



# Numerical Heat Transfer (数值传热学) Introduction to computer-aided project



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# 1. Introduction to heatlines (热线)

The stream function in 2D incompressible flow is defined as:

$$u = \partial \psi / \partial y; v = -\partial \psi / \partial x \longrightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

From 2D steady diffusion-convection process:

We have two total heat fluxes in x and y –direction:

$$J_x = \rho u c_p T - k \frac{\partial T}{\partial x}; J_y = \rho v c_p T - k \frac{\partial T}{\partial y}$$

$$\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} = 0 \longrightarrow u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

$$J_x = \frac{\partial H}{\partial y} = \rho u c_p T - k \frac{\partial T}{\partial x}; J_y = -\frac{\partial H}{\partial x} = \rho v c_p T - k \frac{\partial T}{\partial y} \quad 2/12$$



Function  $H$  satisfies following equation:

$$\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} = \frac{\partial^2 H}{\partial x \partial y} - \frac{\partial^2 H}{\partial y \partial x} = 0$$

The constant-  $H$  line represents the heat transfer routes in diffusion convection problems, and is called **heat lines** by Bejan (1983).

To obtain heat lines two ways may be used:

(1) Solving a Poisson Eq. for  $H$

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = \rho c_p \left[ \frac{\partial}{\partial y} (uT) - \frac{\partial}{\partial x} (vT) \right]$$



(2) To construct the energy flux vector  $\vec{E}$  :

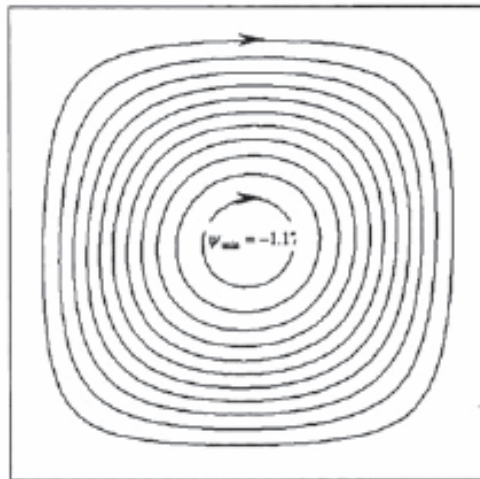
$$\vec{E} = \underbrace{(\rho u c_p T - k \frac{\partial T}{\partial x})}_{\text{X-direction total}} \vec{i} + \underbrace{(\rho v c_p T - k \frac{\partial T}{\partial y})}_{\text{Y-direction total}} \vec{j}$$

X-direction total

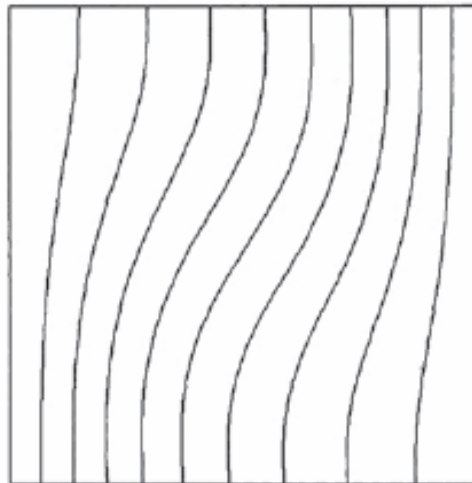
Y-direction total

By use of the numerical results of velocity and temperature distributions the energy flux vector can be easily obtained , hence , obtained the rout of heat transfer in the body without ths solution of the Poisson equation of H.

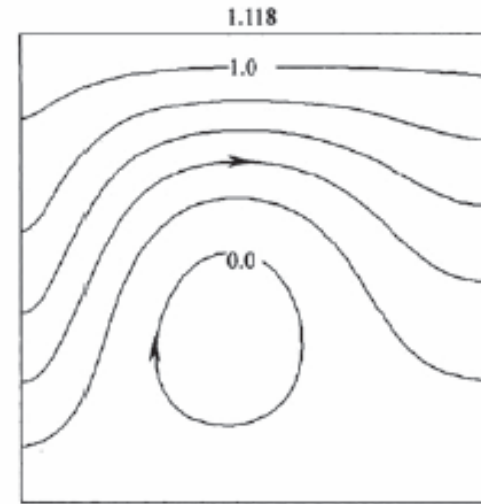
Examples of heatlines:



