Numerical Heat Transfer

(数值传热学)

Chapter 9 Application Examples of the General Code for 2D Elliptical FF & HT Problems

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9-1-1 Physical problem and its math formulation

Known: **Steady heat conduction of constant properties without source term shown in Fig. 1 has following temperature distribution on four boundaries:**

T=x+y+xy

Find: **Temperature distribution within the region.**

Solution: GGE $\frac{\partial(\rho^{*}\Phi)}{\partial t}+div(\rho^{*}\vec{u}\Phi)=div(\Gamma_{\Phi}grad\Phi)+S_{\phi}^{*}$

Laplace equation: $\nabla^{2}\phi=div(grad\,\phi)=0$

Compared with the standard form, it is a diffusion problem with GAMA and source term as follows:

$$\Gamma_{\phi}=\lambda=1, S_{\phi}^{*}=0$$

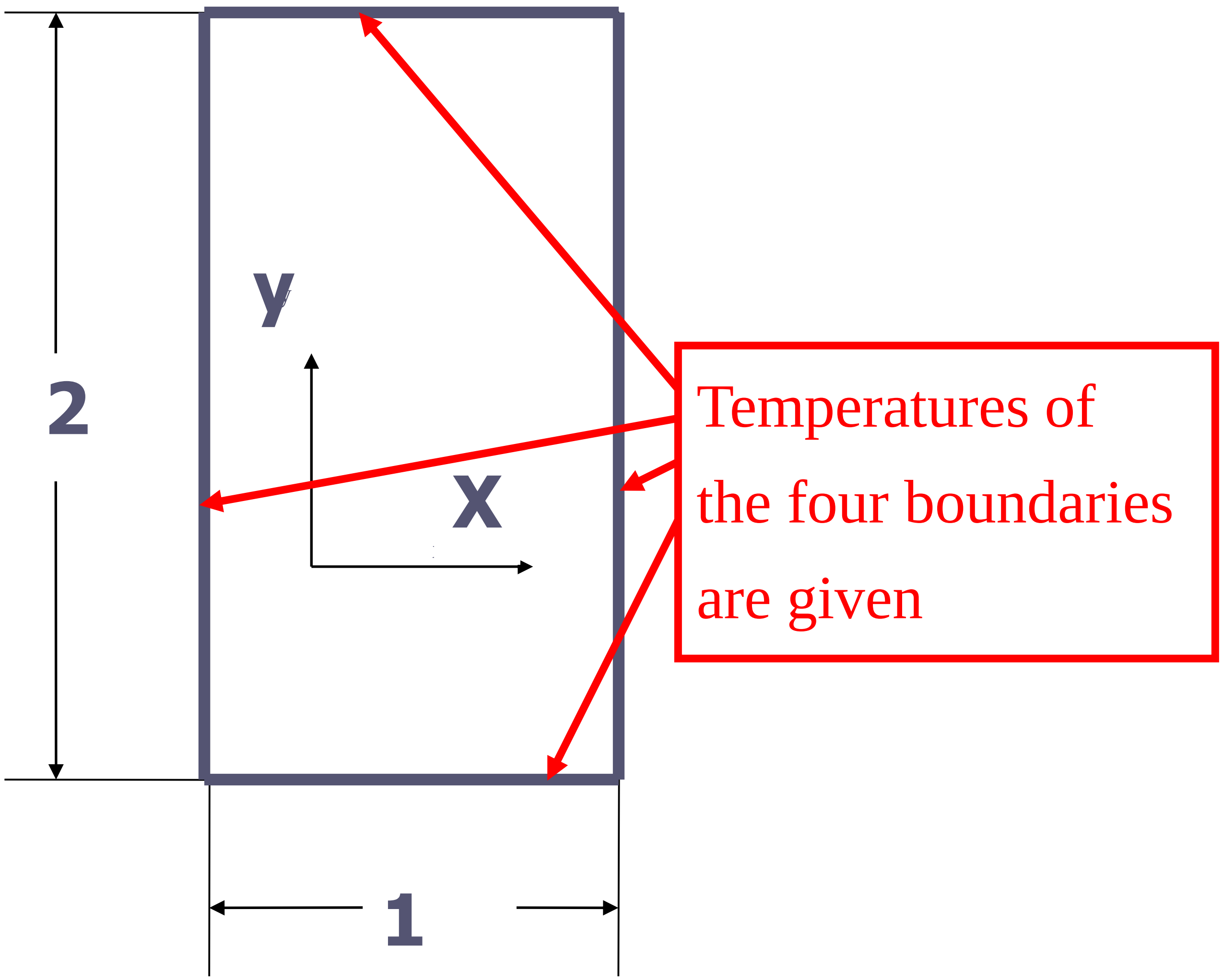


Fig.1 Computational domain

9-1-2 Program reading

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C***********************************************************
      INTEGER*4 I,J
C***********************************************************
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
     SUBROUTINE USER
C************************************************************
     USE START_L
     USE USER_L
     IMPLICIT NONE
C************************************************************
C-------------------------PROBLEM ONE-------------------------------------
C                      Example of USER structure
C************************************************************
     ENTRY GRID
     LAST=10
     LSOLVE(4)=.TRUE.
     LPRINT(4)=.TRUE.
     TITLE(4)='  .TEMP.  '  !  Title for output temperature field.
```

```
PARAMETER(NI=100,NJ=200,NIJ=NI,NFMAX=10,NFX4=NFMAX+4)
!****************************************
CHARACTER*8 TITLE(NFX4)
LOGICAL LSOLVE(NFX4),LPRINT(NFX4),LBLK(NFX4),LSTOP
REAL*8,DIMENSION(NI,NJ,NFX4)::F      !  One 3D function
```

```
      XL=1.
      YL=2.              !  MODE=1 is a default
      L1=7
      M1=7
      CALL UGRID
      RETURN

      ENTRY START
      DO 100 J=1,M1
      DO 101 I=1,L1
      T(I,J)=0.          !For inner region taking zero value.
      IF(I= =1.OR.I= = L1)  T(I,J)=(X(I)+Y(J)+X(I)*Y(J))  !Unchanged B.C.
      IF(J= =1.OR.J= = M1) T(I,J)=(X(I)+Y(J)+X(I)*Y(J))   are given here
101 ENDDO
100 ENDDO
      RETURN
*
      ENTRY DENSE        !  Empty, but keep it
      RETURN
*
      ENTRY BOUND        !  Empty，B.C. has been set up in START
      RETURN
```

```
      ENTRY OUTPUT
      IF(ITER = = 0) THEN          ! The head only needs to be out put once
      PRINT 401          ! Output to screen
      WRITE(8,401)       ! Output through file
401 FORMAT(1X,'  ITER',13X,'T(4,4)',14X,'T(5,3)')
      ELSE
      PRINT 403,    ITER,  T(4,4), T(5,3)   ! Print out two temps. in each
      WRITE(8,403) ITER,T(4,4),T(5,3)       iteration for observation
403 FORMAT(1X,I5,2F20.6)
      ENDIF
      IF(ITER = = LAST) CALL PRINT    ! Out put 2D field after
      RETURN                          getting converged solution.
*
      ENTRY GAMSOR
      IF(ITER = = 0) THEN        ! For constant property problem call once only
      DO 500 J=1,M1
      DO 501 I=1,L1
      GAM(I,J)=1.           ! The zero initial values of Sc, Sp  have been set in
501 ENDDO                   "RESET". Only GAMA is set up here.
500 ENDDO
       ELSE
       ENDIF
      RETURN
      END
```

(0)

9-1-3 Analysis of results

COMPUTATION IN CARTESIAN COORDINATES

****

! Head， resulted from Statement 401 in OUTPUT ENTRY

ITER	T(4,4)	T(5,3)
0	0.000000	0.000000
1	1.999978	1.720364
2	2.000000	1.720001
3	2.000000	1.720000
4	2.000000	1.720000
5	2.000000	1.720000
6	2.000000	1.720000
7	2.000000	1.720000
8	2.000000	1.720000
9	2.000000	1.720000
10	2.000000	1.720000

! Resulted from Statement PRINT 403， WRITE （8,403） in OUTPUT ENTRY

403 FORMAT(1X,I5,2F20.6)

2F20.6

2F-two floating-point number

20.6-Every data take 20 places; after decimal (小数点) there are 6 digits

************************** .TEMP. **************************

J \ I =	1	2	3	4	5	6	7
7	2.00E+00	2.30E+00	2.90E+00	3.50E+00	4.10E+00	4.70E+00	5.00E+00
6	1.80E+00	2.08E+00	2.64E+00	3.20E+00	3.76E+00	4.32E+00	4.60E+00
5	1.40E+00	1.64E+00	2.12E+00	2.60E+00	3.08E+00	3.56E+00	3.80E+00
4	1.00E+00	1.20E+00	1.60E+00	2.00E+00	2.40E+00	2.80E+00	3.00E+00
3	6.00E-01	7.60E-01	1.08E+00	1.40E+00	1.72E+00	2.04E+00	2.20E+00
2	2.00E-01	3.20E-01	5.60E-01	8.00E-01	1.04E+00	1.28E+00	1.40E+00
1	0.00E+00	1.00E-01	3.00E-01	5.00E-01	7.00E-01	9.00E-01	1.00E+00

From initial field

(1)

COMPUTATION IN CARTESIAN COORDINATES
****

The above printed title for coordinate is the results of implementing following statements;

(1) In the GRID of USER we accept the default value of MODE=1;

(2) Format statement at the beginning of SETUP:

1 FORMAT(//15X,'COMPUTATION IN CARTESIAN COORDINATES')

(3) Write statement at the end of SETUP1:

IF(MODE= = 1) WRITE(8,1)

************************** .TEMP. **************************

The above printed title for the temperature field is the results of implementing following statements;

(1) In the GRID of UER: TITLE(4)=' .TEMP. '

(2) In the SUPPLY of main program:

10 FORMAT(1X,26(1H*),3X,A10,3X,26(1H*))

(3) In the ENTRY PRINT: WRITE(8,10) TITLE(NF)

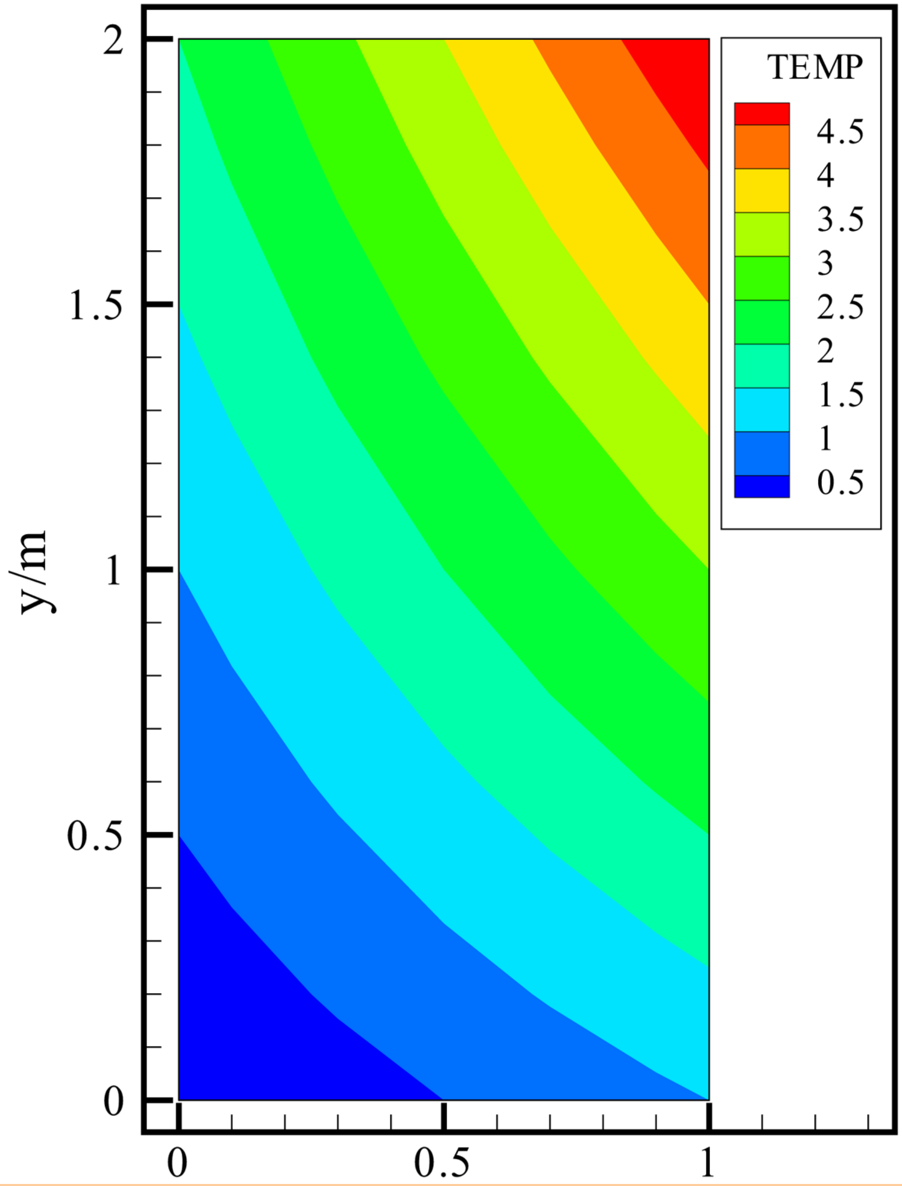


Fig. 2 Isotherms from TECPLOT

9-2 Steady heat conduction in a hollow cylinder
---ASTM for 2nd and 3rd boundary conditions

9-2-1 Physical problem and its math formulation

Known: **Steady heat conduction in a hollow cylinder with variable property and source term shown in Fig. 1 has following boundary conditions:**

Left boundary---given temperature:

T=100(1+y)

Right boundary---convective heat transfer:

Heat transfer coefficient H=5;

Fluid temperature Tf=100.

Top boundary---adiabatic;

Bottom boundary---given heat flux: Q=50

Entire region---non-linear source term:

S=100-0.5T

Thermal conductivity---for most region, $\lambda = 1$

in a local region $\lambda = 0.2(1 + T/100)$

Remarks: **In all examples, physical quantities are only given by their numerical values without units. It is assumed that all units are homogeneous(单位和谐).**

Find: **temperature distribution in the domain.**

Solution:

$$div(\Gamma_{\phi} grad\,\phi) + S_{\phi}^{*} = 0$$

It is a conduction problem with given GAMA and source term: Γ_{ϕ} S_{ϕ}^{*}.

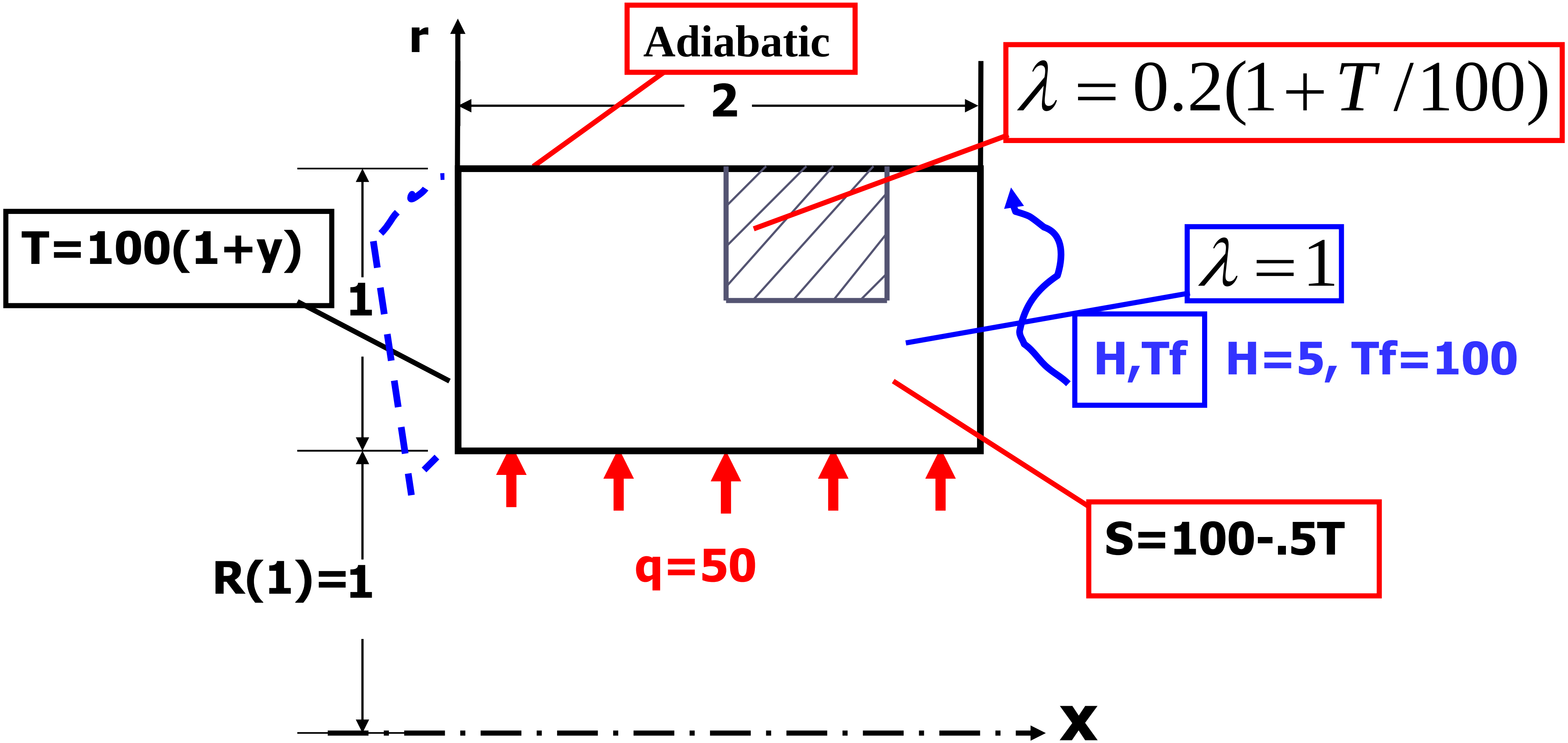


Fig.1 Computational domain

(7)

9-2-2 Program reading

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C**********************************************************
      INTEGER*4 METHOD, I, J
      REAL*8 HTC, TF, GAM1, GY, RES,ARES
C**********************************************************
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
     SUBROUTINE USER
C***********************************************************
     USE START_L
     USE USER_L
     IMPLICIT NONE
C***********************************************************
C-------------------------------PROBLEM TWO------------------------------------
C     Two –dimensional steady-state heat conduction in a hollow cylinder
C-----Implementation of ASTM and comparison with updating method------
C-----------------------------------------------------------------------------------
C***********************************************************
```

