Numerical Heat Transfer
(数值传热学)
Chapter 1 Introduction

Instructor Tao, Wen-Quan

Int. Joint Research Laboratory of Thermal Science & Engineering
Xi'an Jiaotong University
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Brief Introduction to Course

1. Textbook—《数值传热学》, 2nd ed., 2001

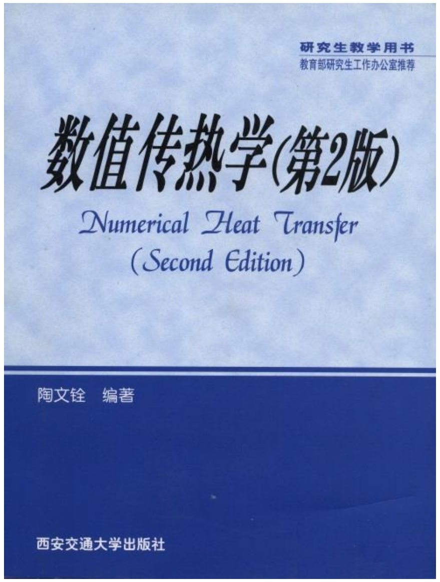
2. Teaching hours— 48 hrs-basic principles, including teaching code ; 12 hrs-FLUENT software

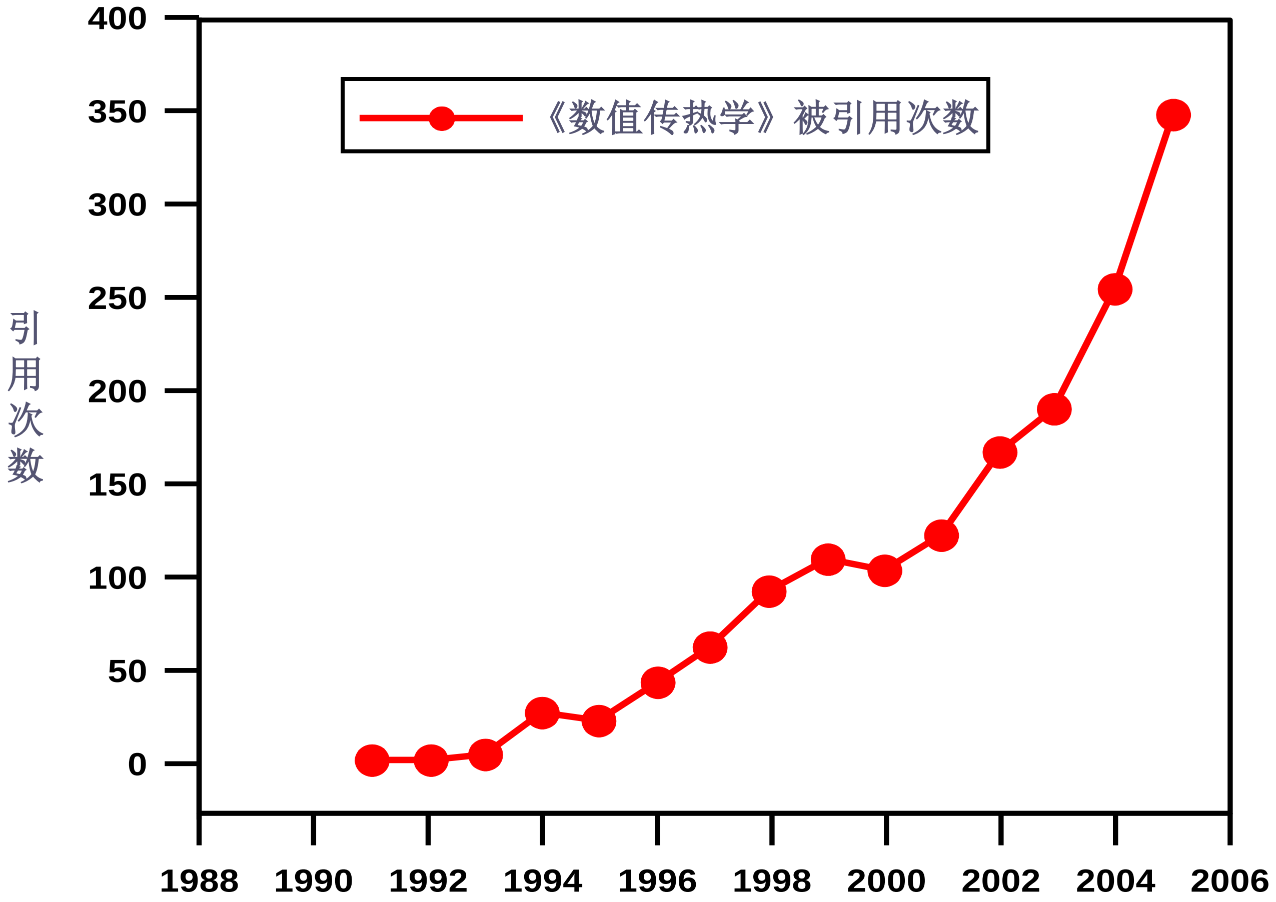
3. Course score(课程成绩)— Home work/Computer-aided project: —50/50

4. Methodology— Open, Participation and Application (开放，参与，应用)

5. Teaching assistants— Ding,Hao(丁昊）, Bai,Fan（白帆）, Wang,Xiao-Juan(王晓娟）,Liu,Ji-Xin(刘霁鑫）

QQ: 585614170





《数值传热学》Citations of the textbook

So far the total citation number home and abroad>9000

Relative International Journal 有关的主要国外期刊

1. Numerical Heat Transfer, Part A- Applications; Part B- Fundamentals
2. International Journal of Numerical Methods in Fluids.
3. Computer & Fluids
4. Journal of Computational Physics
5. International Journal of Numerical Methods in Engineering
6. International Journal of Numerical Methods in Heat and Fluid Flow
7. Computer Methods of Applied Mechanics and Engineering
8. Engineering Computations
9. Progress in Computational Fluid Dynamics
10. Computer Modeling in Engineering & Sciences (CMES)
11. ASME Journal of Heat Transfer
12. International Journal of Heat and Mass Transfer
13. ASME Journal of Fluids Engineering
14. International Journal of Heat and Fluid Flow
15. AIAA Journal

Methods for improving teaching and studying

1. **Speaking simple but clear English with Chinese note (注释) of new terminology (术语) and some difficult words---teacher's side;**
2. **Repeating very important contents by Chinese to enhance (加强) students' understanding (理解)----teacher 's side;**
3. **Enhancing communications between students and teachers thru a QQ-group ;My four assistants will help in this regard----both sides;**
4. **Previewing (预习) PPT of 2016 loaded in our group; website: http//nht.xjtu.edu.cn----students' side.**

Recent reformation for teaching NHT(教学改革)

1. **Teaching theory and code alternatively (交替地):**

   From1983 to 2015, theory and teaching code (教学程序) were sequentially(按顺序地) taught with theory first and code second;

   From the year 2016 following change has been made;

   Chapter 1-Chapter 7----Basic theory for NHT

   Chapter 8, 9--- Teaching code and its applications

   Chapter 10- Turbulence models

   Chapter 11- Grid generation techniques

2. **Teaching introduction to FLUENT(Chapter 12) and its applications (Chapter 13) from 2017.**

Contents of Chapter 1

1.1 Mathematical formulation （数学描述） of heat transfer and fluid flow (HT & FF) problems

1.2 Basic concepts of NHT and its application examples

1.3 Mathematical and physical classification of HT & FF problems and its effects on numerical solution

1.1 Mathematical formulation of heat transfer and fluid flow (HT & FF) problems

1.1.1 Governing equations （控制方程） and their general form

1. Mass conservation

2. Momentum conservation

3. Energy conservation

4. General form

1.1.2 Conditions for unique solution (唯一解)

1.1.3 Example of mathematical formulation

1.1 Mathematical formulation of heat transfer and fluid flow (HT & FF) problems

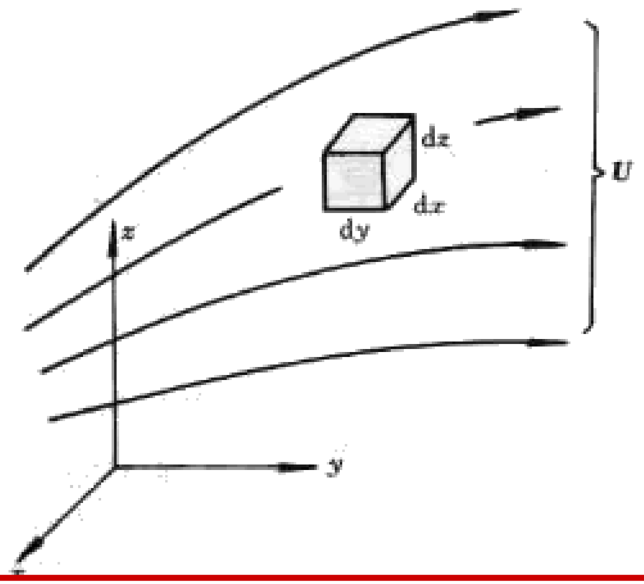
All macro-scale （宏观）HT & FF problems are governed by three conservation laws： mass, momentum and energy conservation law (守恒定律).

The differences between different problems are in: conditions for the unique solution（唯一解）： initial （初始的） & boundary conditions, physical properties and source terms.

1.1.1 Governing equations and their general form

1. Mass conservation

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$



"*div*" is the mathematical symbol for divergence （散度）.

$$\frac{\partial \rho}{\partial t} + div(\rho \vec{U}) = 0$$

$$div(\rho \vec{U}) = \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}$$

For incompressible fluid （不可压缩流体）：

$$div(\vec{U}) = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

called flow without divergence (流动无散条件)。

2. Momentum conservation

Applying the 2nd law of Newton （F=ma) to the elemental control volume（控制容积） shown above in the three-dimensional coordinates:

[Increasing rate of momentum of the CV]= [Summation of external（外部） forces applying on the CV]

Adopting Stokes assumption： stress is linearly proportional to strain(应力与应变成线性关系)， We have：

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}+\frac{\partial(\rho uw)}{\partial z}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}(\bar{\lambda}\,div\vec{U}+2\eta\frac{\partial u}{\partial x})$$

$$+\frac{\partial}{\partial y}[\eta(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})]+\frac{\partial}{\partial z}[\eta(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x})]+\rho F_x$$

η dynamic viscosity ， $\bar{\lambda}$ fluid 2nd molecular viscosity.

It can be shown that the above equation can be reformulated as （改写为） following general form of Navier-Stokes equation:

$$\frac{\partial(\rho u)}{\partial t} + div(\rho u\vec{U}) = div(\eta\, grad u) + S_u$$

Transient term 非稳态项	Convection term对流项	Diffusion term扩散项	Source term源项

u ----dependent variable（因变量） to be solved;

$\vec{U}$ ----fluid velocity vector;

S_u ----source term.

Source term in x-direction:

$$S_u = \frac{\partial}{\partial x}(\eta\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(\eta\frac{\partial v}{\partial x}) + \frac{\partial}{\partial z}(\eta\frac{\partial w}{\partial x}) + \frac{\partial}{\partial x}(\bar{\lambda} div\vec{U}) + \rho F_x - \frac{\partial p}{\partial x}$$

Similarly:

$$S_v = \frac{\partial}{\partial x}(\eta\frac{\partial u}{\partial y}) + \frac{\partial}{\partial y}(\eta\frac{\partial v}{\partial y}) + \frac{\partial}{\partial z}(\eta\frac{\partial w}{\partial y}) + \frac{\partial}{\partial y}(\bar{\lambda} div\vec{U}) + \rho F_y - \frac{\partial p}{\partial y}$$

$$S_w = \frac{\partial}{\partial x}(\eta\frac{\partial u}{\partial z}) + \frac{\partial}{\partial y}(\eta\frac{\partial v}{\partial z}) + \frac{\partial}{\partial z}(\eta\frac{\partial w}{\partial z}) + \frac{\partial}{\partial z}(\bar{\lambda} div\vec{U}) + \rho F_z - \frac{\partial p}{\partial z}$$

For incompressible fluid with constant properties the source term does not contain velocity-related part.

3. Energy conservation

[Increasing rate of internal energy in the CV]= [Net heat going into the CV]+[Work conducted by body forces and surface forces]

Introducing Fourier's law of heat conduction and neglecting the work conducted by forces; Introducing enthalpy（焓）$h = c_p T$, assuming c_p = Constant:

$$\frac{\partial(\rho T)}{\partial t} + div(\rho T \vec{U}) = div(\frac{\lambda}{c_p} grad(T)) + S_T$$

$$grad(T) = \frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k}$$

$$\frac{\lambda}{c_p} \rightarrow \frac{\lambda \eta}{c_p \eta} \rightarrow (\frac{\lambda}{c_p \eta})\eta \rightarrow \frac{\eta}{\Pr}$$

4. General form of the governing equations**

$$\frac{\partial(\rho\phi)}{\partial t}+div(\rho\phi\vec{U})=div(\Gamma_{\phi}^{*}grad(\phi))+S_{\phi}^{*}$$

Transient **Convection** **Diffusion** **Source**

The differences between different variables:

（1） Different boundary and initial conditions；

（2） Different nominal source（名义源项） terms；

（3） Different physical properties (nominal diffusion coefficients, 名义扩散系数)

5. Some remarks（说明）

1. The derived transient 3D Navier-Stokes equations can be applied for both laminar and turbulent flows.

2. When a HT & FF problem is in conjunction with (与…有关）mass transfer process, the component （组份） conservation equation should be included in the governing equations.

3. Although c_p is assumed constant, the above governing equation can also be applied to cases with weakly changed c_p (比热略有变化）.

4. Radiative （辐射）heat transfer is governed by a differential-integral （微分-积分）equation, and its numerical solution will not be dealt with here.

