Numerical Heat Transfer
(数值传热学)
Chapter 10 General Code for 2D Elliptical Fluid Flow and Heat Transfer (1)

Instructor：Fang, Wen-Zhen; Tao, Wen-Quan

Email: fangwenzhen@xjtu.edu.cn

Key Laboratory of Thermo-Fluid Science & Engineering
Xi'an Jiaotong University

2023-Nov-15

Partial differential equations (PDEs) 偏微分方程

discretize

Algebraic equations (ABEqs) 代数方程

$$
\begin{cases}
\dfrac{\partial \rho}{\partial t} + div(\rho \vec{U}) = 0 \\
\dfrac{\partial (\rho u)}{\partial t} + div(\rho u \vec{U}) = div(\mu \, grad \phi) - \dfrac{\partial p}{\partial x} \\
\dfrac{\partial (\rho v)}{\partial t} + div(\rho v \vec{U}) = div(\mu \, grad \phi) - \dfrac{\partial p}{\partial y} \\
\dfrac{\partial (\rho c_p T)}{\partial t} + div(\rho c_p \vec{U} T) = div(\lambda \, grad T) \\
\text{Boundary conditions (BCs)}
\end{cases}
$$

$$a_P \phi_P = a_E \phi_E + a_W \phi_W + a_N \phi_N + a_S \phi_S + b$$

How to discretize?

- Control volume method
- SIMPLER to treat the coupling between velocity and pressure
- ASTM for BCs

Difficulty in:

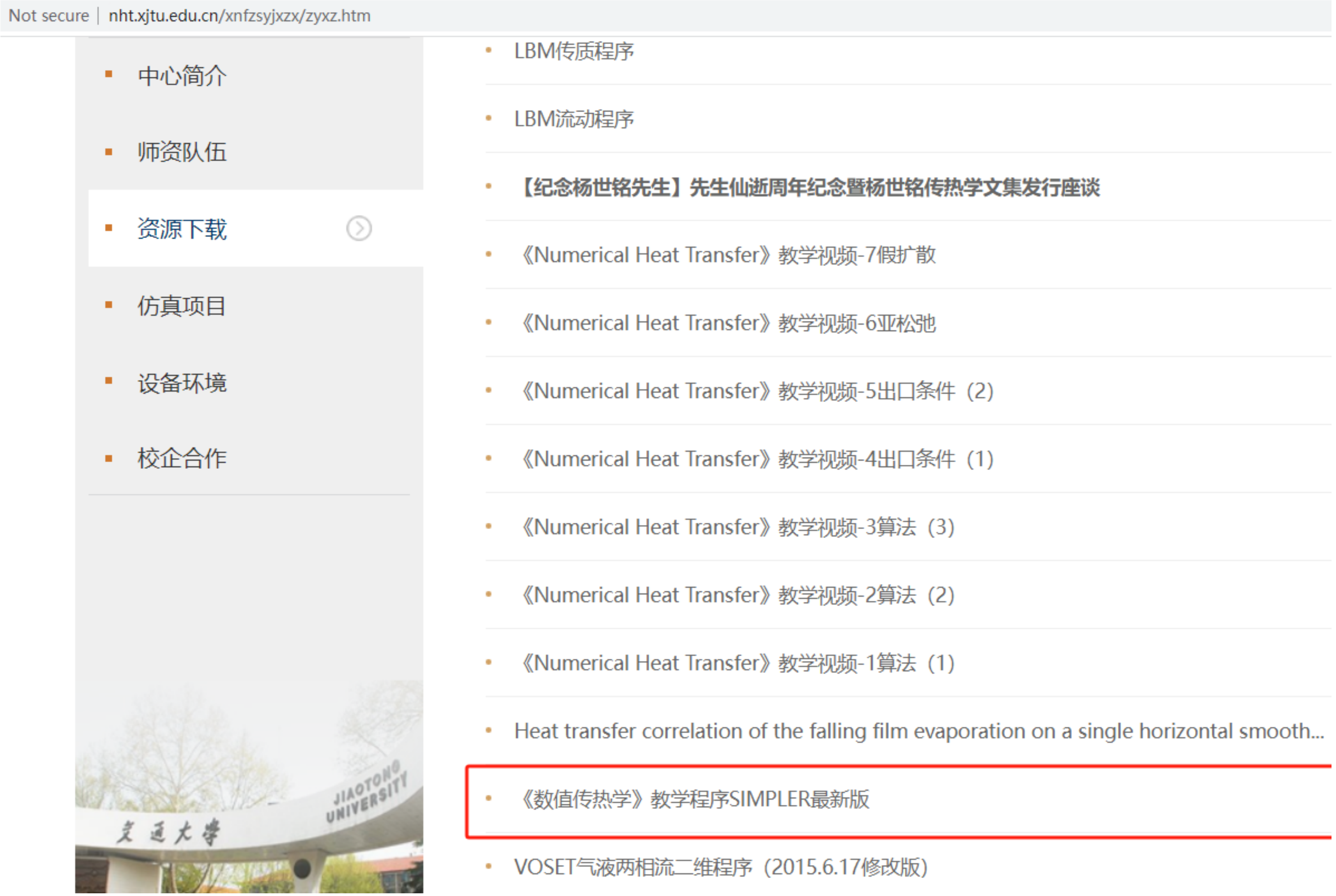
- Non-linear; coupling
- No Eq. for Pressure

How to solve ABEqs?

- Iterative method

SIMPLER Teaching Code

http://nht.xjtu.edu.cn/xnfzsyjxzx/zyxz.htm



8 examples:

- EXAM1
- EXAM2
- EXAM3
- EXAM4
- EXAM5
- EXAM6
- EXAM7
- EXAM8

Exam 1:

- Main95.f
- ONE.F

Exam 2:

- Main95.f
- TWO.F

Chapter 10 General Code for 2D Elliptical Fluid Flow and Heat Transfer Problems

Main95.f

ONE.F

10.1 Format Improvement of the General Governing Equation

The G.E. we just learned in the previous chapters is:

$$\frac{\partial(\rho\phi)}{\partial t}+div(\rho\phi\vec{U})=div(\Gamma_{\phi}^{*}grad\phi)+S_{\phi}^{*} \qquad (1)$$

Equation	ρ	ϕ	$\Gamma_\phi{}^*$	$S_\phi{}^*$
Continuity equation	ρ	1	0	0
Momentum eqn. (x direction)	ρ	u	μ	$\rho f_x-\frac{\partial p}{\partial x}$
Momentum eqn. (y direction)	ρ	v	μ	$\rho f_y-\frac{\partial p}{\partial y}$
Energy equation	ρ	T	λ/c_p	S_T/c_p

$$\frac{\partial(\rho T)}{\partial t}+\frac{\partial(\rho uT)}{\partial x}+\frac{\partial(\rho vT)}{\partial y}=\frac{\partial}{\partial x}\left(\frac{\lambda}{c_p}\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\lambda}{c_p}\frac{\partial T}{\partial y}\right)+\frac{S_T}{c_p}$$

When the fluid heat capacity is not constant, numerical results may not satisfy the conservation condition.

10-1 Analysis for energy equation

A correct energy equation is:

$$\frac{\partial(\rho c_p T)}{\partial t}+\frac{\partial(\rho c_p uT)}{\partial x}+\frac{\partial(\rho c_p vT)}{\partial y}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+S_T. \qquad (2)$$

Reforming into the previous format of G.E.:

$$\frac{\partial(\rho T)}{\partial t}+\frac{\partial(\rho uT)}{\partial x}+\frac{\partial(\rho vT)}{\partial y}=\frac{\partial}{\partial x}\left(\frac{\lambda}{c_p}\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\lambda}{c_p}\frac{\partial T}{\partial y}\right)+\frac{S_T}{c_p}$$

$$-\frac{1}{c_p^2}\left[\rho c_p T\frac{\partial c_p}{\partial t}+\left(\rho c_p uT-\lambda\frac{\partial T}{\partial x}\right)\frac{\partial c_p}{\partial x}+\left(\rho c_p vT-\lambda\frac{\partial T}{\partial y}\right)\frac{\partial c_p}{\partial y}\right]$$

This term was neglected in the previous G.E. (3)

When it can be neglected?

(1) c_p is constant;

（2）Or following three terms simultaneously equal zero:

$$\rho c_p uT - \lambda \frac{\partial T}{\partial x} = 0, \quad \rho c_p vT - \lambda \frac{\partial T}{\partial y} = 0 \quad \text{and} \quad \rho c_p T \frac{\partial c_p}{\partial t} = 0 \qquad (4)$$

（3） Or the sum of the three terms equal zero!

Obviously, such chances are quite limited! Hence, simulation results based on previous G.E. inherently include some errors!

8-2 Improved format of the general G.E.

The framework (框架) of the previous general G.E. is retained (保留), but the diffusion coefficient is resumed to （恢复到） its original value (λ) by introducing a nominal density (ρc_p) as follows:

$$\frac{\partial(\rho^* \phi)}{\partial t} + div(\rho^* \phi \vec{U}) = div(\Gamma_\phi grad \phi) + S_\phi^* \qquad (5)$$

The new form of G.E. for different variables are:

Equation	ρ^*	ϕ	Γ_ϕ	$S_\phi{}^*$
Continuity equation	ρ	1	0	0
Momentum eqn. (x direction)	ρ	u	μ	$\rho f_x - \frac{\partial p}{\partial x}$
Momentum eqn. (y direction)	ρ	v	μ	$\rho f_y - \frac{\partial p}{\partial y}$
Energy equation	ρc_p	T	λ	S_T

That is, now we regard ρc_p in the energy equation as a general density:

$$\frac{\partial(\rho c_p T)}{\partial t}+\frac{\partial(\rho c_p u T)}{\partial x}+\frac{\partial(\rho c_p v T)}{\partial y}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+S_T.$$

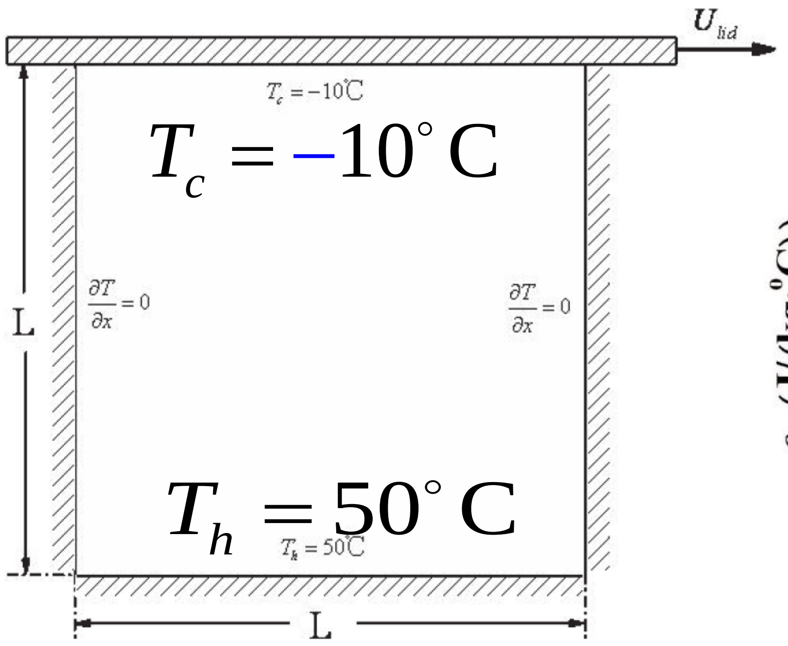
Such a treatment is much better than taking Γ / c_p as a nominal diffusion coefficient and S_T / c_p as a nominal source term .

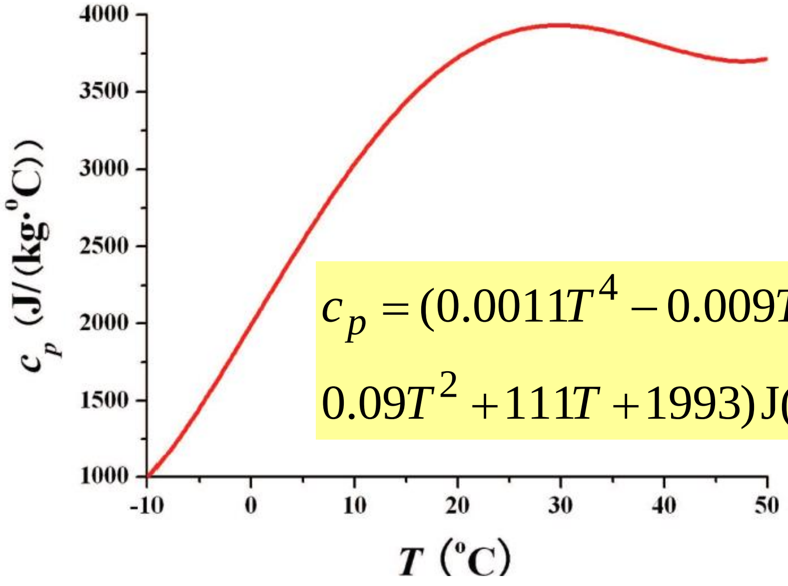
Li W, Yu B, Wang Y , et al. Communications in Computational Physics, 2012, 12(5): 1482-1494

10-3 Comparisons of energy G.E.

