Numerical Heat Transfer
(数值传热学)
Chapter 2 Discretization of Computational Domain and Governing Equations

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2.1 Grid Generation

2.1.1 Task, method and classification

1. Task of domain discretization

Discretizing the computational domain into a number of sub-domains which are not overlapped(重叠) and can completely cover the computational domain.

Five kinds of information can be obtained:

(1) Node（节点）:the position at which the values of dependent variables are solved;

(2) Control volume (控制容积) :the minimum volume at which the conservation law is applied;

(3) Interface（界面）:boundary of two neighboring CVs.

(4) Grid lines（网格线）**: Curves formed by connecting two neighboring nodes.**

(5) Spatial relationship between two neighboring nodes.

The influencing coefficients will be decided in the procedure of equation discretization

2. Classification of domain discretization

(1) According to node relationship: structured vs. unstructured

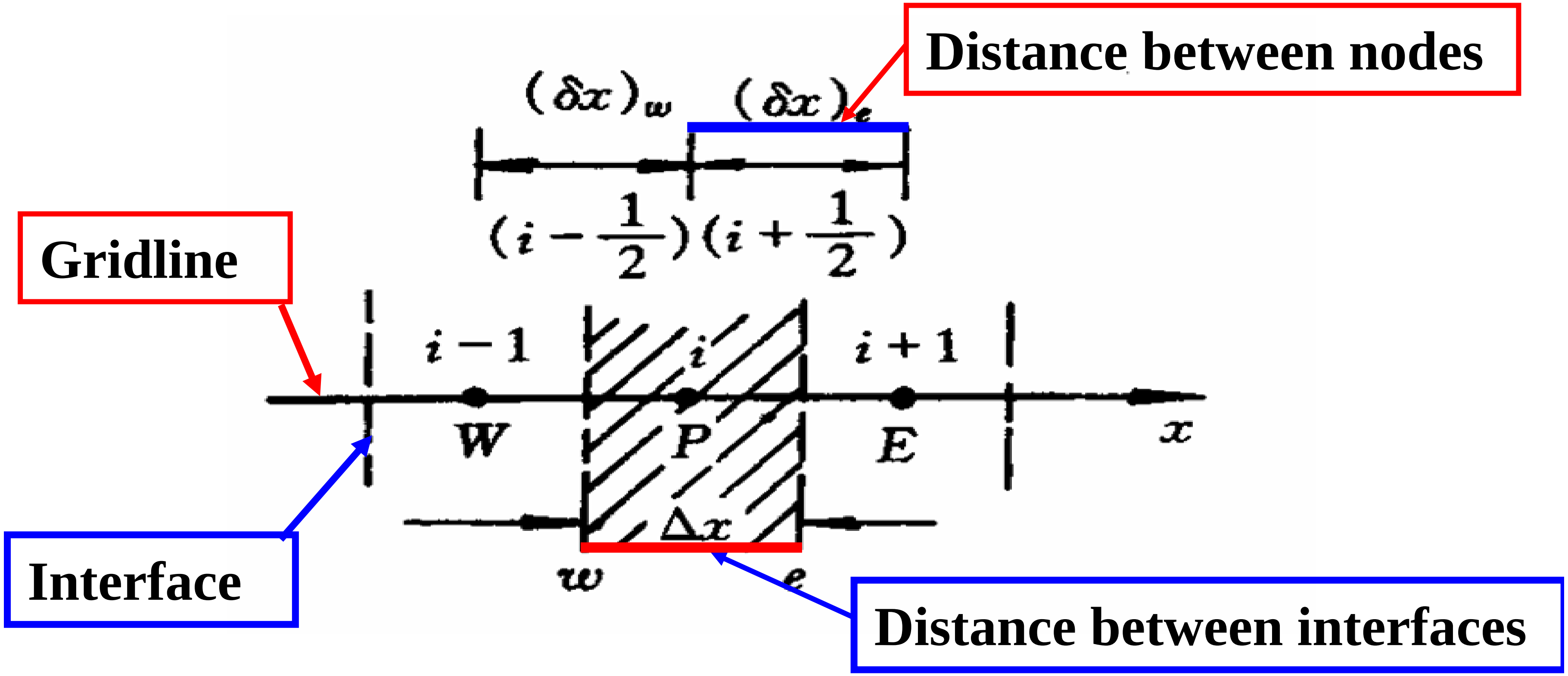
(2) According to node position: inner node vs. outer node

2.1.2 Expression of grid system

Grid line－solid line; Interface-dashed line;

Distance between nodes－δx

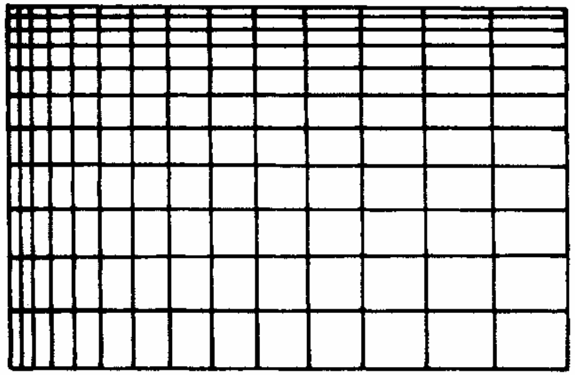
Distance between interfaces－Δx 。



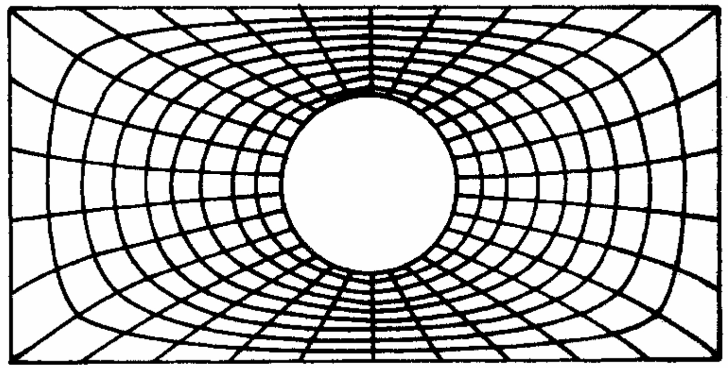
2.1.3 Introduction to different methods of grid generation

（1） **Structured grid (结构化网格)**： Node positions layout is in order, and fixed for the entire domain.

（2） **Unstructured grid (非结构化网格)**： Node position layout is in disorder, and may change from node to node. The generation and storage of the relationship of neighboring nodes are the major concern of grid generation.

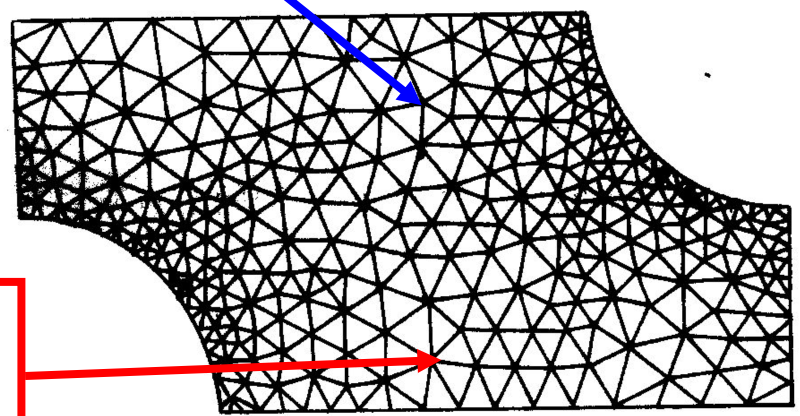


Structured （a)



Structured （b)

