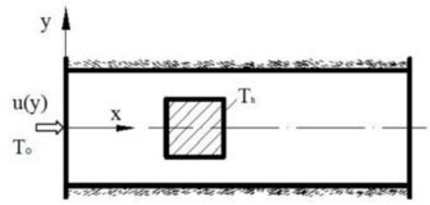
Homework 1 ——《Numerical Heat Transfer》

Problem 1-7

A solid square having uniform temperature T_h is placed at center line of a two-dimensional parallel plate channel as shown in figure below. Flow is fully developed. The upper and lower side of the channel is insulated and therefore you may assume that these sides are adiabatic whereas outlet boundary is far from the solid square. Fluid enters the channel with a uniform temperature, T_{in}= C. Write down the governing equations for steady state, incompressible laminar flow with constant properties.
Also write down boundary conditions for the velocity and temperature for the given domain (It is preferable, at exit boundary, to take first derivative zero). Flow is incompressible and material properties are constants.



Problem 2-4

Using the control volume integration method discretize the 1-D heat conduction equation given below.

$$\frac{1}{r}\frac{1}{dr}(rk\frac{dT}{dr})+S=0\text{, Where S is constant.}$$

Also discretize the non–conservative form, as given below, of 1-D equation by using Taylor series expansion method.

$$k\frac{dT^2}{dr^2}+\frac{k}{r}(\frac{dT}{dr})+S=0$$

Express the both results as : $a_P T_P = a_E T_E + a_W T_W + b$. Where b is known hut contains

T_P , T_E and T_W . Moreover, check for the case of constant properties and uniform grids

that these two results are the same or not?

Problem 2-7

Derive the following equation and analyze the order of truncation error
(Ref.Equation2-7 in the text book)

$$T_{i,1}=\frac{1}{11}(18T_{i,2}-9T_{i,3}+2T_{i,4}\frac{6\Delta y q_B}{\lambda})$$

Problem 2-9

As shown in Fig. 2-11 (see page 45 of the textbook), grids 1,2,3,4 are uniformly distributed. The pressures at grids 1,2 and 3 are known. Derive the finite difference equation for the pressure gradient at the wall with 2nd order accuracy.

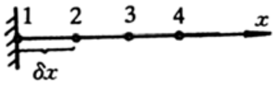


Fig.2-11

Problem 2-12

Derive the difference equation for 2st order derivative $\frac{\partial^2 \phi}{\partial x^2}$ with 4rd order truncation error.