For the same physical problems, adopting the same discretized scheme, algorithm, grid system, solution method, etc, the only difference is in the G.E., Eqs.(1) and (5):

1） Example 1 ---Natural convection in enclosure with a moving lid and variable fluid heat capacity

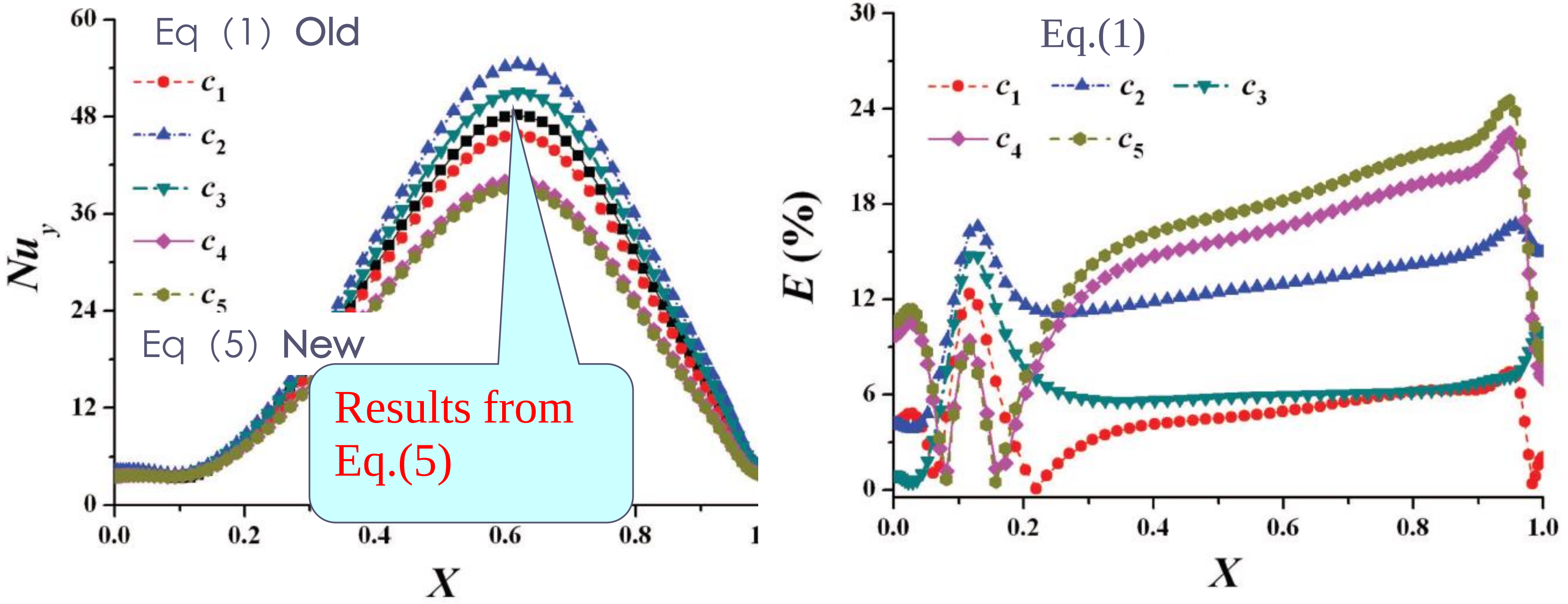




$$c_p = (0.0011T^4 - 0.009T^3 + 0.09T^2 + 111T + 1993)\,\mathrm{J(kg \bullet K)^{-1}}$$

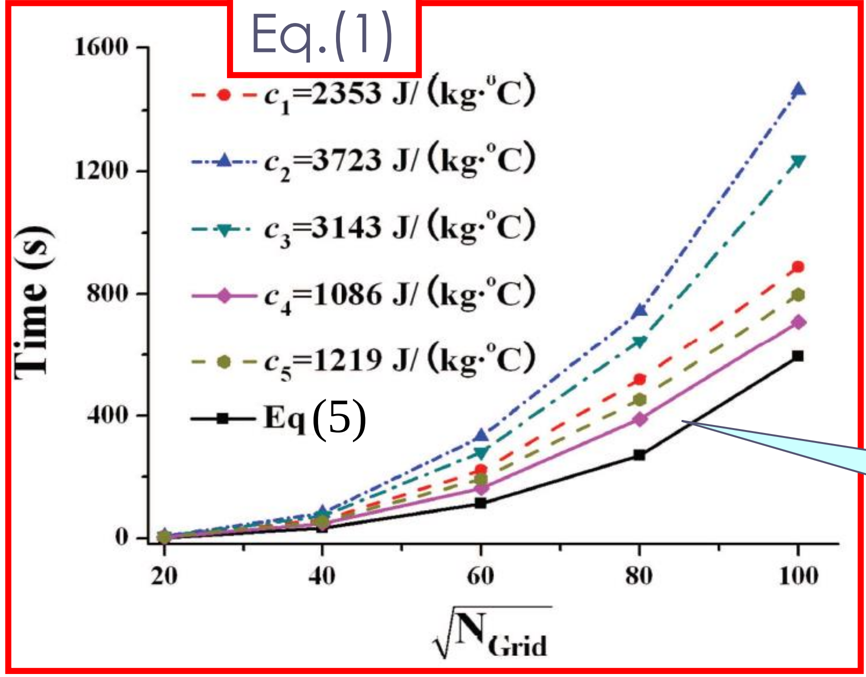
For the new G.E. of Eq.5, fluid heat capacity can be determined by the local temperature (correct one).

For originial G.E. of Eq.1, five average c_p are used, which lead to large deviations with the correct one.



(a) Nusselt number

(b) Relative deviation



Results from Eq.(5)

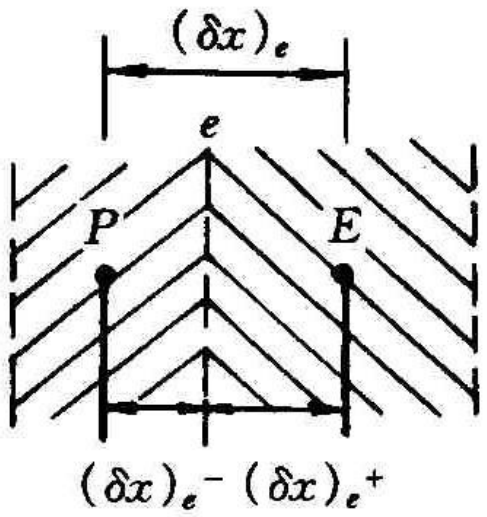
The convergence rate of the new G.E. is significantly higher than that of the previous format.

2）Example 2 ---Conjugated problems(耦合问题)

Harmonic mean should be used to ensure continuity of heat flux. In the original G.E., diffusivity is nominal, which is λ / c_p ; Thus for conjugated problem where fluid and solid are coupled, we should assume c_p are the same for the fluid and solid:

Harmonic mean for conjugated problem

$$\frac{(\delta x)_e}{\lambda / c_{pf}} = \frac{(\delta x)_{e^+}}{\lambda_E / c_{pf}} + \frac{(\delta x)_{e^-}}{\lambda_P / c_{pf}}$$



$$\frac{(\delta x)_e}{\lambda_e} = \frac{(\delta x)_{e^+}}{\lambda_E} + \frac{(\delta x)_{e^-}}{\lambda_P}$$

Harmonic mean

For steady problems, such treatment is OK, while for transient problem, it does not work. For the new G.E., the diffusivity is λ, so there is no such problem at all.

10.2 Numerical Methods Adopted in Teaching Code and Discretization Equations

10.2.1 Major numerical methods adopted

1.Primitive variable method (原始变量法): Dependent variables are u, v, p ;

2.Practice B of domain discretization: first determine the interfaces positions then the node positions;

3.Control volume method for discretization: Conservative convective schemes ;

4.Staggered grids: three systems for u, v and p;

5.Power-law(乘方格式) for convection-diffusion discretization:

But easy to be replaced by CD, FUD or HS; For higher-order schemes, adopting defer correction(延迟修正) method;

6. Linearization for source term:

$$S = S_C + S_P \phi_P \, , \; S_P \le 0$$

7. Harmonic mean for interface diffusivity:

$$\frac{(\delta x)_e}{\lambda_e} = \frac{(\delta x)_{e^+}}{\lambda_E} + \frac{(\delta x)_{e^-}}{\lambda_P}$$

8. **Fully implicit** for transient problems: space derivatives are determined by the end instant of a time step;

9. Boundary conditions treated as 1st kind: adopting ASTM for 2nd and 3rd kinds of boundary conditions.

10. SIMPLER algorithm to treat the coupling between velocity and pressure: at one iteration level, solving two Poisson equations--- pressure equation and pressure correction equation；

11. Iterative method for solving discretized equations：

1) Iterative method for solving algebraic equations (inner iteration) ；

2) Iterative method for nonlinear Eqs. (outer iteration);

Under-relaxation is organized into solution procedure;

12. ADI with block correction to solve algebraic equations.

10.2.2 Discretized equation of three kinds of variables

1. General scalar variable (标量) ϕ

For all other scalar variables (including *T*, etc)

$$\frac{\partial(\rho^*\phi)}{\partial t}+div(\rho^*\phi\vec{U})=div(\Gamma_\phi grad\phi)+S_\phi^*$$

$$a_P\phi_P=a_E\phi_E+a_W\phi_W+a_N\phi_N+a_S\phi_S+b$$

$$a_E=D_eA(|P_{\Delta e}|)+[[-F_e,0 \qquad a_W=D_wA(|P_{\Delta w}|)+[[F_w,0$$

$$a_P=a_E+a_W+a_N+a_S+a_P^0-S_P\Delta V$$

$$a_P^0=\frac{\rho\Delta V}{\Delta t} \qquad b=S_c\Delta V+a_P\phi_P^0$$

Power-law scheme: $A(|P_\Delta|)=\|0,(1-0.1|P_\Delta|^5)\|$

$$P_{\Delta e}=\frac{F_e}{D_e}=\frac{(\rho^* uA)_e}{(\Gamma A/\delta x)_e}=\frac{\rho_e^* u_e(\delta x)_e}{\Gamma_e}=\left(\frac{\rho^* u\delta x}{\Gamma}\right)_e$$

For temperature: $\Gamma_\phi=\lambda$ $\rho^*=\rho c_p$

2. Pressure & pressure correction equation

The derivation is based on mass conservation $\partial(\rho u_i)/\partial x_i=0$

$$[(\rho u)_e-(\rho u)_w]\Delta y+[(\rho v)_n-(\rho v)_s]\Delta x=0$$

(1) For SIMPLER, substituting $u_e=u_e+(A_e/a_e)(p_P-p_E)$

into discretized mass conservation equation:

$$a_P p_P=a_E p_E+a_W p_W+a_N p_N+a_S p_S+b$$

$$b=(\rho A\tilde{u})_w-(\rho A\tilde{u})_e+(\rho A\tilde{v})_s-(\rho A\tilde{v})_n \quad a_E=(\rho Ad)_e, a_P=\sum a_{nb}$$

（2） Substituting $u_e = u_e^* + \frac{A_e}{a_e}\left(p_P' - p_E'\right)$ into mass conservation equation:

$$a_P p_P' = a_E p_W' + a_W p_W' + a_N p_N' + a_S p_S' + b$$

Except *b* term， a_P and $a_{E,W,N,S}$ are the same as *p*-equation.

$$b = \left(\rho A u^*\right)_w - \left(\rho A u^*\right)_e + \left(\rho A v^*\right)_s - \left(\rho A v^*\right)_n$$

Remarks： The adopted mass conservation equation does not include $\partial\rho/\partial t$ and velocity correction neglects density effect. Thus, this code only applicable to incompressible flow.