1.1.2 Conditions for unique solution

1. Initial condition (初始条件) $t = 0,\ \ T = f(x, y, z)$

2. Boundary condition (边界条件)

（1） **First kind (Dirichlet):** $T_B = T_{given}$

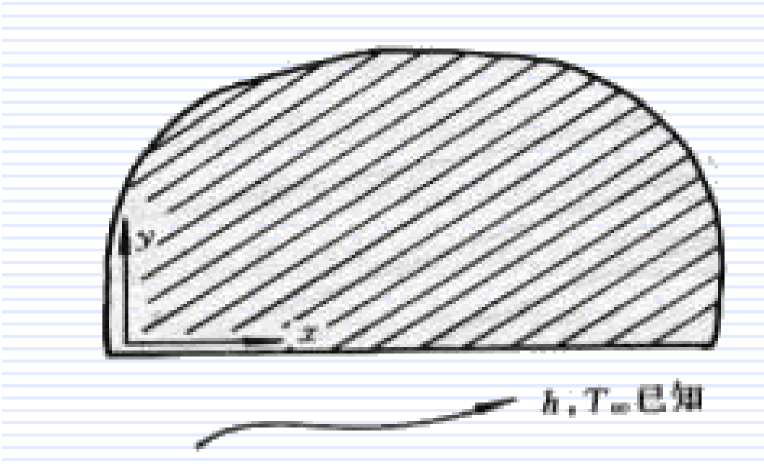
（2） **Second kind (Neumann):** $q_B = -\lambda(\frac{\partial T}{\partial n})_B = q_{given}$

（3） **Third kind (Rubin):** **Specifying（规定） the relationship between boundary value and its first-order normal derivative:**

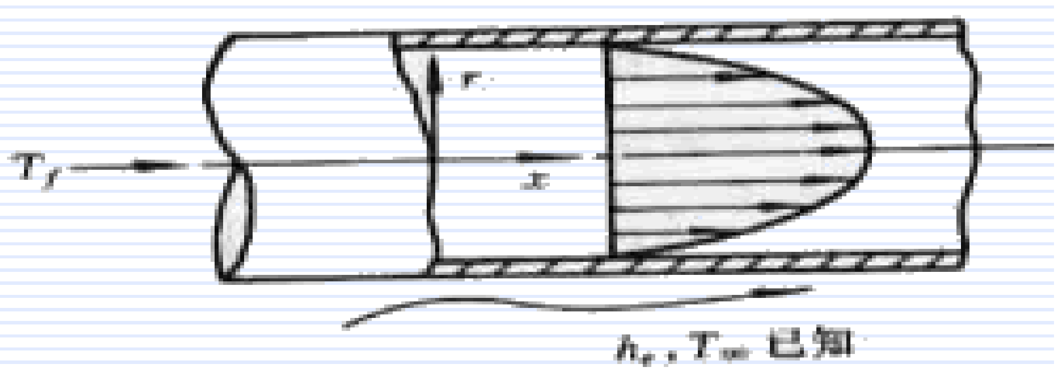
$$-\lambda(\frac{\partial T}{\partial n})_B = h(T_B - T_f)$$

3. Fluid thermo-physical properties and source term of the process.

3^{rd} kind boundary conditions for both solid heat conduction and convective heat transfer problems



(a)



(b)

Heat conduction with 3rd kind B.C. at surface

h, T_∞ *are known*

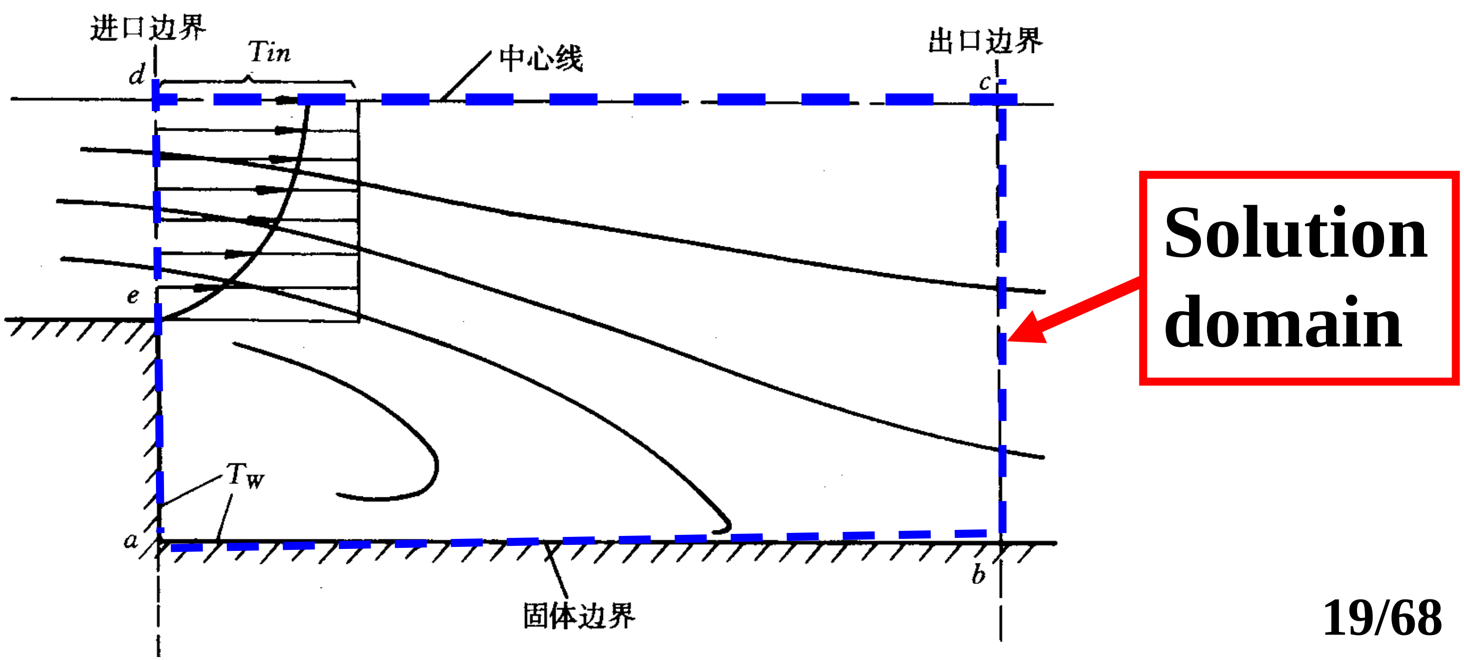
Inner convective heat transfer with 3rd kind condition at wall outside

h_e, T_∞ *are known*

1.1.3 Example of mathematical formulation

1. Problem and assumptions

Convective heat transfer in a sudden expansion region : 2D, steady- state, incompressible fluid, constant properties, neglecting gravity and viscous dissipation（粘性耗散）.



2. Governing equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial (uu)}{\partial x}+\frac{\partial (vu)}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})$$

$$\frac{\partial (uv)}{\partial x}+\frac{\partial (vv)}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})$$

$$\frac{\partial (uT)}{\partial x}+\frac{\partial (vT)}{\partial y}=a(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})$$

$$a=\frac{\lambda}{\rho c_p}$$

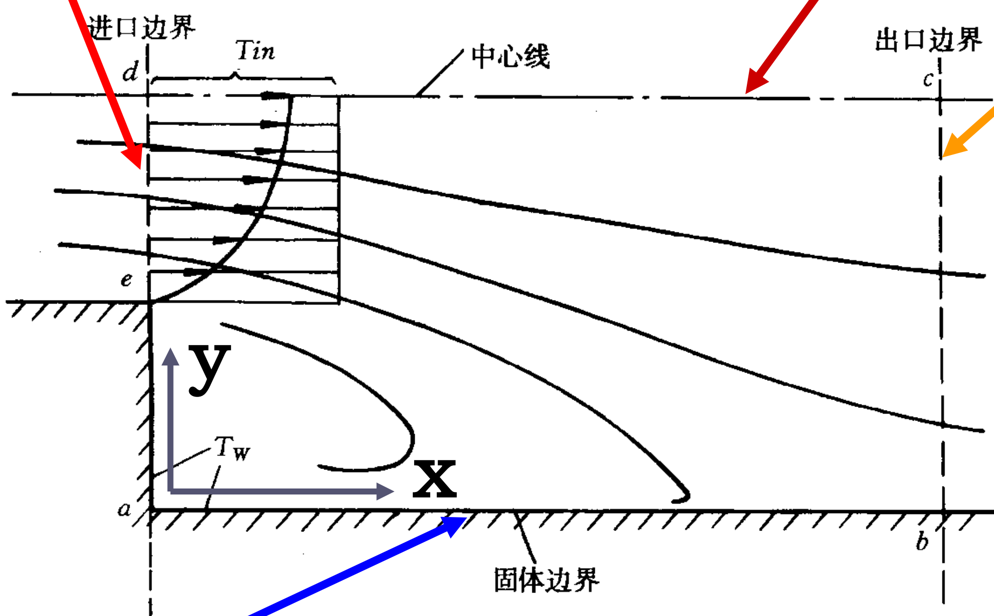
3. Boundary conditions

（1）**Inlet**： specifying variations of u,v,T with y；

（3）**Center line**：

$$\frac{\partial u}{\partial y}=\frac{\partial T}{\partial y}=0;\quad v=0$$

（4）**Outlet**：Mathematically the distributions of u,v,T or their first-order derivatives(导数) are required. Actually, **approximations must be made.**



（2）**Solid B.C.**： No slip（滑移） in velocity, no jump（跳跃） in temp.

Notes to Section 1.1

In the left hand side

$$\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}+\frac{\partial(\rho uw)}{\partial z}=div(\rho u\vec{U})$$

The right hand side :

$$\frac{\partial}{\partial x}(\bar{\lambda}div\vec{U}+2\eta\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}[\eta(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})]+\frac{\partial}{\partial z}[\eta(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x})]+\rho F_x-\frac{\partial p}{\partial x}=$$

$$\underbrace{\frac{\partial}{\partial x}(\eta\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(\eta\frac{\partial u}{\partial y})+\frac{\partial}{\partial z}(\eta\frac{\partial u}{\partial z})}_{div(grad(u))}+\underbrace{\frac{\partial}{\partial x}(\eta\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(\eta\frac{\partial v}{\partial x})+\frac{\partial}{\partial z}(\eta\frac{\partial w}{\partial x})+\frac{\partial}{\partial x}(\bar{\lambda}div\vec{U})}_{S_u}$$

$$\rho F_x-\frac{\partial p}{\partial x}=div(\eta\, grad u)+S_u$$

$$grad(u)=\frac{\partial u}{\partial x}\mathbf{i}+\frac{\partial u}{\partial y}\mathbf{j}+\frac{\partial u}{\partial z}\mathbf{k}$$

$$div(grad(u))=\frac{\partial}{\partial x}(\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(\frac{\partial u}{\partial y})+\frac{\partial}{\partial z}(\frac{\partial u}{\partial z})$$

Thus we have:

$$\frac{\partial(\rho u)}{\partial t}+div(\rho u\vec{U})=div(\eta\, grad u)+S_u$$

Navier-Stokes

Gradient of a scalar (标量的梯度) is a vector:

$$grad(u) = \frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k}$$

Divergence of a vector (矢量的散度) is a scalar:

$$div(grad(u)) = div(\frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k})$$

$$div(grad(u)) = \frac{\partial}{\partial x}(\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(\frac{\partial u}{\partial y}) + \frac{\partial}{\partial z}(\frac{\partial u}{\partial z})$$

$$div(\eta grad(u)) = \frac{\partial}{\partial x}(\eta\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(\eta\frac{\partial u}{\partial y}) + \frac{\partial}{\partial z}(\eta\frac{\partial u}{\partial z})$$

End of Notes to Section 1.1

1.2 Basic concepts of NHT and its application examples

1.2.1 Basic concepts of numerical solutions based on continuum assumption

1.2.2 Classification of numerical solution methods based on continuum assumption

1.2.3 Three fundamental approaches of scientific research and their relationships

1.2.4 Application examples

1.2.5 Some suggestions

1.2 Basic concepts of NHT and its application examples

1.2.1 Basic concepts of numerical solutions based on continuum assumption（连续性假设）**

Replacing the fields of continuum variables (velocity, temp. etc.) by sets (集合） of values at discrete（离散的） points (nodes, 节点) (Discretization of domain,区域离散);

Establishing algebraic equations for these values at the discrete points by some principles（Discretization of equations，方程离散);

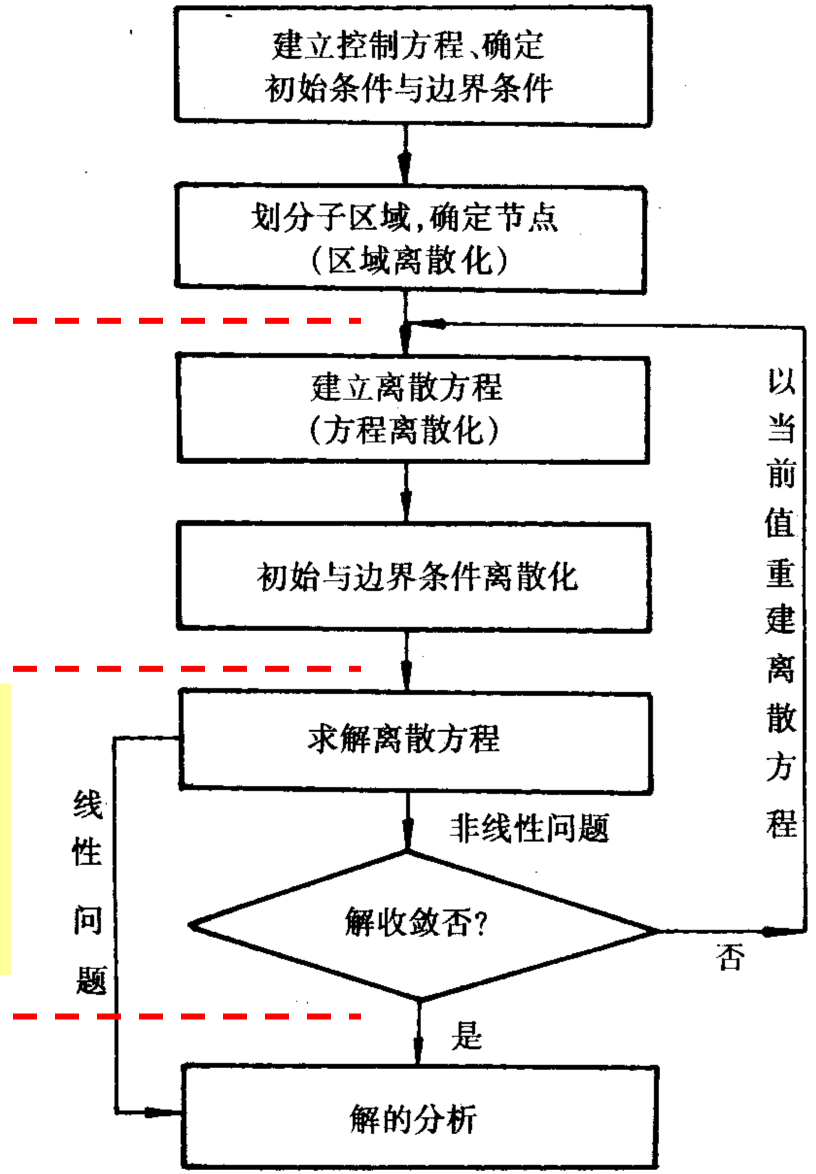
Solving the algebraic equations by computers to get approximate solutions of the continuum variables (Solution of equation，方程求解).

