Chapter # 3

Problem # 3-1

One-dimensional unsteady heat conduction in Dufort-Frankel format as given below:

$$\frac{T_i^{n+1}-T_i^{n-1}}{2\Delta t}=\frac{a}{\Delta x^2}\left(T_{i+1}^n-T_i^{n+1}-T_i^{n-1}+T_{i-1}^n\right)$$

This equation contains three time levels i.e. $(\mathrm{n}-1,\mathrm{n},\mathrm{n}+1)$, so it is called three-level scheme. Write down the expression for truncation error and get the consistency condition.

Problem # 3-3

In the following two-dimensional convection - diffusion equation:

$$\rho\frac{\partial\phi}{\partial t}+\rho(\mathrm{u}\frac{\partial\phi}{\partial x}+\mathrm{v}\frac{\partial\phi}{\partial y})=\Gamma(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2})$$

$\mathrm{u},\mathrm{v},\rho$ and Γ are constants and greater than zero. One of the differential forms of above equation is:

$$\frac{\phi_{i,j}^{n+1}-\phi_{i,j}^n}{\Delta t}+u\frac{\phi_{i,j}^n-\phi_{i-1,j}^n}{\Delta x}+v\frac{\phi_{i,j}^n-\phi_{i,j-1}^n}{\Delta y}=(\frac{\Gamma}{\rho})(\frac{\phi_{i+1,j}^n-2\phi_{i,j}^n+\phi_{i-1,j}^n}{\Delta x^2})+(\frac{\Gamma}{\rho})(\frac{\phi_{i,j+1}^n-2\phi_{i,j}^n+\phi_{i,j-1}^n}{\Delta y^2})$$

Using von Neumann method to verify the stability condition as given below:

$$\Delta\,\mathrm{t}\le\frac{1}{\frac{2a}{\Delta x^2}+\frac{2a}{\Delta y^2}+\frac{u}{\Delta x}+\frac{v}{\Delta x}},\quad a=\Gamma/\rho$$

Problem # 3-4

There is a heat exchanger pipe with fully developed velocity field, where temperature field is described by the following equation

$$u\frac{\partial T}{\partial x}=\frac{\nu}{P_r}\frac{\partial^2 T}{\partial y^2}$$

Using explicit format for discretization of the given equation and find out the stability condition.

Problem # 3-9

Using central difference scheme prove the conservation of the diffusion term.

Problem # 3-10

Second order partial differential scheme for first order derivative (see Table 2-1) is called second-order upwind scheme (taking nodes in coming flow direction). Analyze its transportive property.