3. Momentum equation (taking u as example)

Governing equation:

$$\frac{\partial(\rho u)}{\partial t}+div(\rho\vec{u}u)=div(\mu\, grad u)-\frac{\partial p}{\partial x_i}+S_u$$

Discretized equation:

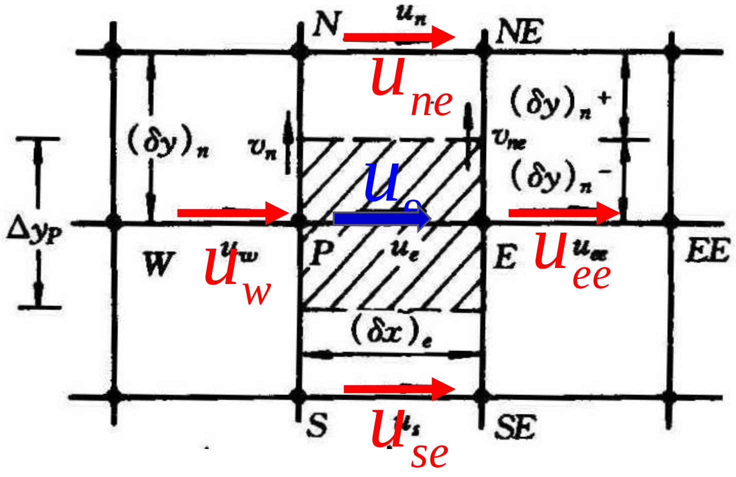
$$a_e u_e = a_{ee}u_{ee}+a_w u_w+a_{ne}u_{ne}+a_{se}u_{se}+b+A_e(p_P-p_E)$$

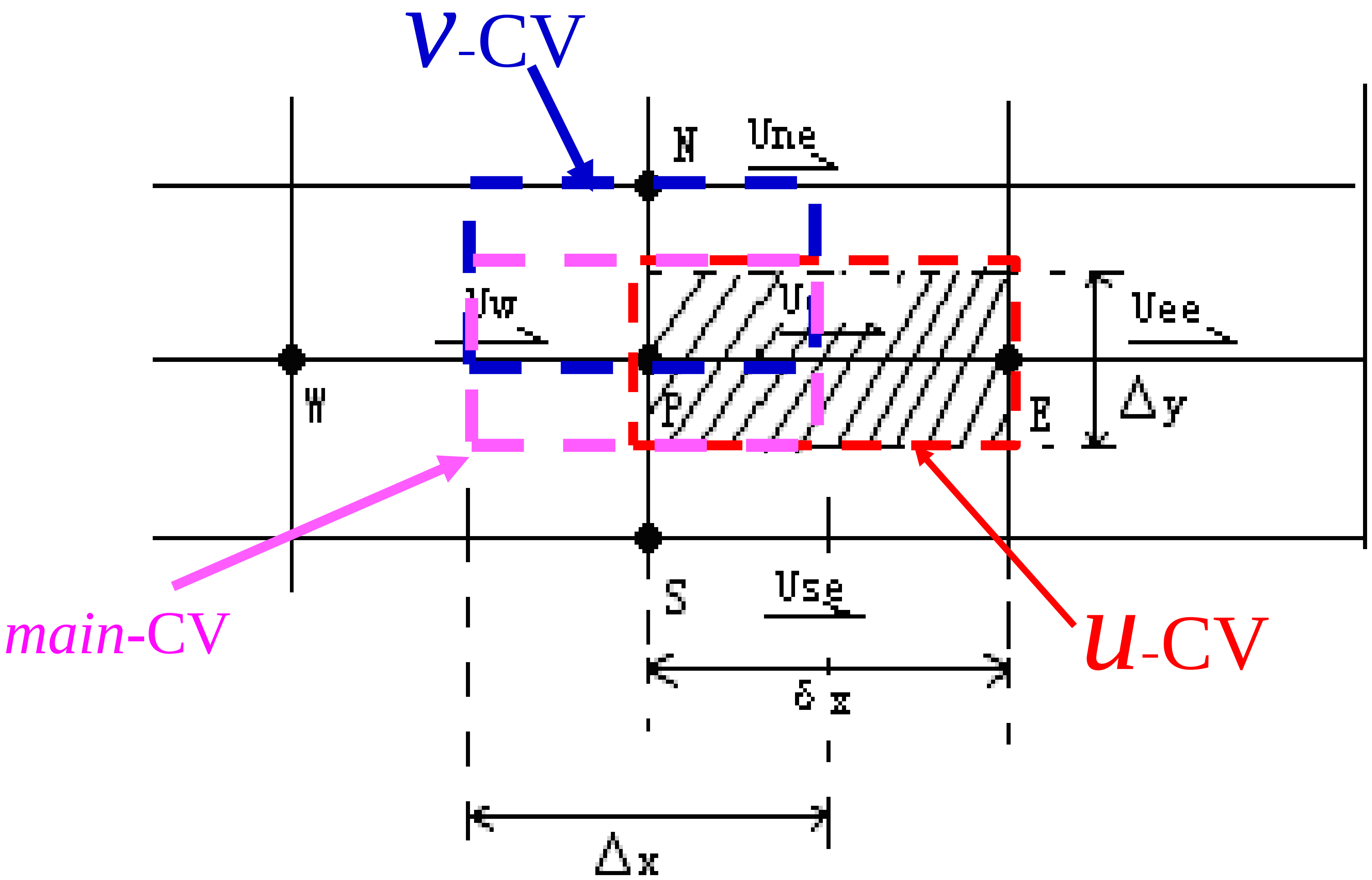
Volume of CV:

For general ϕ scalar variable $\Delta V=\Delta x\cdot\Delta y$

For u : $\Delta V=\delta x\cdot\Delta y$

For v : $\Delta V=\Delta x\cdot\delta y$




v-CV
N
Une
Uw
U
Uee
W
P
E
Δy
S
Use
main-CV
u-CV
δx
Δx

10.2.3 Implementation of under-relaxation

For the solution of non-linear Eqs., changes of variables between two subsequent iterations should not be too large to ensure convergence. Under-relaxation can control the speed of this change:

In the code, the under-relaxation is implemented during the solution procedure, which implies that the obtained solution has been underrelaxed:

$$\phi = \phi_P^0 + \alpha\left[\frac{\sum a_{nb}\phi_{nb} + b}{a_P} - \phi_P^0\right] \qquad (0<\alpha < 1)$$

$$\phi = \phi_P^0 + \alpha[\frac{\sum a_{nb}\phi_{nb} + b}{a_P} - \phi_P^0]$$

Thus：

$$(\frac{a_P}{\alpha})\phi_P = \sum a_{nb}\phi_{nb} + b + (1-\alpha)\frac{a_P}{\alpha}\phi_P^0$$

New a_P, $a_P^{'}$

New b, $b^{'}$

Finally following equation is sent to the solver:

$$a_P'\phi_P = \sum a_{nb}\phi_{nb} + b'$$

the obtained solution has been underrelaxed

10.3 Code Structure and Module Functions

10.3.1 Major features of general code

10.3.2 Entire structure of the code

10.3.3 Basic function of major modules

10.3.4 Functions and limitations of the code

10. 3 Code Structure and Module Functions

10.3.1 Major features of general code (通用程序)

General codes may be classified into two categories: One is commercial codes, the other is developed and used by researchers themselves.

1.Introduction to commercial codes(商用软件)

Commercial software has a good generality **(通用性)**

Most widely adopted ones include:

PHEONICS, COMSOL, FLUENT, CFX , STAR-CD etc.

Except COMSOL, which adopts FEM, the rest adopts the finite volume method (FVM).

2. **The codes developed and used by researcher themselves should also possess some generalities.**

Following techniques are often used:

(1) Adopting module structure

The so called module (模块) is a set of statements, which possess input, output and can implement some functions;

For those who just call(调用) the module do not need to know the content of the module, only need to know what are the input and output of the module.

Subroutine（子程序） in FORTRAN is a kind of module. Module structure has a good readability (可读性), and is convenient for maintenance (维护).

(2) The inherent relationship between different modules should be loose (松宽的), so that changes in one module will not affect other modules.

(3) The code is usually **divided into two parts**: unchanged part and user;

For the application of the code, the unchanged part is kept as is (保持不变); It is the **blackbox** for the users. The user part is closely related to the problem to be solved.

(4) Discretization, scheme and solution procedure should belong to three different modules.

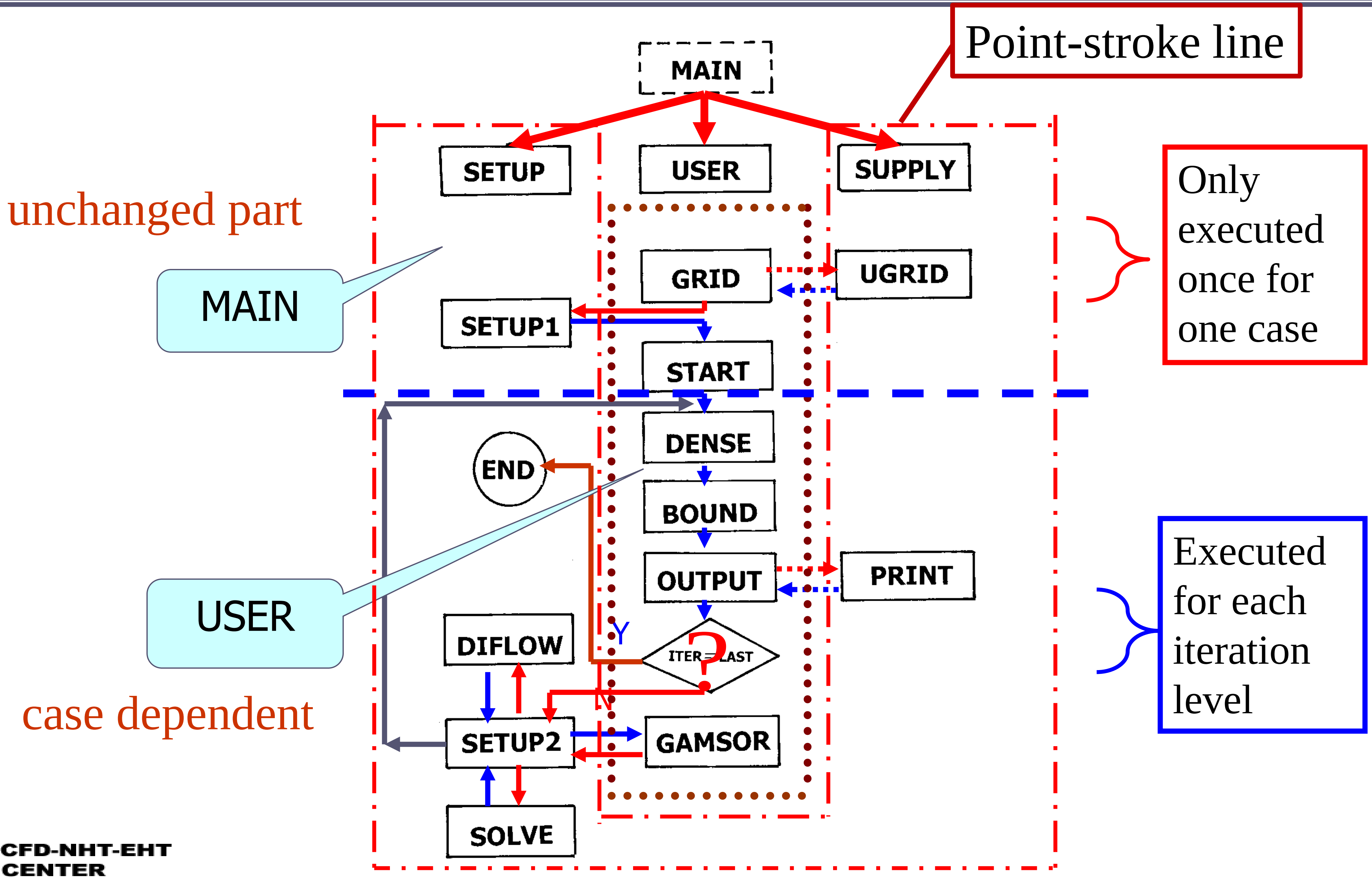
Such a structure is convenient to the studies for the algorithm, scheme and solution methods of the algebraic equations.

(5) For common variables, default values（预置值） should be set up;

(6) A certain pre-process (前处理）and post-process （后处理）functions should be provided.