Discretizing Domain(区域离散)

Discretizing Equations (方程离散)

Solving algebraic equations (方程求解)

Analyzing numerical results (结果分析)



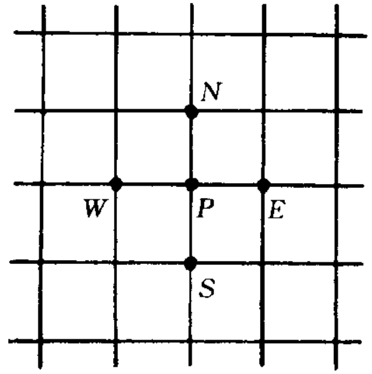
Flow chart （流程图）

1.2.2 Classification of numerical solution methods based on continuum assumption

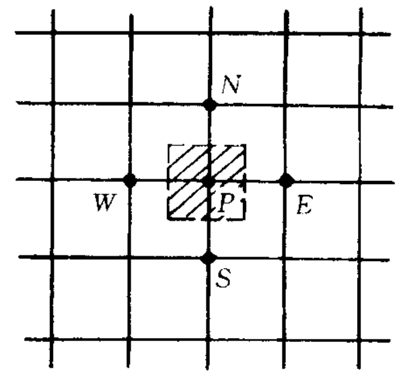
1. Finite difference method（**FDM**）	有限差分法：L F Richardson (1910), A Thom（1940s））
2. Finite volume method（**FVM**）	有限容积法：D B Spalding; S V Patankar
3. Finite element method（**FEM**）	有限元法：O C Zienkiewicz; 冯康(Kang Feng)
4. Finite analytic method（**FAM**）	有限分析法：陈景仁(Ching Jen Chen)
5. Boundary element method（**BEM**）	边界元法：D B Brebbia
6. Spectral analysis method（**SAM**）	（谱分析法）

Comparisons of FDM(a),FVM(b),FEM(c),FAM(d)

FDM
有限差分



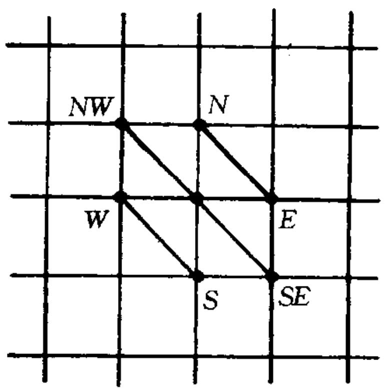
(a)



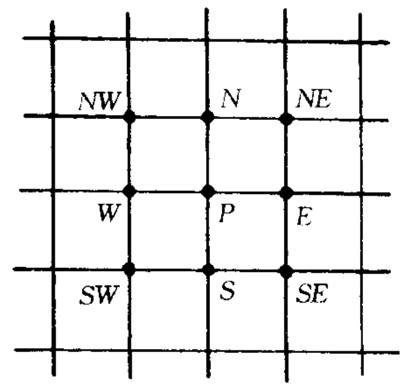
(b)

FVM
有限容积

FEM
有限元



(c)

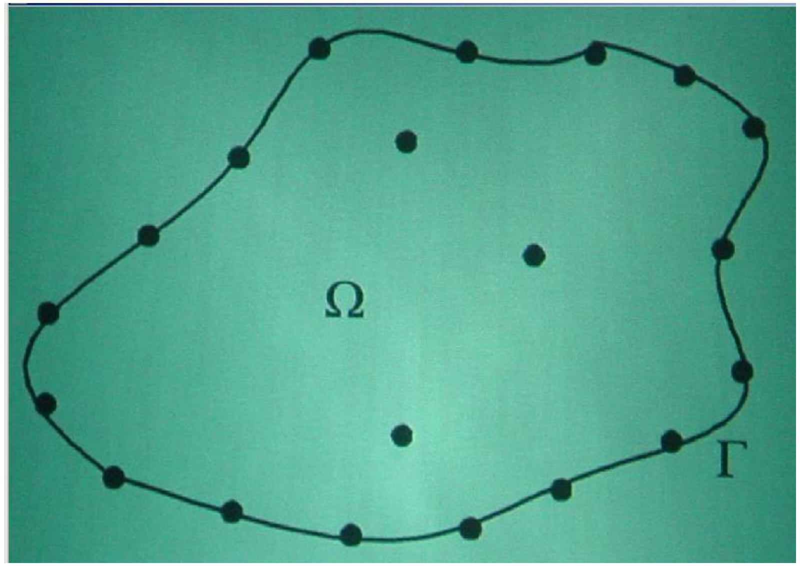


(d)

FAM
有限分析

All these methods need a grid system(网格系统):

1）Determination of grid positions；2）Establishing the influence relationships between grids.

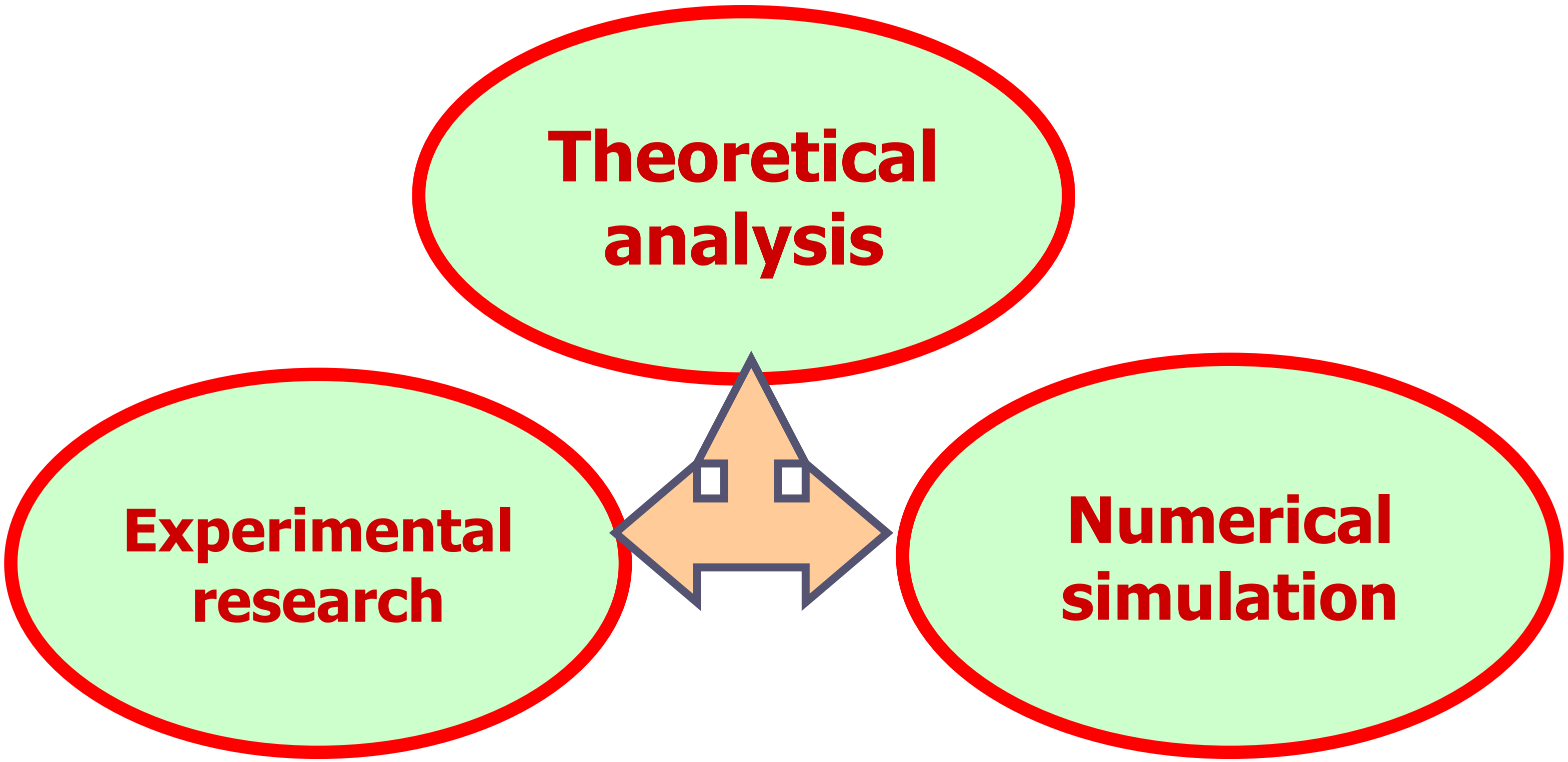


BEM （边界元）requires a basic solution(基准解), which Greatly limits its applications in convective problems.

SAM can only be applied to geometrically simple cases.

Manole、Lage 1990－1992 statistics（统计）： FVM ---47 %； adopted by most commercial softwares; Our statistics of NHT in 2007 even much higher.

1.2.3 Three fundamental approaches（方法） of scientific research and their relationships



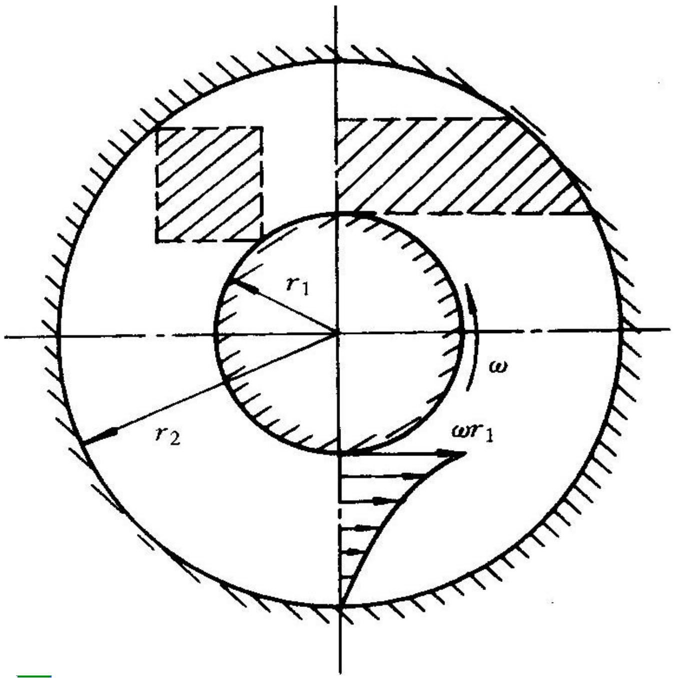
The starting point of numerical simulation for physical problems is Fluid Dynamics. Thus it is often called Computational Fluid Dynamics---CFD.

1. Theoretical analysis

Its importance should not be underestimated（低估）.

It provides comparison basis for the verification（验证） of numerical solutions.

Examples: The analytic solution of velocity from NS equation for following case：



$$\frac{u}{u_1} = \frac{r_1 / r_2}{1-(r_1/r_2)^2} \bullet \frac{1-(r/r_2)^2}{r/r_2}$$

$$u_1 = \varpi r_1$$

2. Experimental study

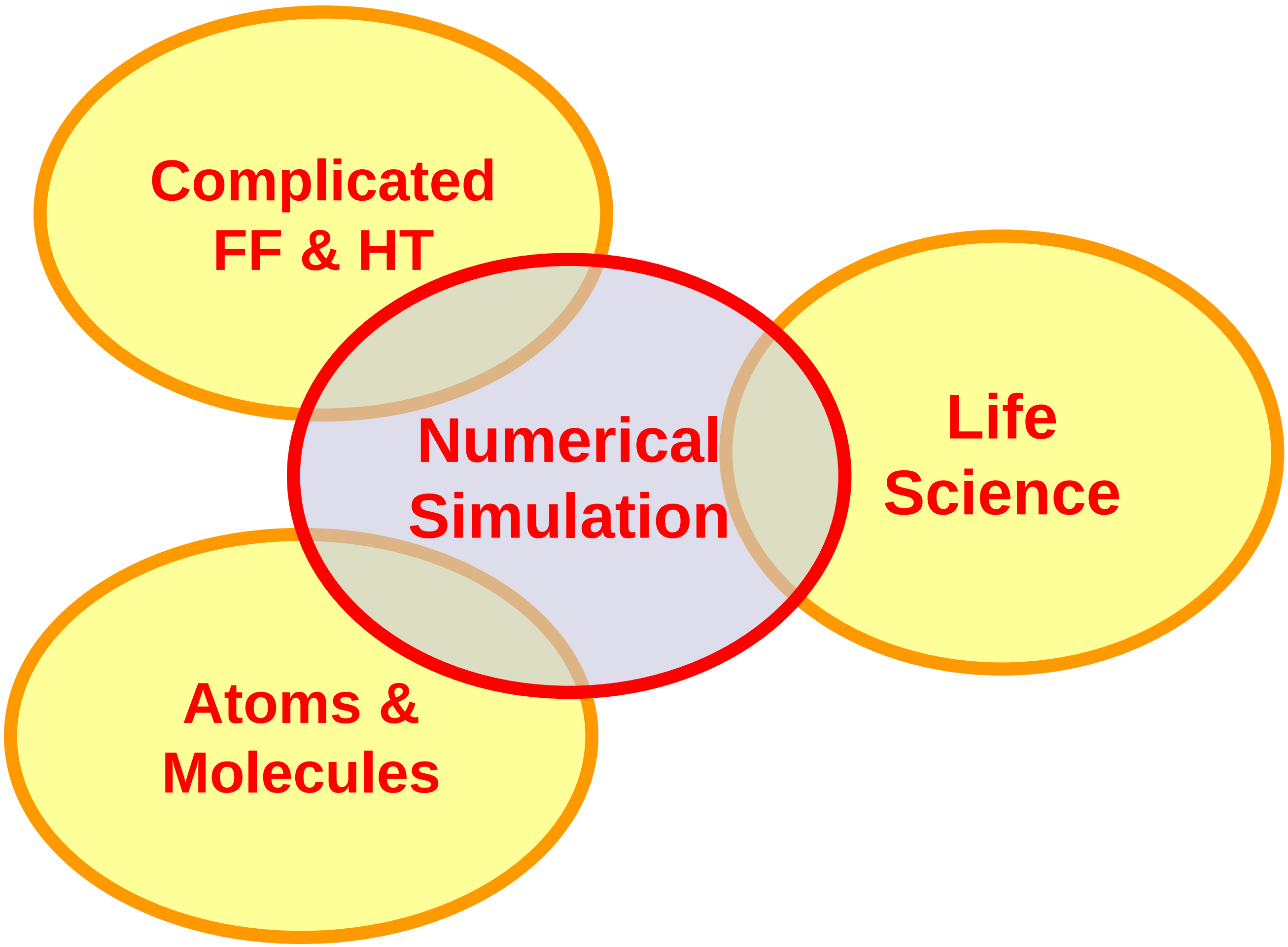
A basic research method: observation； properties measurement; verification of numerical results

3.Numerical simulation

Numerical simulation is an inter-discipline (交叉学科), and plays an important and un-replaceable role in exploring (探索)unknowns, promoting (促进) the development of science & technology, and for the safety of national defense（国防安全）.