5 elements



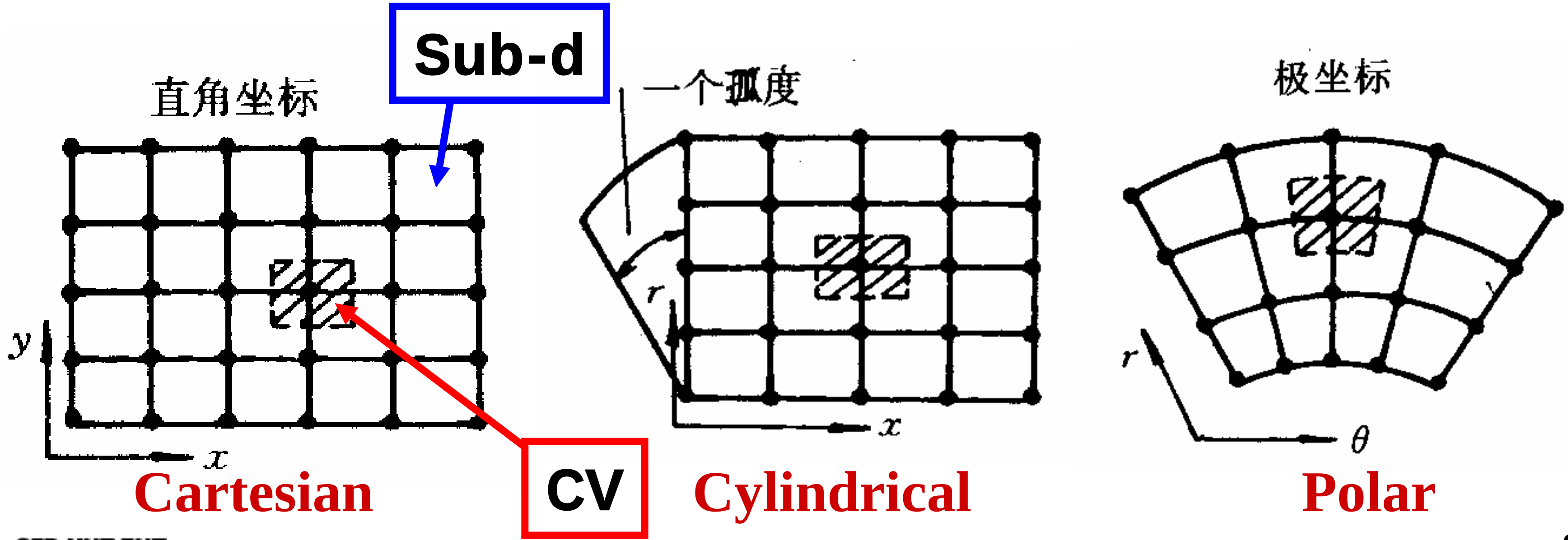
Un-structured

6 neighboring elements

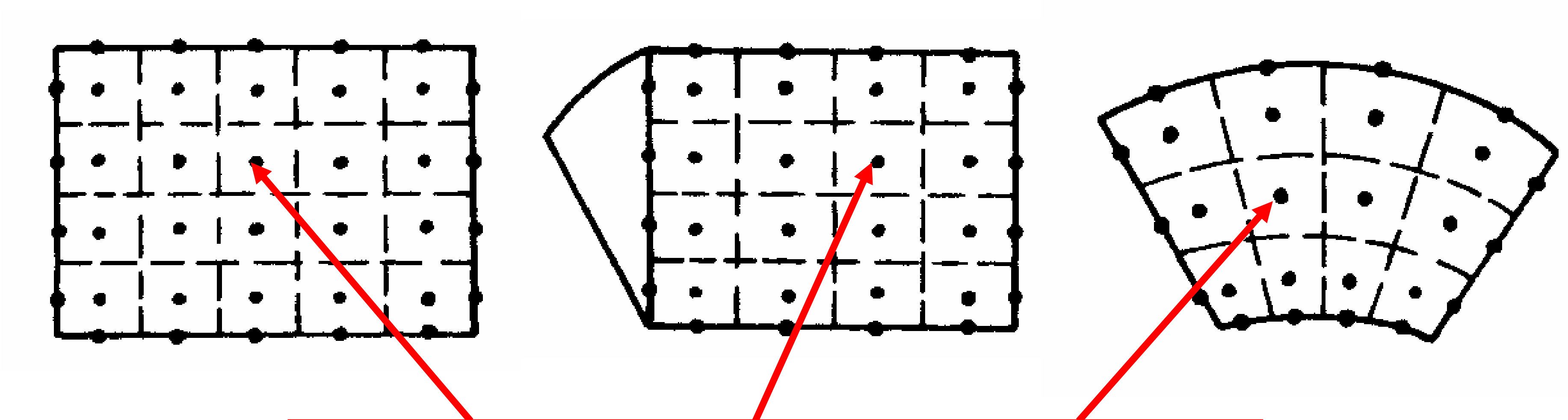
Both structured and unstructured grid layout (节点布置) have two practices.

（3）Outer node and inner node for structured grid

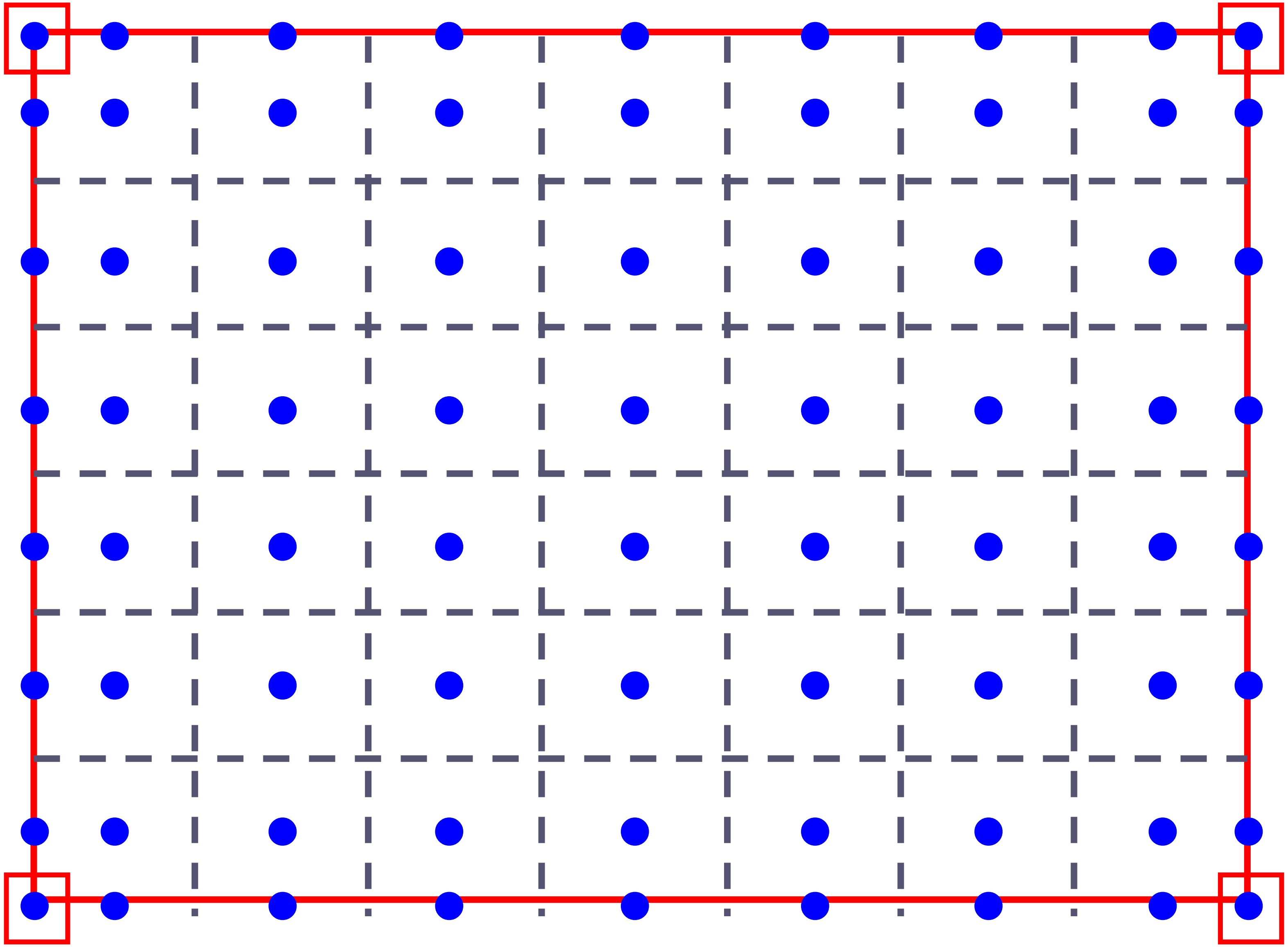
(a) Outer node method：Node is positioned at the vertex of a sub-domain(区域的角顶)；The interface is between two nodes；Generating procedure：Node first and interface second---called Practice A， or cell-vertex method (单元顶点法).



(b) Inner node method：Node is positioned at the center of sub-domain; Sub-domain is identical to control volume; Generating procedure: Interface first ad node second, called Practice B, or cell-centered （单元中心法）.



Sub-domain is the control volume

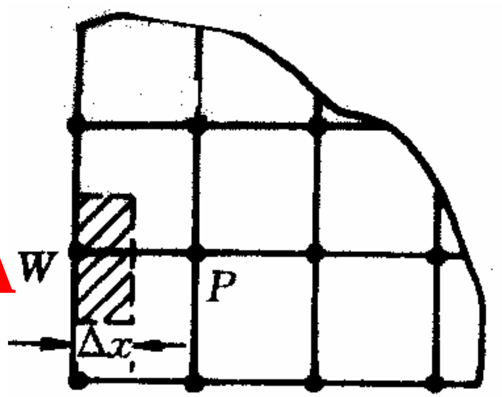


Generating procedure of Practice B

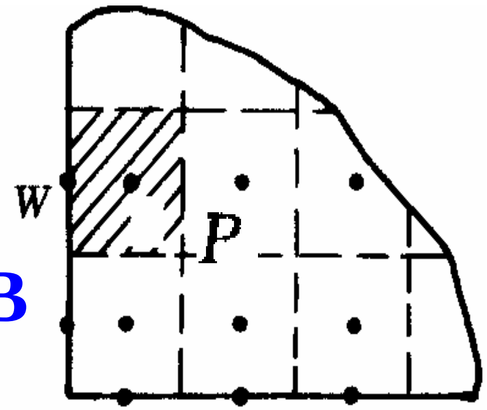
2.1.4 Comparison between Practices A and B

(a) Boundary nodes have different CV.

Practice A



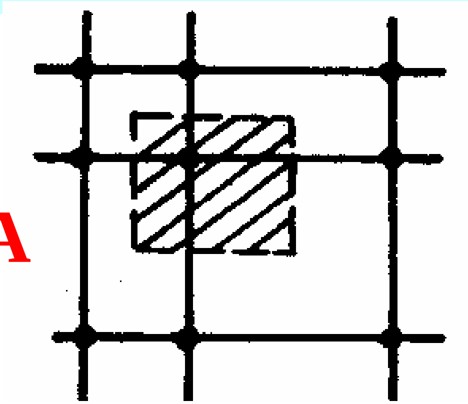
Practice B



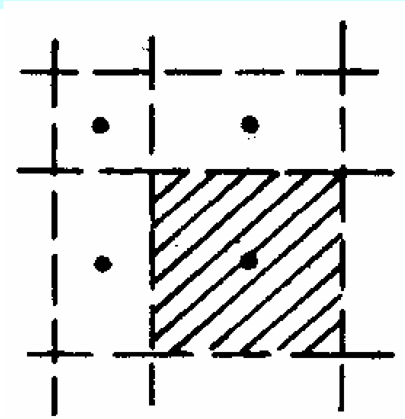
Boundary point has half CV. Boundary poinnt has zero CV

(b) Practice B is more feasible for non-uniform grid layout.