ENTRY GRID

```
TITLE(4)= ‘  .TEMP.  ‘     ! Title for temperature field print out
LSOLVE(4)=.TRUE.
LAST=100
TITLE(13)=‘   .COND.  ‘    ! Title for variable conductivity print out
LPRINT(4)=.TRUE.           ! Regarding GAMA as the13th variable,
LPRINT(13)=.TRUE.          for print out variable conductivity
MODE=2
XL=2.
YL=1.
R(1)=1.
L1=7
M1=7
CALL UGRID
RETURN
```

Specify lengths and node numbers of domain

```
EQUIVALENCE(F(1,1,1),U(1,1)),(F(1,1,2),V(1,1)),(F(1,1,3),PC(1,1))
1, (F(1,1,4),T(1,1))
EQUIVALENCE(F(1,1,11),P(1,1)),(F(1,1,12),RHO(1,1)),(F(1,1,13)
1,GAM(1,1)),(F(1,1,14),CP(1,1))
```

(6)

```
    ENTRY START
    METHOD=1                    ! Boundary temperature updated method;
                                  While METHOD= 2 is ASTM method
    DO 100 J=1,M1
    DO 101 I=1,L1
    T(I,J)=200.
    IF(I == 1) T(I,J)=100.*(1.+Y(J))   ! Specify left boundary temperature
101 ENDDO
100 ENDDO
    HTC=5.
    Q=50.
    TF=100.
    GAM1=1.    ! Set up conductivity value for main body
    RETURN
*
    ENTRY DENSE
    RETURN                      ! Empty, but keep it
```

Specify boundary condition parameters

(5)

```
    ENTRY BOUND
    DO 300 I=2,L2                          ! For METHOD 1, updated temperature
    T(I,M1)=T(I,M2)                        ! For METHOD 2, getting
    T(I,1)=T(I,2)+Q*YDIF(2)/GAM1           boundary temp. after coverged
300 ENDDO
    GY=GAM1/XDIF(L1)                       ! Temporary variable for later application
    DO 301 J=2,M2
    T(L1,J)=(HTC*TF+GY*T(L2,J))/(HTC+GY)   ! East boundary , updated temperature
301 ENDO
    RETURN
```

$$q = \lambda \frac{T(i,1) - T(i,2)}{YDIF(2)}$$

Heat transferring into the region is taken as positive!

$$T(i,1) = T(i,2) + q\frac{YDIF(2)}{\lambda}$$

$$h(T_f - T_{L1}) = \frac{\lambda}{XDIF(L1)}(T_{L1} - T_{L2}) = GY(T_{L1} - T_{L2})$$

$$hT_f + GYT_{L2} = T_{L1}(h + GY)$$

$$T_{L1} = (hT_f + GYT_{L2})/(h + GY)$$

(7)

```
      ENTRY OUTPUT
      IF(ITER= =0) THEN
      PRINT 403, METHOD
      WRITE(8,403) METHOD
403 FORMAT(1X,'   METHOD =',I 1)
      PRINT 401
      WRITE(8,401)
401 FORMAT(1X,'   ITER',11X,'T(4,5)',14X,'T(5,3)')
      ENDIF
      IF (ITER>0) PRINT 402,ITER, T(4,5), T(5.3)
      WRITE(8,402) ITER,T(4,5),T(5,3)
402 FORMAT(1X,I6,2F20.6)
      IF(ITER= =LAST) CALL PRINT
      RETURN
```

METHOD is an indicator for boundary condition treatment for 2nd and 3rd kinds

"I1" shows that the value of METHOD is expressed by an integer with one digit

! Integer has at most six digits;, 2 floating-point data with 6 digits after decimal and total length of 20 places.

```
      ENTRY GAMSOR
      DO 500 J=1,M1
      DO 501 I=1,L1          ! Specify GAMA for whole domain
      GAM(I,J)=GAM1            GAMA =Lamda
  501 ENDDO
  500 ENDDO
      DO 503 J=4,7
      DO 504 I=4,5           ! Specify variable conductivity
      GAM(I,J)=0.2*(1.+T(I,J)/100.)
  504 ENDDO
  503 ENDDO
      DO 510 J=2,M2
      DO 511 I=2,L2          ! Specify source term   S=100-.5T
      CON(I,J)=100.
      AP(I,J)=-.5
  511 ENDDO
  510 ENDDO
```

(4)

```
IF(METHOD= =1) RETURN ! Following is for ASTM
    DO 520 I=2,L2
    GAM(I,M1)=0.
    GAM(I,1)=0.
    CON(I,2)=CON(I,2)+Q*R(1)/ARX(2)
                     !  Accumulative
520 ENDDO
    RES=1./HTC+1./GY
    ARES=1./(RES*XCV(L2))
    DO 521 J=2,M2
    GAM(L1,J)=0. !  Adiabatic for ASTM
    CON(L2,J)=CON(L2,J)+ARES*TF
    AP(L2,J)=AP(L2,J)-ARES
521 ENDDO
                     !Accumulative
    RETURN
    END
```

Adiabatic (GAM(I,M1)=0., GAM(I,1)=0.)