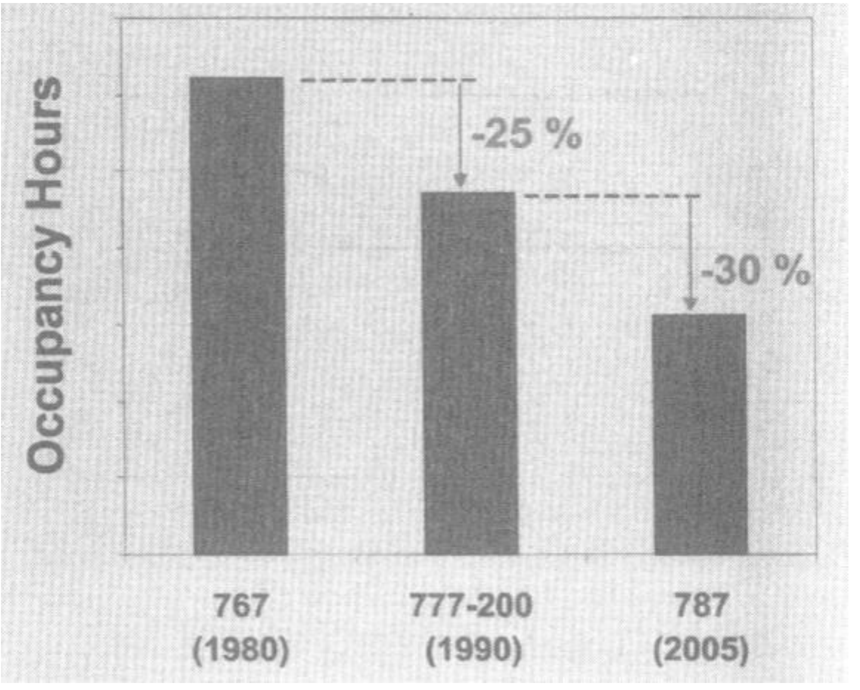
With the rapid development of computer hardware（硬件）, its importance and function will become greater and greater.

Historically, in 1985 the West Europe listed the first commercial software-PHEONICS as the one which was not allowed to sell to the communist countries.



In 2005 the USA President Advisory Board put forward a suggestion to the president that in order to keep competitive power （竞争力） of USA in the world it should develops scientific computation.

In the year of 2006 the director of design department of Boeing , M. Grarett , reported to the US Congress（国会） indicating that the high performance computers have completely changed the way of designing Boeing airplane.

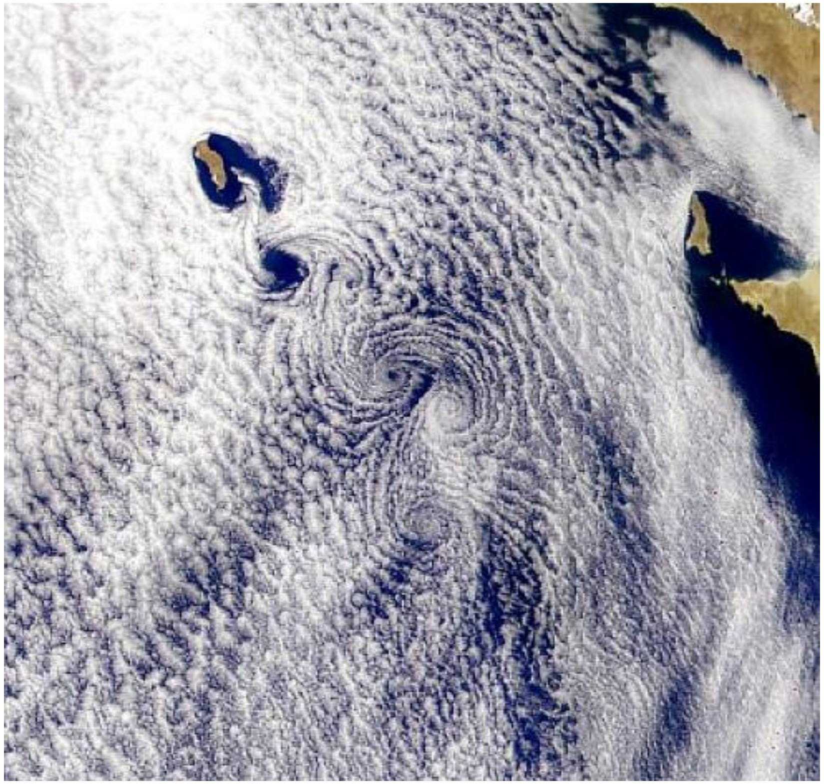


Numerical simulation plays an important role in the design of Boeing airplane

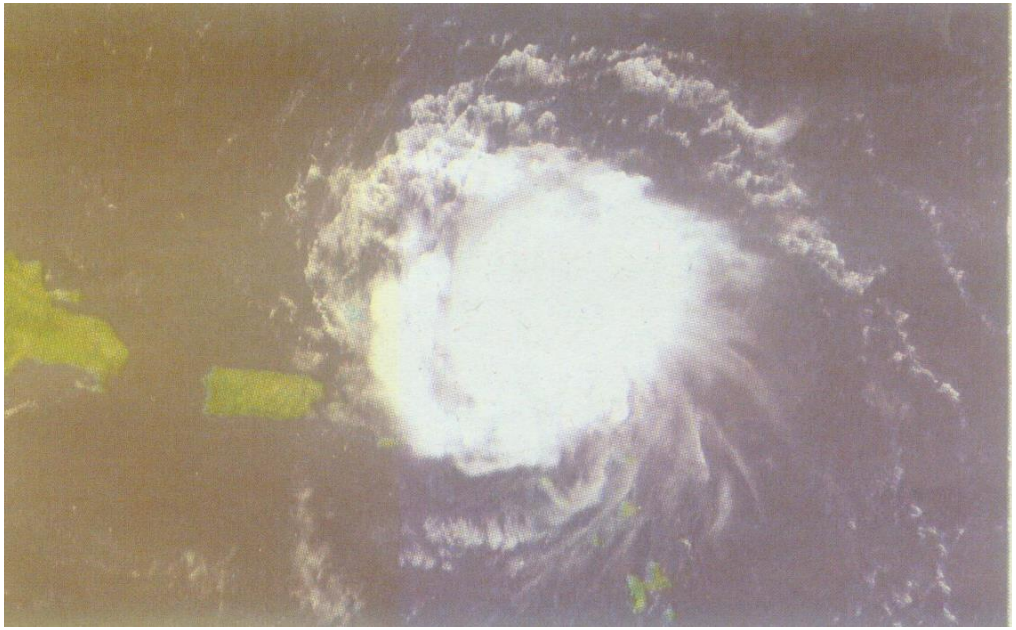
1.2.4 Application examples

Example 1： Weather forecast－

Num. solution is the only way.

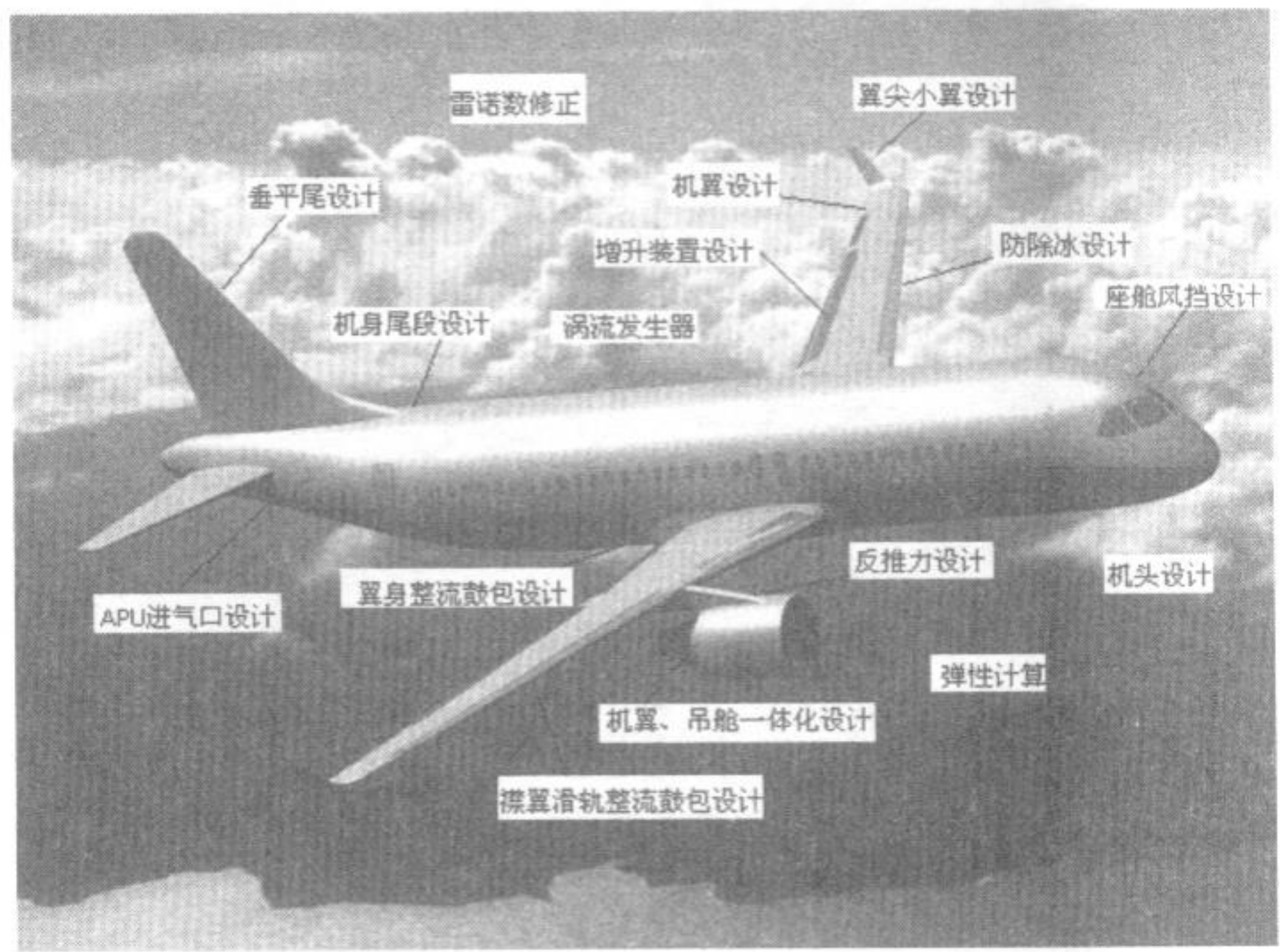


Large scale vortex



Cloud Atlas sent back by a meteorological satellite

Example 2：Aeronautical & aerospace （航空航天）engineering



Example 3：Hydraulic construction（水利建设）

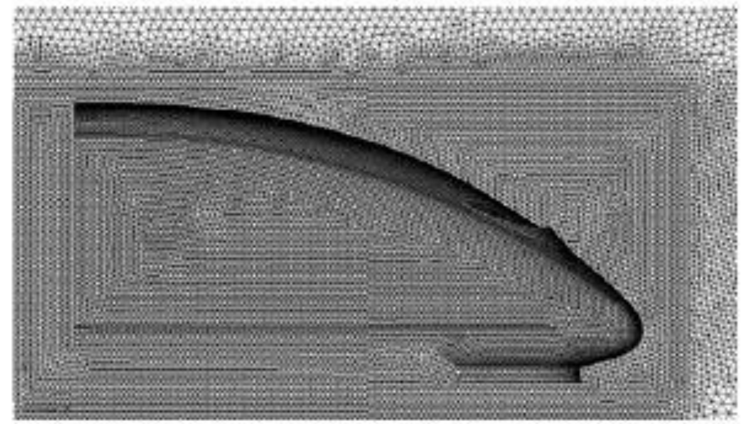
The mud （泥）and sand（沙） content of our Yellow River is about 35 kg/m^3, ranking No. 1 in the world, leading to following unpleasant situation: the ground floor of some cities is lower than the riverbed（河床） of Yellow River:Kai Feng-13m lower, Xin Xiang- 20m lower

In 2002 the idea of three yellow rivers was proposed：

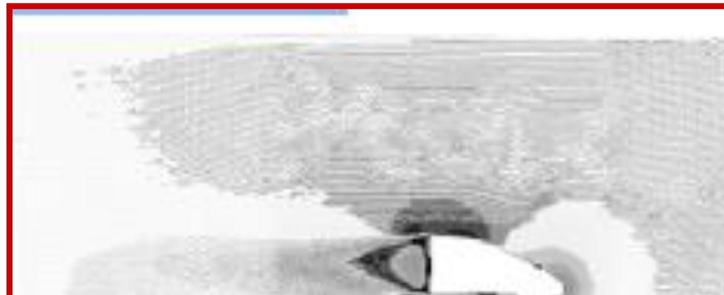
(1) Original YR；
(2) Numerical YR;
(3) Model YR.