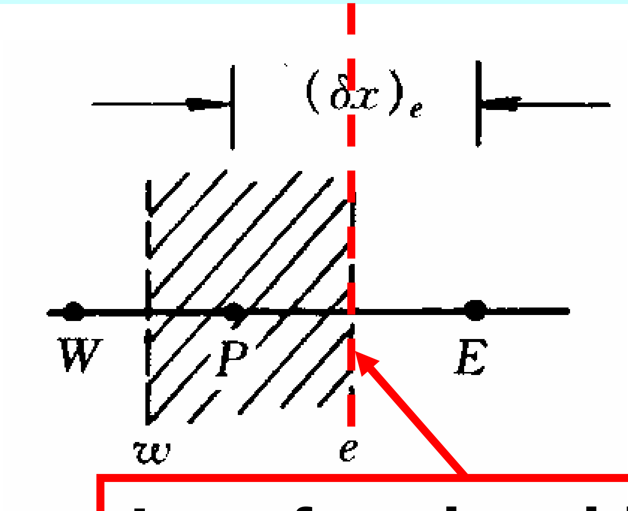
Practice A



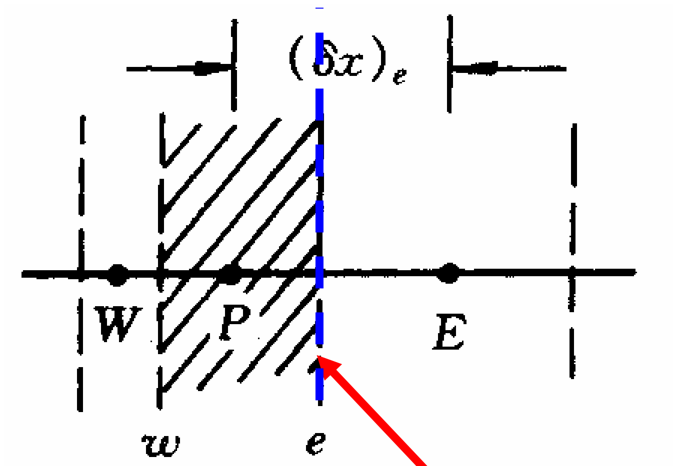
Practice B



(c) For non-uniform grid layout, Practice A can guarantee the discretization accuracy of interface derivatives.



Interface in middle



Interface is biased（偏置）

$$\left(\frac{\partial \phi}{\partial x}\right)_e \cong \frac{\phi_E - \phi_P}{(\delta x)_e}$$

2nd-order accuracy

$$\left(\frac{\partial \phi}{\partial x}\right)_e \cong \frac{\phi_E - \phi_P}{(\delta x)_e}$$

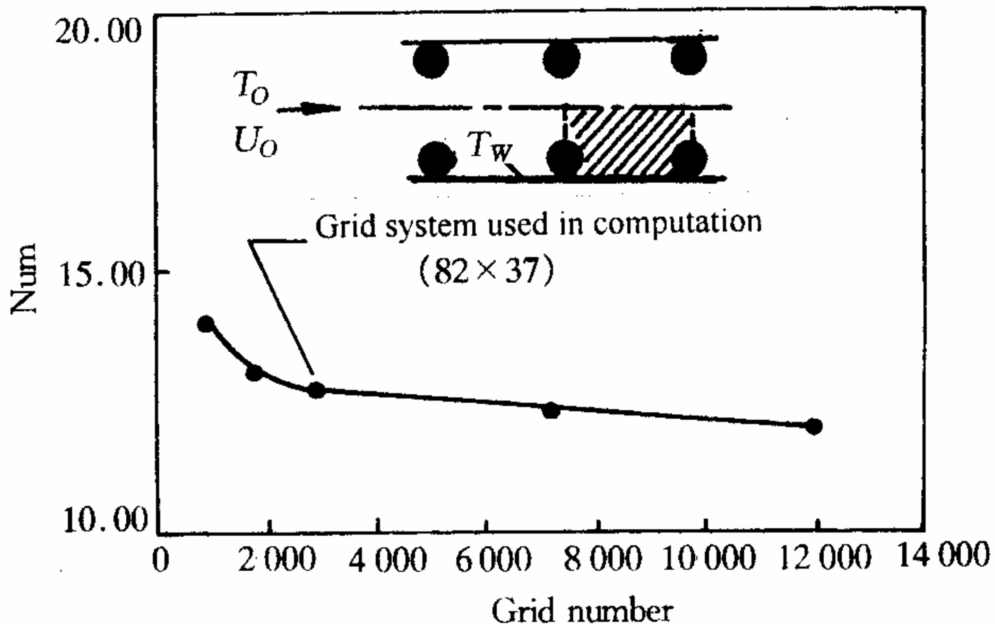
Lower than 2nd order accuracy

2.1.5 Grid-independent solutions

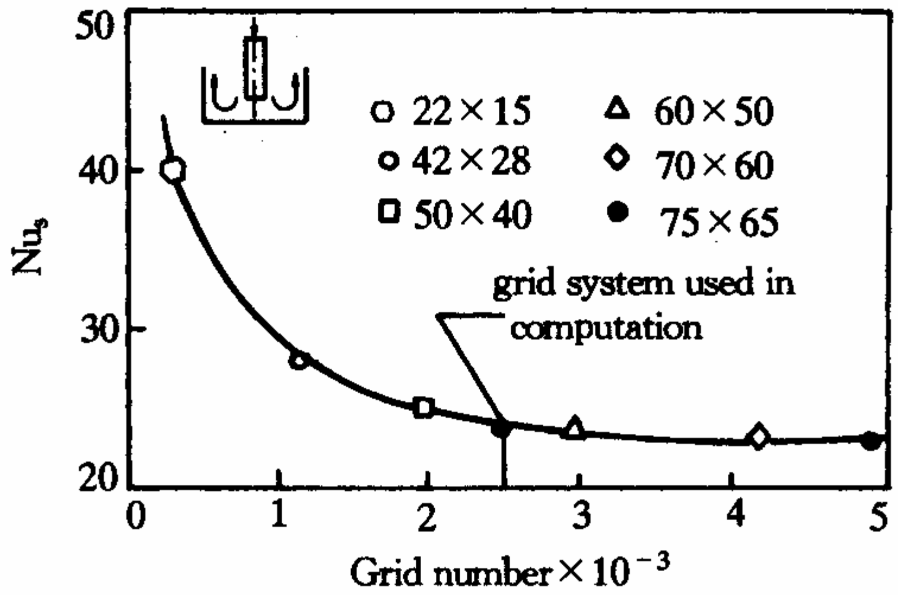
Grid generation is an iterative procedure; Debugging (调试) and comparison are often needed. For a complicated geometry grid generation may take a major part of total computational time.

Grid generation method has been developed as a sub-field of numerical solutions (Grid generation techniques).

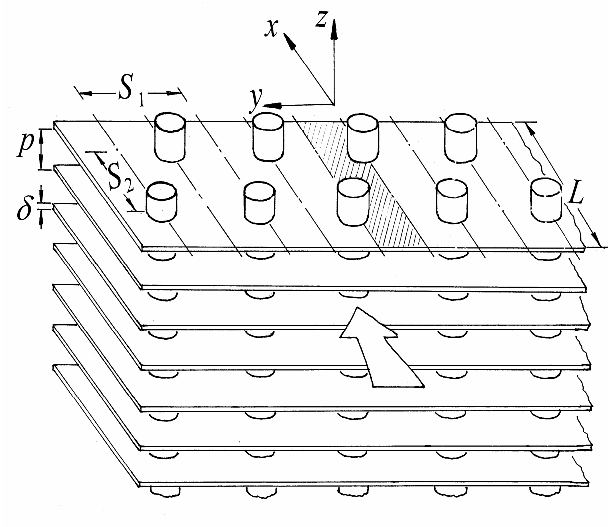
The appropriate grid fineness(细密程度) is such that the numerical solutions are nearly independent on the grid numbers. Such numerical solutions are called grid-independent solutions. This is required for publication.

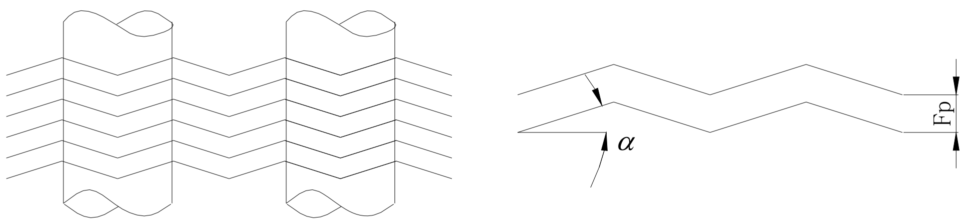


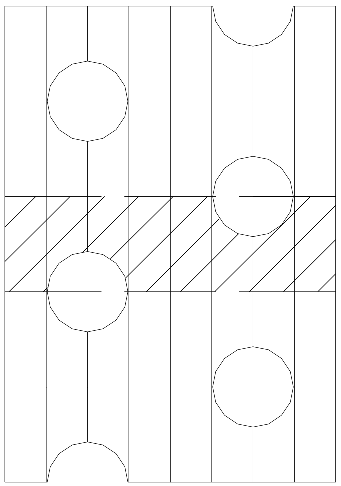
Int. Journal Heat & Fluid Flow, 1993, 14(3):246-253。

