# Chapter # 6

**Problem # 6-1**

As mentioned in section 6.1, the problem of segregated algorithm for fluid flow, there is no independent governing equation for pressure. In order to deal with the problem of the coupling between the pressure and velocity, SIMPLE and a series of algorithms are introduced. But, on the other hand, the pressure Poisson equation can be derived from the momentum equation and continuity equation, for example, as shown below is the equation of two-dimensional rectangular coordinates for incompressible fluid:

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=2\left[\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial y}\right)-\left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right)\right]$$

Somebody thinks that we can get the pressure equation and momentum equation simultaneous to solve the flow, namely, solving the $u$ equation, the $v$ equation and pressure equation in turn ($u$ , $v$ are known, and can be the source term of the pressure equation). This is the one iteration of the separation method, thus, without using SIMPLE algorithm. Try to derive the pressure Poisson equation and comment on the view.

**Problem # 6-4**

Considering the two-dimensional flow in figure 6-11, whereas $u_w = 50$ , $v_s = 20$ , $p_N = 0$ $p_E = 10$ are known , assuming the flow is in steady state and the density is constant. The discrete equation for $u_e$ , $v_n$ is:

$u_e = p_P - p_E$ ; $v_n = 0.7(p_P - p_N)$

Using the SIMPLE algorithm to get the value of $u_e$ , $v_n$ and $p_P$ .

**Problem # 6-5**

A piping system as shown in figure 6-33 is applied to pump fluid from node 1 to node 2, 3, 4, 5, 6, 7. The pressure of node 1,2,4,5 is given in the parentheses. The flow rate between two nodes can be calculated by using formula $Q = C(\Delta p)$ ,where $\Delta p$ is the pressure difference between the two nodes and $C$ is the hydraulic conductivity. For simplicity, the conductivity of two adjacent nodes is shown by using the letter on the midpoint of the two nodes as a subscript. Such as $C_D$ could be expressed as the hydraulic conductivity between the nodes 3, 6. Where $C_A = 0.4$ , $C_B = 0.2$ , $C_C = 0.1$ , $C_D = 0.2$ , $C_E = 0.1$, $C_F = 0.2$ are known. The flow rate between the 6, 7 is $Q_F = 20$. All quantities are in same unit system. Try to use similar SIMPLE algorithm to determine the value of $p_3$, $p_6$, $Q_A$, $Q_B$, $Q_C$, $Q_D$, $Q_D$ and $p_7$ ( Hint : firstly assume $p_3^*$ , $p_6^*$ to get the flow rate and then calculate the pressure correction value using the mass conservation between node 3, 6)

**Problem # 6-6**

Consider one-dimensional flow within the porous medium. Governing equations are $c|u|u + \frac{dp}{dx} = 0$ and $\frac{d(uA)}{dx} = 0$ , where $c$ is constant, $A$ is the effective area. For a discrete system shown in figure 6- 12, $\Delta x = 1$ ( uniform grid ) , $C_B = 0.4$ , $C_C = 0.2$ $A_B = 2$ , $A_C = 3$ , $p_1 = 140$ , $p_3 = 30$ are known. All units are coordinated. Use SIMPLE algorithm to get the value of $p_2$ , $u_B$ , $u_C$

**Problem # 6-7**

Drive the expression for velocity source term in cylindrical axisymmetric coordinates and Polar coordinates as shown in table 6-1.