Right wall (RES … AP(L2,J)=AP(L2,J)-ARES)

South B: $S_{c,ad}=\frac{qA}{\Delta V}=\frac{q\bullet XCV(i)\bullet R(1)}{ARX(2)\bullet XCV(i)}=\frac{q\bullet R(1)}{ARX(2)}$

$\frac{A}{\Delta V}=\frac{ARX(j)}{ARX(j)\bullet XCV(i)}=\frac{1}{XCV(i)}$

$S_{c,ad}=\frac{A}{\Delta V}\frac{T_f}{\delta x/\Gamma+1/h}=$

$=\frac{1}{XCV(i)}\frac{1}{\delta x/\Gamma+1/h}T_f$

RES ($\delta x/\Gamma+1/h$)

ARES ($\frac{1}{XCV(i)}\frac{1}{\delta x/\Gamma+1/h}$)

$S_{P,ad}=-\frac{1}{XCV(i)}\frac{1}{\delta x/\Gamma+1/h}$

9-2-3 Results analysis

COMPUTATION FOR AXISYMMETRICAL SITUATION

****

METHOD =1

ITER	T(4,5)	T(5,3)
0	200.000000	200.000000
1	196.503891	193.806549
2	194.450150	190.325912
3	192.184113	187.114395
4	189.861618	184.072250
5	187.567535	181.222870
6	185.361771	178.597488
7	183.282364	176.208923
8	181.350449	174.055115
9	179.575180	172.125107
10	177.957458	170.403229

Initial field

(1)

11	176.492798	168.871887
12	175.173325	167.513016
13	173.989273	166.309189
14	172.930008	165.243973
15	171.984665	164.302246
16	171.142624	163.470215
17	170.393753	162.735428
18	169.728561	162.086731
19	169.138290	161.514206
20	168.614944	161.008957
21	168.151245	160.563156
22	167.740601	160.169846
23	167.377090	159.822830
24	167.055481	159.516693
25	166.770981	159.246658

26	166.519409	159.008408
27	166.296982	158.798203
28	166.100388	158.612778
29	165.926620	158.449173
30	165.773102	158.304855
31	165.637451	158.177505
32	165.517609	158.065186
33	165.411758	157.966049
34	165.318222	157.878601
35	165.235626	157.801422
36	165.162720	157.733337
37	165.098282	157.673233
38	165.041412	157.620209
39	164.991196	157.573425
40	164.946838	157.532135
41	164.907684	157.495712
42	164.873108	157.463547

43	164.842590	157.435181
44	164.815643	157.410141
45	164.791870	157.388062
46	164.770844	157.368561
47	164.752319	157.351334
48	164.735947	157.336151
49	164.721497	157.322754
50	164.708740	157.310913
51	164.697495	157.300476
52	164.687561	157.291245
53	164.678772	157.283127
54	164.671051	157.275940
55	164.664200	157.269608
56	164.658157	157.264008
57	164.652847	157.259094
58	164.648148	157.254730
59	164.643982	157.250885
60	164.640289	157.247482

61	164.637070	157.244492
62	164.634201	157.241837
63	164.631683	157.239502
64	164.629471	157.237442
65	164.627502	157.235626
66	164.625778	157.234024
67	164.624268	157.232590
68	164.622894	157.231339
69	164.621689	157.230225
70	164.620636	157.229279
71	164.619736	157.228409
72	164.618896	157.227646
73	164.618179	157.226990
74	164.617538	157.226379
75	164.616974	157.225861
76	164.616486	157.225418
77	164.616058	157.225021
78	164.615662	157.224655
79	164.615341	157.224350
80	164.615036	157.224060

(O)

81	164.614746	157.223816
82	164.614517	157.223587
83	164.614304	157.223389
84	164.614120	157.223236
85	164.613968	157.223068
86	164.613815	157.222931
87	164.613693	157.222839
88	164.613571	157.222717
89	164.613495	157.222641
90	164.613403	157.222549
91	164.613312	157.222488
92	164.613251	157.222412
93	164.613205	157.222382
94	164.613159	157.222321
95	164.613113	157.222275
96	164.613037	157.222229
97	164.613007	157.222214
98	164.612976	157.222168
99	164.612946	157.222153
100	164.612930	157.222137

The 1st three digits after decimal unchanged during 5 iterations!

(1)

*************************** TEMP ***************************

J \ I =	1	2	3	4	5	6	7
7	2.00E+02	1.75E+02	1.70E+02	1.64E+02	1.48E+02	1.25E+02	2.00E+02
6	1.90E+02	1.75E+02	1.70E+02	1.64E+02	1.48E+02	1.25E+02	1.12E+02
5	1.70E+02	1.69E+02	1.69E+02	1.65E+02	1.49E+02	1.26E+02	1.13E+02
4	1.50E+02	1.60E+02	1.68E+02	1.66E+02	1.52E+02	1.28E+02	1.14E+02
3	1.30E+02	1.52E+02	1.68E+02	1.70E+02	1.57E+02	1.33E+02	1.16E+02
2	1.10E+02	1.49E+02	1.72E+02	1.75E+02	1.63E+02	1.39E+02	1.19E+02
1	1.00E+02	1.54E+02	1.77E+02	1.80E+02	1.68E+02	1.44E+02	2.00E+02

************************* COND *************************

J \ I =	1	2	3	4	5	6	7
7	1.00E+00	1.00E+00	1.00E+00	5.28E-01	4.95E-01	1.00E+00	1.00E+00
6	1.00E+00	1.00E+00	1.00E+00	5.28E-01	4.95E-01	1.00E+00	1.00E+00
5	1.00E+00	1.00E+00	1.00E+00	5.29E-01	4.98E-01	1.00E+00	1.00E+00
4	1.00E+00	1.00E+00	1.00E+00	5.33E-01	5.05E-01	1.00E+00	1.00E+00
3	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00
2	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00
1	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00

COMPUTATION FOR AXISYMMETRICAL SITUATION

****

METHOD =2

ITER	T(4,5)	T(5,3)
0	200.000000	200.000000
1	163.633240	156.107574
2	164.603409	157.204285
3	164.612839	157.222092
4	164.612747	157.221954
5	164.612747	157.221954
6	164.612747	157.221954
7	164.612747	157.221970
8	164.612747	157.221954
9	164.612747	157.221970
10	164.612747	157.221954
11	164.612747	157.221970
12	164.612747	157.221954

In order to keep the 1st three digits after decimal unchanged during 5 iterations, Method 1 needs 90 iterations, while Method 2 only needs 8 iterations! Speed of convergence of Method 2 is 10 times of Method 1!

(2)

13	164.612747	157.221970
14	164.612747	157.221954
15	164.612747	157.221970
16	164.612747	157.221954
17	164.612747	157.221970
18	164.612747	157.221954
19	164.612747	157.221970
20	164.612747	157.221954
21	164.612747	157.221970
22	164.612747	157.221954
23	164.612747	157.221970
24	164.612747	157.221954
25	164.612747	157.221970
26	164.612747	157.221954
27	164.612747	157.221970
28	164.612747	157.221954
29	164.612747	157.221970
30	164.612747	157.221954

31	164.612747	157.221970
32	164.612747	157.221954
33	164.612747	157.221970
34	164.612747	157.221954
35	164.612747	157.221970
36	164.612747	157.221954
37	164.612747	157.221970
38	164.612747	157.221954
39	164.612747	157.221970
40	164.612747	157.221954
41	164.612747	157.221970
42	164.612747	157.221954
43	164.612747	157.221970
44	164.612747	157.221954
45	164.612747	157.221970
46	164.612747	157.221954
47	164.612747	157.221970
48	164.612747	157.221954

49	164.612747	157.221970
50	164.612747	157.221954
51	164.612747	157.221970
52	164.612747	157.221954
53	164.612747	157.221970
54	164.612747	157.221954
55	164.612747	157.221970
56	164.612747	157.221954
57	164.612747	157.221970
58	164.612747	157.221954
59	164.612747	157.221970
60	164.612747	157.221954
61	164.612747	157.221970
62	164.612747	157.221954
63	164.612747	157.221970
64	164.612747	157.221954
65	164.612747	157.221970
66	164.612747	157.221954

67	164.612747	157.221970
68	164.612747	157.221954
69	164.612747	157.221970
70	164.612747	157.221954
71	164.612747	157.221970
72	164.612747	157.221954
73	164.612747	157.221970
74	164.612747	157.221954
75	164.612747	157.221970
76	164.612747	157.221954
77	164.612747	157.221970
78	164.612747	157.221954
79	164.612747	157.221970
80	164.612747	157.221954
81	164.612747	157.221970
82	164.612747	157.221954
83	164.612747	157.221970
84	164.612747	157.221954

85	164.612747	157.221970
86	164.612747	157.221954
87	164.612747	157.221970
88	164.612747	157.221954
89	164.612747	157.221970
90	164.612747	157.221954
91	164.612747	157.221970
92	164.612747	157.221954
93	164.612747	157.221970
94	164.612747	157.221954
95	164.612747	157.221970
96	164.612747	157.221954
97	164.612747	157.221970
98	164.612747	157.221954
99	164.612747	157.221970
100	164.612747	157.221954

For diffusion probelms further iterations after getting the converged solution will not change the results! But it is not for convective problems!

！ For METHOD＝2， all boundary temperatures will be used only after getting converged solution. In order to save time following IF statement may be added before DO –loop 300 ：

IF(METHOD= =2 .AND. ITER < LAST) RETURN

In ENTRY BOUND:

T(I,M1)=T(I,M2)
T(I,1)=T(I,2)+Q*YDIF(2)/GAM1
T(L1,J)=(HTC*TF+GY*T(L2,J))/(HTC+GY)