**Stream lines**

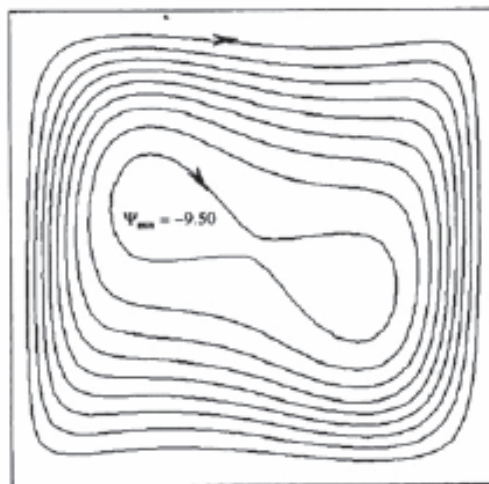


**Isotherms**

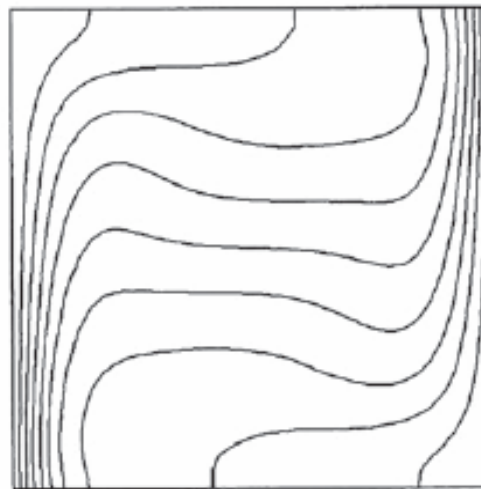


**Heatlines**

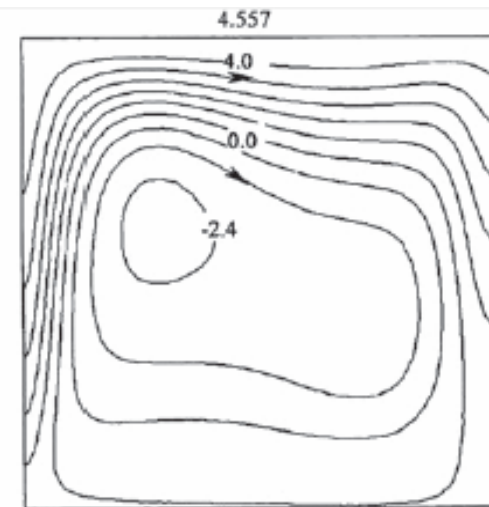
(a)  $Ra=1000$ ,  $Pr=0.71$



**Stream lines**

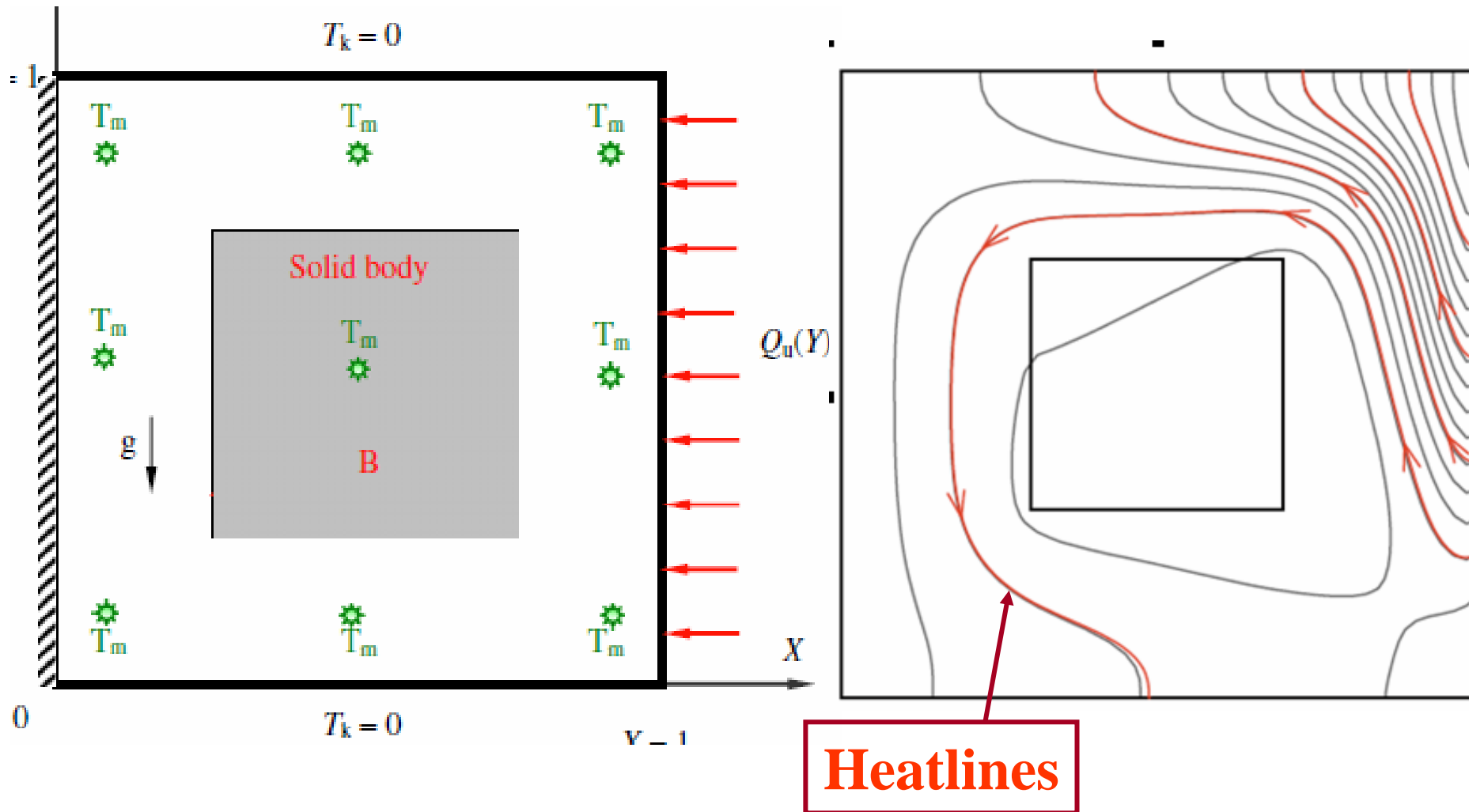


**Isotherms**



**Heatlines**

(b)  $Ra=10000$ ,  $Pr=0.71$



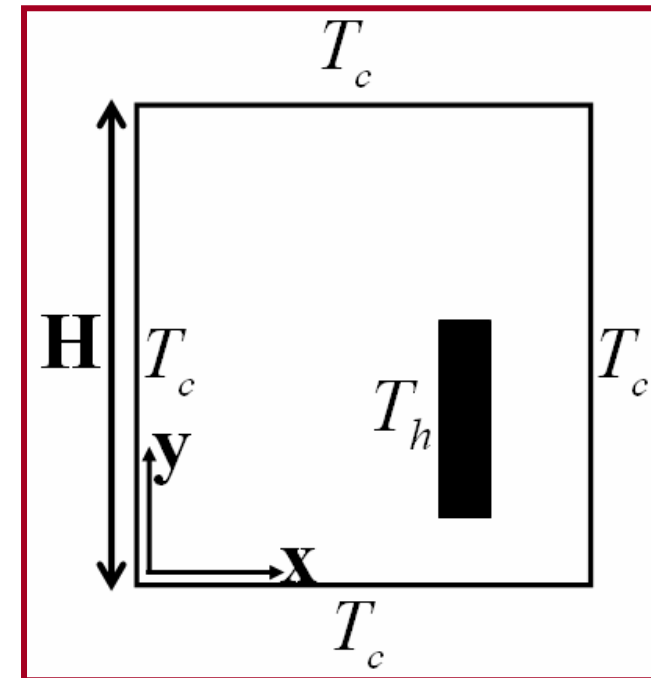


## 2. Project description

In a 2D square enclosure shown below there is an isolated plate with constant temperature  $T_h$ ; The inner walls of the enclosure are at the same temperature  $T_c$ . Air is filled in the enclosure. Rayleigh number is defined as:

$$Ra = \left( \frac{g \alpha \Delta T H^3}{\nu^2} \right) Pr$$

**Solve fluid velocity and temperature fields in the enclosure for  $Ra=10000$ ; Draw the diagrams of stream lines, isotherms and heatlines. The dimensions and position of the inner plate are determined by student.**