10.3.2 The entire structure of the teaching code

Our teaching code belong to this category. It divides into two parts: **Main Program (主程序)** and **User** （用户）.


Point-stroke line
MAIN
SETUP
USER
SUPPLY
unchanged part
MAIN
GRID
UGRID
SETUP1
START
Only executed once for one case
DENSE
END
BOUND
OUTPUT
PRINT
USER
Y
ITER=LAST
?
N
DIFLOW
case dependent
SETUP2
GAMSOR
SOLVE
Executed for each iteration level

10.3.3 Basic functions (功能) of major modules

1．“MODULE”

The ”MODULE” is a terminology (术语) which is for being quoted (被引用) in FORTRAN 90/95 .

```
MODULE START_L
 PARAMETER(NI=100,NJ=200,NIJ=NI,NFMAX=10,NFX4=NFMAX+4)
*****************************************************
CHARACTER*8 TITLE(NFX4)
LOGICAL LSOLVE(NFX4),LPRINT(NFX4),LBLK(NFX4),LSTOP
REAL*8,DIMENSION(NI,NJ,NFX4)::F
REAL*8,DIMENSION(NI,NJ,6)::COF,COFU,COFV,COFP
REAL*8,DIMENSION(NI,NJ)::P,RHO,GAM,CP,CON,AIP,AIM,AJP,
```

The major feature of MODULE is that there is no any executive statement wherein (**在MODULE 中没有任何执行语句**).

Its major function is to be quoted (被引用） by other units of the program----When it is quoted all the contents in it will be copied to the unit quoting it, and all the quoting units share the same memory.

Its functions include:

(1) Packaging data (封装数据);
(2) Initializing data (数据初始化);
(3) Declaring type of data (声明数据类型).

MODULE structure is as follows：

```
MODULE module_name
●●●●●●
●●●●●●
●●●●●●
END MODULE
```

When a MODULE is going to be used:

USE module_ name
IMPLICIT NONE

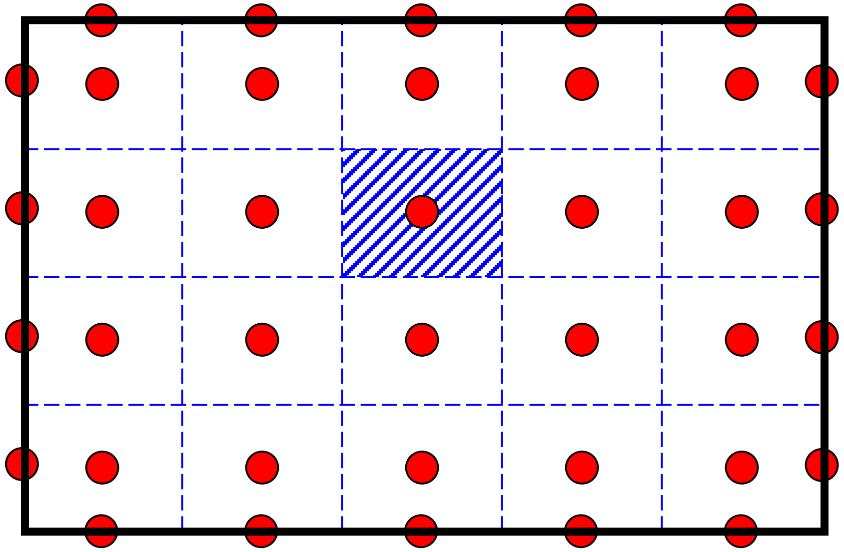
This means the type of all the variables in the module should be clarified individually (每个变量的类型必须逐一说明) , and variables should be declared. We cannot use the implicit rules.

2．MAIN （different form Main Program）

(1) Set up entire flow chart;

(2) Judge whether terminating the execution of the code.

3．GRID—Grid generation



(1) **Select coordinate**： MODE＝1, 2, 3 stands for *x*-*y*, *r*-*x* and *r*-θ

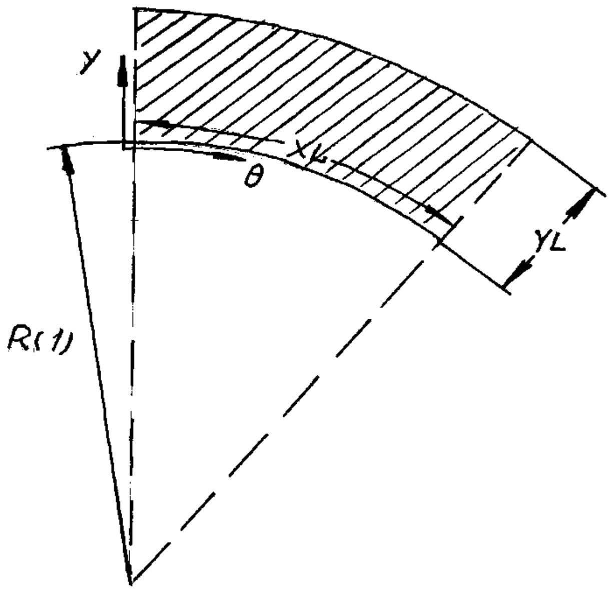
(2) Set up length in x, y direction by XL, YL, respectively;

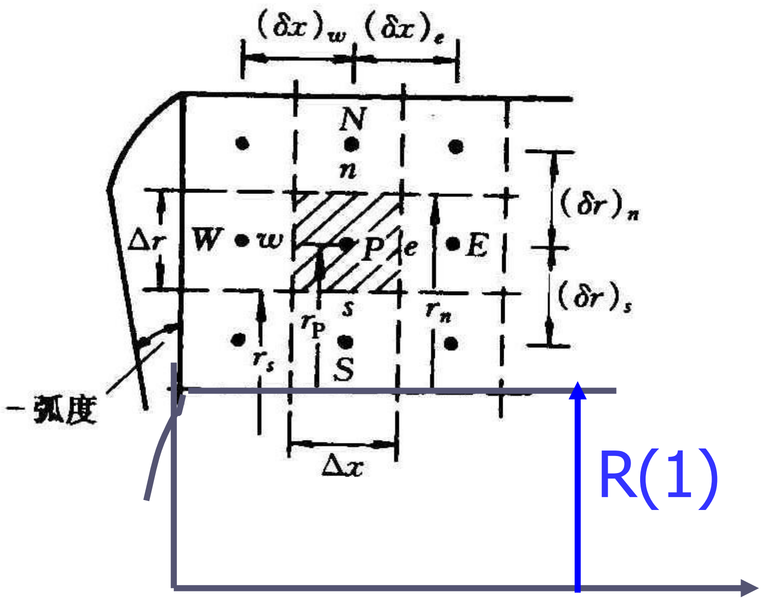
(3) Set up number of nodes in x and y directions by L1, M1, respectively；

(4) Set up interface positions of the main CV in x,y direction, respectively, by

XU(i), YV(j), i=2, L1, j=2,M1 (Practice B)；

(5) Set up the starting radius R(1) for MODE≠1





Polar coordinate Cylindrical symmetric coordinate

(6) Call UGRID to generate interface position for uniform grid system.

4．UGRID

Generate interface positions according to pre-specified XL, YL and L1, M1.

5．SETUP 1

1) Set up 1-D arrays of geometric parameters which remain unchanged during iteration:

(1) Set up node positions by X(i), Y(j), i=1··L1, j=1··M1,

(2) Generate **width of main CV** by XCV(i), YCV(j),
i=2···L2, j=2···M2;

(3) Determine distance between two neighboring nodes by XDIF(i), YDIF(j), i=2···L1, j=2···M1,

XDIF(i)=X(i)−X(i-1),

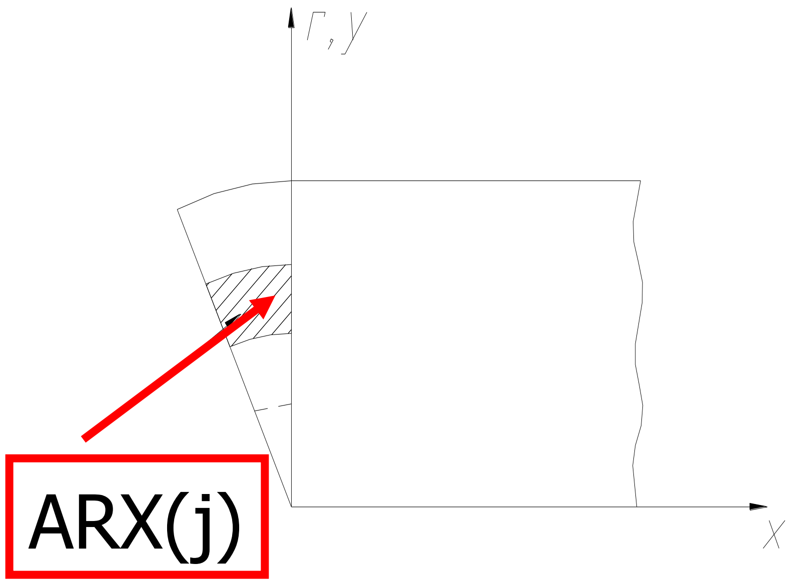
YDIF(j)=Y(j)−Y(j-1).

(4) Generate width of u, v CVs, respectively, by：

XCVS(i), i=3···L2， YCVS(j), j=3···M2；

(5) Set up radius R(j) in Y direction and scaling factor SX(j) in X direction ；

(6) Calculate surface area normal to X direction： ARX(j)



(7) Set up XCVI(i)， XCVIP(i)， which correspond to $(\delta x)_{e^-}, (\delta x)_{e^+}$, respectively;

(8) Establish interpolation functions, such as FX(i), FXM(i), etc.

2) Set up initial values of u, v, p, p’, RHO(i, j), AP(i, j), CON(i,j), CP(i, j); Except RHO(i,j) and CP(i,j), the initial values of all others are zero;

6．START

(1) Specify initial values for unsteady problems;

(2) Assume initial values of iteration for steady-state problems; Give boundary conditions which do not change during iteration.

Above four modules (GRID,UGRID,SETUP1,START) are executed only once during simulation of one case.

7．DENSE

Specify fluid density; For a constant-density problem, it can be empty(空块), but should be kept as is. For energy equation, the definition of the nominal density is conducted in the Main Program. If the actual density is a function of temperature, it should be defined in this module.

8. BOUND

Set up boundary conditions for all variables.

9. OUTPUT

(1) For every outer iteration, output some representative results for observation of convergence;

Solution procedure with a fixed set of coefficients of ABEqs is called one outer iteration. After finishing one level iteration, the coefficients are updated and the next outer iteration begins. The value of the indicator ITER is added by 1.

(2) Compute some special 2nd quantities: h, q, Nu, f ,etc.;

(3) Call PRINT， output 2-D fields.

10．PRINT

Output simulation results.

11．SETUP2 （Key module of Main Program）

（1）Call **GAMSOR** to determine Γ_{ϕ}, S_P, S_C ;

（2）Call **DIFLOW** to determine $A(|P_{\Delta}|)$ for the scheme;

（3）Set up the discretized coefficients denoted by

$$AIP(I,J), AIM(I,J)\ AJM(i,j), AJP(i,j),\ AP(i,j)$$

$CON(i,j)$---a_E, a_W, a_S, a_N, a_P and b in the lecture

（4）Call SOLVE to solve algebraic equations;

（5）Update indicator: ITER＝ITER+1.

12．GAMSOR

$$\frac{\partial(\rho^*\Phi)}{\partial t} + div(\rho^*\vec{u}\Phi) = div(\Gamma_\Phi grad\Phi) + S_\phi^*$$

(1) Determine Γ_ϕ for different variables:

$$u\ ,v - \mu(\text{or}\ \eta)\ ;T - \lambda$$

(2) Store source terms S_P, S_C of different variables into correspondent $AP(i, j)$,$CON(i, j)$, respectively.

(3) Set up **additional** source terms for boundary CVs with 2nd or 3rd condition

$CON(I,J)$ ← Original data + $CON(I,J)$ $S_{c,ad}$

$AP(I,J)$ ← Original data + $AP(I,J)$ $S_{p,ad}$

Add accumulated !

13．DIFLOW

Determine $A(|P_\Delta|)$ based on D and F. $P_\Delta = \frac{F}{D} = \frac{\rho^* uA}{\Gamma A/\delta x}$

14．SOLVE

Adopt ADI line iteration + block correction to solve algebraic equations;

Inner iteration is controlled by an variable NTIMES(NF) , usually within 1 to 6;

Outer iteration is controlled by indicator ITER, which may reach 10^3—10^5, depending on cases.