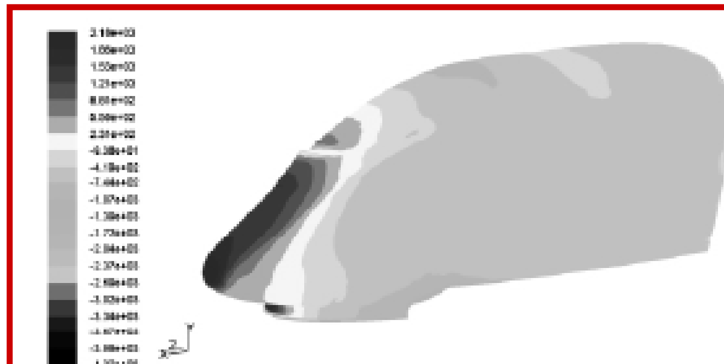
Through 8 times of modeling and simulation the height of the riverbed was averagely decreased by 1.5 m.

Example 4: Design of head shape of high-speed train



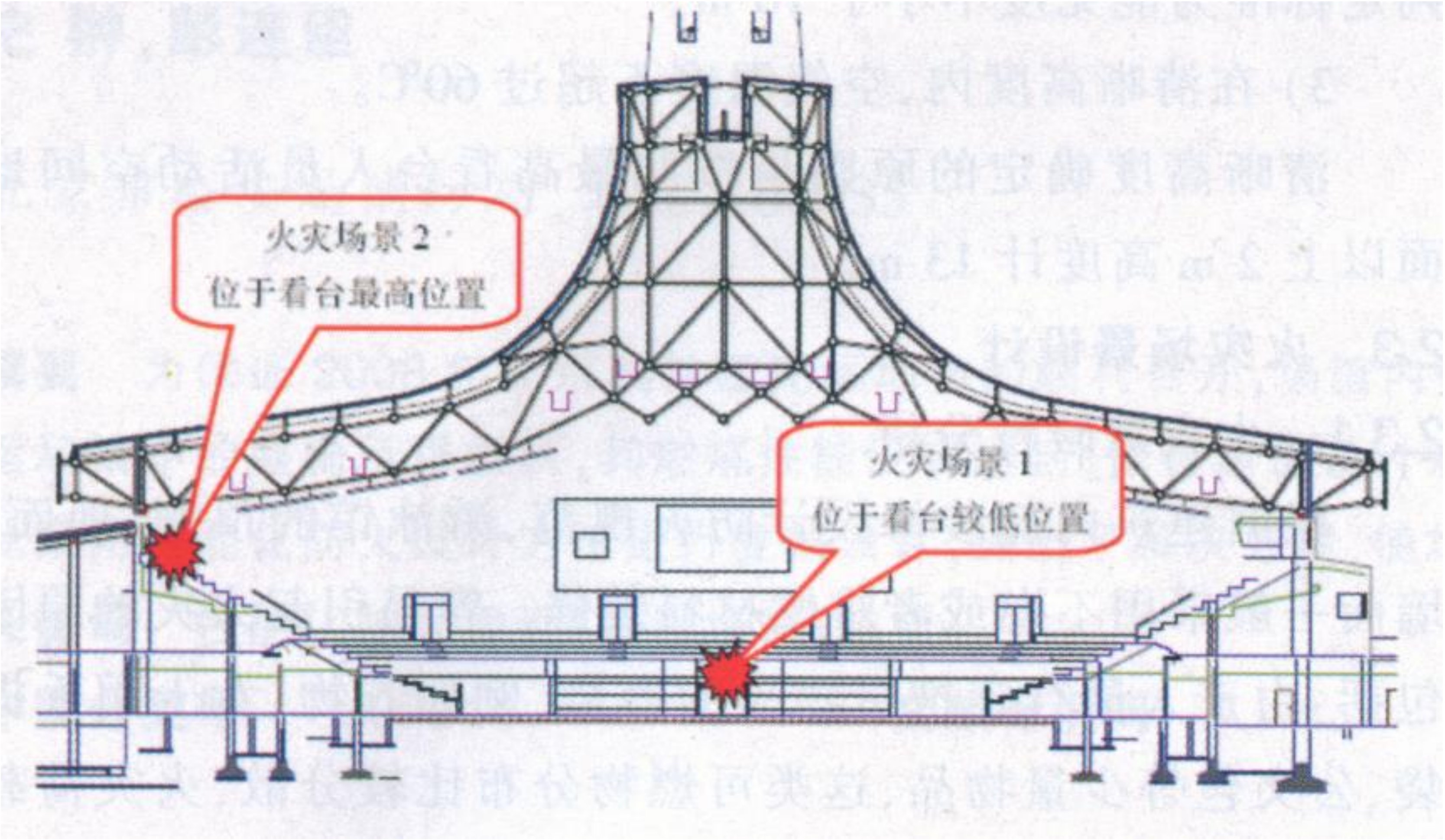
The front head shape of the high speed train is of great importance for its aerodynamic performance（空气动力学特性）. Numerical wind tunnel is widely used to optimize the front head shape.



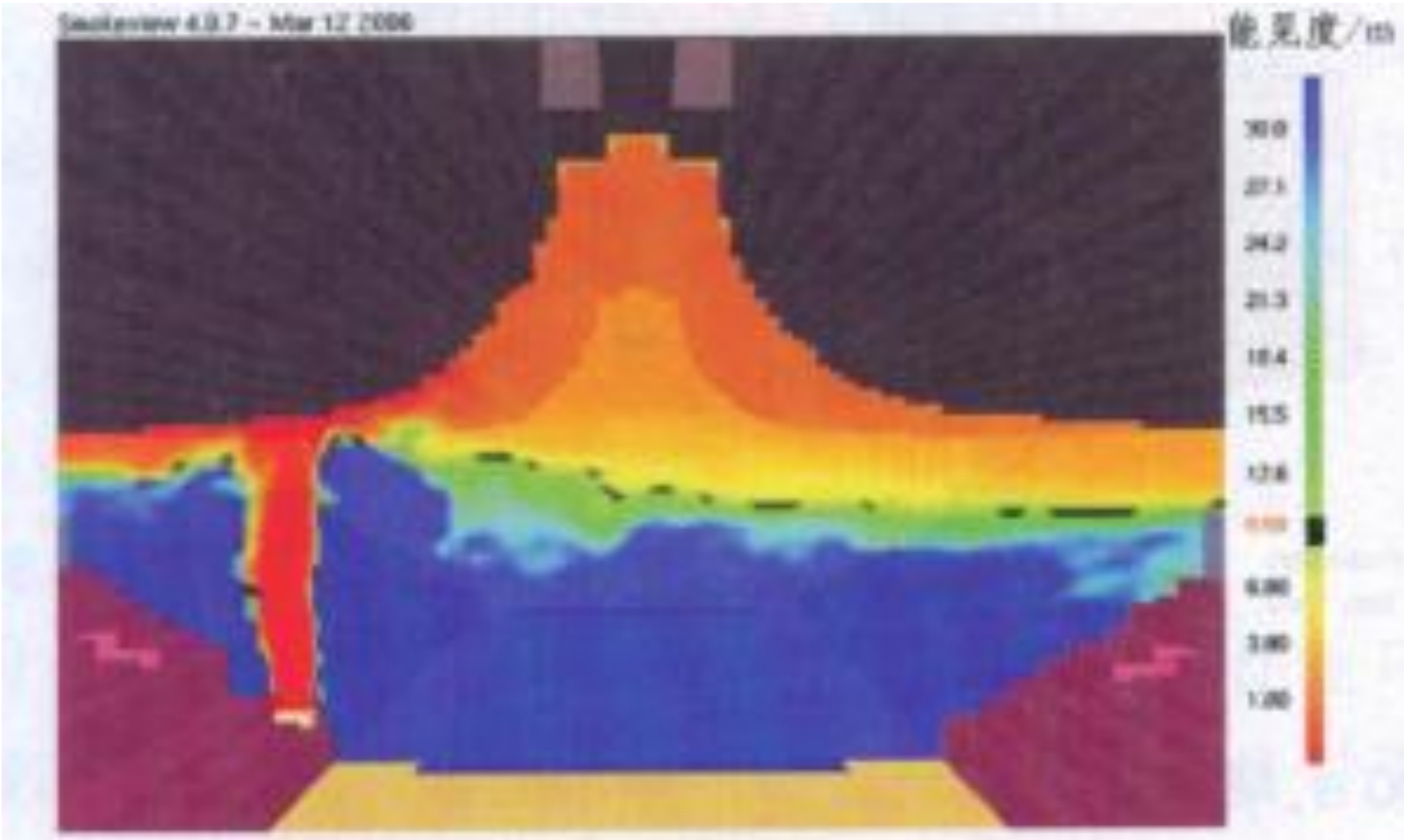


Example 5：Prediction of fire disaster（火灾） for Olympic Gymnasium（体育馆）in 2008

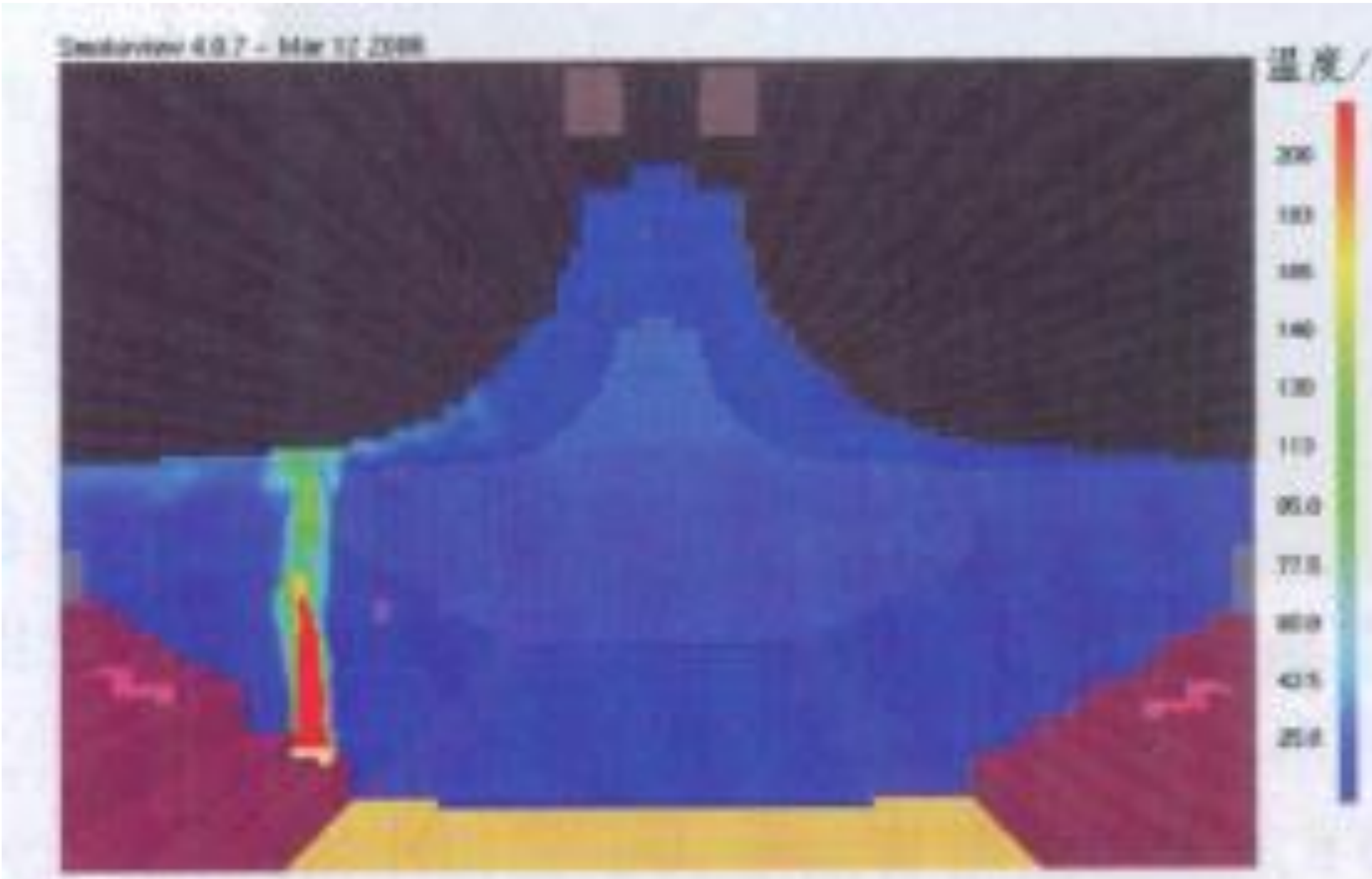
In the construction of gym :the chair material cannot meet the requirement of fireproof（防火）. A decision should be made asap on whether such material can be used. Numerical simulation was adopted.