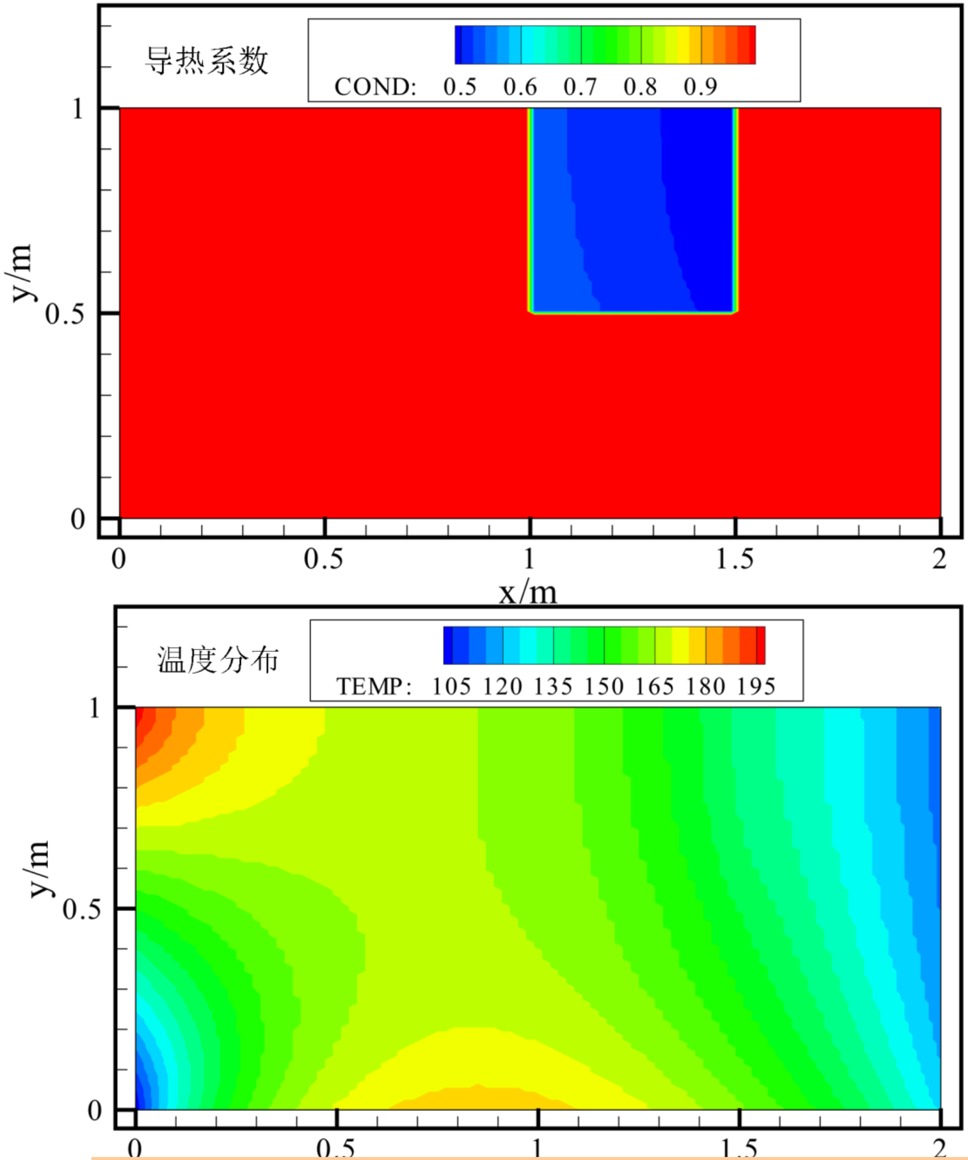


Fig.2 Computational results

Uniform heat flux south boundary

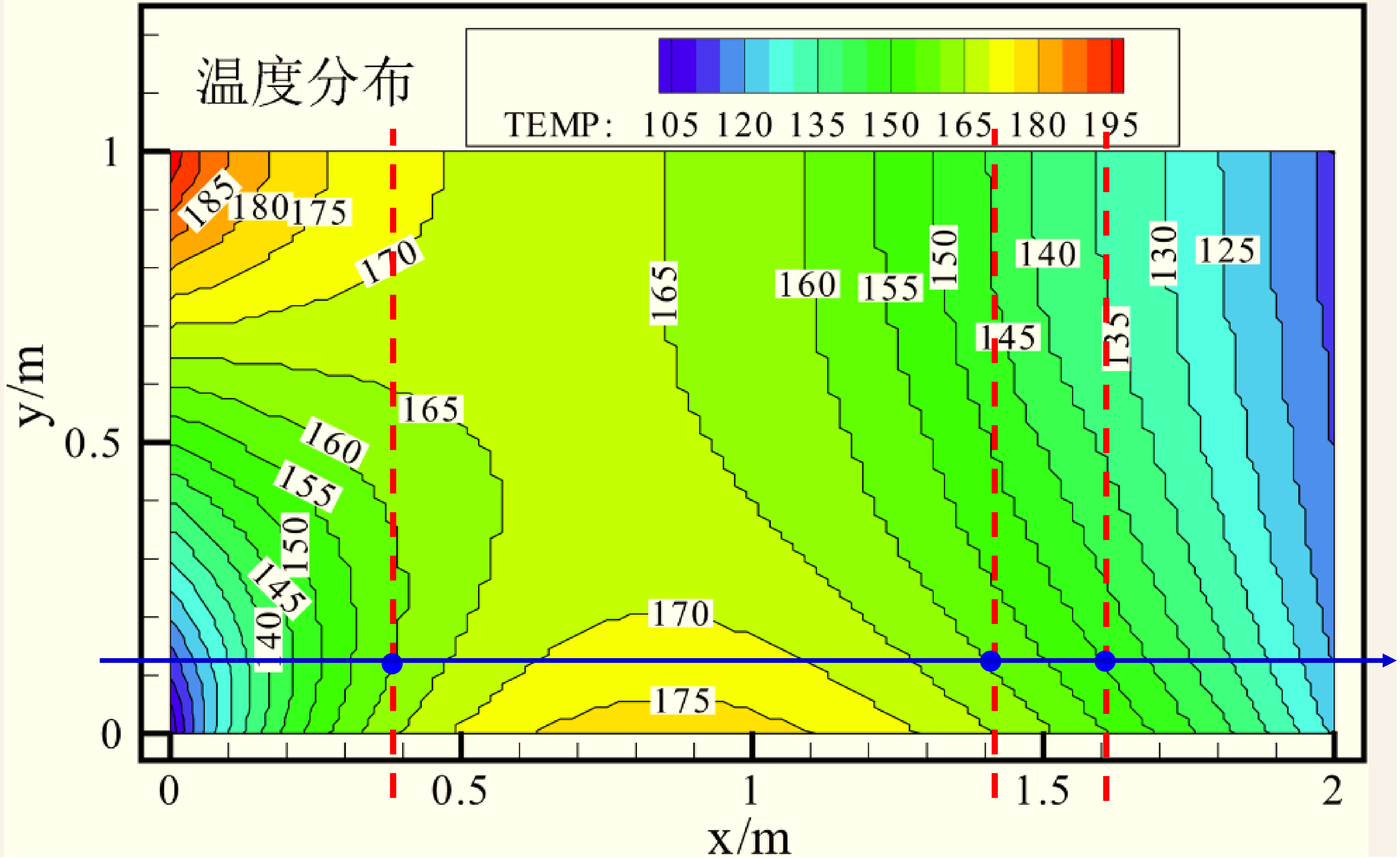


Fig.3 Isotherms

9-3 Example 3 Fully-developed heat transfer in a square duct －Numerical techniques for FDHT

9-3-1 Physical problem and its math formulation

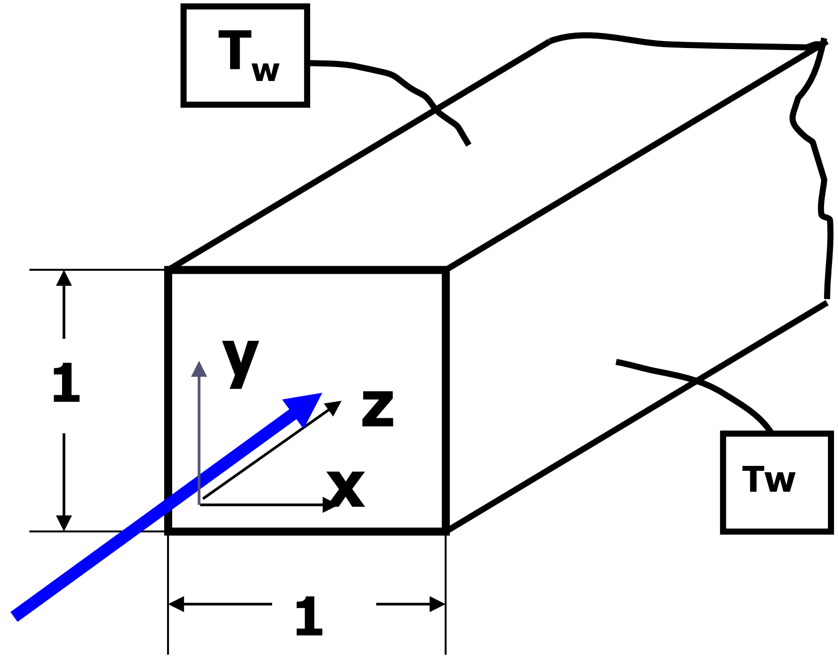
Known： Fully developed laminar heat transfer of fluid with constant properties (Fig. 1).

Find： Velocity and temperature distribution in cross section and fRe and Nu.

Solution: For fully developed laminar flow in a straight duct， cross-sectional velocity components are zero, and the axial velocity is governed by following eq.：

$$\eta\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\frac{dp}{dz}=0$$

Fig. 1 Schematic diagram of physical problem



Compared with the standard form w- eq. is of **conduction type** and following results are obtained:

$$\Gamma_{\phi} = \eta \qquad S_C = -dp/dz$$

Governing eq. for fluid temperature:

$$\rho c_p w \frac{\partial T}{\partial z} = \frac{\partial}{\partial x}(\lambda \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(\lambda \frac{\partial T}{\partial y})$$

Note: u=0, v=0

9-3-2 Numerical methods

(1) Dimensionless temperature

Defining dimensionless temperature $\Theta = \dfrac{T - T_w}{T_b - T_w}$

Then: $T = \Theta(T_b - T_w) + T_w,\quad \dfrac{\partial T}{\partial z} = \Theta \dfrac{dT_b}{dz}$

Energy eq. is transformed into following conduction eq. with source term:

$$\frac{\partial}{\partial x}(\lambda \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(\lambda \frac{\partial T}{\partial y}) - \rho c_p w \Theta \frac{dT_b}{dz} = 0$$

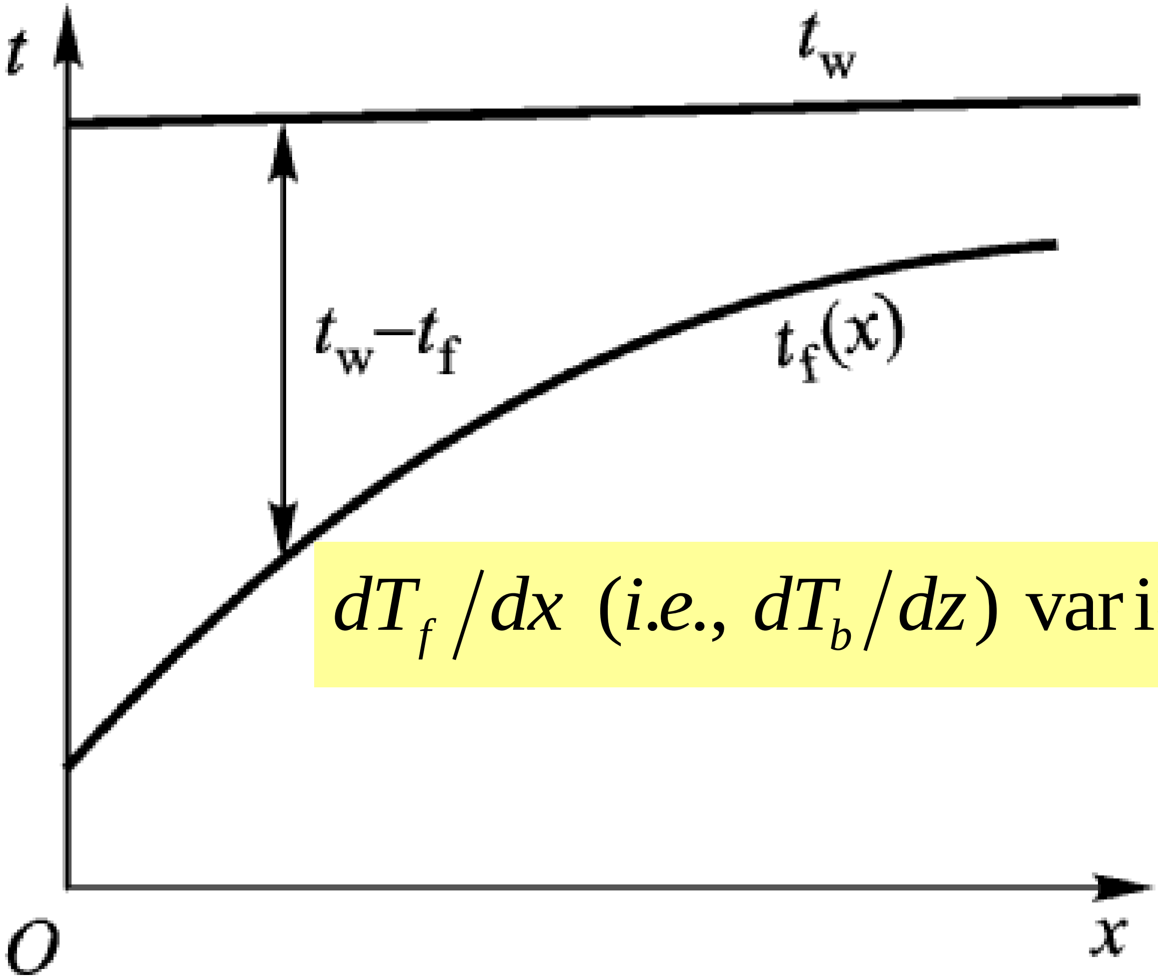
Compared with the standard form: $\Gamma_\phi = \lambda$ $\quad S_C = -\rho c_p w \Theta \dfrac{dT_b}{dz}$

(2) Numerical methods

1.This problem is governed by two conduction-type equations with source term;

2.The two equations are partially coupled: Velocity is in the source term of temperature; However, temperature is not included in w-equation. Thus w-eq. should be solved first;

3.For uniform wall temperature case， dT_b/dz does not equal constant and an assumed value can be used for simulation; The dimensionless temperature (which is included in the source term of temperature) should be updated during iteration.



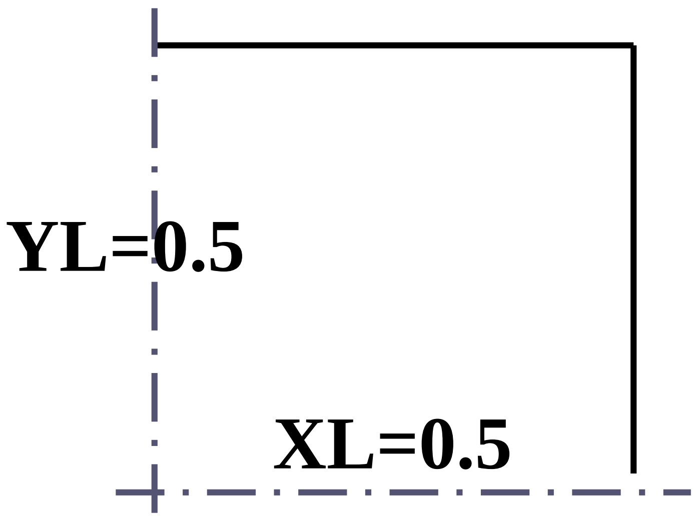
dT_f / dx ($i.e.$, dT_b / dz) varies with x

Fig. 2 Streamwise variation of fluid temperature at uniform wall temperature condition

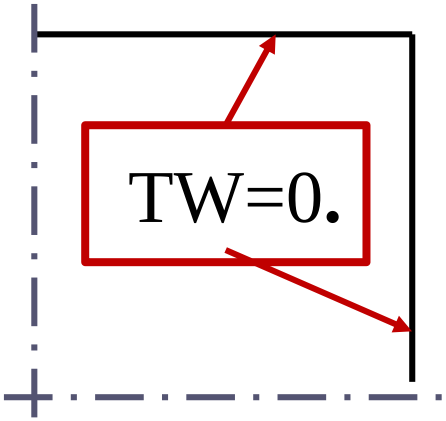
9-3-3 Program reading

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C*****************************************************************
      INTEGER*4 I,J
      REAL*8 AMU, DEN, RHOCP, DPDZ, DTBDZ, ASUM, TSUM, AR,
    1 WR, WBAR, TB, DH, RE, FRE, ANU, TW, QW, THETA, DTDZ
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
     SUBROUTINE USER
C******************************************************************
     USE START_L
     USE USER_L
     IMPLICIT NONE
C******************************************************************
C------------------------------PROBLEM THREE-----------------------------------
C     Fully developed laminar fluid flow and heat transfer in a square duct
C--------------------------------------------------------------------------------
C*****************************************************************
```

```
ENTRY GRID
TITLE(4)=‘  .THETA.  '   ! Title of dimensionless temperature for output
TITLE(5)='. W/WBAR. '    ！ Title of dimensionless velocity for output
LSOLVE(5)=.TRUE.         ！ W solved first，temperature
LPRINT(4)=.TRUE.         is not solved temporary
LPRINT(5)=.TRUE.
LAST=22
XL=0.5                   ！ Symmetric，only 1/4
YL=0.5                   domain needs to be solved
L1=7
M1=7
CALL UGRID
RETURN
```