10.3.4 Functions and limitations of the code

Three functions (功能):

(1) It can solve the incompressible fluid flow and heat transfer problems in three 2D coordinates;

(2) It can solve 10 dependent variables consecutively(连续地） and print out 14 variables consecutively；

(3) It can solve both dimensional and dimensionless governing equations.

Limitations (限制)

(1) It is not convenient to simulate transient problems with high non-linearity；

(2) It is not convenient to simulate problems with irregular domain;

(3) It can not simulate compressible fluid flow.

10. 4 Grid System

10.4.1 Regulations for three coordinates

10.4.2 Numbering system for geometric parameters and variables

10.4.3 Composite picture of coordinates

10.4.4 Explanations of pressure field simulation

10. 4 Grid System

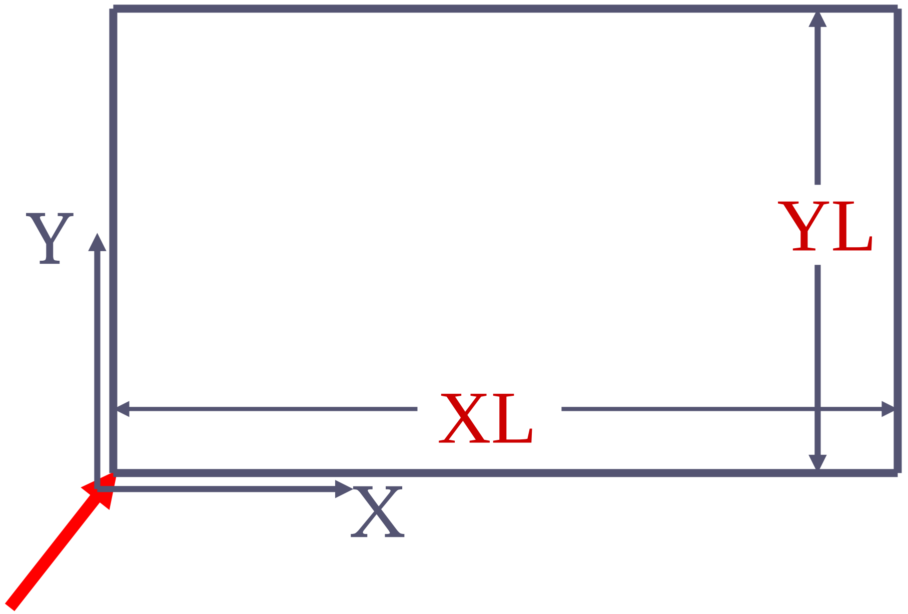
10.4.1 Regulations for three coordinates

1. Cartesian coordinate

(1) MODE＝1；

(2) Unit thickness in *Z*-direction；

(3) Origin of the coordinate locates in the left-bottom position.



X-Y coordinate

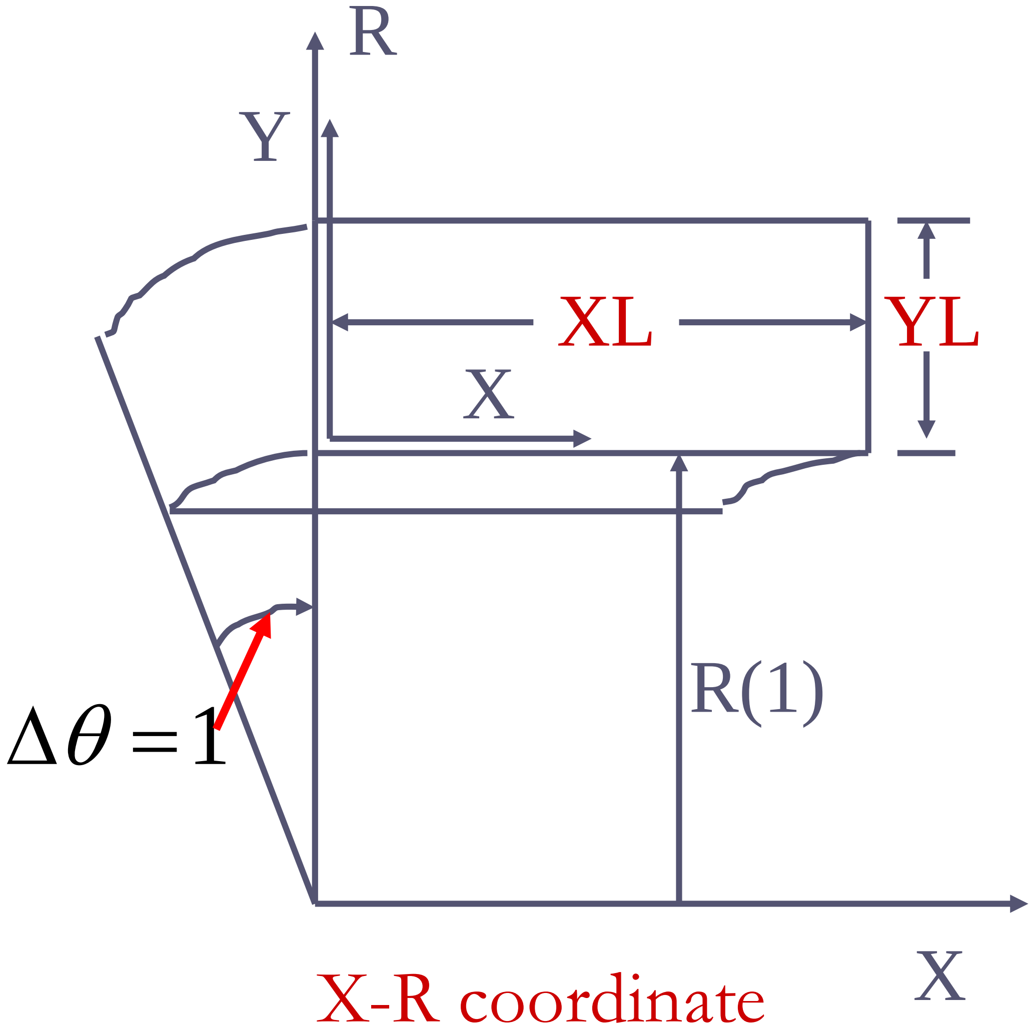
2. Symmetric cylindrical coordinate

(1) MODE＝2；

(2) Simulation is conducted for $\Delta\theta = 1$ radian (弧度);

(3) R(j) starts from symmetric axis

(4) R(1) should be given.



X-R coordinate

3. Polar coordinate

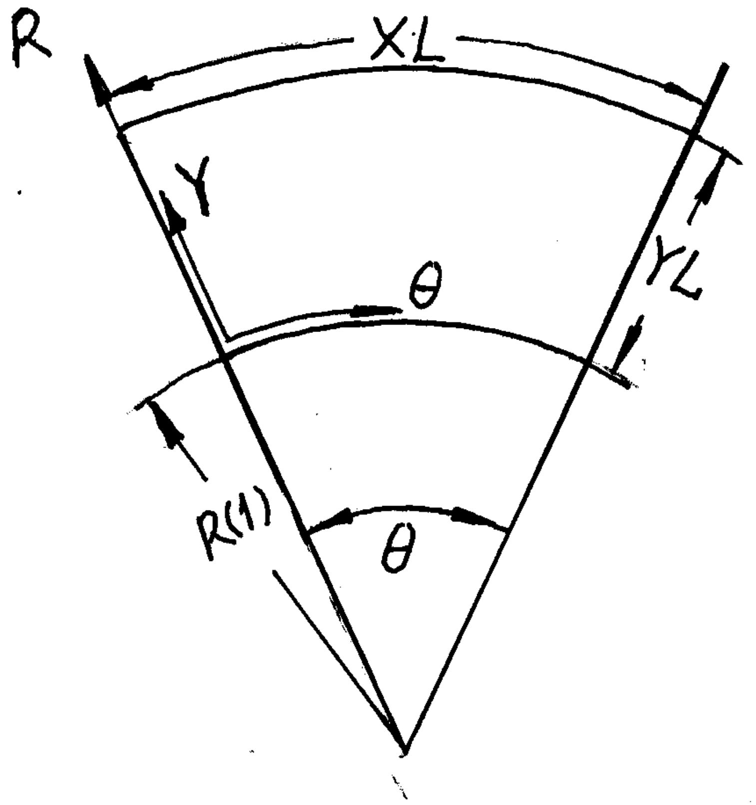
(1) MODE＝3;

(2) Unit thickness in z-direction;

(3) R(j) starts from circle center;

(4) R(1) should be given;

(5) Angle θ should be less than 2π



Theta-R coordinate

10.4.2 Numbering system for geometric parameters and variables

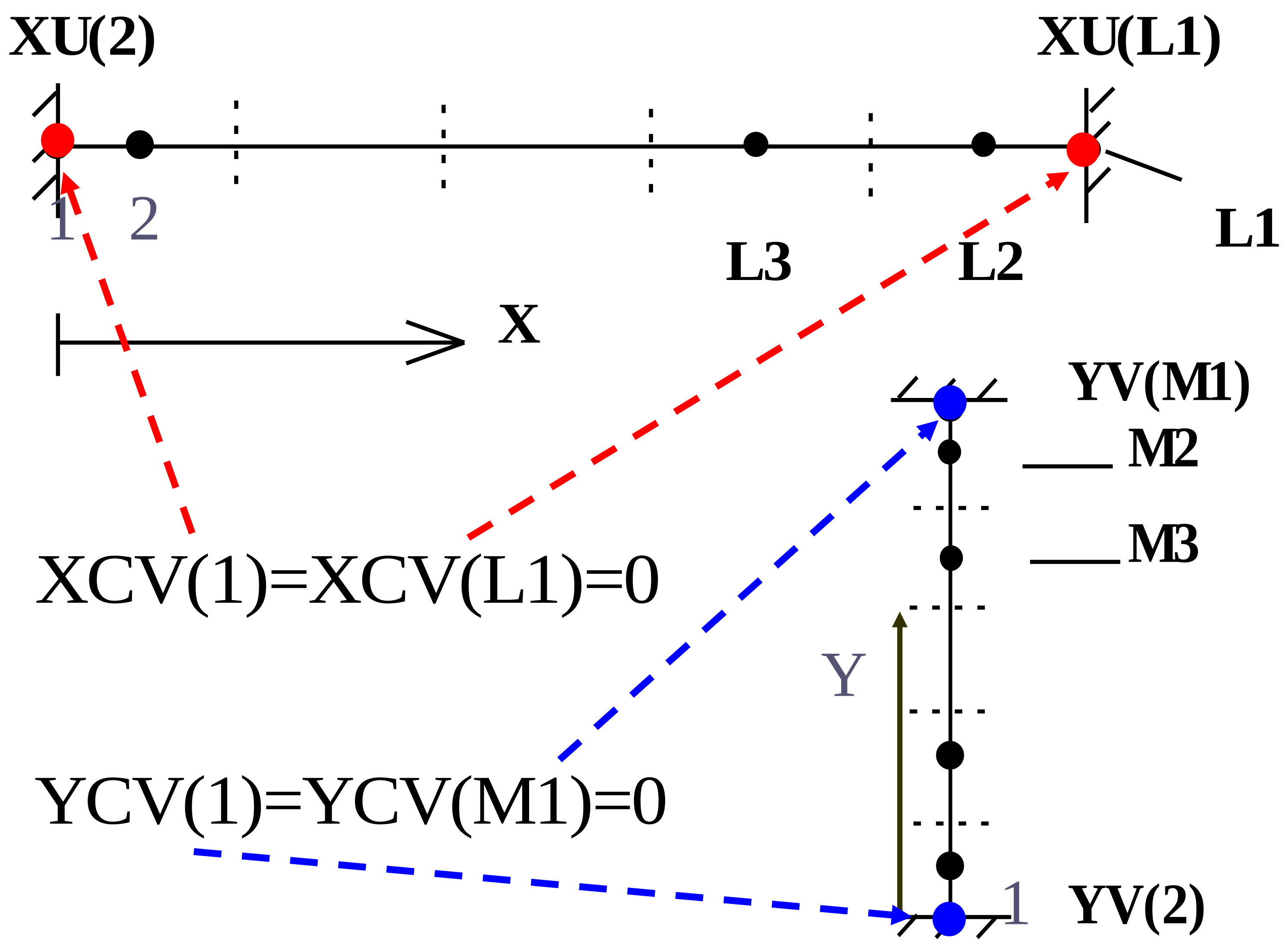
1.Interfaces of main CV： XU(i), i=2,……L1,