### 3. Requirement for course paper

1. Students are required to write a technical paper based on the simulation results. The paper should be written in the format of Journal of Xi'an Jiaotong University.
2. Please print paper on A4 papers and bind them by staple(订书钉) with your name and school number on the cover page.
3. You may apply commercial software, such as FLUENT, or our teaching code to solve the problem. If you adopt our teaching code please attach your revised USER at the end of your paper. Please indicate what kind of code you are using to solve the problem. **Use of teaching code is encouraged!**





4. Your paper is regarded as an technical paper of numerical study. It should include, but not limited to, following sections:
  - 1) Introduction;
  - 2) Physical problem and mathematical formulation;
  - 3) Numerical methods, including grid system, discretization scheme, convergence condition, etc.
  - 4) Results and discussion.;
  - 5) Conclusion
5. Please hand in you paper before the end of May, 2016 to Room 204 of East No 3 Building.

**It is a good practice. Please do it yourself!**



## 对课程论文的要求

1. 课程论文应按照西安交大学报（自然科学版）论文的要求撰写，具体要求见西安交大学报每期的封里，但论文的长度可以放宽；凡新的推导力求详细；**鼓励采用本教学程序求解；**
2. 课程论文及程序一律用A4纸打出；
3. 课程论文应包括：1) 引言；2) 物理问题及控制方程；3) 数值方法；4) 求解结果及讨论；5) 结论；
4. 在2016年5月30日前交课程论文及程序到**东三楼204室**，届时提前将在网上发出通知。



作业未交齐者务必于近期内交到北二楼8818.

## 致 谢

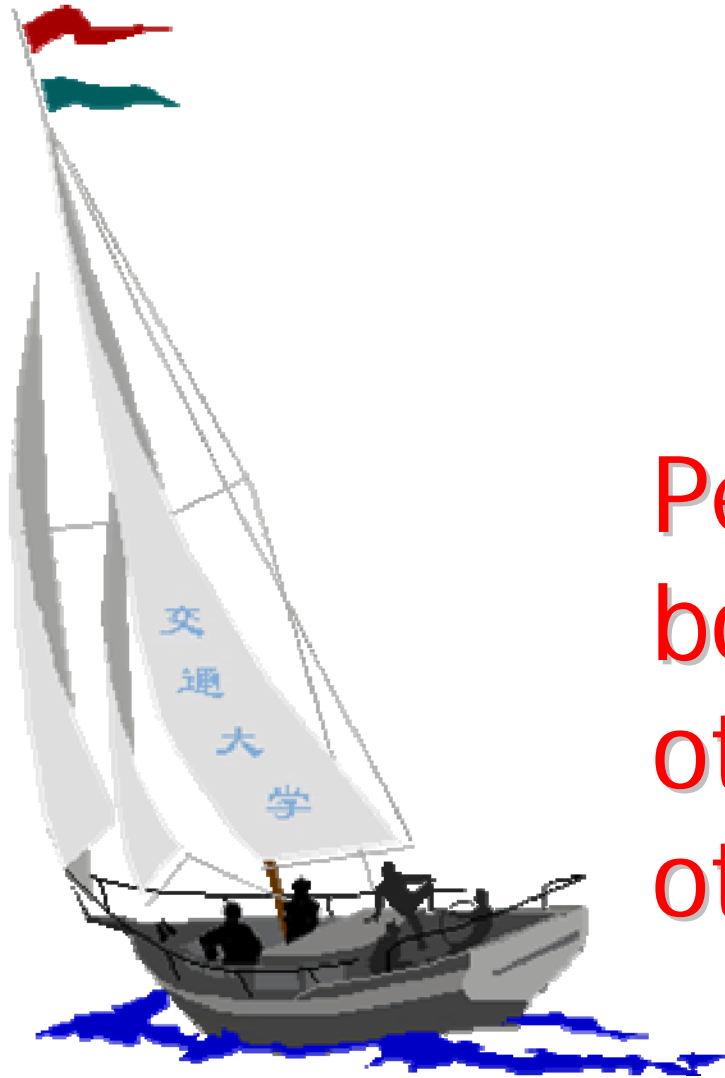
谢谢博士研究生蒲航，马斌斌，徐晋，李楠协助批改作业和创建QQ群！

谢谢大家的耐心与支持！



**Wish you have a very  
successful new year !**

**祝各位新年愉快，  
考试成功！**



# 同舟共济 渡彼岸!

People in the same  
boat help each  
other to cross to the  
other bank, where....

