

## Homework 2 — 《Numerical Heat Transfer》

### Problem 3-1

One-dimensional unsteady heat conduction in Dufort-Frankel format as given below:

$$\frac{T_i^{n+1} - T_i^{n-1}}{2\Delta t} = \frac{a}{\Delta x^2} (T_{i+1}^n - T_i^{n+1} - T_i^{n-1} + T_{i-1}^n)$$

This equation contains three time levels i.e. (n-1,n,n+1). so it is called three-level scheme. Write down the expression for truncation error and get the consistency condition.

### Problem 3-2

An implicit central difference scheme for 1-D unsteady convection –diffusion equation is as follows:

$$\rho \frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} + \rho u \frac{\phi_{i+1}^{n+1} - \phi_{i-1}^{n+1}}{2\Delta x} = \Gamma \frac{\phi_{i+1}^{n+1} - 2\phi_i^{n+1} + \phi_{i-1}^{n+1}}{\Delta x^2}$$

$u, \rho, \Gamma$  are constants and greater than zero.

Verify that:

- (1) The scheme possess consistence;                      (2) It is unconditionally stable.

### Problem 3-4

There is a heat exchanger pipe with fully developed velocity field, where temperature field is described by the following equation

$$u \frac{\partial T}{\partial x} = \frac{v}{P_r} \frac{\partial^2 T}{\partial y^2}$$

Using explicit format for discretization of the given equation and find out the stability condition.

### Problem 3-7

Verify that the downwind difference scheme of the convective term always propagates the disturbance to the upwind direction.

### Problem 3-10

Second order partial differential scheme for first order derivative (see Table 2-1) is called second-order upwind scheme (taking nodes in coming flow direction). Analyze its transportive property.