YV(j), j=2,……M1

2.Main nodes： Last three nodes in X-direction： L1, L2, L3; in Y-direction: M1, M2, M3

3.Width of main CV： XCV(i), i = 2,……L2;

YCV(j), j=2,……M2


XU(2)
XU(L1)
1
2
L3
L2
L1
X
YV(M1)
M2
M3
Y
1
YV(2)
XCV(1)=XCV(L1)=0
YCV(1)=YCV(M1)=0

4.Distances between nodes:

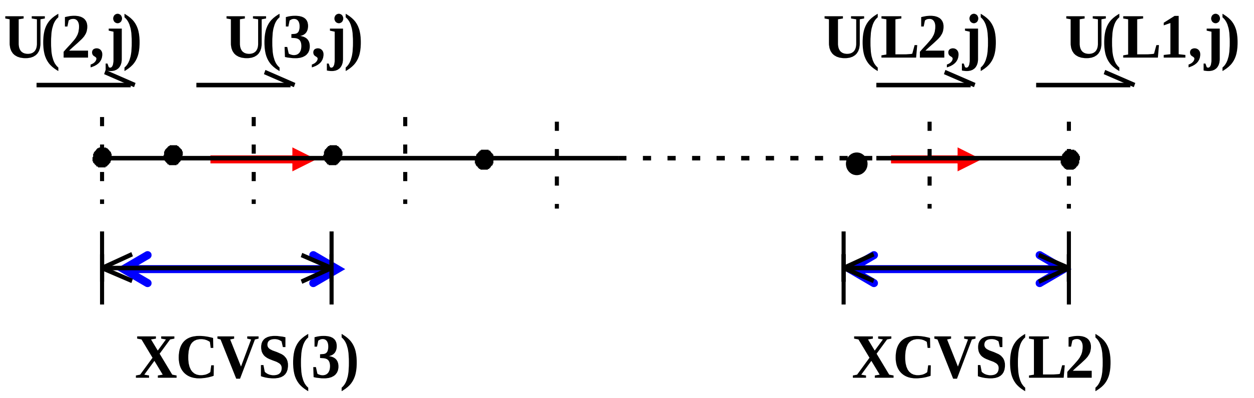
$$XDIF(i) = X(i) - X(i-1)\,,\ \ i = 2,......L1$$

$$YDIF(j) = Y(j) - Y(j-1),\ \ \ j = 2,......M1$$

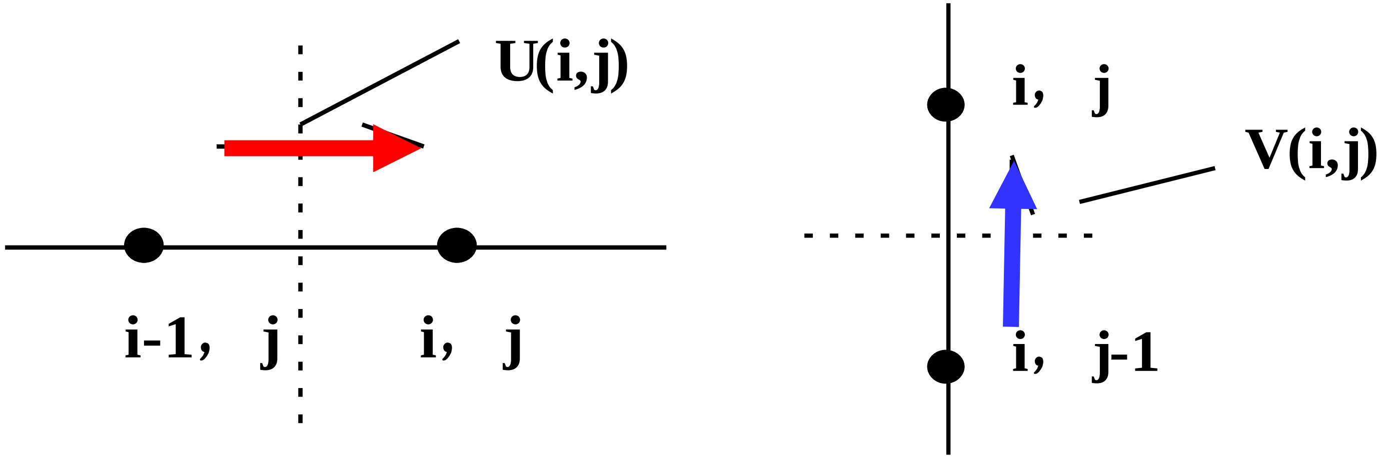
5.Widths of velocity CVs :

For component u: XCVS(i), i=3,……L2;

For component v: YCVS(j), j=3,……M2



6.Velocity numbering： the node number towards which the velocity arrow directs is the number of velocity



7.Starting points of solution region of ABEqs.

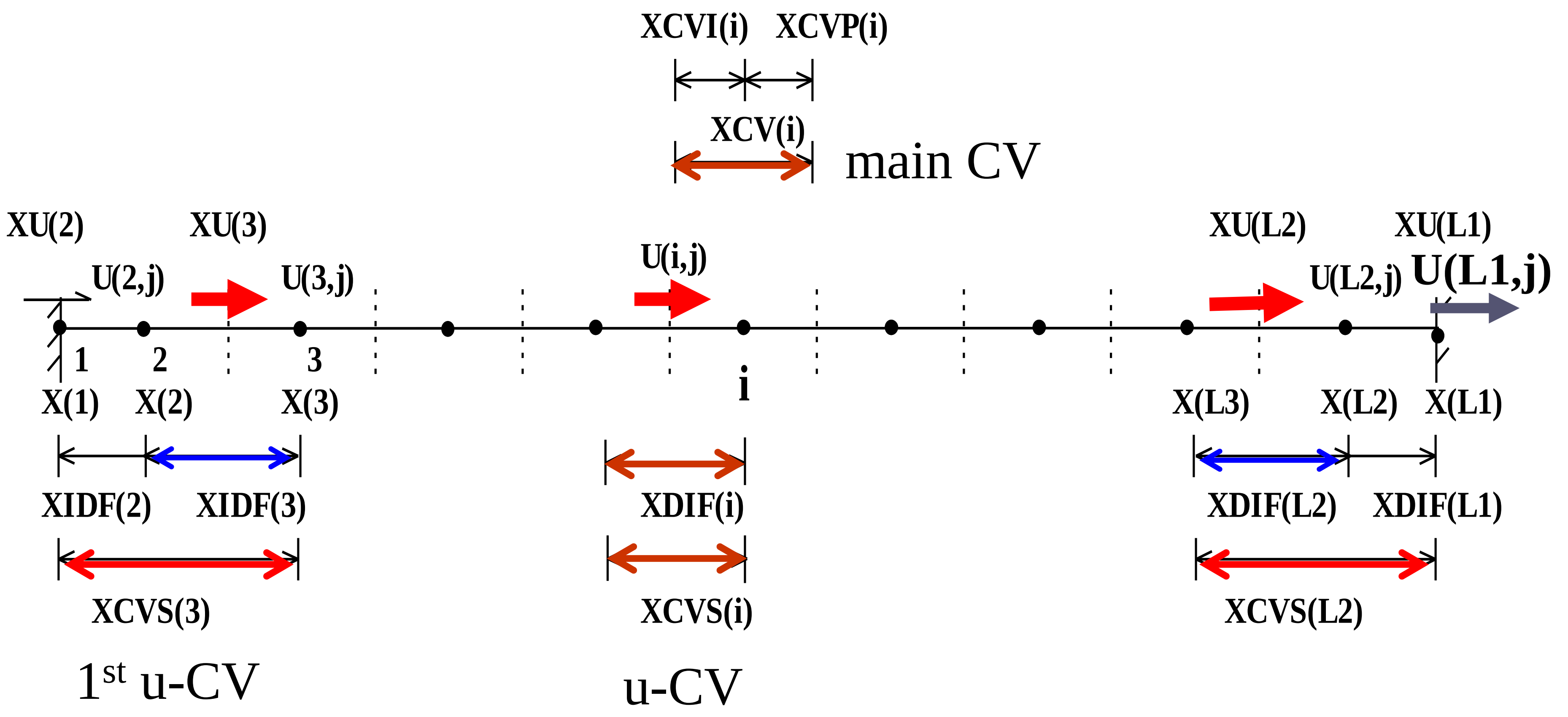
Because all boundaries are treated as 1st kind, solution is conducted within the inner region; **the starting nodes of solutions for *u, v, p* are different**, denoted in the code by FORTRAN variables IST， JST.

IST，JST represent the number of starting node in X,Y iteration, respectively:

Variable	IST	JST
$\phi, p, p^{'}$	2	2
u	3	2
v	2	3

10.4.3 Composite picture of coordinates

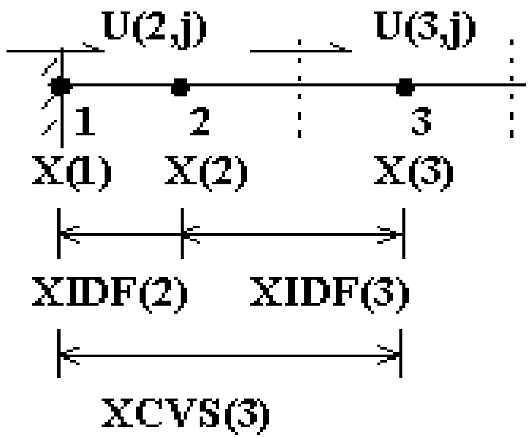
Composite figure in X-direction



Similarly in Y-direction.

10.4.4 Explanations of pressure field simulation

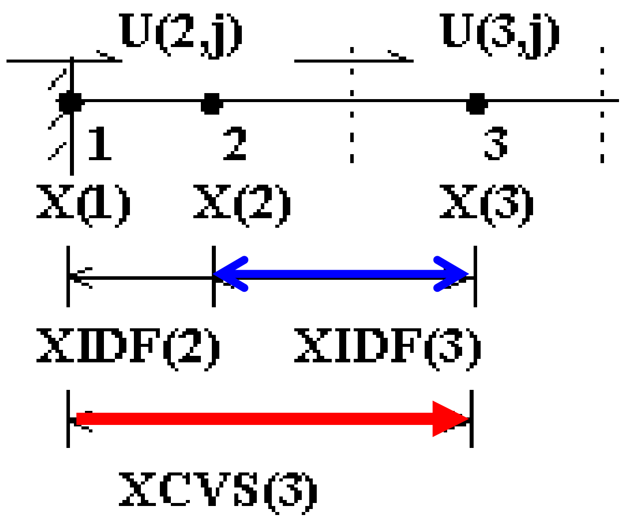
1. The boundary pressure is extrapolated after obtaining converged solutions.



2.Pressure difference for *u*(3, j) during iteration：

For *u*(3,j) its width is XCVS(3) and pressure difference [*p*(3,j)-*p*(1,j)] should be used in momentum equation; However, *p*(1,j) is not known during iteration, following method is used to overcome the difficulty: using a magnified area to compensate reduced pressure difference：

$$\left[\frac{VOL(3,j)}{XDIF(3)}\right](p(2,j)-p(3,j))$$



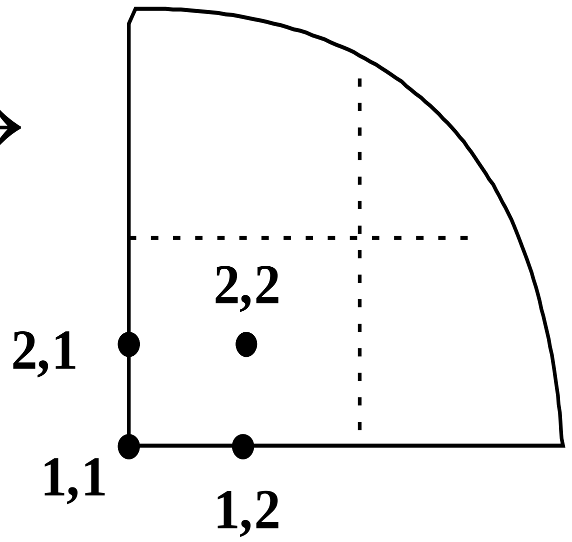
In the inner region, XCVS(i)=XDIF(i)， hence VOL(i,j)/ XDIF(i) equals the area where pressure acts; While near the boundary, XDIF(3) is less than XCVS(3). Thus this treatment is equivalent to linear interpolation, from [p(3,j)-p(2,j)] to get [p(3,j)-p(1,j)] :

$$p(3,j)-p(1,j)\cong(p(3,j)-p(2,j))\left[\frac{XCVS(3)}{XDIF(3)}\right]$$

3.Pressures at four corners are not used during simulation; However, for output requirement, they are calculated as follows:

$$p(1,1)+p(2,2)=p(1,2)+p(2,1)\rightarrow$$

$$p(1,1)=p(2,1)+p(1,2)-p(2,2)$$



Interpolation for Corner pressure
角点压力的插值

4.For incompressible flow, pressure value is relative to some reference; **For output purpose, a reference point is specified by (IPREF,JPREF)**：

$$p(i,j)=p(i,j)-p(IPREF,JPREF)$$

In the code, the default values are 1 for both IPREF and JPREF.