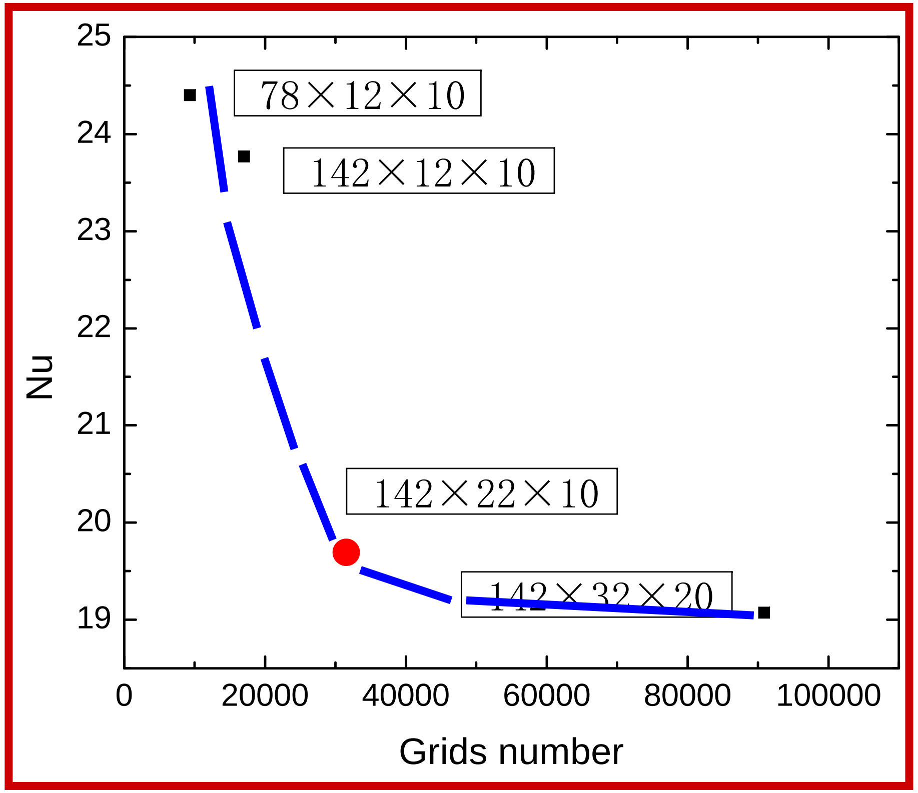


Int. Journal Numerical Methods in Fluids, 1998, 28: 1371-1387。









International Journal of Heat Mass Transfer, 2007, 50:1163-1175

2.2 Taylor Expansion and Polynomial Fitting for equation discretization

2.2.1 1-D model equation

2.2.2 Taylor expansion and polynomial fitting methods

2.2.3 FD form of 1-D model equation

2.2.4 FD form of polynomial fitting

2.2 Taylor Expansion and Polynomial Fitting for Equation discretization

2.2.1 1-D model equation（一维模型方程）

1-D model equation has four typical terms ： transient term, convection term, diffusion term and source term. It is specially designed for discussion of discretization methods.

Non-cons.

$$\frac{\partial(\rho\phi)}{\partial t}+\rho u\frac{\partial\phi}{\partial x}=\frac{\partial}{\partial x}(\Gamma\frac{\partial\phi}{\partial x})+S_\phi$$

For FDM

Conserva-tive

$$\frac{\partial(\rho\phi)}{\partial t}+\frac{\partial(\rho u\phi)}{\partial x}=\frac{\partial}{\partial x}(\Gamma\frac{\partial\phi}{\partial x})+S_\phi$$

For FVM

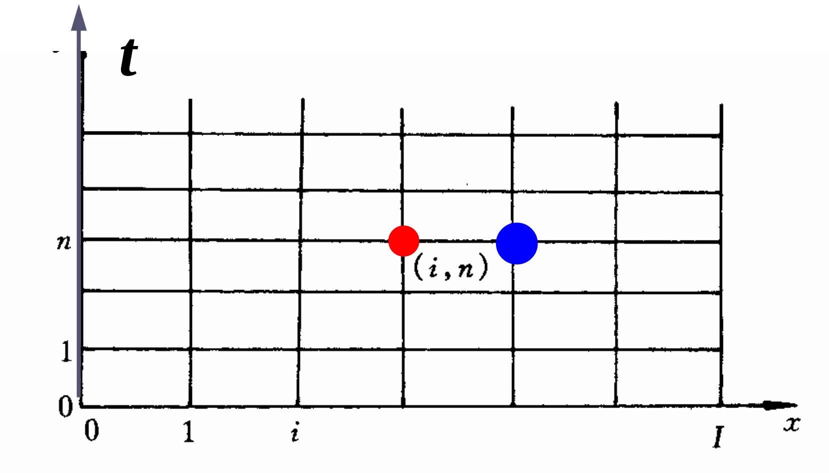
Trans **Conv.** **Diffus.** **Source**

Small but complete---"麻雀虽小，五脏俱全！"

2.2.2 Taylor expansion for FD of derivatives

1. FD form of 1st order derivative

Expanding $\phi(x,t)$ at (i+1,n) with respect to point (i,n):



$$\phi(i+1,n)=\phi(i,n)+\frac{\partial\phi}{\partial x})_{i,n}\Delta x+\frac{\partial^2\phi}{\partial x^2})_{i,n}\frac{\Delta x^2}{2!}+.....$$

$$\frac{\partial\phi}{\partial x})_{i,n}=\frac{\phi(i+1,n)-\phi(i,n)}{\Delta x}-\frac{\Delta x}{2}(\frac{\partial^2\phi}{\partial x^2})_{i,n}+...$$

$$\frac{\partial \phi}{\partial x})_{i,n} = \frac{\phi(i+1,n)-\phi(i,n)}{\Delta x} + O(\Delta x)$$

n

i

$O(\Delta x)$ Is called **truncation error(截断误差):**

With $\Delta x \to 0$ **replacing** $\frac{\partial \phi}{\partial x})_{i,n}$ **by** $\frac{\phi(i+1,n)-\phi(i,n)}{\Delta x}$

will lead to an error $\leq$ $K\Delta x$ where K is in dependent of Δx

The exponent of Δx **is called order of TE(截差的阶数).**

Replacing analytical solution $\phi(i,n)$ **by approximate value** ϕ **, yields:**

Forward difference:

(向前差分)

$$\frac{\partial \phi}{\partial x})_{i,n} \cong \frac{\phi_{i+1}^{n} - \phi_{i}^{n}}{\Delta x}, O(\Delta x)$$

Backward difference:
（向前差分）

$$\left.\frac{\partial \phi}{\partial x}\right)_{i,n} \cong \frac{\phi_i^n - \phi_{i-1}^n}{\Delta x}, O(\Delta x)$$

Central difference:

$$\left.\frac{\partial \phi}{\partial x}\right)_{i,n} \cong \frac{\phi_{i+1}^n - \phi_{i-1}^n}{2\Delta x}, O(\Delta x^2)$$

2. Different FD forms of 1st ad 2nd order derivatives

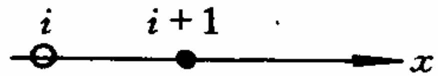
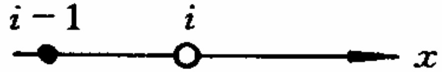
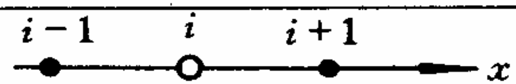
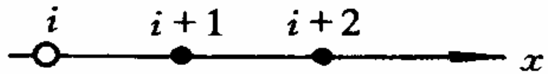
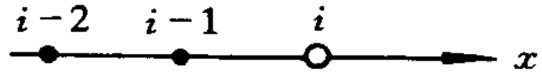
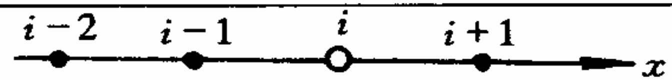
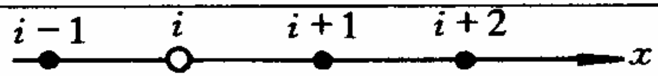
Stencil (格式图案) of FD expression

$$\frac{\phi_{i+1}^n - \phi_i^n}{\Delta x}$$

○ **For the node where FD form is constructed**

● **For nodes which are used in the construction**

Table 2 in the textbook

导数	差分表示式	格式图案	截差
$\left.\frac{\partial\phi}{\partial x}\right)_{i,n}$	$\frac{\phi_{i+1}^n-\phi_i^n}{\Delta x}$		$O(\Delta x)$
	$\frac{\phi_i^n-\phi_{i-1}^n}{\Delta x}$		$O(\Delta x)$
	$\frac{\phi_{i+1}^n-\phi_{i-1}^n}{2\Delta x}$		$O(\Delta x^2)$
	$\frac{-3\phi_i^n+4\phi_{i+1}^n-\phi_{i+2}^n}{2\Delta x}$		$O(\Delta x^2)$
	$\frac{3\phi_i^n-4\phi_{i-1}^n+\phi_{i-2}^n}{2\Delta x}$		$O(\Delta x^2)$
	$\frac{4\phi_{i+1}^n+6\phi_i^n-12\phi_{i-1}^n+2\phi_{i-2}^n}{12\Delta x}$		$O(\Delta x^3)$
	$\frac{-2\phi_{i+2}^n+12\phi_{i+1}^n-6\phi_i^n-4\phi_{i-1}^n}{12\Delta x}$		$O(\Delta x^3)$
	$\frac{\phi_{i-2}^n-8\phi_{i-1}^n+8\phi_{i+1}^n-\phi_{i+2}^n}{12\Delta x}$		$O(\Delta x^4)$

导数	差分表示式	格式图案	截差
$\left(\frac{\partial^2\phi}{\partial x^2}\right)_{i,n}$	$\frac{\phi_i^n - 2\phi_{i+1}^n + \phi_{i+2}^n}{\Delta x^2}$	i (○), $i+1$ (●), $i+2$ (●) → x	$O(\Delta x)$
	$\frac{\phi_i^n - 2\phi_{i-1}^n + \phi_{i-2}^n}{\Delta x^2}$	$i-2$ (●), $i-1$ (●), i (○) → x	$O(\Delta x)$
	$\frac{\phi_{i+1}^n - 2\phi_i^n + \phi_{i-1}^n}{\Delta x^2}$	$i-1$ (●), i (○), $i+1$ (●) → x	$O(\Delta x^2)$
	$(-\phi_{i-2}^n + 16\phi_{i-1}^n - 30\phi_i^n + 16\phi_{i+1}^n - \phi_{i+2}^n)/12\Delta x^2$	$i-2$ (●), $i-1$ (●), i (○), $i+1$ (●), $i+2$ (●) → x	$O(\Delta x^4)$

Rule of thumb（大拇指原则）for judging correction of a FD form

Rule of Thumb

(1) Dimension (量纲) should be consistent(和谐);

(2) Zero derivatives of any order for a uniform field.

