3.854E-01 | 0.000E+00 |
| 6 | 6.454E-01 | -4.768E-07 | 2.304E+01 | 3.889E-01 | 0.000E+00 |
| 7 | 2.911E-01 | 7.153E-07 | 2.342E+01 | 3.894E-01 | 0.000E+00 |
| 8 | 1.338E-01 | -3.278E-07 | 2.346E+01 | 3.900E-01 | 0.000E+00 |
| 9 | 6.046E-02 | -5.364E-07 | 2.352E+01 | 3.900E-01 | 0.000E+00 |
| 10 | 2.868E-02 | -5.364E-07 | 2.352E+01 | 3.900E-01 | 0.000E+00 |
| 11 | 1.286E-02 | -4.321E-07 | 2.353E+01 | 3.900E-01 | 0.000E+00 |

| ! ITER | SMAX | SSUM | V(6,4) | T(2,6) | F.RE |
|---|---|---|---|---|---|
| 12 | 6.224E-03 | 2.850E-07 | 2.353E+01 | 3.901E-01 | 0.000E+00 |
| 13 | 3.349E-03 | -3.660E-07 | 2.353E+01 | 3.901E-01 | 0.000E+00 |
| 14 | 1.544E-03 | 1.974E-07 | 2.353E+01 | 3.901E-01 | 0.000E+00 |
| 15 | 8.407E-04 | -2.626E-07 | 2.353E+01 | 3.901E-01 | 0.000E+00 |
| 16 | 3.686E-04 | -1.118E-08 | 2.353E+01 | 3.901E-01 | 0.000E+00 |
| 17 | 1.961E-04 | 1.043E-07 | 2.353E+01 | 3.901E-01 | 0.000E+00 |
| 18 | 7.963E-05 | 2.775E-07 | 2.353E+01 | 3.901E-01 | 0.000E+00 |
| 19 | 4.327E-05 | 3.166E-08 | 2.353E+01 | 3.901E-01 | 0.000E+00 |
| 20 | 2.098E-05 | -1.825E-07 | 2.353E+01 | 3.901E-01 | 6.000E+01 |
| 21 | 2.098E-05 | -1.825E-07 | 2.353E+01 | 3.901E-01 | 5.323E+01 |
| 22 | 2.098E-05 | -1.825E-07 | 2.353E+01 | 3.901E-01 | 5.238E+01 |
| 23 | 2.098E-05 | -1.825E-07 | 2.353E+01 | 3.901E-01 | 5.236E+01 |
| 24 | 2.098E-05 | -1.825E-07 | 2.353E+01 | 3.901E-01 | 5.236E+01 |
| 25 | 2.098E-05 | -1.825E-07 | 2.353E+01 | 3.901E-01 | 5.236E+01 |

**! Solving *w*-eq.**

************************** .VEL U. **************************

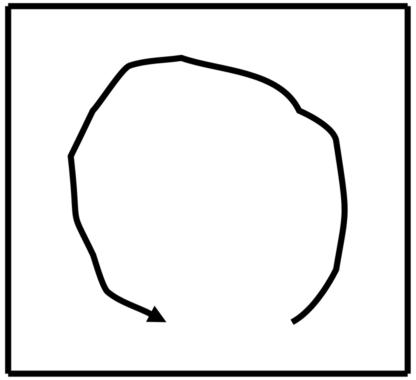
| I = | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| J | | | | | | |
| 7 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| 6 | 0.00E+00 | -1.52E+01 | -1.78E+01 | -1.77E+01 | -1.31E+01 | 0.00E+00 |
| 5 | 0.00E+00 | -8.36E+00 | -1.40E+01 | -1.40E+01 | -9.70E+00 | 0.00E+00 |
| 4 | 0.00E+00 | 7.76E-01 | 8.31E-02 | -8.31E-0 2 | -7.76E-01 | 0.00E+00 |
| 3 | 0.00E+00 | 9.70E+00 | 1.40E+01 | 1.40E+01 | 8.36E+00 | 0.00E+00 |
| 2 | 0.00E+00 | 1.31E+01 | 1.77E+01 | 1.78E+01 | 1.52E+01 | 0.00E+00 |
| 1 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |



Natural convection
in cross section

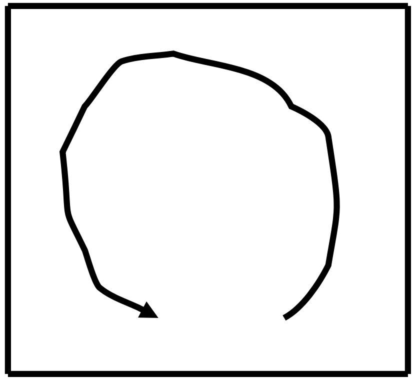
*************************** .VEL V. ***************************

| I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| J | | | | | | | |
| 7 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| 6 | 0.00E+00 | -1.52E+01 | -2.64E+00 | 1.01E-01 | 4.66E+00 | 1.31E+01 | 0.00E+00 |
| 5 | 0.00E+00 | -2.35E+01 | -8.26E+00 | 8.31E-02 | 8.96E+00 | 2.28E+01 | 0.00E+00 |
| 4 | 0.00E+00 | -2.28E+01 | -8.96E+00 | -8.31E-02 | 8.26E+00 | 2.35E+01 | 0.00E+00 |
| 3 | 0.00E+00 | -1.31E+01 | -4.66E+00 | -1.01E-01 | 2.64E+00 | 1.52E+01 | 0.00E+00 |
| 2 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |



*************************** .STR FN. ***************************

| J \ I = | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 7 | 0.00E+00 | -3.91E-07 | 2.60E-07 | 1.16E-07 | 1.26E-08 | 0.00E+00 |
| 6 | 0.00E+00 | 3.03E+00 | 3.56E+00 | 3.54E+00 | 2.61E+00 | 0.00E+00 |
| 5 | 0.00E+00 | 4.71E+00 | 6.36E+00 | 6.34E+00 | 4.55E+00 | 0.00E+00 |
| 4 | 0.00E+00 | 4.55E+00 | 6.34E+00 | 6.36E+00 | 4.71E+00 | 0.00E+00 |
| 3 | 0.00E+00 | 2.61E+00 | 3.54E+00 | 3.56E+00 | 3.03E+00 | 0.00E+00 |
| 2 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |



Stream functions of the four walls are zero

************************** . TEMP . **************************

No decoration

| J \ I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 7 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 1.00E+00 |
| 6 | 0.00E+00 | 3.90E-01 | 7.20E-01 | 7.86E-01 | 8.33E-01 | 9.11E-01 | 1.00E+00 |
| 5 | 0.00E+00 | 3.25E-01 | 6.21E-01 | 6.77E-01 | 7.15E-01 | 8.43E-01 | 1.00E+00 |
| 4 | 0.00E+00 | 2.41E-01 | 4.58E-01 | 5.00E-01 | 5.42E-01 | 7.59E-01 | 1.00E+00 |
| 3 | 0.00E+00 | 1.57E-01 | 2.85E-0 1 | 3.23E-01 | 3.79E-01 | 6.75E-0 1 | 1.00E+00 |
| 2 | 0.00E+00 | 8.92E-02 | 1.67E-01 | 2.14E-01 | 2.80E-01 | 6.10E-0 1 | 1.00E+00 |
| 1 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 1.00E+00 |

Cold wall

Hot wall

No decoration

************************** .W/WBAR. **************************

| J \ I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 7 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| 6 | 0.00E+00 | 4.96E-01 | 7.74E-01 | 7.73E-01 | 6.99E-01 | 4.72E-01 | 0.00E+00 |
| 5 | 0.00E+00 | 7.89E-01 | 1.50E+00 | 1.54E+00 | 1.34E+00 | 7.52E-01 | 0.00E+00 |
| 4 | 0.00E+00 | 8.21E-01 | 1.63E+00 | **1.85E+00** | 1.63E+00 | 8.21E-01 | 0.00E+00 |
| 3 | 0.00E+00 | 7.52E-01 | 1.34E+00 | 1.54E+00 | 1.50E+00 | 7.89E-01 | 0.00E+00 |
| 2 | 0.00E+00 | 4.72E-01 | 6.99E-01 | 7.73E-01 | 7.74E-01 | 4.96E-01 | 0.00E+00 |
| 1 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |

*w* velocity of the four walls are zero

************************** PRESSURE **************************

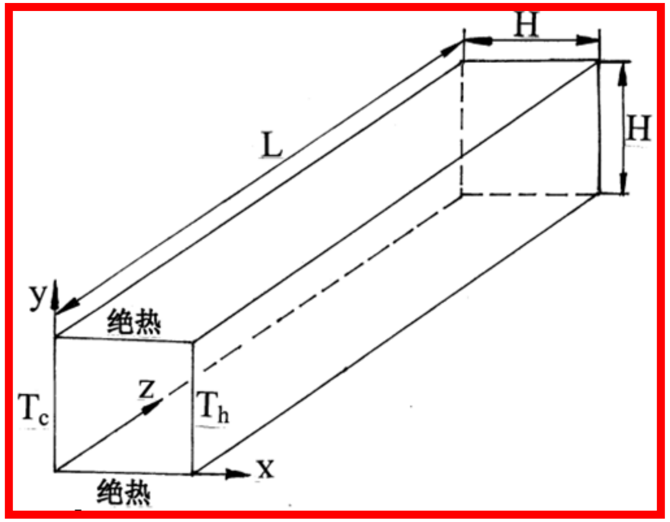
| J \ I = | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 7 | 3.64E+03 | 3.73E+03 | 4.05E+03 | 4.33E+03 | 4.67E+03 | 4.89E+03 | 5.00E+03 |
| 6 | 3.09E+03 | 3.18E+03 | 3.36E+03 | 3.56E+03 | 3.84E+03 | 4.05E+03 | 4.16E+03 |
| 5 | 2.14E+03 | 2.09E+03 | 1.99E+03 | 2.02E+03 | 2.17E+03 | 2.36E+03 | 2.46E+03 |
| 4 | 1.10E+03 | 1.02E+03 | 8.42E+02 | 7.85E+02 | 8.42E+02 | 1.02E+03 | 1.10E+03 |
| 3 | 4.58E+02 | 3.63E+02 | 1.73E+02 | 2.45E+01 | -7.31E+00 | 9.20E+01 | 1.42E+02 |
| 2 | 1.56E+02 | 5.04E+01 | -1.61E+02 | -4.37E+02 | -6.35E+02 | -8.17E+02 | -9.08E+02 |
| 1 | 0.00E+00 | -1.06E+02 | -3.28E+02 | -6.67E+02 | -9.49E+02 | -1.27E+03 | -1.36E+03 |

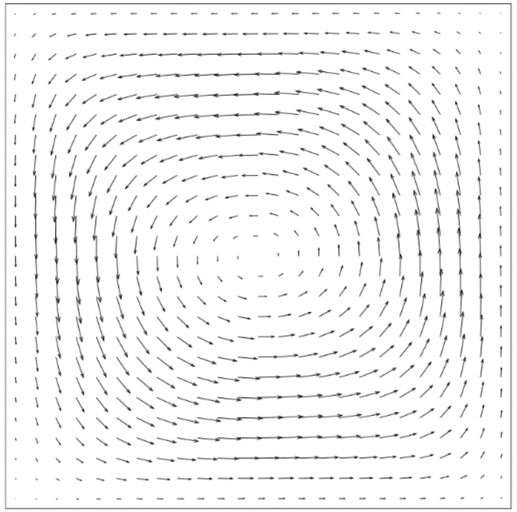
Pmax (5.00E+03)

Pmin (-1.36E+03)