Fire prediction for indoor swimming pool



Predicted visibility（能见度）after 900 seconds of fire outbreak

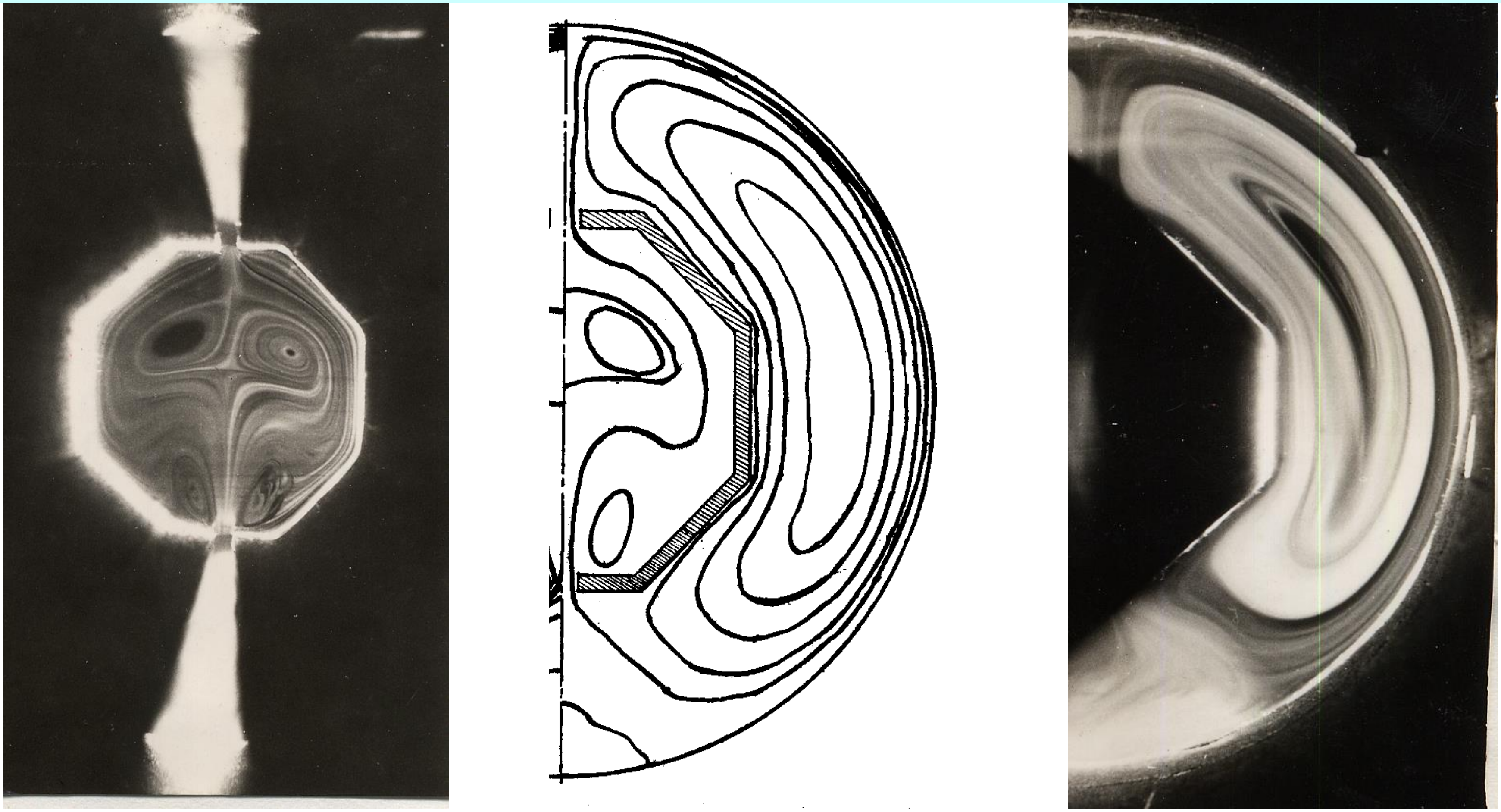


Predicted gas temperature distribution after 900 seconds of fire outbreak

Example 6：House safety－Fire prediction

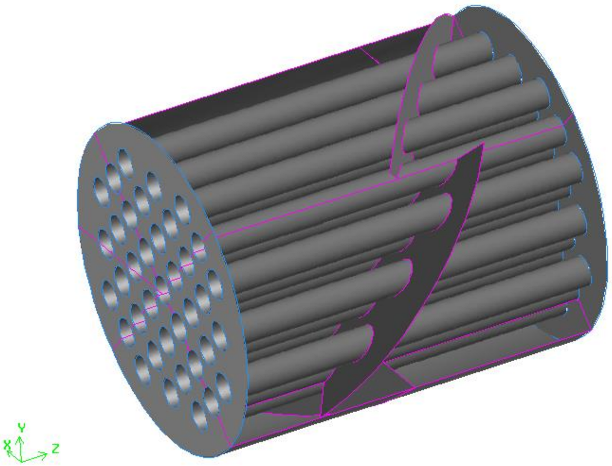
The major purpose is to guarantee（保证） that once fire outbreak（火灾） occurs, the persons living in the house can safely leave for outside within a certain amount of time.

Example 7：Heat transfer characteristics of large electric current bar（电流母线）

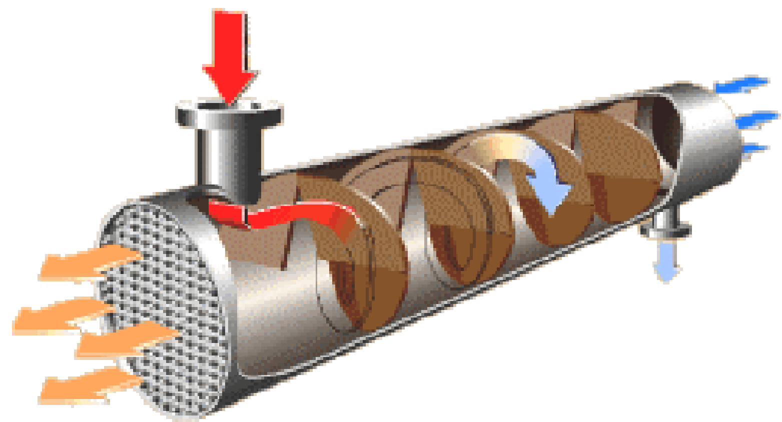


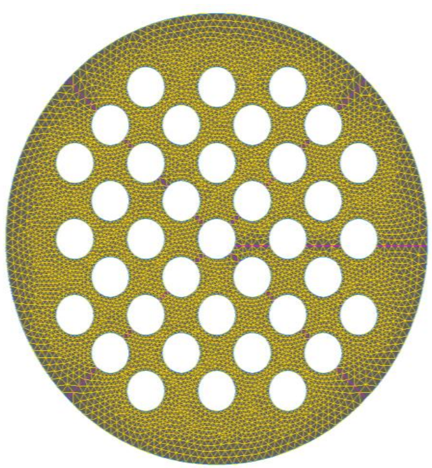
Comparisons of predicted and simulated flow fields

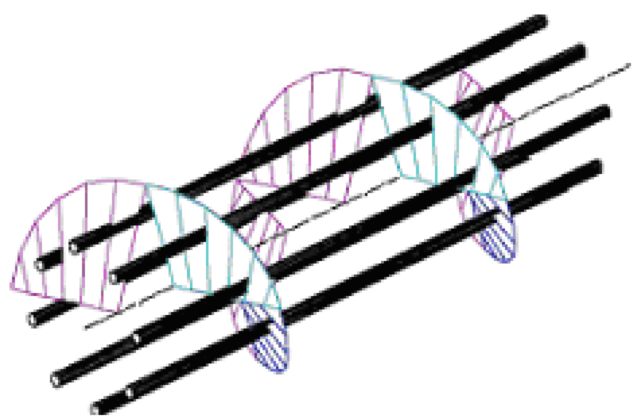
Example 8：Shell-side simulation of helical baffle(螺旋折流板） heat exchanger

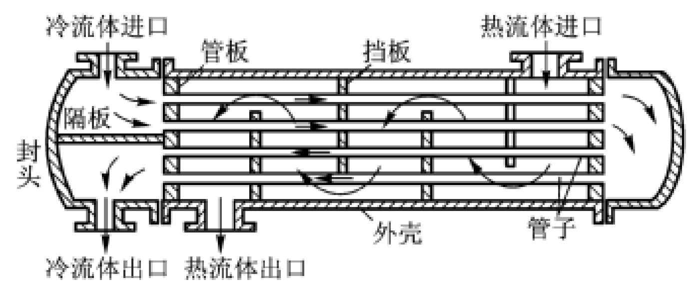


$$\begin{matrix} 1.34 \\ 2.73 \end{matrix} \times 10^{6}$$

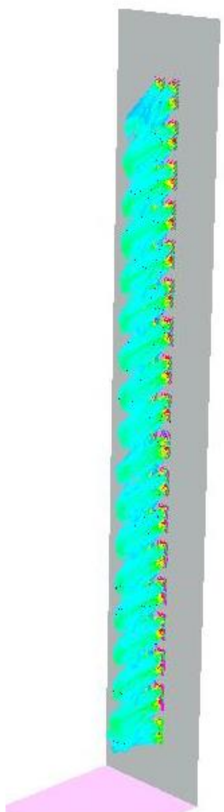




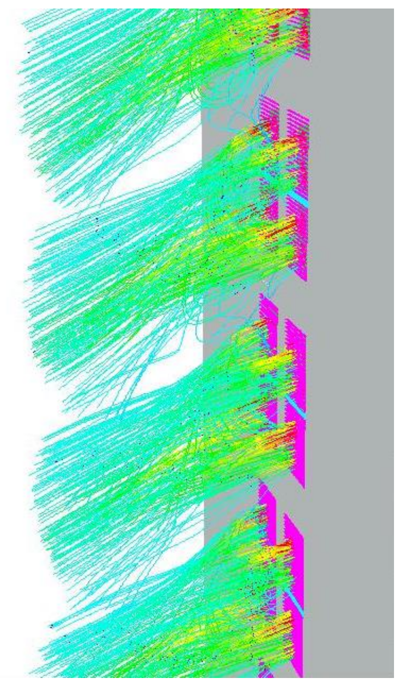




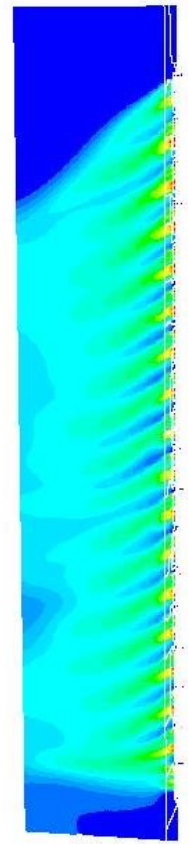
Example 9：Velocity and temperature distribution simulation to improve design of air-conditioning system(淮安市茂业时代广场大楼项目空调系统气流分析)



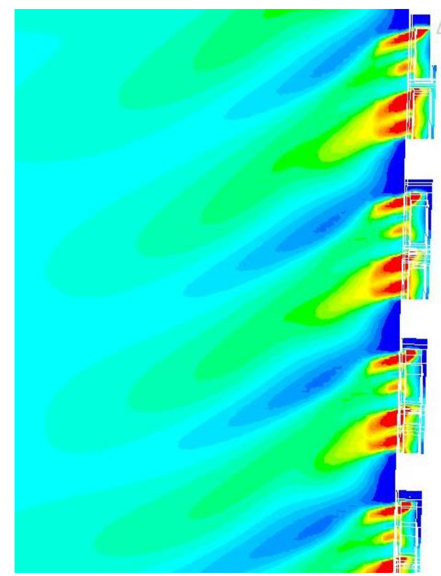
Out flow streamlines



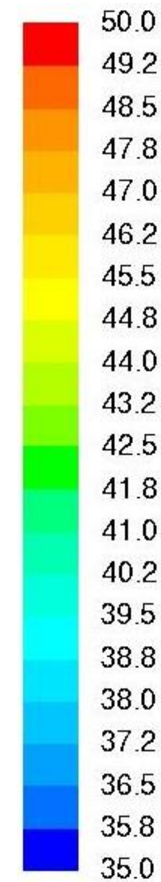
Out flow stream lines of 13F,14F,15F



Outflow air temperature clouds

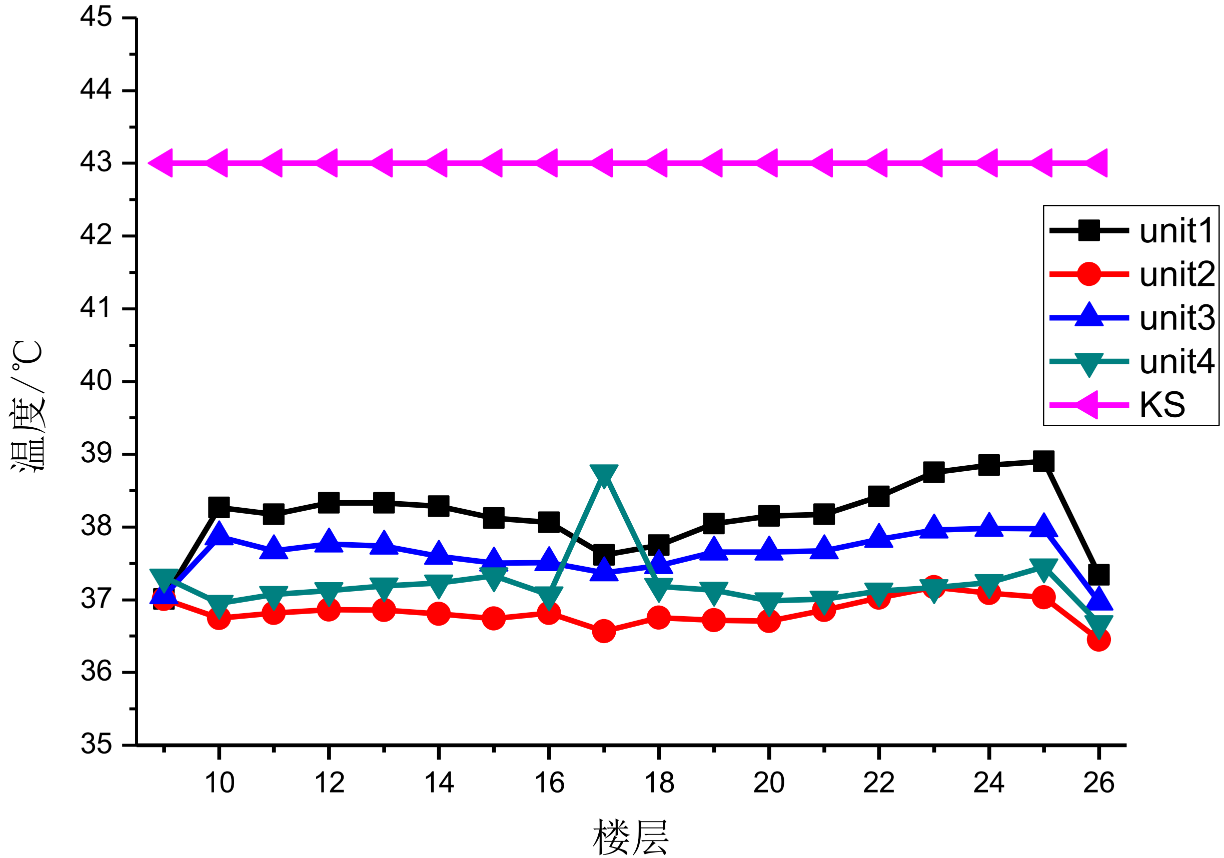


Temperature distributions of 13F,14F,15F,16F



Temperature scale：℃

Effect of floor number on the inflow temperature of air for condenser



The allowed upper limit is 43 ℃, thus the design is OK.