Brief review of 2015-09-16 lecture key points

1. Classification of governing equations

1）From mathematical view point

(1) Elliptic, parabolic and hyperbolic

(2) Relationship to numerical solution method

2）From physical view point

(1) Conservative vs. non-conservative

(2) Key to conservative form： convective term is expressed by divergence.

Conservative form is suggested to derive the discretization equation.

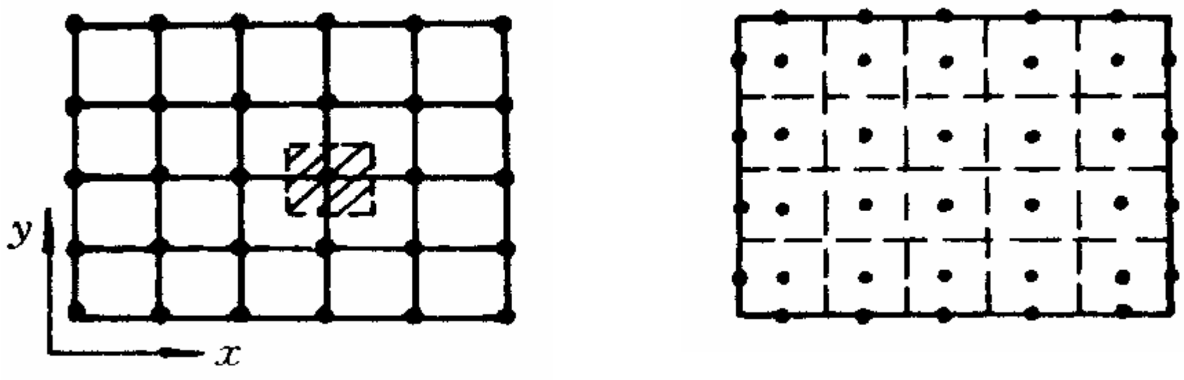
2. Domain discretization

Results： grid (node); interface; grid line; control volume

3. Cell-vertex or cell-center

（1） Cell-vertex method； A: node first, interface second

（2） Cell-center method； B: interface first, node second



4. One-D model equation

Non-conservative: $\frac{\partial(\rho\phi)}{\partial t}+\rho u\frac{\partial\phi}{\partial x}=\frac{\partial}{\partial x}(\Gamma\frac{\partial\phi}{\partial x})+S_\phi$

Conservative : $$\frac{\partial(\rho\phi)}{\partial t}+\frac{\partial(\rho u\phi)}{\partial x}=\frac{\partial}{\partial x}(\Gamma\frac{\partial\phi}{\partial x})+S_{\phi}$$

5. Equation discretization----Taylor series expansion

By Taylor series expansion :

$$\frac{\partial\phi}{\partial x})_{i,n}\cong\frac{\phi_{i+1}^{n}-\phi_{i}^{n}}{\Delta x},O(\Delta x)$$

Forward difference

$$\frac{\partial\phi}{\partial x})_{i,n}\cong\frac{\phi_{i}^{n}-\phi_{i-1}^{n}}{\Delta x},O(\Delta x)$$

Backward difference

$$\frac{\partial\phi}{\partial x})_{i,n}\cong\frac{\phi_{i+1}^{n}-\phi_{i-1}^{n}}{2\Delta x},O(\Delta x^{2})$$

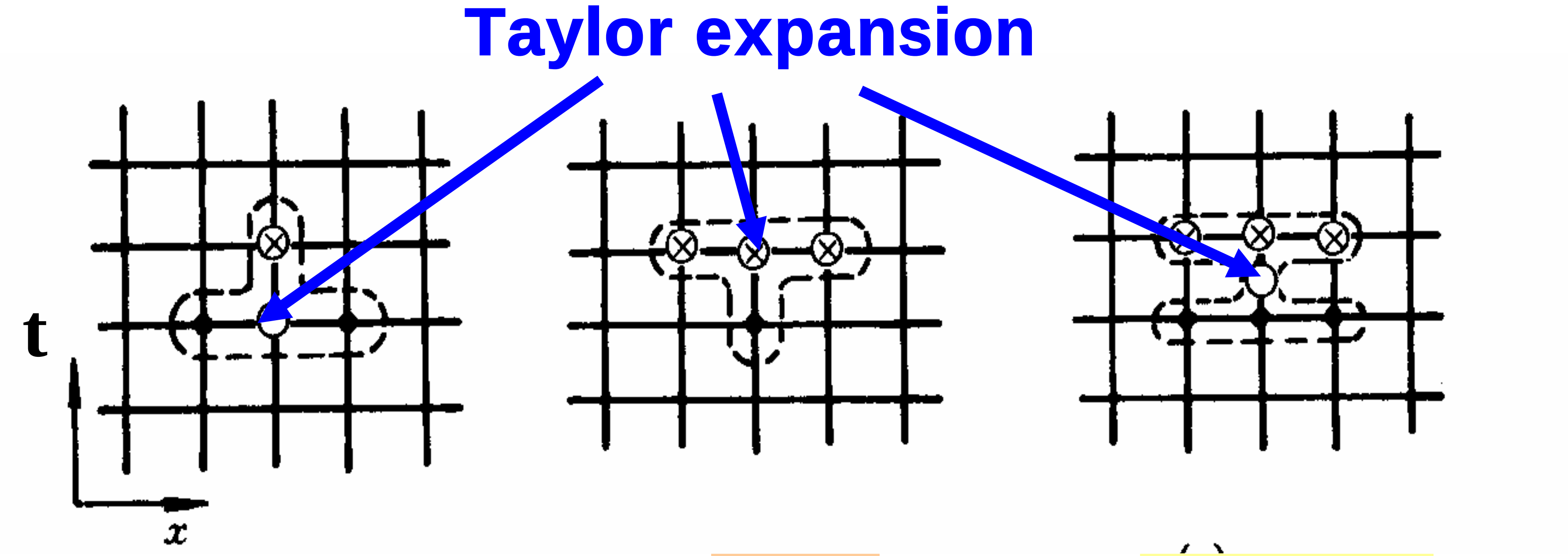
Central difference

$$\frac{\partial^{2}\phi}{\partial x^{2}})_{i,n}\cong\frac{\phi_{i+1}^{n}-2\phi_{i}+\phi_{i-1}^{n}}{\Delta x},O(\Delta x^{2})$$

Central difference

2.2.3 Discretized form of 1-D model equation

1. Time level at which spatial derivatives are determined



显式 explicit	隐式 implicit	C-N格式 Crank-Nicolson
$O(\Delta t)$	$O(\Delta t)$	$O(\Delta t^2)$

2. Explicit scheme of 1-D model equation

Analytical form

$$\rho\frac{\phi(i,n+1)-\phi(i,n)}{\Delta t}+\rho u\frac{\phi(i+1,n)-\phi(i-1,n)}{2\Delta x}=$$
$$\Gamma\frac{\phi(i+1,n)-2\phi(i,n)+\phi(i-1,n)}{\Delta x^2}+S(i,n)+HOT$$

Finite difference form

$$\rho\frac{\phi_i^{n+1}-\phi_i^n}{\Delta t}+\rho u\frac{\phi_{i+1}^n-\phi_{i-1}^n}{2\Delta x}=$$
$$\Gamma\frac{\phi_{i+1}^n-2\phi_i^n+\phi_{i-1}^n}{\Delta x^2}+S_i^n,O(\Delta t,\Delta x^2)$$

TE. of FD equation

2.2.4 Polynomial fitting for FD of derivatives

Assuming a local profile (型线)for the function studied:

- **Local linear function－leading to 1st-order FD expressions**

$$\phi(x_0 + \Delta x, t) \cong a + bx$$

Set the origin (原点) at x_0, yields:

$$\phi_i^n = a,\ \phi_{i+1}^n = a + b\Delta x,$$

$$\frac{\partial \phi}{\partial x} \cong b = \frac{\phi_{i+1}^n - a}{\Delta x} = \frac{\phi_{i+1}^n - \phi_i^n}{\Delta x}$$

2. Local quadratic function —leads to 2nd order FD expressions

$$\phi(x_0+\Delta x,t)\cong a+bx+cx^2$$

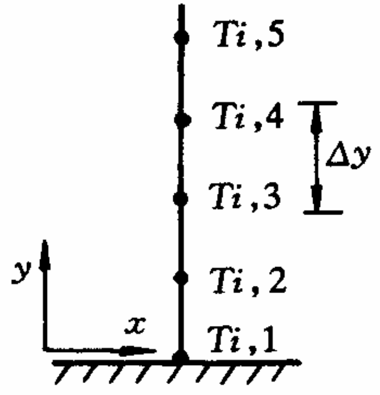
Set the origin (原点) at x_0, yields:

$$\phi_i^n=a,\quad \phi_{i+1}^n=a+b\Delta x+c\Delta x^2,\quad \phi_{i-1}^n=a-b\Delta x+c\Delta x^2$$

$$b=\frac{\phi_{i+1}^n-\phi_{i-1}^n}{2\Delta x},\qquad c=\frac{\phi_{i+1}^n-2\phi_i^n+\phi_{i-1}^n}{2\Delta x^2}$$

$$\frac{\partial\phi}{\partial x}\cong b=\frac{\phi_{i+1}^n-\phi_{i-1}^n}{2\Delta x},\qquad \frac{\partial^2\phi}{\partial x^2}\cong 2c=\frac{\phi_{i+1}^n-2\phi_i^n+\phi_{i-1}^n}{\Delta x^2},$$

3. Polynomial fitting used for treatment of B.C.



[Exam.2－1] **Known**: $T_{i,1}, T_{i,2}, T_{i,3}$

Find: wall heat flux in y-direction with 2nd-order accuracy.