```
ENTRY START
TW=0.
DO 100 J=1,M1
DO 100 I=1,L1
W(I,J)=0.
T(I,J)=1.          ! Set up initial fields
T(I,M1)=TW
T(L1,J)=TW         ! Set up wall temp. for east and top walls
100 CONTINUE
AMU=1.
DEN=1.             ! Set up properties; Dynamic viscosity=1
COND=1.              (very large)，to ensure laminar flow.
CP=1.
RHOCP=DEN*CP
DPDZ=-100.         ! This value must be less than zero
DTBDZ=5.           ! Fluid is heated. The value is arbitrary assumed
RETURN
```



```
      ENTRY DENSE
      RETURN
*
      ENTRY BOUND
      ASUM=0.
      WSUM=0.
      TSUM=0.
      DO 300 J=2,M2
      DO 301 I=2,L2
      AR=XCV(I)*YCV(J)
      WR=W(I,J)*AR
      WSUM=WSUM+WR
      ASUM=ASUM+AR
      TSUM=TSUM+WR*T(I,J)
  301 ENDDO
  300 ENDDO
```

! Empty, but keep it (ENTRY DENSE ... RETURN)

! Initial values for summation (ASUM=0., WSUM=0., TSUM=0.)

Element area (AR=XCV(I)*YCV(J))

WSUM=WSUM+WR ← $\iint w(i,j)dA_{i,j}$

ASUM=ASUM+AR ← $\iint dA_i$

TSUM=TSUM+WR*T(I,J) ← $\iint w(i,j)(T(i,j)dA_{i,j}$

! Average velocity

$$T_b = \frac{\iint w(i,j)(T(i,j)dA_{i,j}}{\iint w(i,j)dA_{i,j}}$$

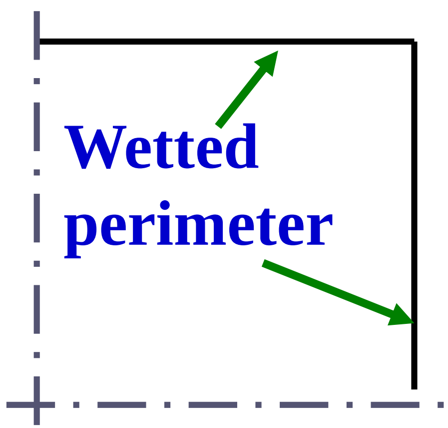
```
WBAR=WSUM/ASUM
TB=TSUM/(WSUM+1.E-30)
DH=4.*XL*YL/(XL+YL)
RE=DEN*WBAR*DH/AMU
FRE=-2.*DPDZ*DH/(DEN*WBAR**2+1.E-30)*RE
QW=DTBDZ*RHOCP*WSUM/(XL+YL)
ANU=QW*DH/(COND*(TW-TB)+1.E-30)
```

! To avoid overflow, a small value is added.

$$f\,\mathrm{Re} = -\frac{(dp/dx)D_h}{\frac{1}{2}\rho w_m^2}\mathrm{Re}$$

$$q_w = \frac{dT_b}{dz}\rho c_p \sum (w_{i,j}A_{i,j})\frac{1}{XL+YL}$$

$$Nu = \frac{hD_h}{\lambda} = \frac{D_h}{\lambda}\frac{q}{\Delta T}$$



```
      IF(ITER>10)   LSOLVE(5)=.FALSE.
      LSOLVE(4)=.TRUE.
      CONTINUE
      RETURN
*
      ENTRY OUTPUT
      IF(ITER= =0) THEN
      PRINT 401
      WRITE(8,401)
401 FORMAT(1X,' ITER',12X,'F.RE',17X,'NU')
      ELSE
      PRINT 402, ITER,FRE,ANU
      WRITE(8,402) ITER,FRE,ANU
402 FORMAT(1X,I6,1P2E20.4)
       ENDIF
      IF(ITER./=LAST) RETURN
      DO 410 J=1,M1
      DO 411 I=1,L1
      W(I,J)=W(I,J)/WBAR
      T(I,J)=(T(I,J)-TW)/(TB-TW)
411 ENDDO
410 ENDDO
      CALL PRINT
      RETURN
```

! Switch of solved variable, very useful technique

In one module, if there is only one IF statement, it can be used without THEN and ENDIF.

1P2E20.4， Scientific expression of data

! Dimensionless to make the result more general