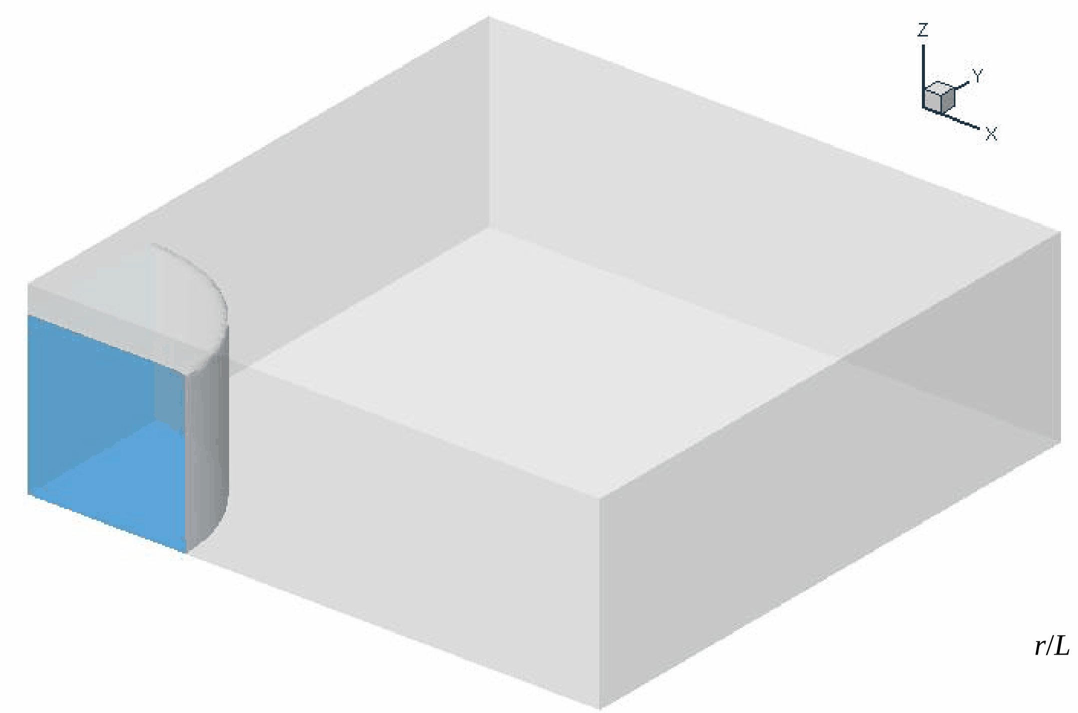
Example 10： Simulation of multiphase flow(多相流)

Visualization(可视化) of numerically predicted results：

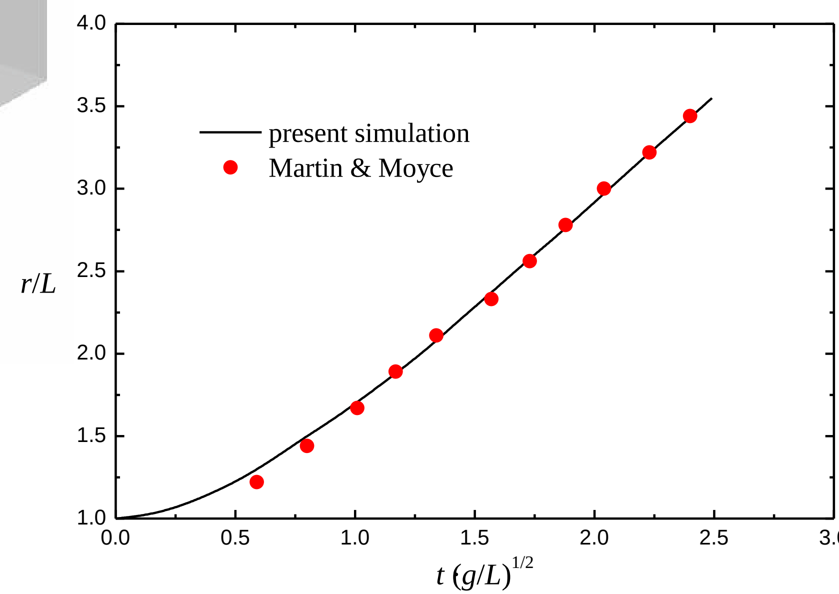
1. Breaking down of a dam (溃坝).

2. Film boiling heat transfer (膜态沸腾）；

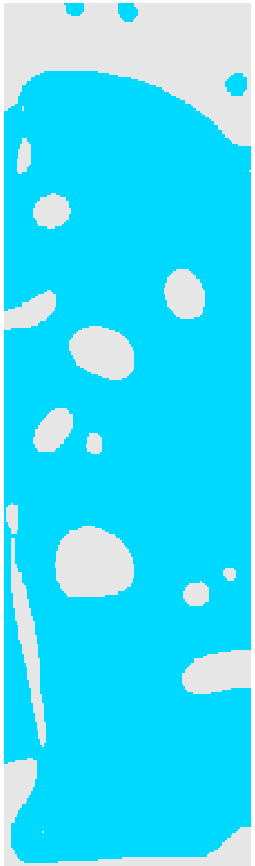
3. Nucleate boiling in shallow liquid layer（浅液层中的核态沸腾)



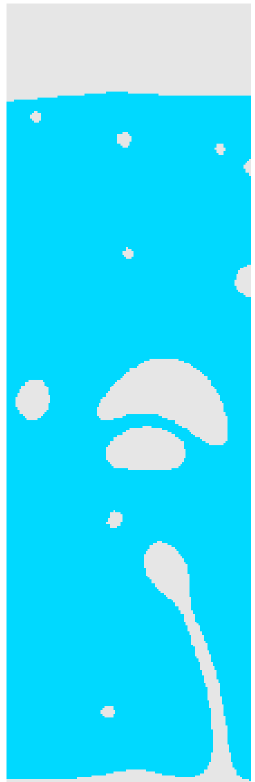
Evolution（演变）process of interface



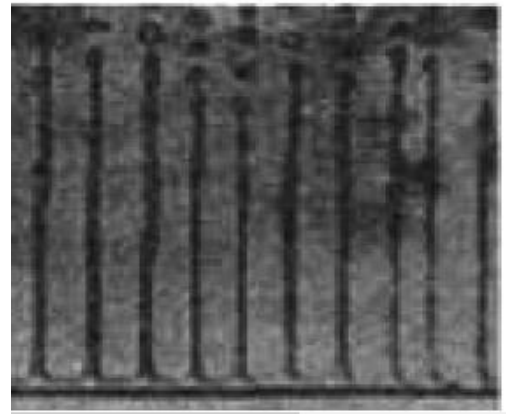
Base radius vs. time



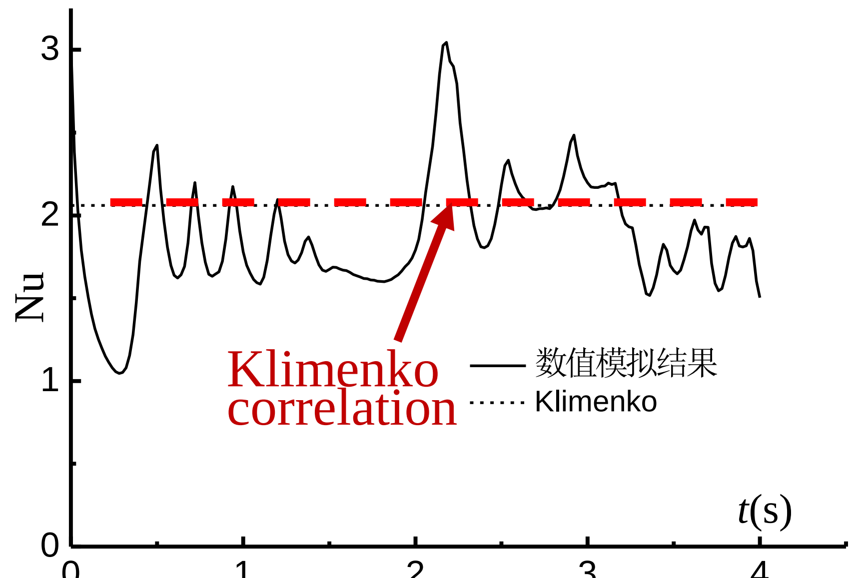
4K



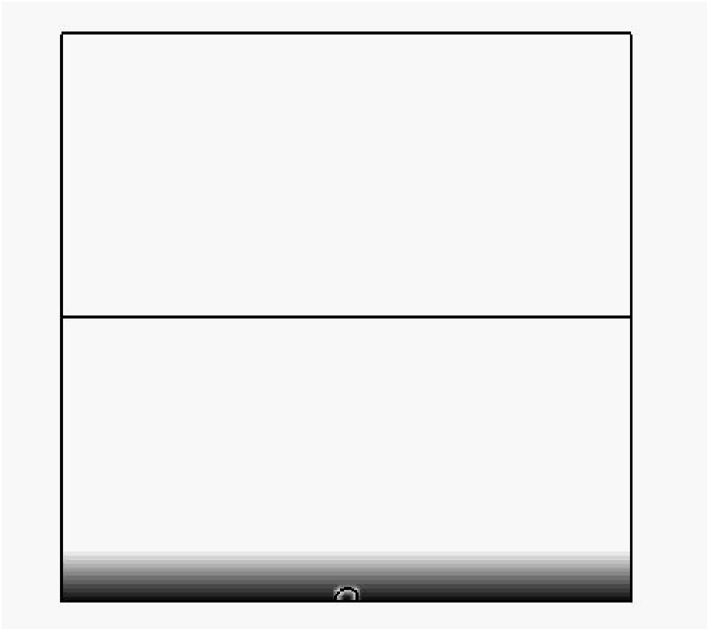
5K



Experiment by Reimann and Grigull (1975)

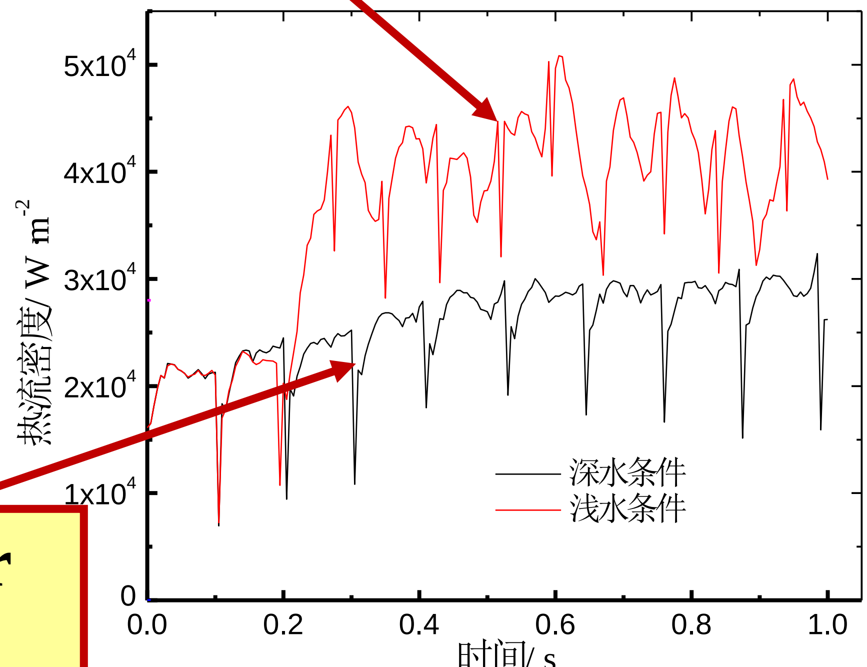


Guo D.Z., Sun D.L.,Li Z.Y., Tao W.Q., Phase Change Heat Transfer Simulation for Boiling Bubbles Arising from a Vapor Film by VOSET Method. *Numerical Heat Transfer, Part A: Applications*, 2011, 59: 857-881



Strong disturbance caused by bubble upward motion greatly enhances HT!

Shallow liquid layer(浅液层)



Deep liquid layer （深液层）

Lin K., Tao W.Q., Numerical simulation of nucleate boiling in shallow liquid, Computers & Fluids, https://doi.org/10.10.16/j.compfluid.2016.12.026

Summary

It is now widely accepted that an appropriate combination of theoretical analysis, experimental study and numerical simulation is the best approach for modern scientific research.

With the further development of computer hardware and numerical algorithm（算法）, the importance of numerical simulation will become more and more significant! （20170911）

Brief review of 2017-09-11 lecture key points

1. General governing eqs. and boundary conditions of FF & HT problems

$$\frac{\partial(\rho\phi)}{\partial t}+div(\rho\phi\vec{U})=div(\Gamma_{\phi}^{*}grad\,\phi)+S_{\phi}^{*}$$

Three kinds of B.C.:

（1） 1st： boundary value is known;

（2） 2nd： boundary heat flux is known

（3） 3rd： relationship between boundary value and boundary 1st-order normal derivative（导数） is known

$$-\lambda\left(\frac{\partial T}{\partial n}\right)_B=h(T_B-T_f)$$

2. Major concept of numerical simulation of HT & FF problems

Domain discretization	----replacing the continuum domain by a number of discrete points, called node or grid, at which the values of velocity, temp., etc., are to be solved;
↓	
Equation discretization	----replacing the governing equations (PDEs) by a number of algebraic equations for the nodes;
↓	
Solution of algebraic eqs.	----solving the algebraic equations of the nodes by a computer.

The differences in the three procedures（过程） lead to different numerical methods based on the continuum assumption.