10. 5 Techniques Adopted in the Code

10.5.1 Ten dependent variables can be solved and 14 variables can be printed out

10.5.2 Iteration for nonlinear steady problem is treated the same as marching process of linear unsteady problem

10.5.3 Methods for saving memories

10.5.4 Methods for saving computational times

10. 5 Techniques Adopted in the Code

10.5.1 Ten dependent variables can be solved and 14 variables can be printed out

1. Define a simple variable NF, its maximum value is **10** (**NFMAX**) ; NF from 1 to 4 represents *u*, *v*, *p'* and *T*, respectively; NF≥5 can represent other variables defined by user.

2. Define a 3-D array *F*(NI,NJ,NFX4), NFX4=14; p, ρ, Γ, and c_p are regarded as the 11th, 12th , 13th and 14th variable, respectively.

3. Define two logic arrays： LSOLVE(NF) and LPRINT(NF)， and their default values are .FALSE.; In USER, if the value of LSOLVE(NF), say for NF＝1, is set as .T.， then in SETUP 2 this variable is solved.

4. In SETUP2，Visit NF from 1 to NFMAX in order;

When some value of NF is visited and **LSOLVE(NF)=.T.， then this variable is solved**；

Similarly in PRINT SUBROUTINE NF is visited from 1 to NFX4(=14) in order， as long as LPRINT(NF) ＝.T.， the variable is printed out.

10.5.2 Iteration for nonlinear steady problem is treated the same as marching process of linear unsteady problem

Finishing one outer iteration for nonlinear steady problem (ITER=ITER+1) is equivalent to one time step forward for unsteady linear problem (*T*=*T*+DT).

In order to guarantee the convergence of iteration for solving ABEqs., several cycles are included in one outer iteration and the cycle number is adjustable.

1. ITER＋1 implies finishing one outer iteration, which is equivalent to *T*＝*T*＋DT， forward one time step.

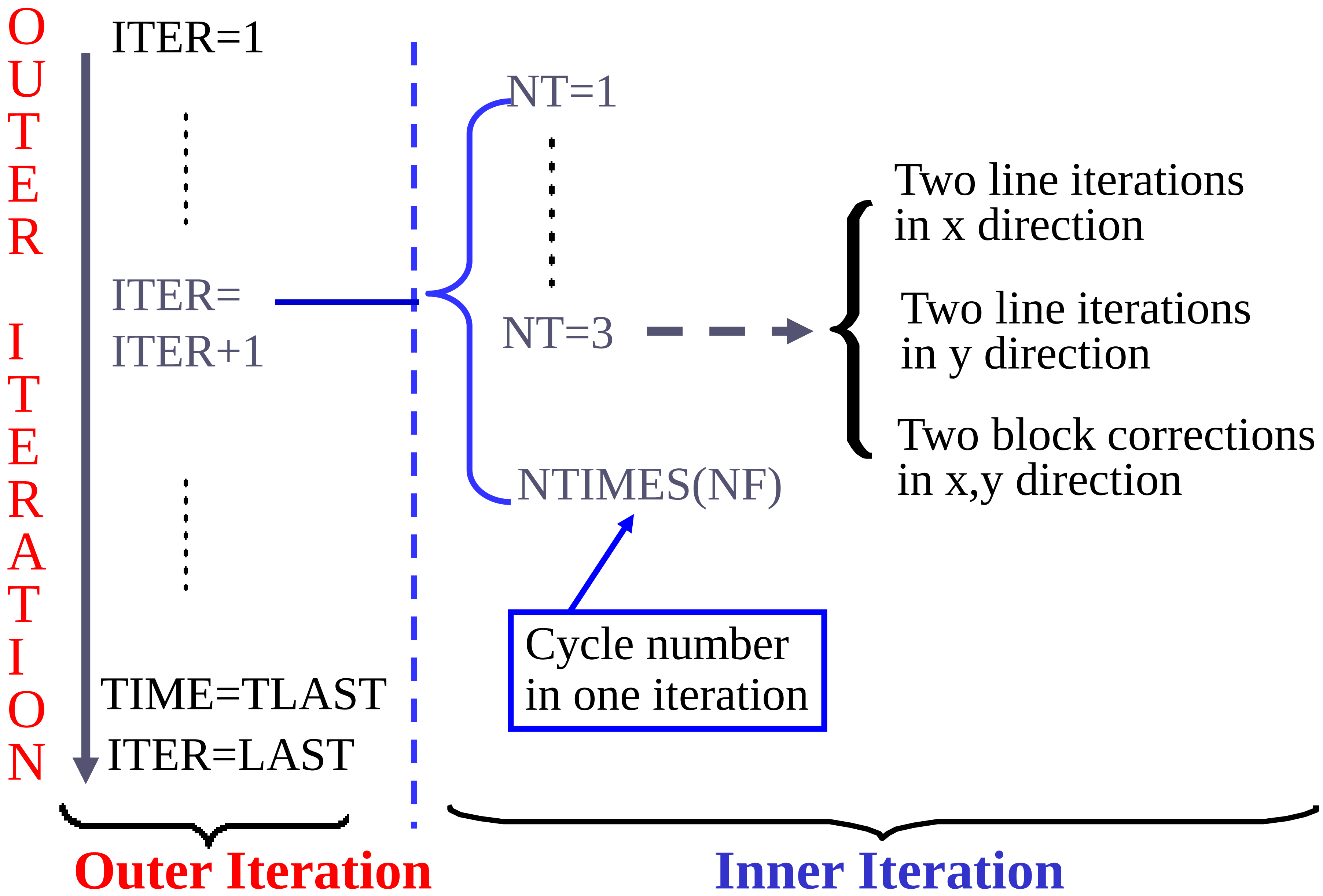
2. Set up an array, NTIMES(NF)， for indicating cycle number in one iteration. Its default value is one. If NTIMES(2)＝4， then the algebraic equations of the second variable (NF=2) will be iteratively solved by four cycles.

3. Every cycle includes 6 solution practices:

In X, Y-direction two block corrections;

In X-direction back-and forth (来回) line iterations;

In Y-direction, back-and forth line iterations .


OUTER ITERATION
ITER=1
ITER= ITER+1
TIME=TLAST
ITER=LAST
NT=1
NT=3
NTIMES(NF)
Two line iterations in x direction
Two line iterations in y direction
Two block corrections in x,y direction
Cycle number in one iteration
Outer Iteration
Inner Iteration

4. Principles for selecting value of NTIMES(NF)

(1) Steady and nonlinear problems:

NTIMES(NF) takes values of 1～2, because the coefficients are to be further updated;

(2) Unsteady ad linear problems:

NTIMES(NF) may take a larger value, say 4～5, to ensure that for every outer iteration the solution of algebraic equations are converged.

The present code is not recommended to apply for solving unsteady and nonlinear problems: For such problem, the coefficients within one time step should be updated, while in the present code within one time step coefficients remain unchanged. Then the time step has to be small enough.

10.5.3 Methods for saving memories

1.Adopt multiple inlet statement ENTRY

Function of ENTRY:

All ENTRYs within the same subroutine can share all MODULE located in the beginning part of the subroutine, and each ENTRY can be called individually.

USER SUBROUTINE

```
USE START_L
USE USER_L
IMPLICIT NONE

ENTRY START
………..
RETURN

ENTRY GRID
…………
RETURN
```

2. A compromise is made between memory and computational time

All one dimensional geometric parameters have their own individual array, totally 23 arrays, including $x(i)$ and $y(j)$;

All two dimensional parameters, including $\Delta V(i, j)$ are not stored: when needed they are temporary calculated and then they are deleted, rather than stored.

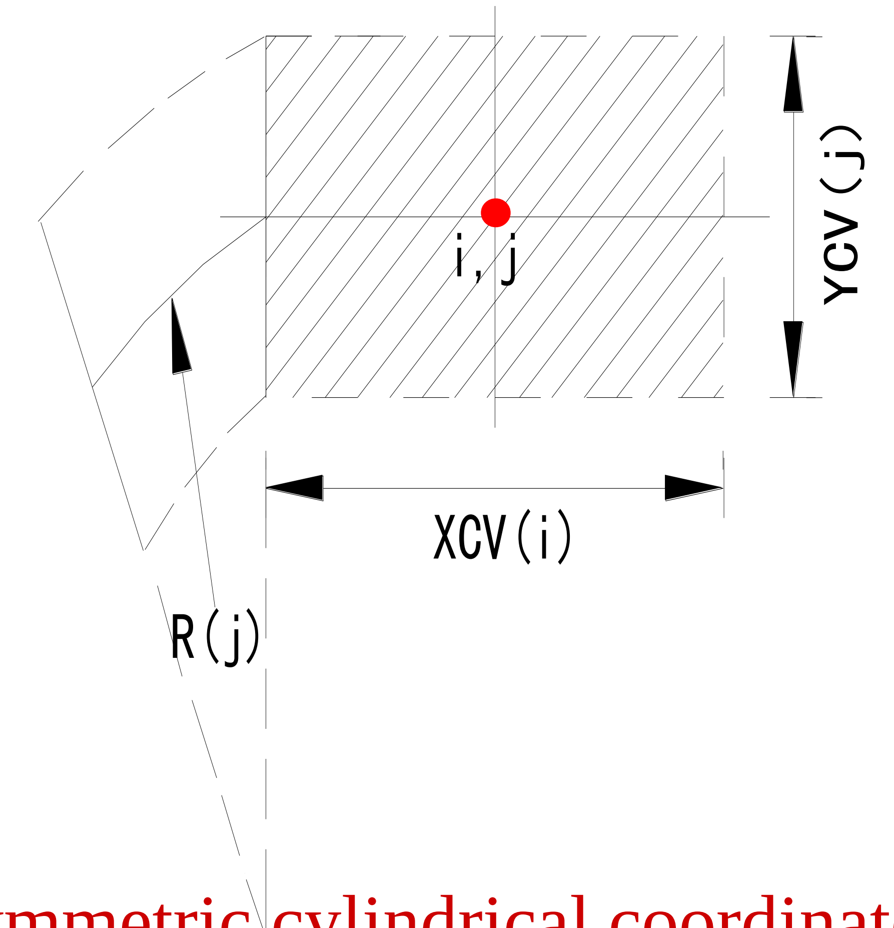
For 2D case, when node numbers in x,y direction are as large as L1=M1= 40, then the memory of one 2-D array is equivalent to 40 1-D arrays.