Solution: Assuming a quadratic temp. function at y＝0

$$T(x,y) = a + by + cy^2, \quad O(\Delta y^3)$$

$$T_{i,1} = a,\ T_{i,2} = a + b\Delta y + c\Delta y^2, T_{i,3} = a + 2b\Delta y + 4c\Delta y^2$$

Yield:

$$b = \frac{-3T_{i,1} + 4T_{i,2} - T_{i,3}}{2\Delta y}$$

Then:

$$q_b = -\lambda \frac{\partial T}{\partial y})_{y=0} \cong \text{-}\lambda b = \frac{\lambda}{2\Delta y}(3T_{i,1} - 4T_{i,2} + T_{i,3}) \quad ,O(\Delta y^2)$$

2.3 Control Volume and Heat Balance Methods for Equation Discretization

2.3 Control Volume and Heat Balance Methods for Equation Discretization

2.3.1 Procedures for implementing CV method

1. Integrating conservative PDE over a CV

2. Selecting profiles for dependent variable and its 1st derivative

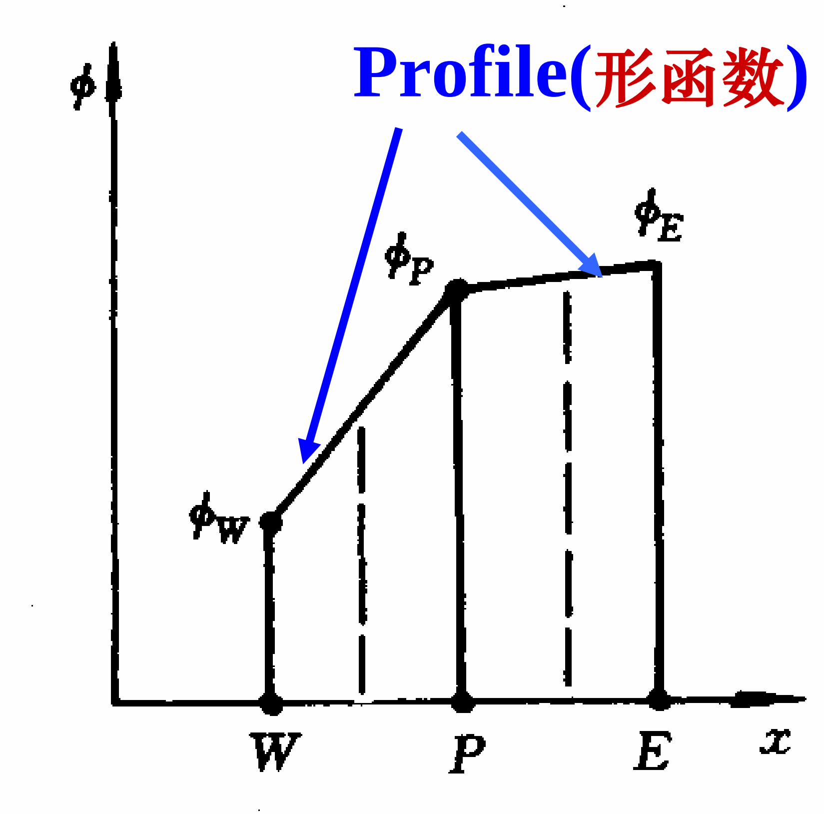
Profile－a local variation pattern of DV with space coordinate

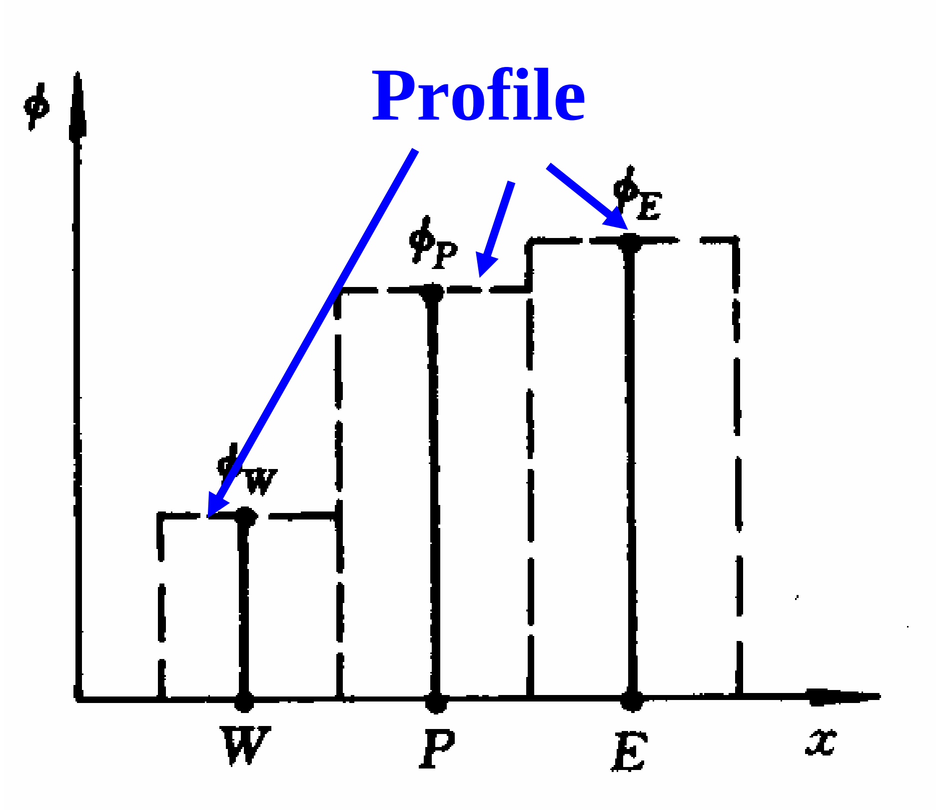
3. Completing integral and rearranging algebraic equations

2.3.2 Two conventional profiles (shape function)

Originally profile is to be solved; here it is to be assumed

Variation with spatial coordinate

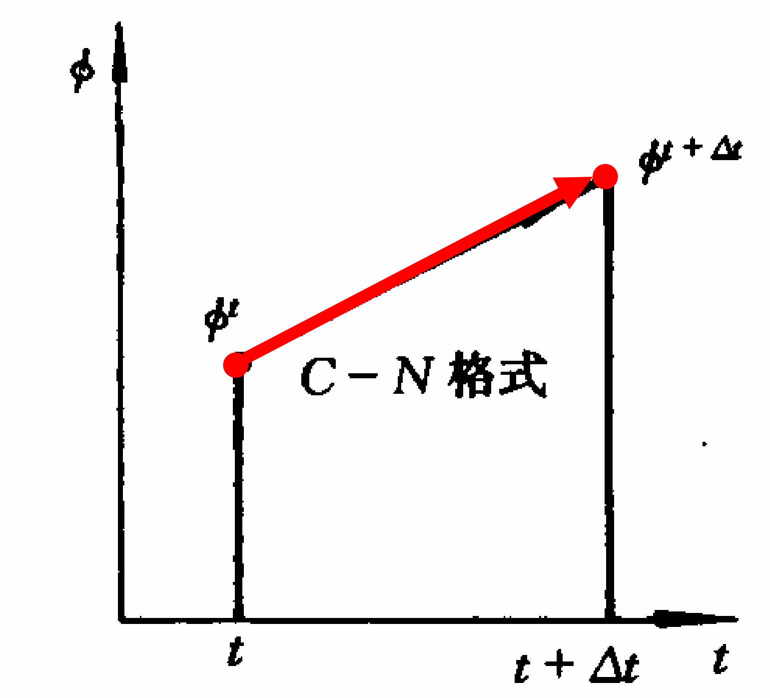


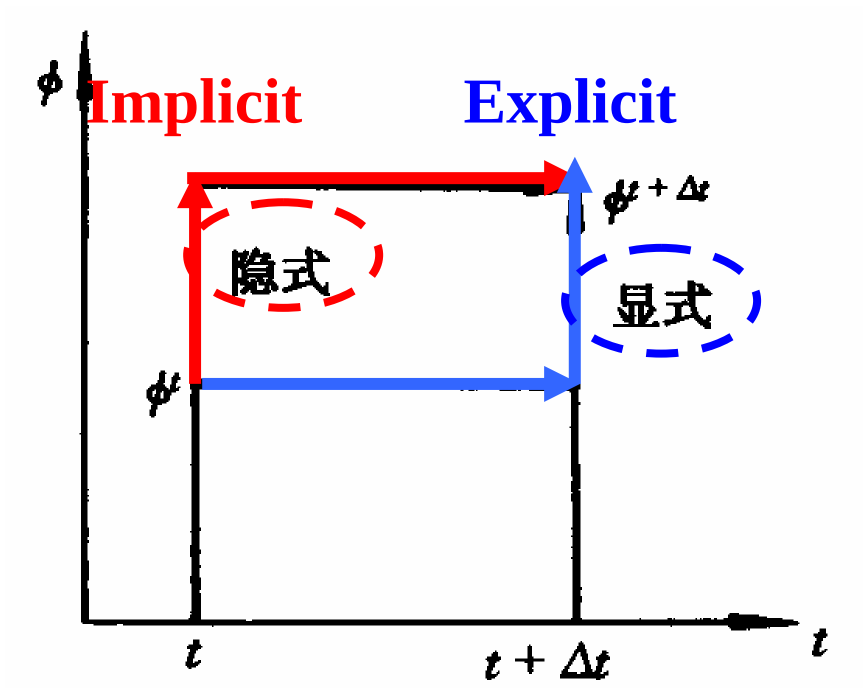


分段线性 阶梯逼近

piece-wise linear **step-wise approximation**

Variation with time





分段线性 阶梯逼近

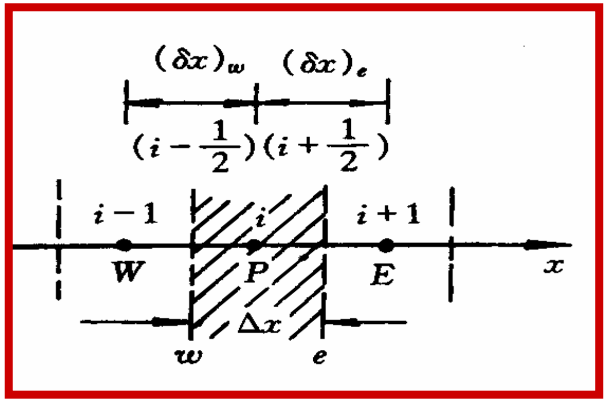
piece-wise linear step-wise approximation