1.2.5 Some Suggestions

1. Understanding numerical methods from basic characteristics of physical process;

2. Mastering complete picture and knowing every details (明其全，析其微) for any numerical method;

3. Practicing simulation method by a computer;

4. Trying hard to analyze simulation results: rationality (合理性) and regularity (规律性);

5. Adopting CSW(商业软件) in conjuction with self-developed code (与自编程序相结合).

1.3 Mathematical and physical classification (分类) of HT & FF problems and its effects on numerical solution

1.3.1 From mathematical viewpoint (观点)

1. General form of 2nd-order PDE (偏微分方程) with two independent variables (二元)

2. Basic features (特点) of three types of PDEs

3. Relationship to numerical solution method

1.3.2 From physical viewpoint

Conservative (守恒型) and non-conservative

1.3 Mathematical and physical classification of FF & HT Problems and its effects on numerical solutions

1.3.1 From mathematical viewpoint

1. General formulation of 2nd order PDEs with two IDVs

$$a\phi_{xx} + b\phi_{xy} + c\phi_{yy} + d\phi_x + e\phi_y + f\phi = g(x,y)$$

a, b, c, d, e, f can be function of x, y, ϕ

$$b^2 - 4ac \begin{cases} < 0, & \text{Elliptic} \\ = 0, & \text{Parabolic} \\ > 0, & \text{Hyperbolic} \end{cases}$$

- < 0, **Elliptic** 椭圆型（回流型）
- $= 0$, **Parabolic** 抛物型（边界层）
- > 0, **Hyperbolic** 双曲型

2. Basic feature of three types of PDEs

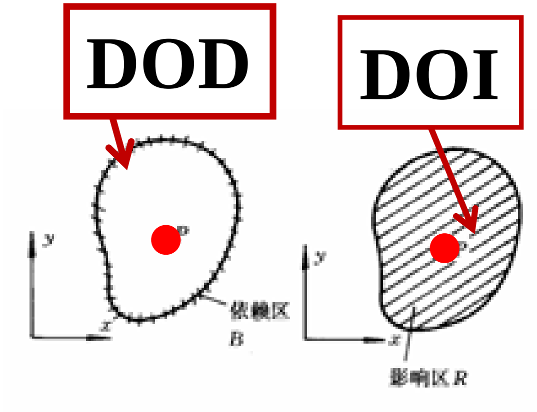
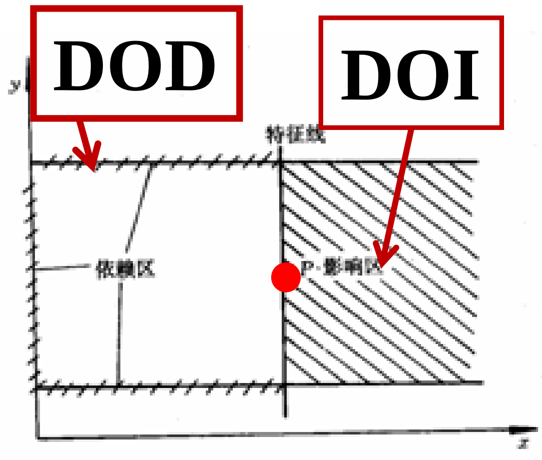
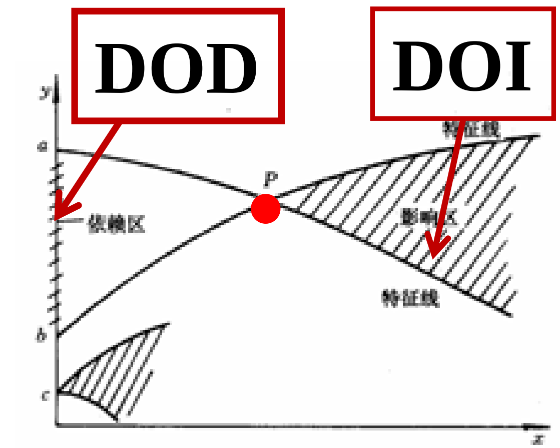
$b^2 - 4ac < 0,$ **having no real characteristic line;** **(没有实的特征线)**

$b^2 - 4ac = 0,$ **having one real characteristic line;**

$b^2 - 4ac > 0,$ **having two real characteristic lines**

leading to the difference in domain of dependence (DOD,依赖区) and domain of influence (DOI,影响区);

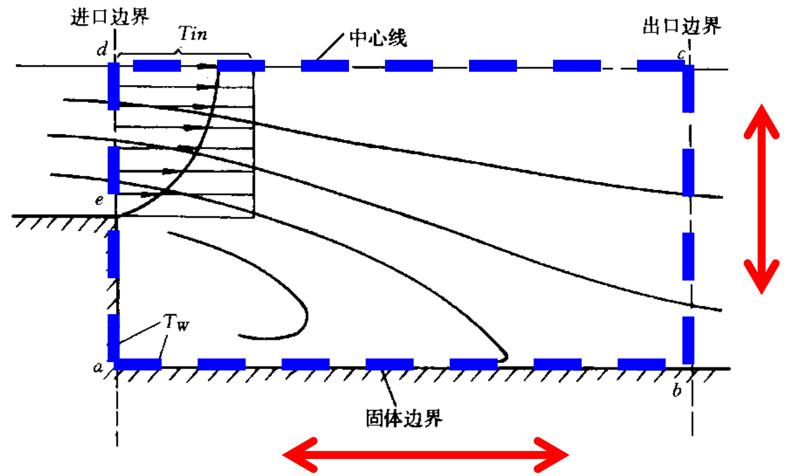
For 2-D case, DOD of a node is a line which determines the value of a dependent variable at the node; DOI of a node is an area within which the values of dependent variable are affected by the node.

Elliptic	Parabolic	Hyperbolic
		
$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$ 稳态导热 $(a=1, b=0, c=1)$	$\frac{\partial T}{\partial t}=a\frac{\partial^2 T}{\partial y^2}$ 非稳态导热 $(a=0, b=0, c=a)$	$\frac{1}{a}\frac{\partial T}{\partial t}+\frac{1}{c^2}\frac{\partial^2 T}{\partial t^2}=\frac{\partial^2 T}{\partial y^2}$ 非傅里叶导热 $(a=1/c^2, b=0, c=-1)$
$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$	$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}$	$\frac{\partial^2 \phi}{\partial t^2}=C^2\frac{\partial^2 \phi}{\partial y^2}$ $(a=1, b=0, c=-C^2)$

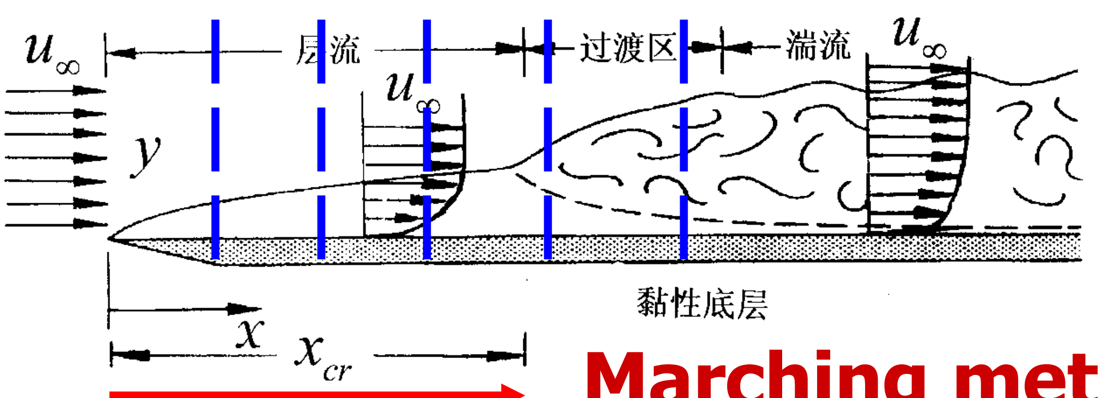
$$a\phi_{xx}+b\phi_{xy}+c\phi_{yy}\ldots$$

3. Relationship with numerical methods

（1）**Elliptic**：flow **with recirculation（回流）**，solution should be conducted simultaneously for the whole domain；



（2）**Parabolic**： flow **without recirculation** ，solution can be conducted by marching method, greatly saving computing time!。



Marching method

1.3.2 From physical viewpoint

1. Conservative (守恒型） vs. non-conservative（非守恒型）：

Conservative： those governing equations whose convective term is expressed by divergence form(散度形式） are called conservative governing equation .

Non-conservative： those governing equation whose convective term is not expressed by divergence form is called non-conservative governing equation .

These two concepts are only for numerical solution.

2. Conservative GE can guarantee the conservation of physical quantity (mass, momentum ,energy , etc.) within a finite (有限大小) volume.

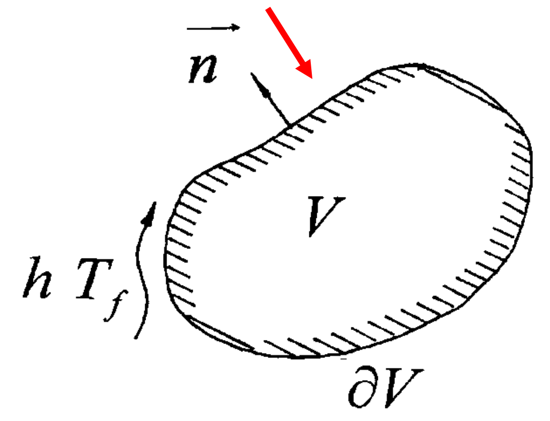
$$\frac{\partial(\rho c_p T)}{\partial t} + div(\rho c_p T\vec{U}) = div(\lambda gradT) + S_T c_p$$

$$\frac{\partial}{\partial t}\int_V (\rho c_p T)dv = -\int_V div(\rho c_p T\vec{U})dv + \int_V div(\lambda gradT)dv + \int_V S_T c_p dv$$

From Gauss theorem

$$\int_V div(\rho c_p T\vec{U})dv = \int_{\partial V} (\rho c_p T\vec{U}) \bullet \vec{n}dA$$

$$\int_V div(\lambda gradT)dv = \int_{\partial V} (\lambda gradT) \bullet \vec{n}dA$$



Dot product（点积）

$$\frac{\partial}{\partial t}\int_V (\rho c_p T)dv = -\int_{\partial V} (\rho c_p T\vec{U}) \bullet \vec{n}dA + \int_{\partial V} (\lambda gradT) \bullet \vec{n}dA + \int_V (S_T c_p)dv$$

Increment（增量） of internal energy	Energy into the region by fluid flow	Energy into the region by conduction	Energy generated by source

Exactly an expression of energy conservation!

Key to conservative form：convective term is expressed by divergence.

3. Generally conservation is expected. Discretization eqs. are suggested to be derived from conservative PDE.

4. Conservative or non-conservative are referred to（指）a finite space（有限空间）; For a differential volume（微分容积） they are identical（恒等的）!

Summary**

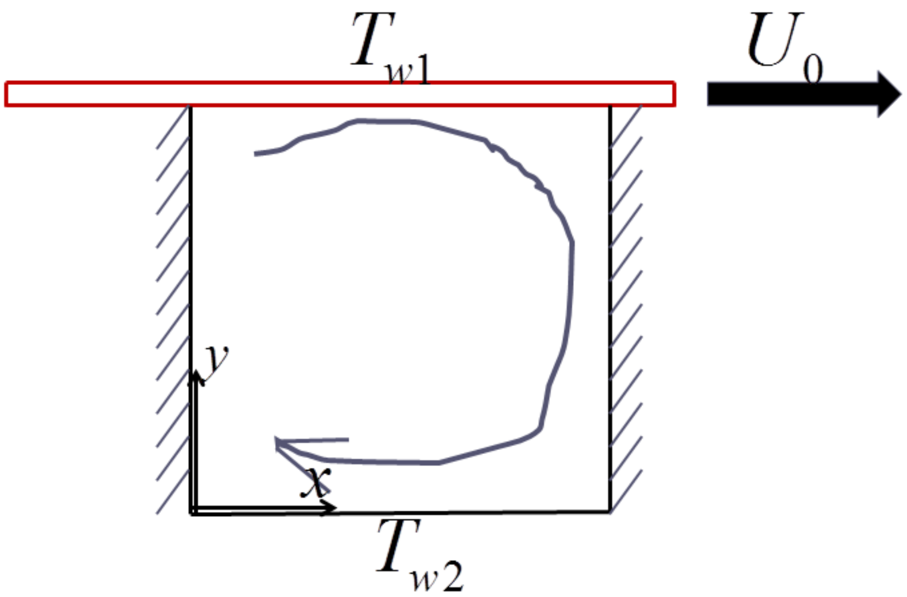
1.The governing eqs. of HT and FF are of 2nd order PDE:

$$a\phi_{xx} + b\phi_{xy} + c\phi_{yy} + d\phi_x + e\phi_y + f\phi = g(x,y)$$

and depending on the value of $(b^2 - 4ac)$, it can be elliptic, parabolic and hyperbolic;

The HT and FF problems of incompressible fluid are either elliptic, or parabolicc;

2. If the convective term of a governing eq. is expressed by the divergent form it is consevative, other-wise it is non-conservative; Discretization eqs. are suggested to be derived from conservative PDE.



Home Work 1

An infinite long solid plate with uniform temperature T_{w1} is moving at the top of a square cavity as shown in the figure. The left and right walls are adiabatic(绝热), while the bottom wall is at temperature T_{w2}. The effect of gravity can be neglected. Write down the mathematical formulation for the FF & HT in the cavity assuming steady state, laminar flow with constant properties.

Hand in with the home work of Chapter 2.

The pdf file of each chapter will be posted at our group website!

Erratum（勘误表）

1. 第3页中间：$-2/3$ 应改为 $-2/3\,\eta$

2. 第3页倒数第3行：$-\frac{\partial p}{\partial x}$ 应改为 $-\frac{\partial p}{\partial x}+\rho F_x$

   倒数第1，2行仿此修改。

3. 第4页倒数第3行：$\lambda div\mathbf{U}$ 应改为 $\lambda(div\mathbf{U})^2$

4. 第7页 式(1-18)中右端：ρ 应改为 p

5. 第9页倒数第3、4行右端：扩散项前的系数应为 ν

6. 式(1-6),(1-8)中漏了重力项。

本组网页地址：http://nht.xjtu.edu.cn 欢迎访问！

For convenience of discussion , a **qq-group** has been set up：

585614170

同舟共济

渡彼岸！

People in the same boat help each other to cross to the other bank, where….