```
      ENTRY GAMSOR
      DO 500 I=1,L1
      DO 500 J=1,M1
      GAM(I,J)=AMU
      IF(NF= = 4) GAM(I,J)=COND        ! GAMA for temperature
      GAM(I,1)=0.                      ! Symmetric=adiabatic for both V and T.
      GAM(1,J)=0.
500 CONTINUE
      IF(NF.EQ.4) GOTO 511
      DO 510 J=2,M2
      DO 510 I=2,L2
      CON(I,J)=-DPDZ                   ! Source term of W
 510 CONTINUE
      RETURN
 511 DO 520 J=2,M2
      DO 520 I=2,L2
      THEAT=(T(I,J)-TW)/(TB-TW+1.E-30) ! Updating dimensionless temp.
      DTDZ=THEAT*DTBDZ                 ! Source term of temp.
520 CON(I,J)=-RHOCP*W(I,J)*DTDZ
      RETURN
      END
```

$$S_C = -\rho c_p w \Theta \frac{dT_b}{dz}$$

9-3-4 Results analysis

COMPUTATION IN CARTESIAN COORDINATES

****

ITER	F.RE	NU
0	0.0000E+00	0.0000E+00
1	6.5168E+01	-3.8363E+00
2	5.6545E+01	-4.4212E+00
3	5.5151E+01	-4.5330E+00
4	5.4891E+01	-4.5545E+00
5	5.4841E+01	-4.5587E+00
6	5.4831E+01	-4.5595E+00
7	5.4829E+01	-4.5596E+00
8	5.4829E+01	-4.5596E+00
9	5.4829E+01	-4.5596E+00
10	5.4829E+01	-4.5596E+00
11	5.4829E+01	4.5875E+00
12	5.4829E+01	3.3408E+00
13	5.4829E+01	3.0894E+00
14	5.4829E+01	3.0361E+00
15	5.4829E+01	3.0257E+00

Energy eq. has not been solved. The values are meaningless

1P2E20.4

Switch of solved variable

16	5.4829E+01	3.0240E+00
17	5.4829E+01	3.0238E+00
18	5.4829E+01	3.0237E+00
19	5.4829E+01	3.0237E+00
20	5.4829E+01	3.0238E+00
21	5.4829E+01	3.0238E+00
22	5.4829E+01	3.0238E+00

Four digits after decimal remain unchanged in successive 6 iterations

***************************   .W/WBAR.   ****************************

J \ I =	1	2	3	4	5	6	7
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
6	0.00E+00	4.58E-01	4.34E-01	3.83E-01	2.95E-01	1.44E-01	0.00E+00
5	0.00E+00	1.12E+00	1.06E+00	9.12E-01	6.72E-01	2.95E-01	0.00E+00
4	0.00E+00	1.58E+00	1.48E+00	1.26E+00	9.12E-01	3.83E-01	0.00E+00
3	0.00E+00	1.87E+00	1.74E+00	1.48E+00	1.06E+00	4.34E-01	0.00E+00
2	0.00E+00	2.00E+00	1.87E+00	1.58E+00	1.12E+00	4.58E-01	0.00E+00
1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

No decoration before output (未作修饰)

**************************** .THETA. ****************************

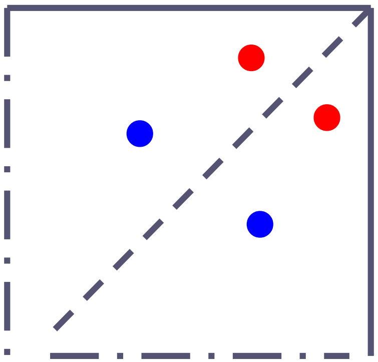
I =	1	2	3	4	5	6	7
J							
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
6	-6.63E-01	2.41E-01	2.14E-01	1.65E-01	1.02E-01	3.41E-02	0.00E+00
5	-6.63E-01	7.38E-01	6.53E-01	5.00E-01	3.07E-01	1.02E-01	0.00E+00
4	-6.63E-01	1.22E+00	1.08E+00	8.19E-01	5.00E-01	1.65E-01	0.00E+00
3	-6.63E-01	1.61E+00	1.42E+00	1.08E+00	6.53E-01	2.14E-01	0.00E+00
2	-6.63E-01	1.84E+00	1.61E+00	1.22E+00	7.38E-01	2.41E-01	0.00E+00
1	-6.63E-01	-6.63E-01	-6.63E-01	-6.63E-01	-6.63E-01	-6.63E-01	0.00E+00

No decoration(未作修饰)

Decoration: before output, set:

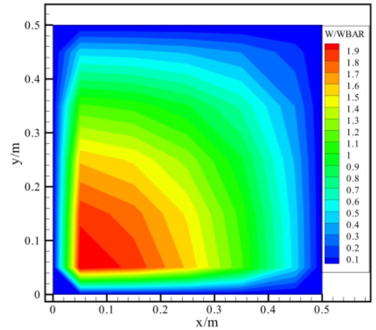
THETA (1， j) = THETA(2,j)

THETA (i,1) = THETA(i,2)

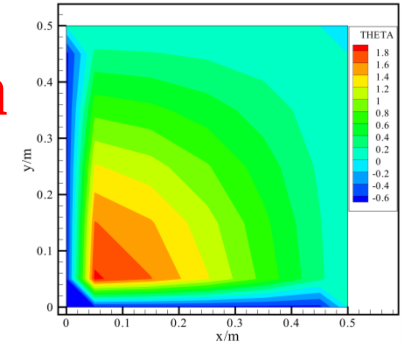


Symmetry about diagonal

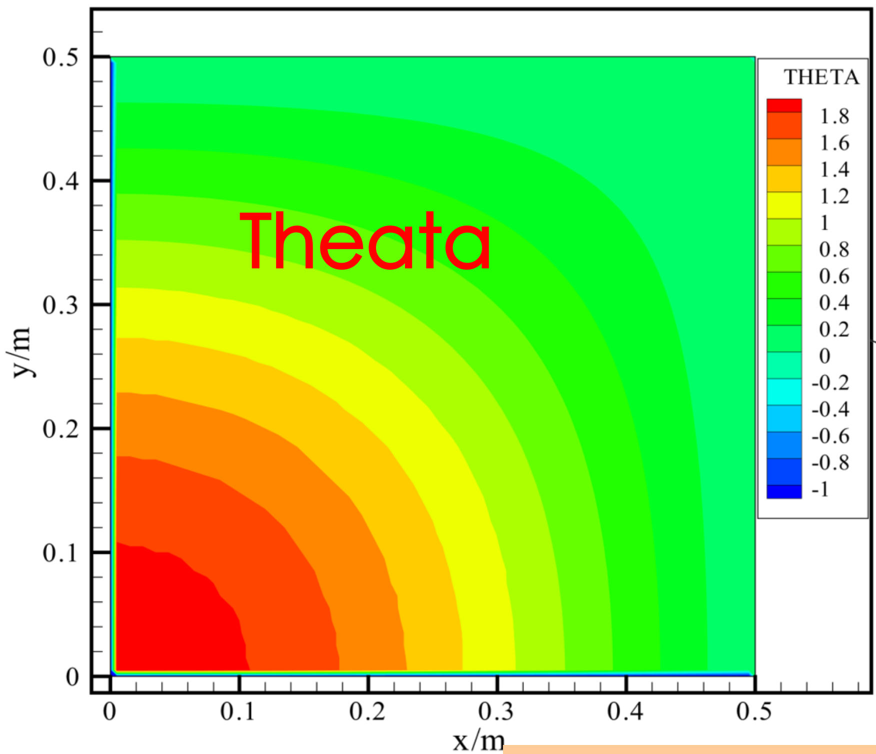
No decoration



No decoration



With decoration



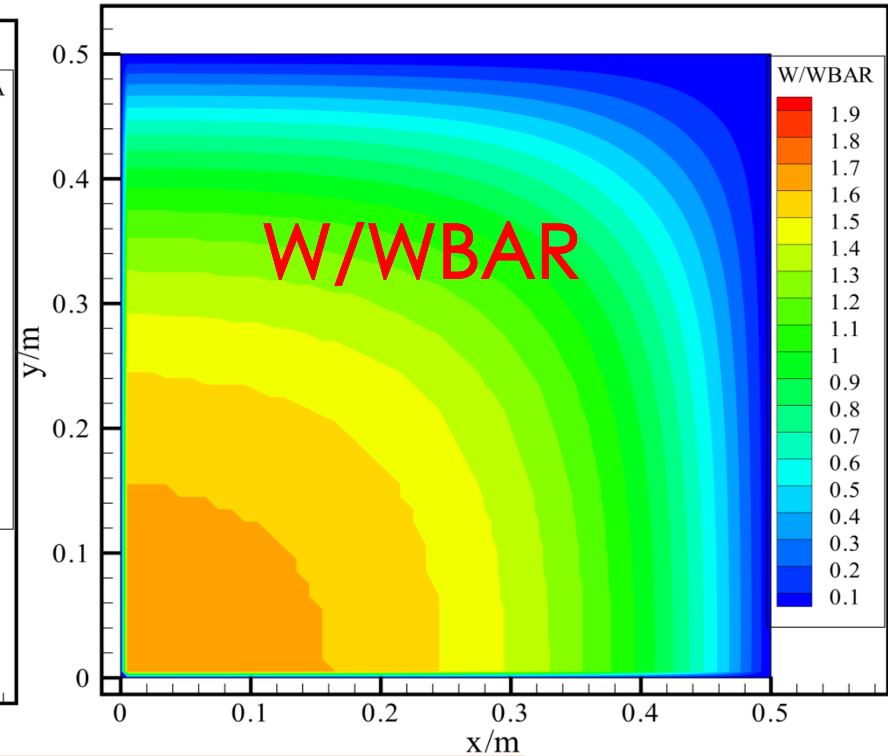


Fig. 3 Results of Problem 3

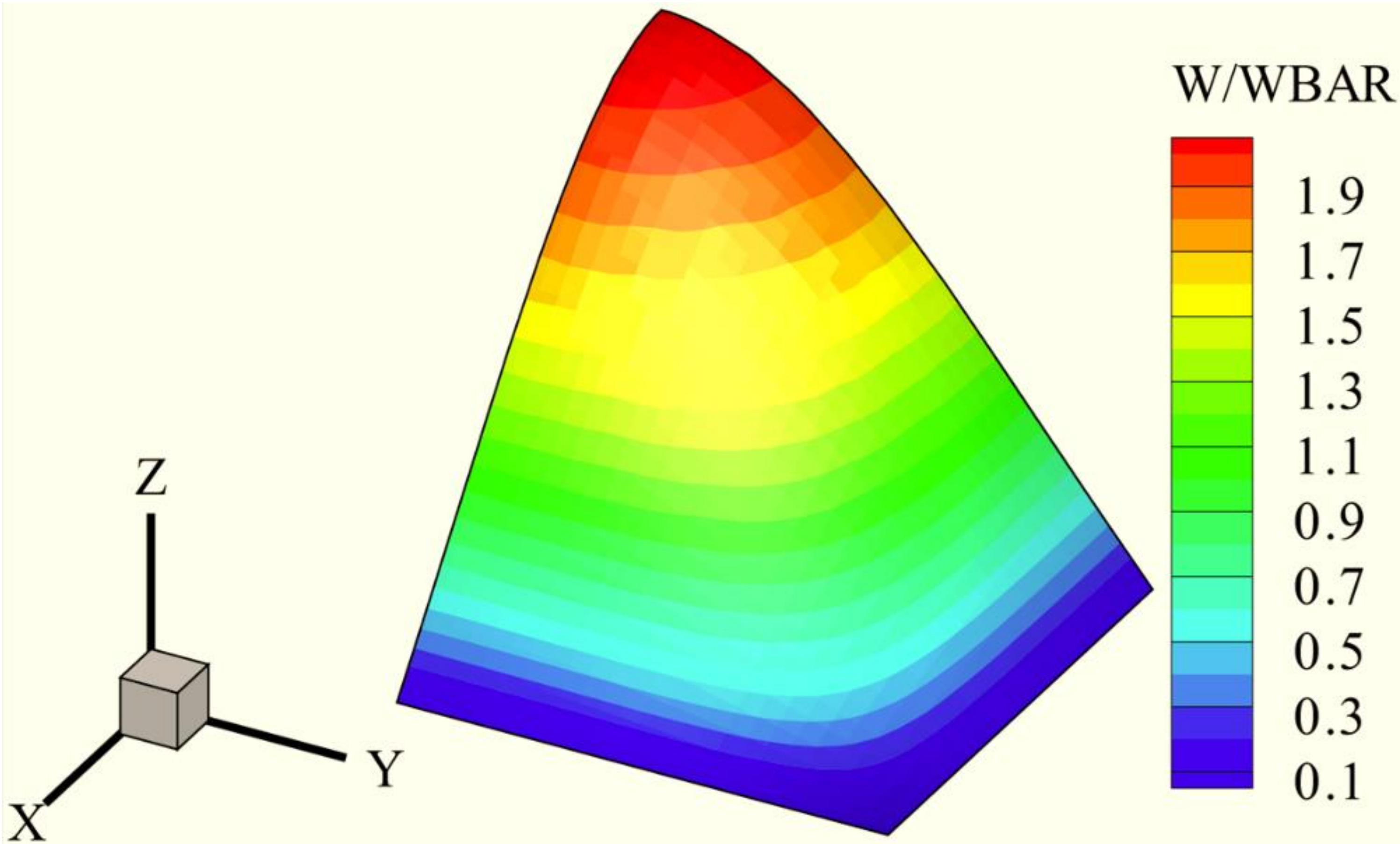


Fig. 4 Pictorial (立体)view of axial velocity distribution

9-4 Fully developed heat transfer in annular space with straight fin at inner wall
－Numerical methods for conjugated problems

9-4-1 Physical Problem and its math formulation

Known： Laminar heat transfer with constant properties in annular space with straight fins at inner wall (Fig. 1). Its outer wall is adiabatic, while inner wall temperature is circumferentially uniform（周向均匀壁温）； $R_1=1$, $R_2=2$, the angle between two successive fins equals 30^o.Ratio of fin thermal conductivity over fluid one is ten.

Find： Cross-sectional distributions of velocity and temperature, and fRe、Nu.

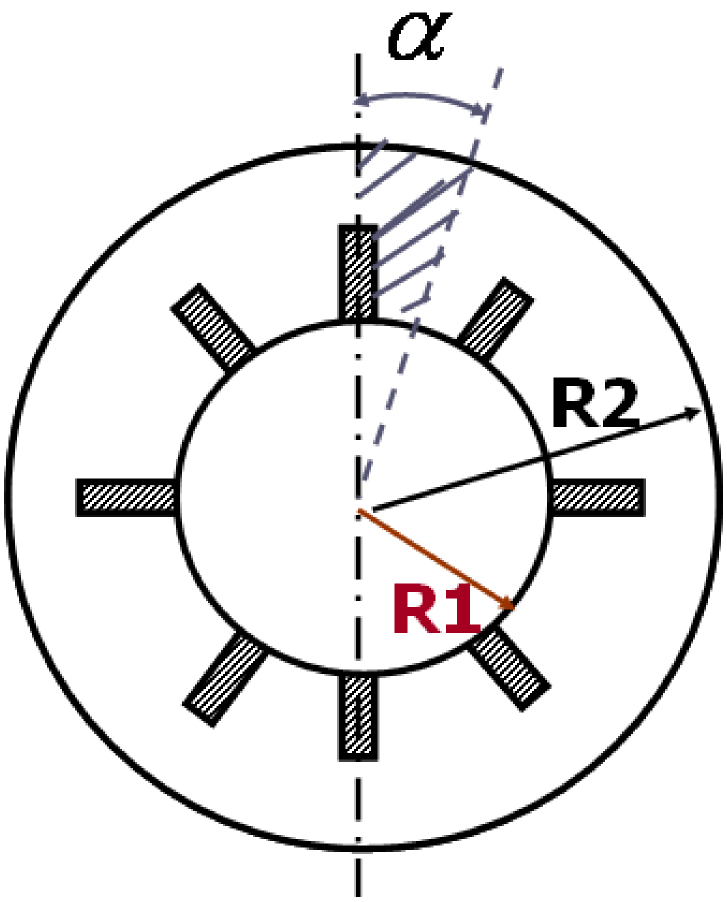
Solution： The governing eq. for axial velocity:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\eta\frac{\partial w}{\partial r})+\frac{1}{r}\frac{\partial}{\partial\theta}(\frac{\eta}{r}\frac{\partial w}{\partial\theta})-\frac{dp}{dz}=0$$

（Polar coordinate)

$div(\eta grad w)$

Source term



$$\frac{\lambda_{fin}}{\lambda_{fluid}}=10$$

$$\frac{R_2}{R_1}=2$$

$$\alpha=15^{\circ}$$

Fig.1 Cross section view of Problem 4

The governing eq. of temperature in the fully developed region：

$$div(\lambda gradT) - \rho c_p w \frac{dT}{dz} = 0$$

Source term

9-4-2 Numerical methods

（1） This problem is governed by two conduction-type equations with source term；

（2） Velocity is not coupled with temperature, and can be solved first;

（3） The fin can be regarded as a special fluid with a very large viscosity; hence the entire flow region can be solved simultaneously---conjugated problem(耦合问题）；

(4) The half of the region between two successive fins can be taken as computational domain because of symmetry；

5） In calculation of cross sectional temperature distribution it can assume that at the whole section $\partial T/\partial z = C$

6） It is assumed that the fin surface coincides with radius.

7） The fin and fluid temperatures are solved at same time (simultaneously) --- conjugated problem (耦合问题)

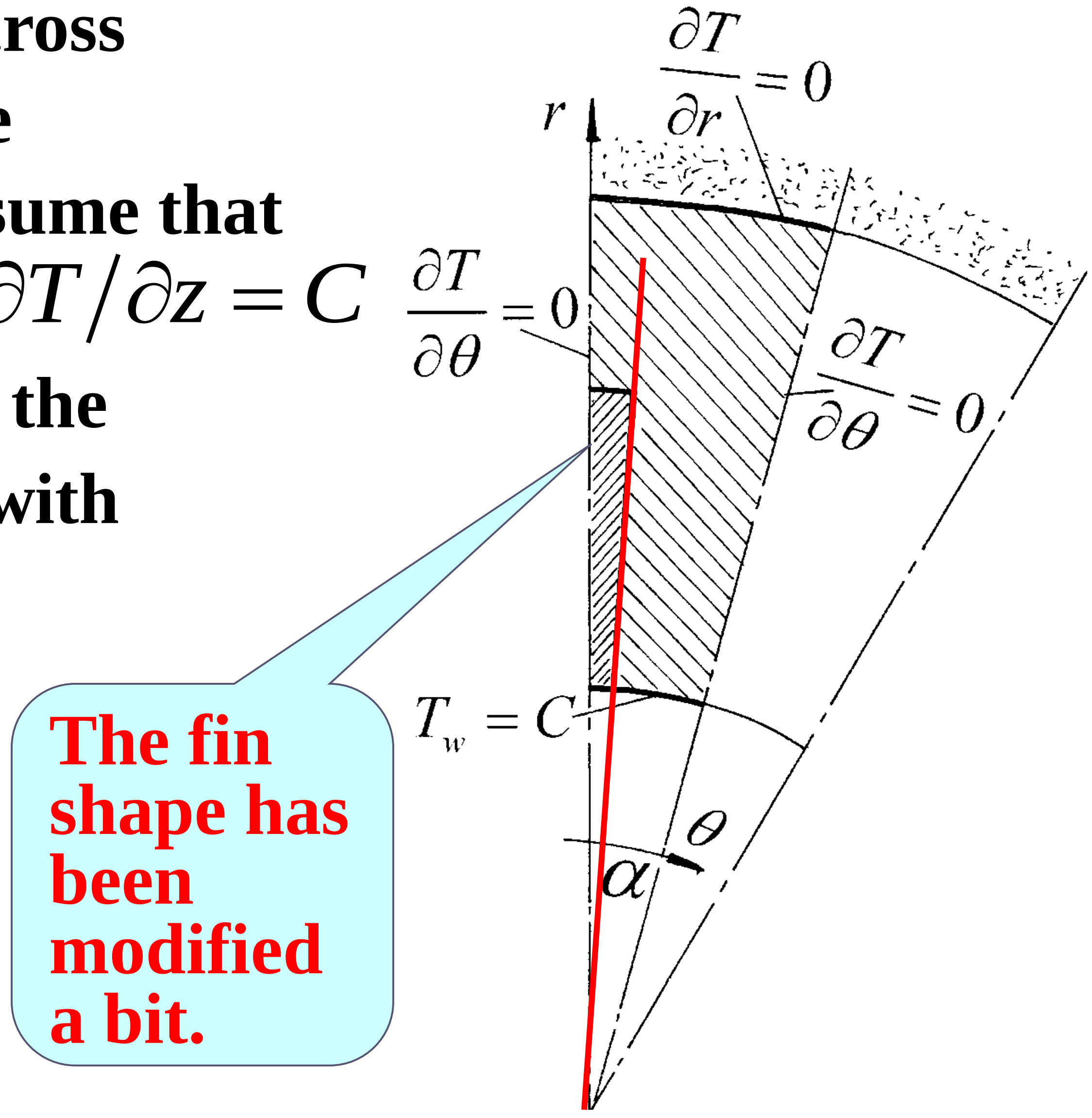


Fig. 2 Computational domain

9-4-3 Program reading

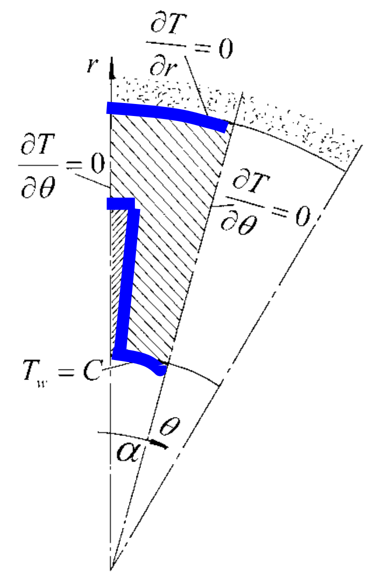
```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C*************************************************************
      INTEGER*4 I, J
      REAL*8 PI, TW, AMU, DPDZ, COND, RHOCP, DTDZ, WSUM, ASUM,
     1 TSUM, AR, WBAR, WP, DH, RE, FRE,TBULK, HTP, HTC, ANU
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
     SUBROUTINE USER
C*************************************************************
     USE START_L
     USE USER_L
     IMPLICIT NONE
C*************************************************************
C---------------------------PROBLEM FOUR--------------------------------
C     Fully developed laminar fluid flow and heat transfer in annular duct with
C----------------------------longitudinal fins on inner tube---------------------------
C*************************************************************
```