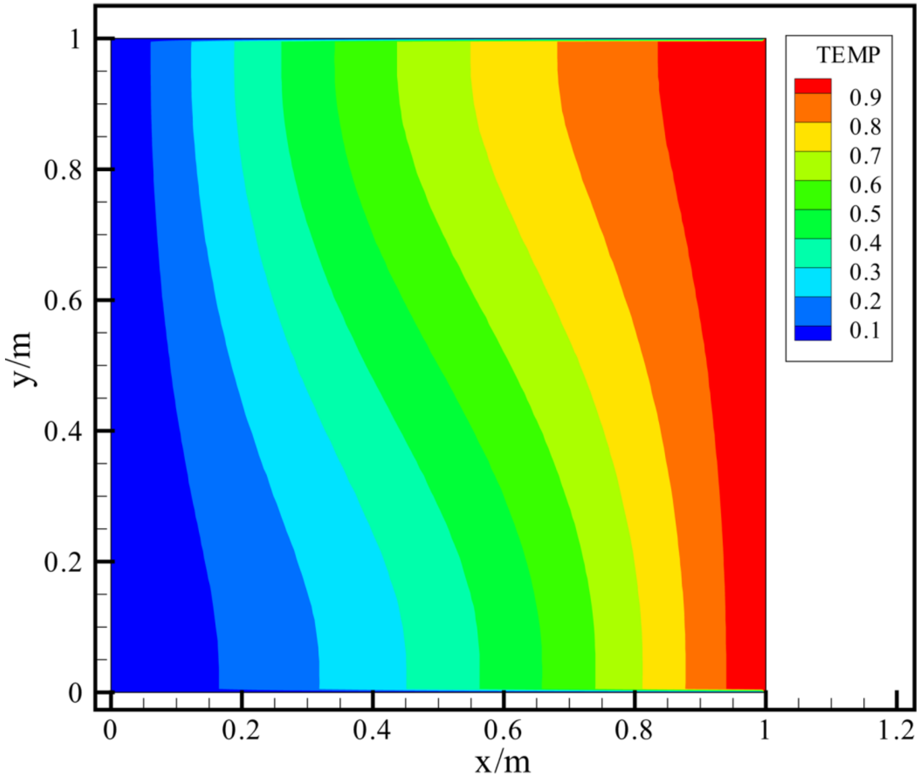
Pressure reference point (0.00E+00)

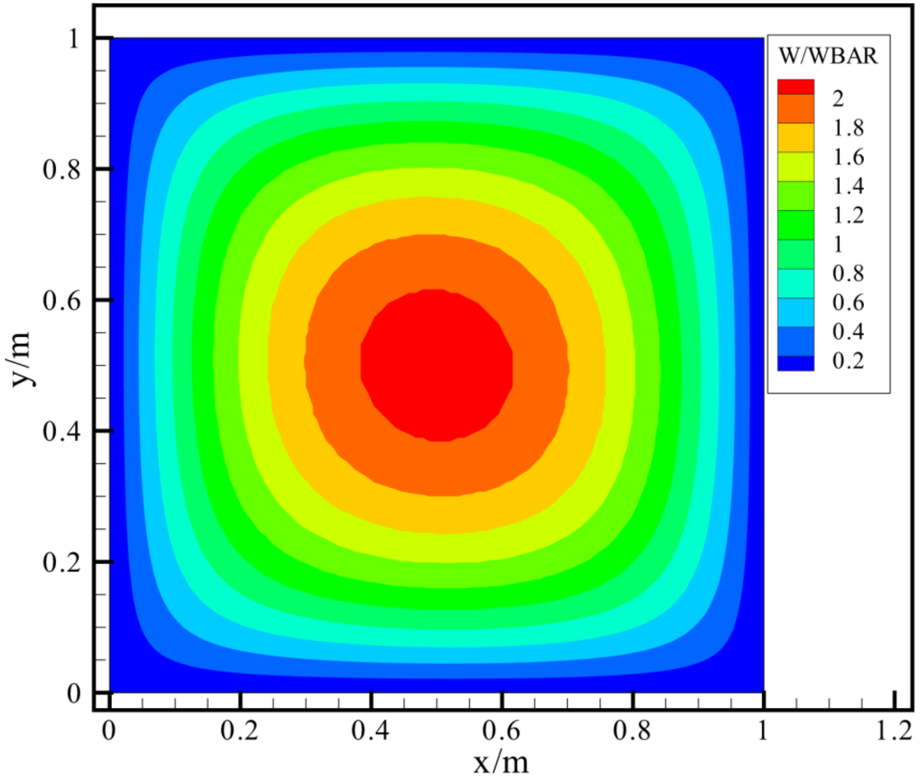
Fig. 2 Results of Problem 6



Temp.




Axial velocity

本组网页地址：http://nht.xjtu.edu.cn 欢迎访问！

*Teaching PPT will be loaded on our website*


同舟共济

渡彼岸！

**People in the same boat help each other to cross to the other bank, where....**