2.3.3 Discretization of 1-D model eq. by CV method

Integrating conservative GE over a CV within [t, $t+\Delta t$],

$$\frac{\partial(\rho\phi)}{\partial t}+\frac{\partial(\rho u\phi)}{\partial x}=\frac{\partial}{\partial x}(\Gamma\frac{\partial\phi}{\partial x})+S_\phi$$

yields:



$$\rho\int_w^e(\phi^{t+\Delta t}-\phi^t)dx+\rho\int_t^{t+\Delta t}[(u\phi)_e-(u\phi)_w]dt=$$

$$\Gamma\int_t^{t+\Delta t}[(\frac{\partial\phi}{\partial x})_e-(\frac{\partial\phi}{\partial x})_w]dt+\int_t^{t+\Delta t}\int_w^e S_\phi dxdt$$

To complete the integral we need the profiles of the dependent variable and its 1st derivative.

1. Transient term

Assuming the step-wise approximation for ϕ with space:

$$\rho\int_{w}^{e}(\phi^{t+\Delta t}-\phi^{t})dx=\rho(\phi_P^{t+\Delta t}-\phi_P^{t})\Delta x$$

2. Convective term

Assuming the explicit step-wise approximation for ϕ **with time:**

$$\rho\int_{t}^{t+\Delta t}[(u\phi)_e-(u\phi)_w]dt=\rho[(u\phi)_e^t-(u\phi)_w^t]\Delta t$$

Further, assuming linear-wise variation of ϕ with space

$$\rho[(u\phi)_e^t-(u\phi)_w^t]\Delta t=\rho u\Delta t(\frac{\phi_E+\phi_P}{2}-\frac{\phi_P+\phi_W}{2})=\rho u\Delta t\frac{\phi_E-\phi_W}{2}$$

Uniform grid

3. Diffusion term

Assuming $\frac{\partial\phi}{\partial x}$ exhibits explicit step-wise variation with space:

$$\Gamma\int_t^{t+\Delta t}[(\frac{\partial\phi}{\partial x})_e-(\frac{\partial\phi}{\partial x})_w]dt=\Gamma[(\frac{\partial\phi}{\partial x})_e^t-(\frac{\partial\phi}{\partial x})_w^t]\Delta t$$

Further, assuming linear-wise variation of ϕ with space

$$\Gamma[(\frac{\partial\phi}{\partial x})_e^t-(\frac{\partial\phi}{\partial x})_w^t]\Delta t=\Gamma\Delta t[\frac{\phi_E-\phi_P}{(\Delta x)_e}-\frac{\phi_P-\phi_W}{(\Delta x)_w}]$$

uniform →

$$=\Gamma\Delta t\frac{\phi_E-2\phi_P+\phi_W}{\Delta x}$$

4. Source term

Assuming explicit step wise with time and step-wise variation with space:

$$\int_t^{t+\Delta t}\int_w^e S dx dt=\overline{S}^t(\Delta x)_P\Delta t$$

$\overline{S}$ **---averaged one over space.**

Dividing both sides by $\Delta t \Delta x$

$$\rho \frac{\phi_P^{t+\Delta t} - \phi_P^t}{\Delta t} + \rho u \frac{\phi_E^t - \phi_W^t}{2\Delta x} =$$

$$\Gamma \frac{\phi_E^t - 2\phi_P^t + \phi_W^t}{\Delta x^2} + \overline{S}^t, O(\Delta t, \Delta x^2)$$

For the uniform grid system, the results are the same as that from Taylor expansion, which reads:

$$\rho \frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} + \rho u \frac{\phi_{i+1}^n - \phi_{i-1}^n}{2\Delta x} =$$

$$\Gamma \frac{\phi_{i+1}^n - 2\phi_i^n + \phi_{i-1}^n}{\Delta x^2} + S_i^n, O(\Delta t, \Delta x^2)$$

2.3.4 Discussion on profile assumptions in FVM

1. In FVM the only purpose of profile is to derive the discretization equations; Once they are established, the function of profile is fulfilled.

2. The selection criterion of profile is easy to be implemented and good numerical characteristics; Consistency (协调)is not required.

3. In FVM profile is indeed the scheme（差分格式）.

2.3.5 Equation discretization by balance method

1. Major concept：Applying the conservation law directly to a CV, and viewing its node as the representative of the CV.

2. 1-Ddiffusion-convection problem with ST

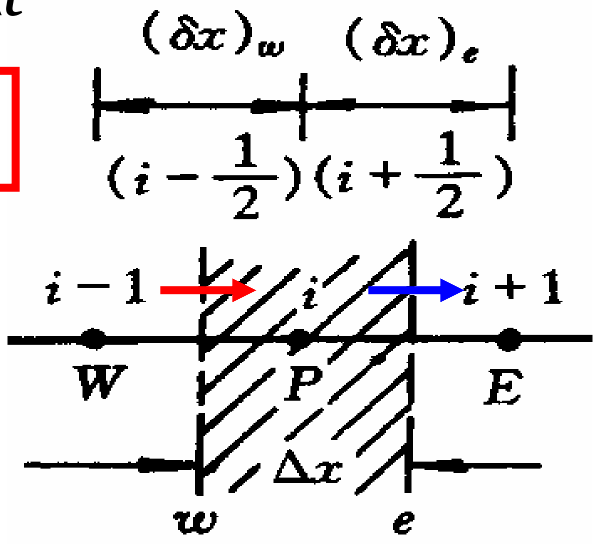
Writing down balance equation for Δx at initial instant

$$\rho(\phi_P^{t+\Delta t}-\phi_P^t)\Delta x=\rho[(u\phi)_w^t-(u\phi)_e^t]\Delta t$$

Transient Convection

$$+[(\frac{\partial\phi}{\partial x})_e^t-(\frac{\partial\phi}{\partial x})_w^t]\Delta t+\overline{S}^t\Delta x\Delta t$$

Diffusion Source



By selecting the profile of dependent variable ϕ with space, the discretization equation can be obtained.

2.3.6 Comparisons of two ways

Content	FDM	FVM
1. Error analysis	Easy	Not easy，via FDM
2. Physical concept	Not clear	Clear
3. Variable length step(变步长)	Not easy	Easy
4. Conservation feature of ABEs	Not guaranteed	May be guaranteed

FVM has been the 1st choice of most CSW.

First Home Work

1－7（补充不可压，常物性的条件）

2－4， 2－7， 2－10， 2－11

Please hand in on Sept.28,2015
Full score of each home work is 10 credits.
Totally 8 home works

Please complete your home work independently!!!

Problem 2-4

Using the control volume integration method discretize the 1-D heat conduction equation given below.

$$\frac{1}{r}\frac{1}{dr}\left(rk\frac{dT}{dr}\right)+S=0$$, where S is constant.

Also discretize the non-conservative form, as given below, of 1-D equation by using Taylor series expansion method.

$$k\frac{d^2T}{dr^2}+\frac{k}{r}\left(\frac{dT}{dr}\right)+S=0$$

Express the both results as: $a_P T_P = a_E T_E + a_W T_W + b$

where 'b' is known but not contains T_P, T_E and T_W. Moreover, check for the case of constant properties and uniform grids that these two results are the same or not?

Problem 2-7

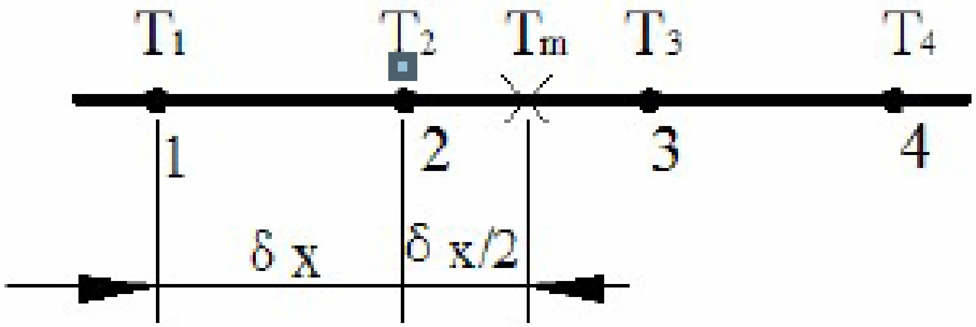
Derive the following equation and analyze the order of truncation error (Ref. Equation 2-7 in the text book).

$$T_{i,1}=\frac{1}{11}(18T_{i,2}-9T_{i,3}+2\,\mathrm{T}_{i,4}+\frac{6\Delta y q_B}{\lambda})$$

Problem 2-10

Figure given below shows four equally spaced nodes 1, 2, 3 and 4, and each node has a constant value of temperature. Use the so-called double parabolic interpolation to get the value T_m for the node lying at the middle between two nodes 2, 3. In first step get the value of T_{m1} by using parabolic (binomial) interpolation from the values of node 1, 2 and 3. In the 2nd step get the value of T_{m2} by using same method from the values of node 2, 3 and 4. Then take $T_m = (T_{m1} + T_{m2})/2$

Derive the expression for T_m and analyze the order of truncation error



Problem 2-11

Derive the expression for 1st order derivative

$\frac{\partial \phi}{\partial x}$ with 3rd order truncation error.