Volume calculation of 3 CVs of cylindrical coordinate:

$$VOL = XCV(i) * \underline{YCV(j) * R(j)} = XCV(i) * \underline{YCVR(j)}$$

Area normal to flow direction

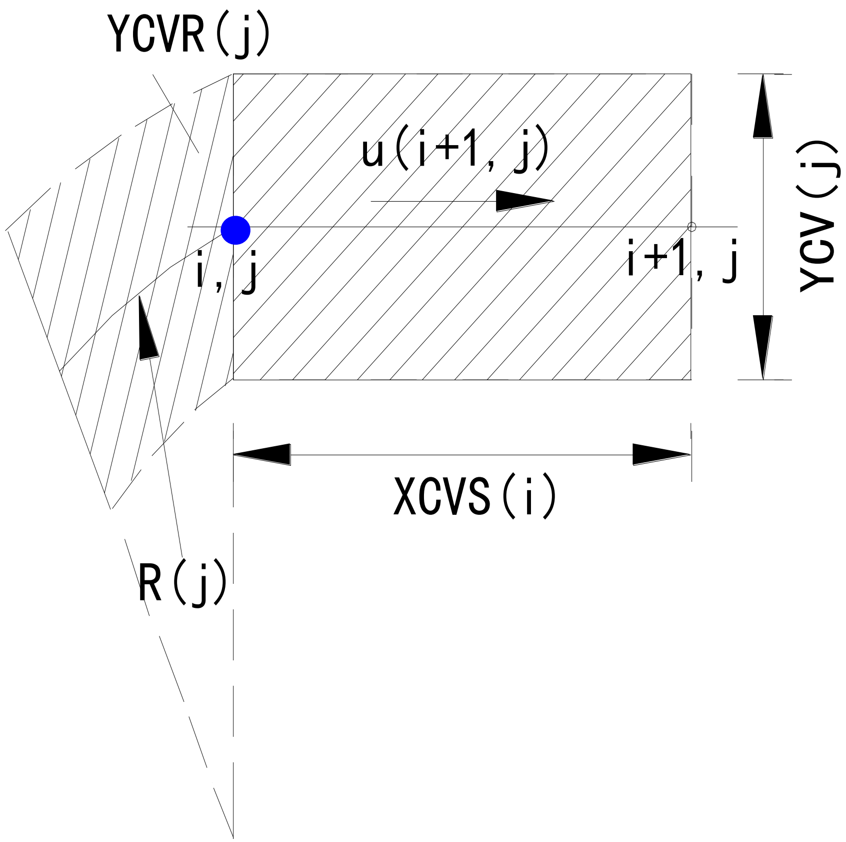
Main CV:

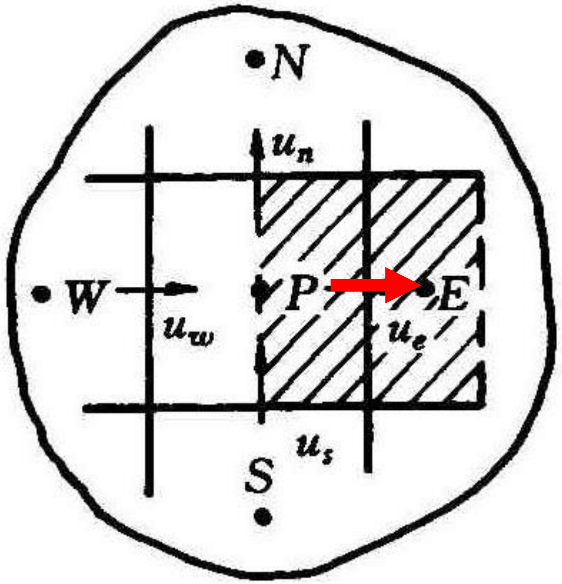


Symmetric cylindrical coordinate

u-CV：

$$VOL = XCVS(i) * \underline{YCV(j) * R(j)} = XCVS(i) * \underline{YCVR(j)}$$

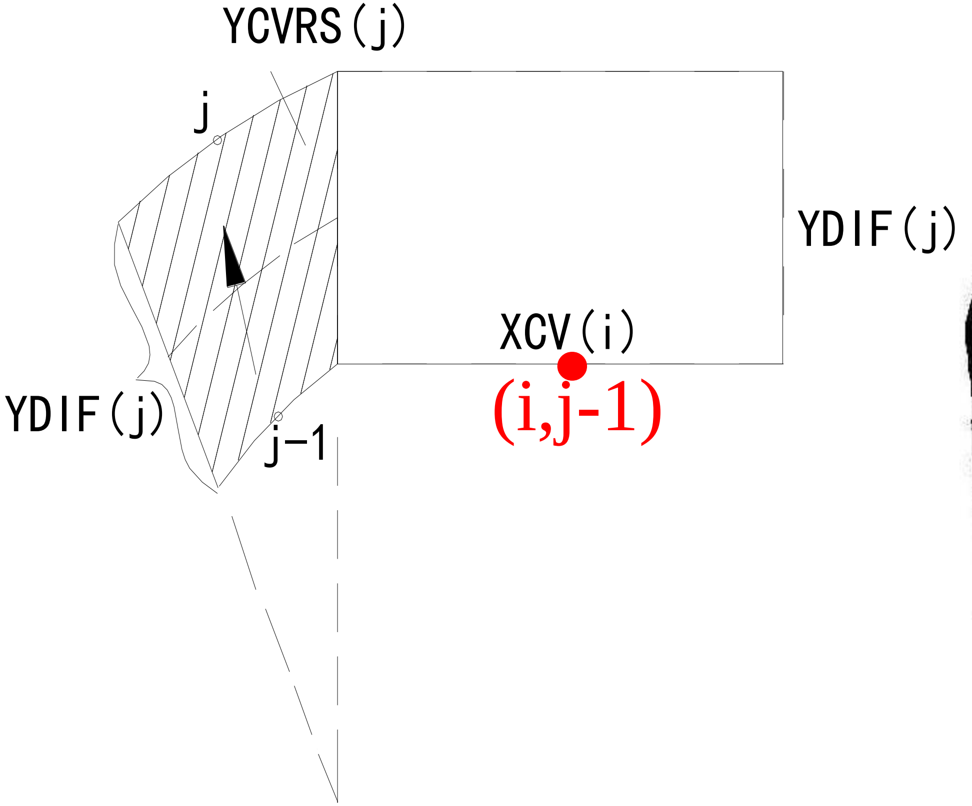


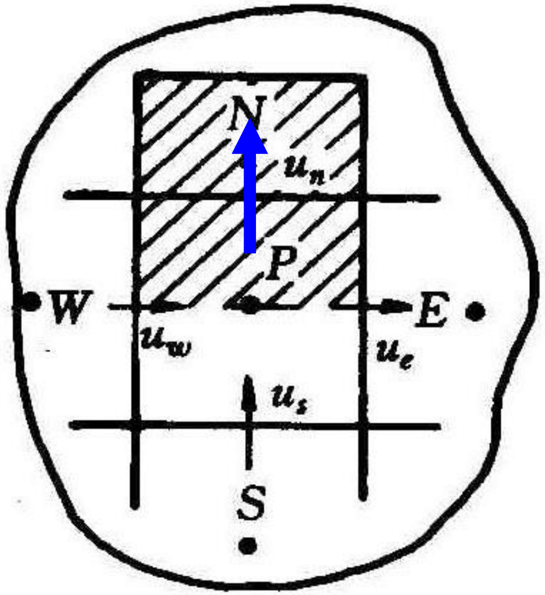


Symmetric cylindrical coordinate, u-CV

v-CV:

$$VOL = XCV(i) * YDIF(j) * \frac{R(j)+R(j-1)}{2} = XCV(i) * YCVRS(j)$$





Symmetric cylindrical coordinate, v-CV

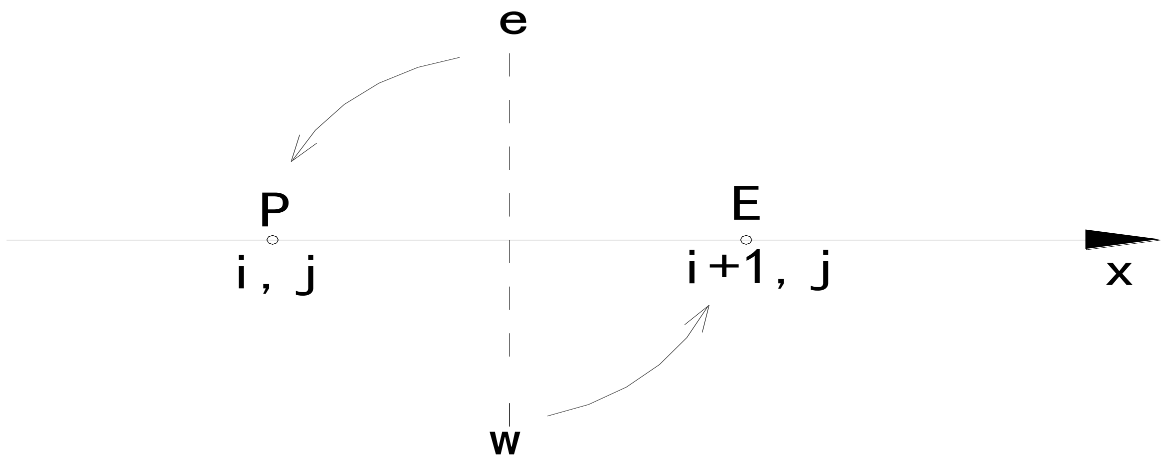
3.Terminate outer iteration by specifying LAST, rather than by comparison of two subsequent iterations, thus saving one 3-D array. However, the appropriate value of LAST should be determined during iteration.

10.5.4 Methods for saving computational times

1.Data transfer between different units of the code by using MODULE. It is the most efficient way for data transfer.

2.Take advantage of relationship between coefficients, saving computational time .

It has been shown that for the five 3-point schemes:



$$a_E(i,j)=D_eA(|P_{\Delta e}|)+[|-F_e,0|]$$

$$a_W(i+1,j)=D_wA(|P_{\Delta w}|)+[|F_w,0|]$$

$$a_W(i+1,j)-a_E(i,j)=[|F_w,0|]-[|-F_e,0|]=\underline{F_e}$$

Thus $a_E(i, j)$ can be easily obtained from $a_W(i+1, j)$

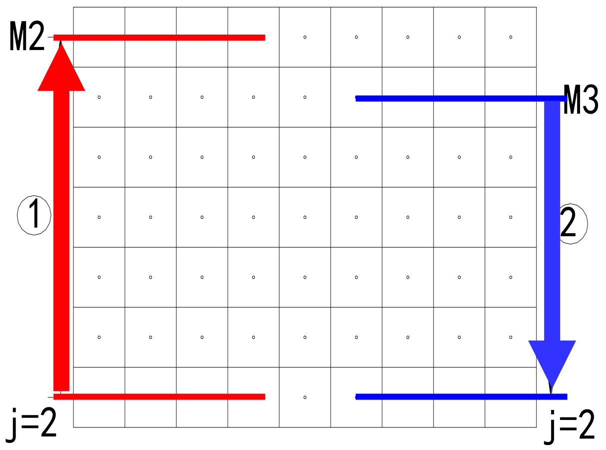
$$a_E(i, j) = a_W(i+1, j) - FLOW$$

Similarly:

$$a_N(i, j) = a_S(i, j+1) - FLOW$$

3.Time saving during ADI-line iteration

When scanning from bottom to top along y, J=2 to M2; then back scanning can start from M3 rather from M2, because the line of M2 is just solved in the upward scanning and all the coefficients and constants in that line remain unchanged.



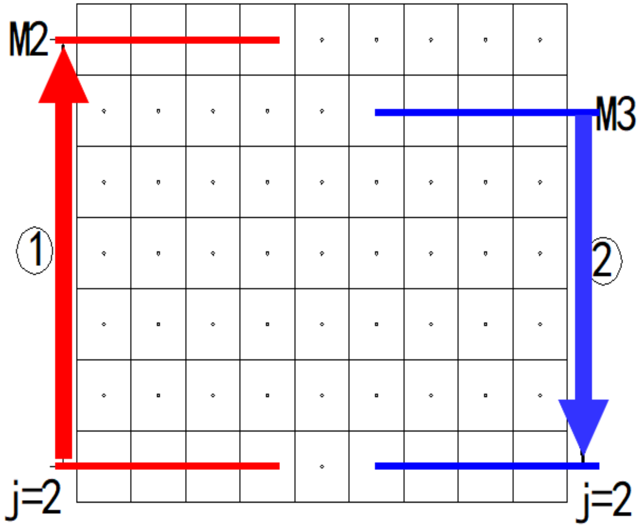
Coefficients are not changed during one level iteration, the key is the constant term *b*: for the line M2 in the upward scanning:

$$b = b' + a_N \Phi_{given}(i, M1) + a_S \Phi^*(i, M3)$$

In the downward scanning, the b term of line M2 does not change, while for the line M3 b-term has been changed after the solution of line M2:

$$b = b' + a_N \Phi^*(i, M2) + a_s \Phi^*(i, M3-1)$$

In the downward scanning $\Phi^*(i, M2)$ has been updated during upward scanning. Hence this line should be solved.



Announcement

In the study of application of FLUENT, our class will be divided into two teaching groups: Fundamental and Intermediate.

They are presented at the same time. Every student should make your choice. Please send your choice to the teaching assistance group in this week. We have to arrange an additional classroom from the graduate school.

本组网页地址：http://nht.xjtu.edu.cn 欢迎访问！

Teaching PPT will be loaded on our website

同舟共济

渡彼岸！

People in the same boat help each other to cross to the other bank, where....