```
ENTRY GRID
TITLE(4)='.THETA.'
TITLE(5)='.W/WBAR.'
LSOLVE(5)=.TRUE.
LPRINT(4)=.TRUE.        ! Velocity solved first, temperature next
LPRINT(5)=.TRUE.
LAST=6
NTIMES(4)=4             ! Both equations are linear, NTIMES
NTIMES(5)=4               may take larger values to decrease
                          outer iteration times.
MODE=3 ! Polar coordinate
PI=3.14159              ! Transform from degree to radian
THL=15.*PI/180. (从度转化为弧度)
YL=1.
R(1)=1.    ! Specify the bottom radius
L1=7
M1=7
CALL UGRID
RETURN
```

```
ENTRY START
TW=1.   ! Set up cross sectional wall temperature
DO 100 J=1,M1
DO 101 I=1,L1
F(I,J,4)=TW
F(I,J,5)=0.       ! Initial fields of velocity and temperature
101 ENDDO
100 ENDDO
AMU=1.  ! Very large viscosity to ensure laminar flow
DPDZ=-2000. ! Pressure gradient should be less than zero
RHOCP=1.
COND=1.
DTDZ=100. ! Set up axial gradient of fluid temperature
RETURN
*
ENTRY DENSE       ! Empty, but keep it.
RETURN
```

$$AR(\text{面积元}) = YCV(j)*R(j)*XCV(i)$$
$$= YCV(j)*R(j)*THCV(i)$$
$$= YCVR(j)*THCV(i)$$

```
ENTRY BOUND
ASUM=0.
WSUM=0.
TSUM=0.                 ! Initial values for summation
DO 300 J=2,M2
DO 301 I=2,L2
IF(I>2.OR.I=2 .AND.J>4) THEN     ! Exclude(排除)solid region for flow area
AR=YCVR(J)*THCV(I)
WSUM=WSUM+F(I,J,5)*AR
TSUM=TSUM+AR*F(I,J,4)*F(I,J,5)
ASUM=ASUM+AR ! Flow area
ENDIF
301 ENDDO
300 ENDDO               ! Mean velocity
WBAR=WSUM/ASUM
WP=(R(1)+R(M1))*THL+(1.+THCV(2))*(RMN(5)-R(1))   ! Length of wetted perimeter(润湿边界的周长)
DH=4.*ASUM/WP
RE=RHOCON*WBAR*DH/AMU
FRE=-2.*DPDZ*DH/(RHOCON*WBAR**2+1.E-30)*RE
```



$$f\,\mathrm{Re} = \frac{-(dp/dx)D_h}{(1/2)\rho w_m^2}\mathrm{Re}$$

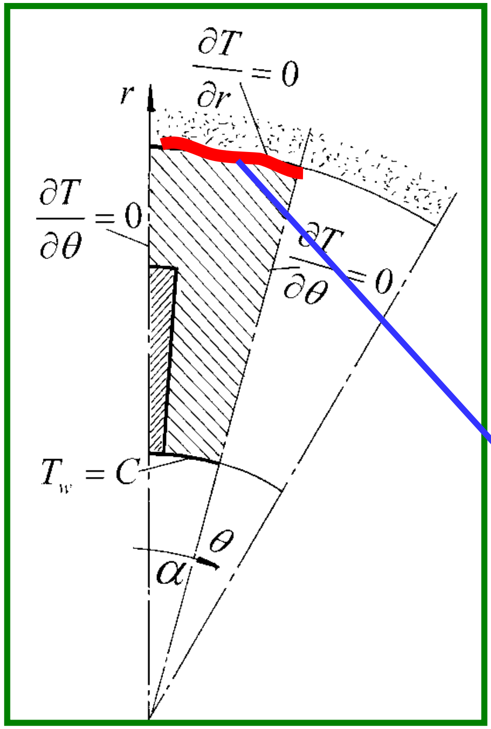
```
TBULK=TSUM/(WSUM+1.E-30)  ! Mean temperature
HTP=WP-R(M1)*THL  ! Length of perimeter for heat transfer
HTC=RHOCP*WSUM*DTDZ/((TW-TBULK+1.E-30)*HTP)
ANU=HTC*DH/COND
IF(ITER<3) RETURN
LSOLVE(5)=.TRUE.
LSOLVE(4)=.FALSE.
RETURN
```

! $Nu = hD_e / \lambda$

Switch solution variable

$$q = \rho c_p (W_m A \frac{\partial T}{\partial z}) \bullet 1/(HTP \bullet 1)$$

$$h = q/(T_w - T_b)$$

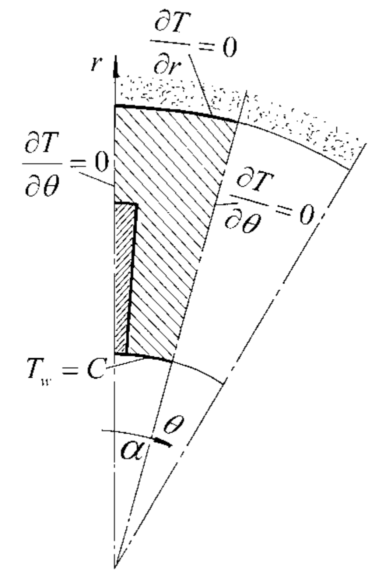


! This length is adiabatic, hence should be excluded in HTP.

```
      ENTRY OUTPUT
      IF(ITER= =0) THEN
      PRINT 401
      WRITE(8,401)
401 FORMAT(1X,' ITER',12X,'F.RE',17X,'NU')
      ELSE
      PRINT 402, ITER, FRE, ANU
      WRITE(8,402) ITER,FRE,ANU
402 FORMAT(1X,I6,1P2E20.4)
      ENDIF
      IF(ITER/=LAST) RETURN
      DO 410 J=1,M1
      DO 411 I=1,L1
      F(I,J,5)=F(I,J,5)/WBAR
      F(I,J,4)=(F(I,J,4)-TW)/(TBULK-TW+1.E-30)
411 ENDDO
410 ENDDO
      CALL PRINT
      RETURN
```

! Output of dimensionless results

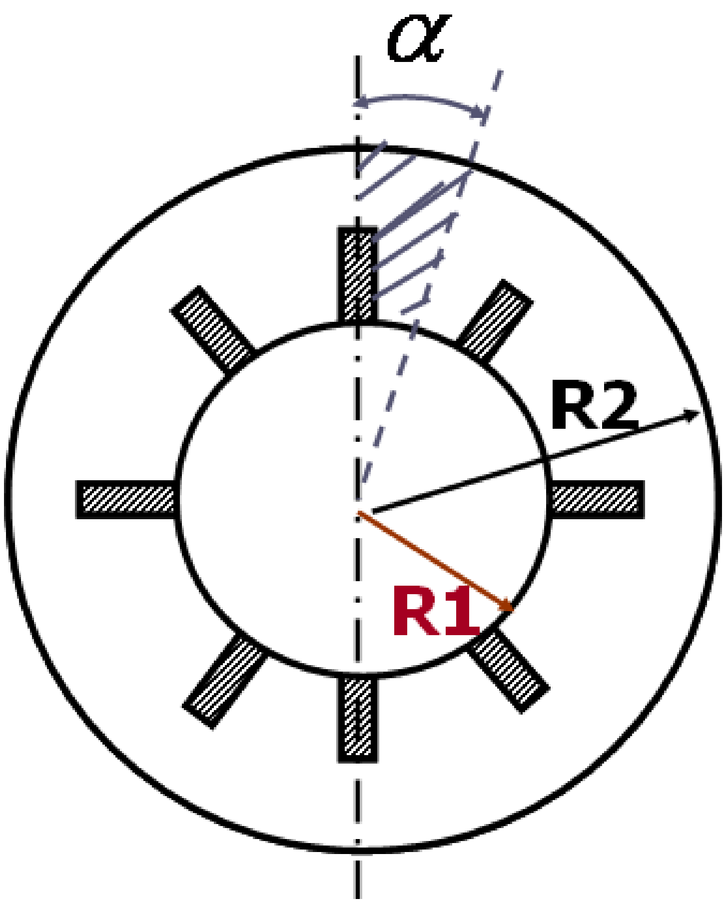
$$\Theta = \frac{T - T_w}{T_b - T_w}; \ \Theta_w = \frac{T_w - T_w}{T_b - T_w} = 0$$