(Ref. Table 2-1, taking two nodes in upstream direction

and one node in downstream direction)

The homework should be hand in on next Monday, September 28 !

同舟共济
渡彼岸！

People in the same boat help each other to cross to the other bank, where....

Remarks on how to learn Numerical Heat Transfer

1. **Skimming through the Chinese textbook before class (课前浏览预习课文);**
2. **Presenting contents by simple and clear English sentences;**
   Adding more Chinese explanatory notes in PPT (增加中文注释);
3. **Contacting the teaching assistant team whenever you have questions and comments(与助教队伍加强交流);**
   They are very helpful and friendly;
   They have their own experiences on how to learn the course.

Announcement

Professor S.P. Vanka of UIUC in USA is invited to present a series lectures on using GPU to accelerate computing speed of heat transfer and fluid flow problems . Interesting students are welcome to attend the lecture:

Place: 2nd flow of Eastern 3rd building
(东三楼二楼东汽报告厅）

Times: Tuesday morning (10:00AM) and afternoon (3:00 PM)
Wednesday afternoon (3:00PM)
Thursday morning (10:00 AM) and afternoon (3:00 PM)

精确形式

$$\rho\frac{\phi(i,n+1)-\phi(i,n)}{\Delta t}+\rho u\frac{\phi(i+1,n)-\phi(i-1,n)}{2\Delta x}=$$
$$\Gamma\frac{\phi(i+1,n)-2\phi(i,n)+\phi(i-1,n)}{\Delta x^2}+S(i,n)+HOT$$

差分表达式

$$\rho\frac{\phi_i^{n+1}-\phi_i^n}{\Delta t}+\rho u\frac{\phi_{i+1}^n-\phi_{i-1}^n}{2\Delta x}=$$
$$\Gamma\frac{\phi_{i+1}^n-2\phi_i^n+\phi_{i-1}^n}{\Delta x^2}+S_i^n,O(\Delta t,\Delta x^2)$$

(3) 多项式拟合法：对局部型线做出假定，用临近节点之值表示型线中的系数： $T(x,y)=a+by+cy^2$

9月10号课程内容概要

1. 区域离散化：节点，控制容积，界面，网格线，内接点法与外节点法, 网格独立解

2. 建立离散方程基本方法：

(1) 一维模型方程（ 1-D model equation ）

非守恒型
$$\frac{\partial(\rho\phi)}{\partial t}+\rho u\frac{\partial\phi}{\partial t}=\frac{\partial}{\partial x}(\Gamma\frac{\partial\phi}{\partial x})+S_\phi$$

守恒型
$$\frac{\partial(\rho\phi)}{\partial t}+\frac{\partial(\rho u\phi)}{\partial t}=\frac{\partial}{\partial x}(\Gamma\frac{\partial\phi}{\partial x})+S_\phi$$

(2) Taylor展开法：将导数用其差分表示式代替；

精确形式

$$\rho\frac{\phi(i,n+1)-\phi(i,n)}{\Delta t}+\rho u\frac{\phi(i+1,n)-\phi(i-1,n)}{2\Delta x}=$$
$$\Gamma\frac{\phi(i+1,n)-2\phi(i,n)+\phi(i-1,n)}{\Delta x^2}+S(i,n)+HOT$$

差分表达式

$$\rho\frac{\phi_i^{n+1}-\phi_i^n}{\Delta t}+\rho u\frac{\phi_{i+1}^n-\phi_{i-1}^n}{2\Delta x}=$$
$$\Gamma\frac{\phi_{i+1}^n-2\phi_i^n+\phi_{i-1}^n}{\Delta x^2}+S_i^n,O(\Delta t,\Delta x^2)$$

8. 用Taylor展开法离散1-D模型方程

微分形式

$$\frac{\partial(\rho\phi)}{\partial t}+\rho u\frac{\partial\phi}{\partial x}=\frac{\partial}{\partial x}(\Gamma\frac{\partial\phi}{\partial x})+S_\phi$$

(i,n)点的精确形式

$$\rho\frac{\phi(i,n+1)-\phi(i,n)}{\Delta t}+\rho u\frac{\phi(i+1,n)-\phi(i-1,n)}{2\Delta x}=$$

$$\Gamma\frac{\phi(i+1,n)-2\phi(i,n)+\phi(i-1,n)}{\Delta x^2}+S(i,n)+HOT$$

(i,n)点的离散形式

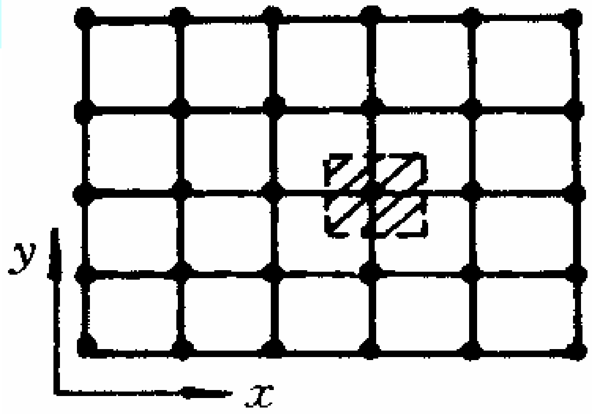
$$\rho\frac{\phi_i^{n+1}-\phi_i^n}{\Delta t}+\rho u\frac{\phi_{i+1}^n-\phi_{i-1}^n}{2\Delta x}=$$

$$\Gamma\frac{\phi_{i+1}^n-2\phi_i^n+\phi_{i-1}^n}{\Delta x^2}+S_i^n,O(\Delta t,\Delta x^2)$$

(2) **Relationship to numerical solution method**

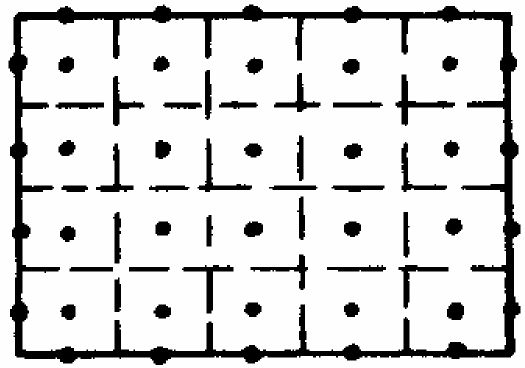
Conservative GE can guarantee the applicability of conservation law for numerical solution.

2. Domain discretization



(1) Node (grid), interface, control volume;

(2) Structure grid vs. unstructured grid.



(3) Practice A:

Node first and interface second;

Practice B:

Interface first, node second.

3. 1-D model equation

Non-conservative $$\frac{\partial(\rho\phi)}{\partial t}+\rho u\frac{\partial\phi}{\partial t}=\frac{\partial}{\partial x}(\Gamma\frac{\partial\phi}{\partial x})+S_\phi$$

Conservative $$\frac{\partial(\rho\phi)}{\partial t}+\frac{\partial(\rho u\phi)}{\partial t}=\frac{\partial}{\partial x}(\Gamma\frac{\partial\phi}{\partial x})+S_\phi$$

4. Taylor series expansion for discretizing 1D ME.

$$\rho\frac{\phi_i^{n+1}-\phi_i^n}{\Delta t}+\rho u\frac{\phi_{i+1}^n-\phi_{i-1}^n}{2\Delta x}=\Gamma\frac{\phi_{i+1}^n-2\phi_i^n+\phi_{i-1}^n}{\Delta x^2}+S_i^n,O(\Delta t,\Delta x^2)$$

5. Polynomial fitting for obtaining of finite difference of derivatives

Local linear function-----1st order discretization

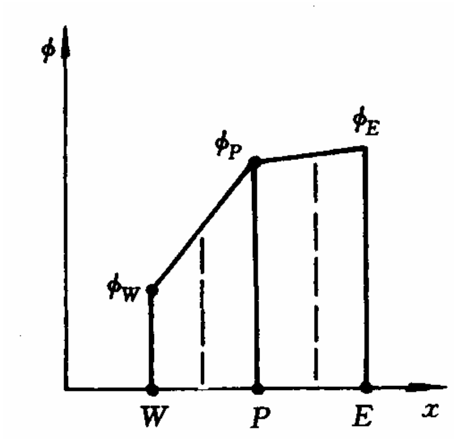
Local quadratic function-----2nd order discretization.

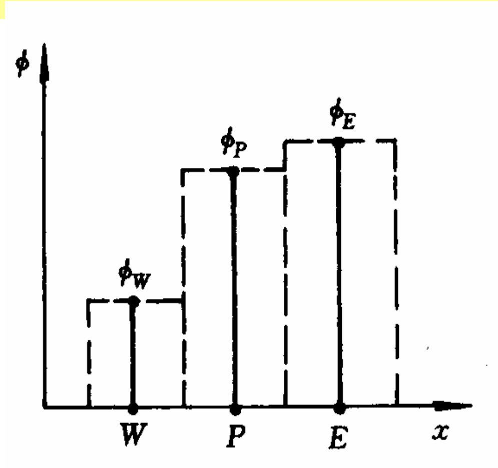
9. 控制容积积分法离散实施步骤

做积分 → 变化型线假设 → 积分进行到底

10. 两种常用型线

变量的随空间变化的型线





变量的随空间变化的型线