```
ENTRY GAMSOR
  DO 500 I=1,L1
  DO 501 J=1,M1
  GAM(I,J)=AMU
  IF(NF= =4) GAM(I,J)=COND !  GAMA for temperature
  GAM(1,J)=0.
  GAM(L1,J)=0.               !  Symmetric=adiabatic
  IF(NF= =4) GAM(I,M1)=0.    !  North boundary is adiabatic
  IF(J<=4) GAM(2,J)=1.E10    !Fin is regarded as fluid  with large viscosity
  IF(NF= =4.AND.J<=4) GAM(2,J)=10.*COND
                             !  Fin conductivity
501 ENDDO
500 ENDDO
  DO 510 J=2,M2
  DO 511 I=2,L2
  CON(I,J)=-DPDZ             !  Source term of W-eq., less than zero
  IF(NF= =4) CON(I,J)=-DTDZ*F(I,J,4)*RHOCP
511 ENDDO
510 ENDDO
  RETURN
  END
```

! Source of Temperature eq.

$$-\rho c_p w \frac{dT}{dz}$$



$$\frac{\lambda_{fin}}{\lambda_{fluid}} = 10$$

$$\frac{R_2}{R_1} = 2$$

$$\alpha = 15^{\circ}$$

Fig.1 Cross section view of Problem 4

9-4-4 Results analysis

COMPUTATION IN POLAR COORDINATES
****

ITER	F.RE	NU
0	0.0000E+00	0.0000E+00
1	6.5484E+01	1.9787E+10
2	6.5484E+01	2.3588E+33
3	6.5484E+01	2.3588E+33
4	6.5484E+01	1.5098E+00
5	6.5484E+01	1.5098E+00
6	6.5484E+01	1.5098E+00

Solving flow only

！ NTIMES=4 ， only one outer iteration solution is converged

！ NTIMES=4 ， only one outer iteration solution is converged

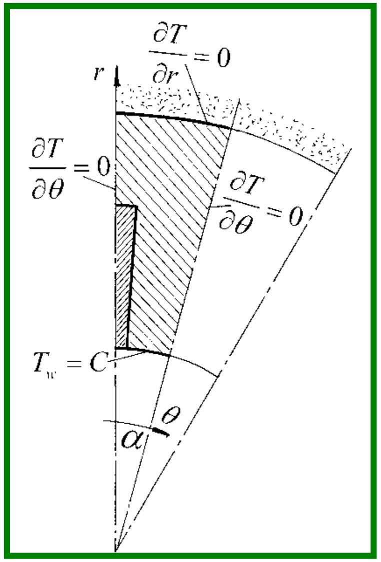
(E)

************************** .W/WBAR. **************************

I =	1	2	3	4	5	6	7
J							
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
6	0.00E+00	8.18E-01	8.50E-01	8.91E-01	9.25E-01	9.43E-01	0.00E+00
5	0.00E+00	1.10E+00	1.30E+00	1.50E+00	1.64E+00	1.72E+00	0.00E+00
4	0.00E+00	4.37E-09	4 .57E-01	1.05E+00	1.41E+00	1.58E+00	0.00E+00
3	0.00E+00	3.34E-09	3.01E-01	7.45E-01	1.03E+00	1.18E+00	0.00E+00
2	0.00E+00	1.43E-09	1.63E-01	3.91E-01	5.36E-01	6.06E-01	0.00E+00
1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

W=0 of fin region

Symmetric line, not decorated.



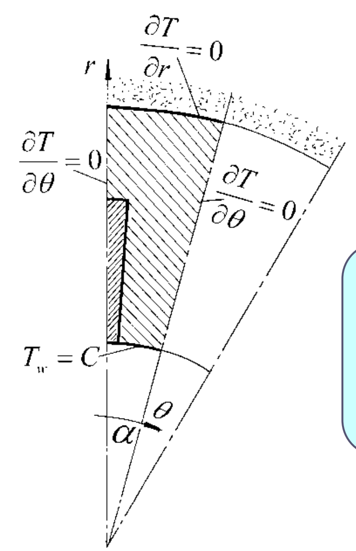
Symmetric line, not decorated

************************** .THETA. ***************************

Adiabatic, not decorated

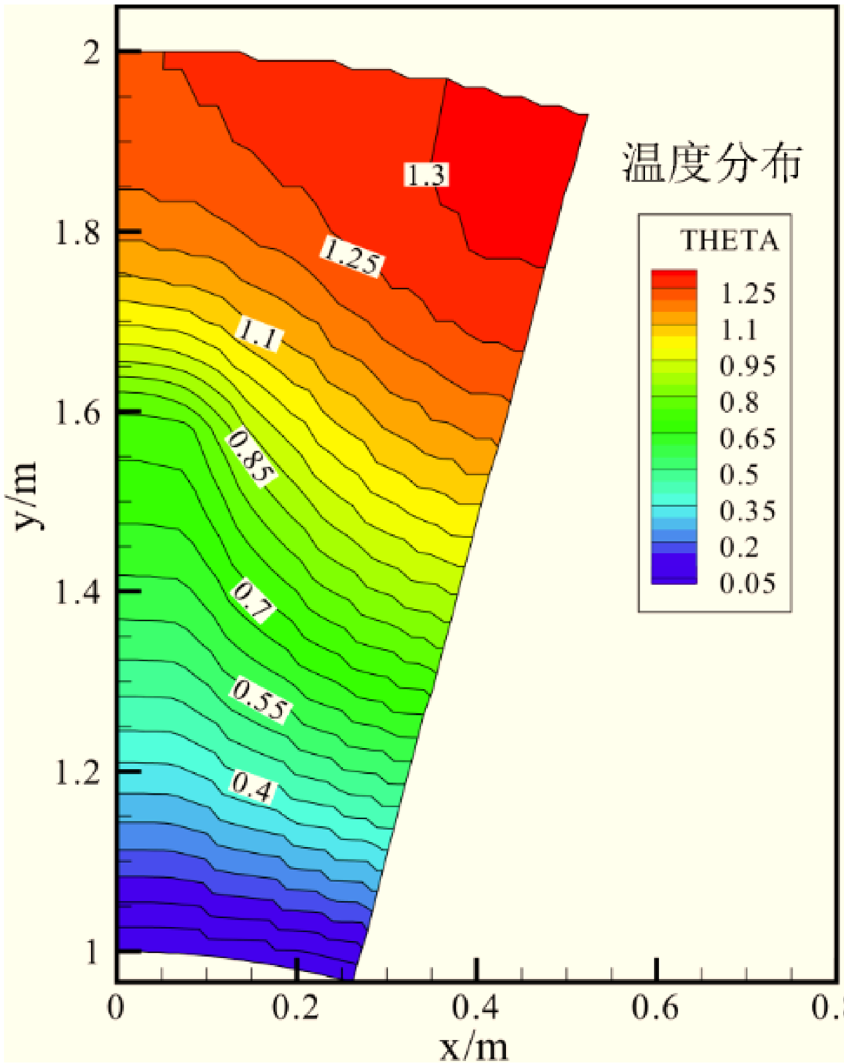
J \ I =	1	2	3	4	5	6	7
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
6	0.00E+00	1.24E+00	1.26E+00	1.28E+00	1.30E+00	1.31E+00	0.00E+00
5	0.00E+00	1.03E+00	1.09E+00	1.15E+00	1.19E+00	1.21E+00	0.00E+00
4	0.00E+00	6.34E-01	7.15E-01	8.24E-01	8.96E-01	9.32E-01	0.00E+00
3	0.00E+00	4.48E-01	4.80E-01	5.36E-01	5.78E-01	6.00E-01	0.00E+00
2	0.00E+00	1.76E-01	1.86E-01	2.04E-01	2 .18E-01	2.26E-01	0.00E+00
1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

$$\Theta = \frac{T - T_w}{T_b - T_w} = 0$$



Symmetric line, not decorated.

Symmetric line, not decorated



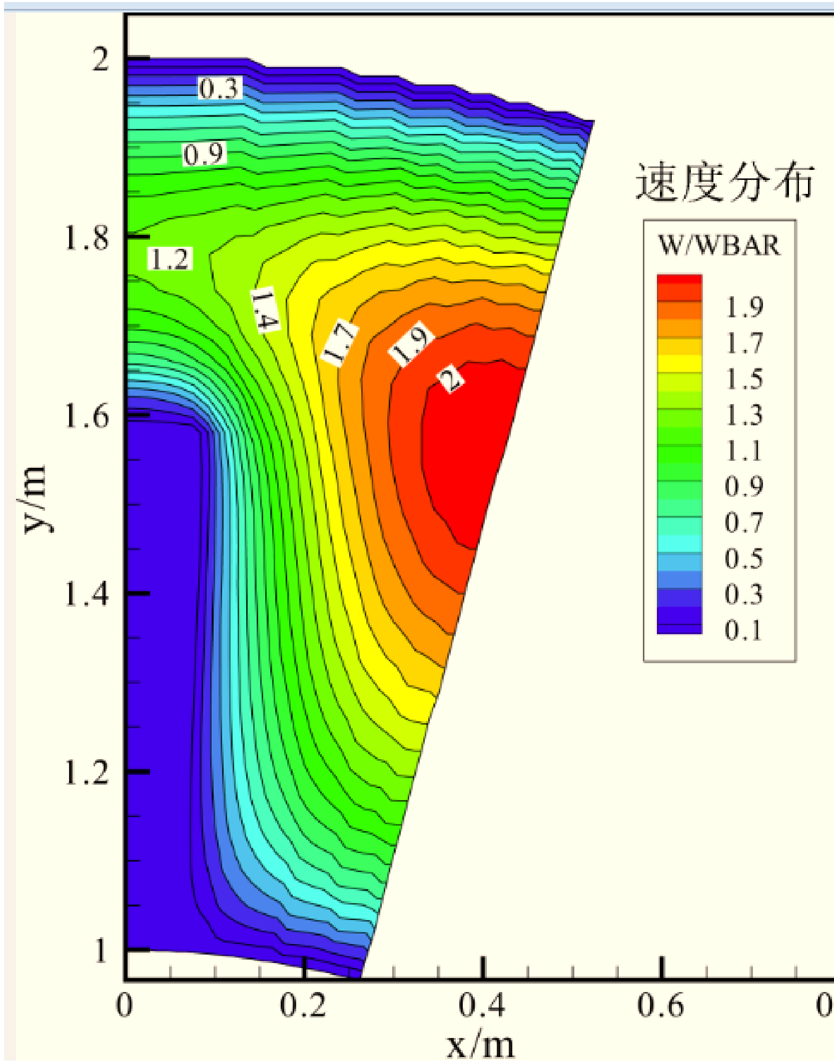


Fig.3 Result of Problem 4

9-5 Fluid flow and heat transfer in a 2-D sudden expansion---Solution of Navier Stokes equation

9-5-1 Physical problem and its math formulation

Known: **Laminar flow and heat transfer in a parallel duct shown in Fig. 1 : Uniform inlet velocity,** $V_{in}=100$, **and uniform inlet temperature,** $T_{in}=50$; **Duct wall are at uniform temperature,** $T_w=300$. **Fluid** Pr=0.7, **molecular dynamic viscosity** $\mu=1$, **density varies according to:**

$$\rho = \rho_{ref} \frac{T_{ref}}{T}$$

where referenced density ρref=1, and referenced temperature Tref=300.

Find: **Distributions of velocity, temperature, density and fluid pressure in the duct.**

$V_{in}=100$

$T_{in}=500$

$T_w=300$

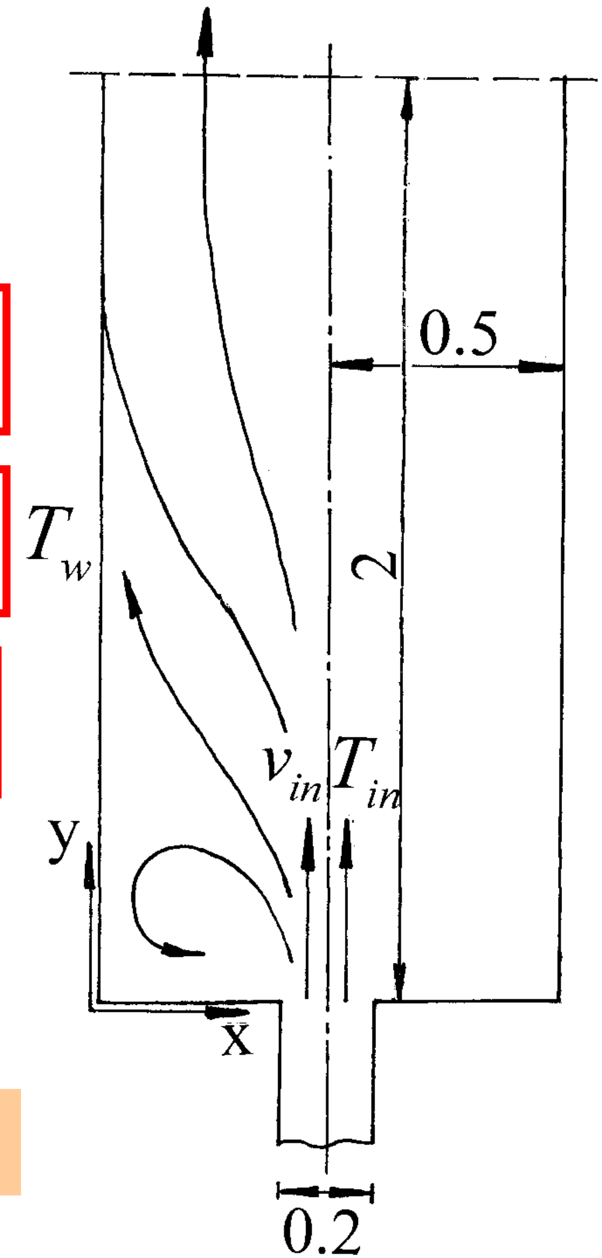


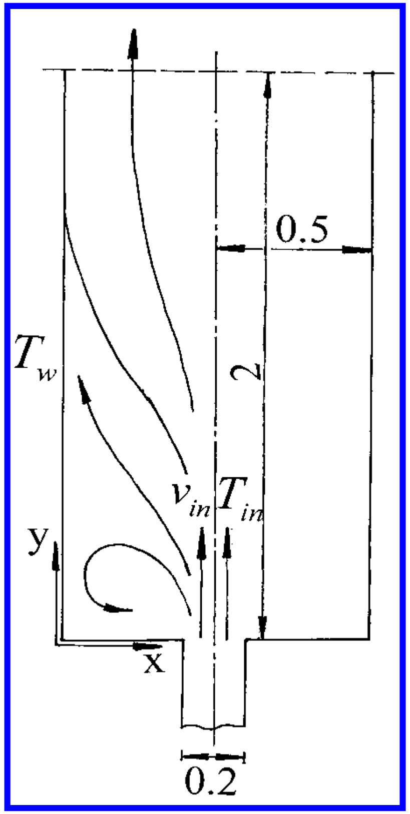
Fig. 1 of Problem 5

The governing equations of velocity and temperature:

$$u: \quad div(\rho \vec{u} u) = -\frac{\partial p}{\partial x} + div(\eta\, grad u) + 0$$

$$v: \quad div(\rho \vec{u} v) = -\frac{\partial p}{\partial y} + div(\eta\, grad v) + 0$$

$$T: \quad div(\rho c_p \vec{u} T) = div(\lambda\, grad T) + 0$$



Boundary conditions:

At symmetric line: $u = 0; \frac{\partial v}{\partial x} = 0; \frac{\partial T}{\partial x} = 0$

At inlet: u, v, T **are specified;**

At solid wall: $u = v = 0; T = T_w$

9-5-2 Numerical methods

(1) This is an open-flow system. Determination of normal velocity at the outlet boundary for open flow field is important: Set the outlet in region without recirculation, adopt local one-way method with total mass conservation;

(2) Convergence condition for flow field iteration: SSUM, SMAX less than pre-specified values or 4 to 5 digits remain unchanged during 5 to 10 successive iterations;

(3) Variation of density with temperature is specified in ENTRY DENSE.

9-5-3 Program reading

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C*****************************************************************
      INTEGER*4 I,J
      REAL*8 TIN, TW, VIN, VOUT, PR, AMU, COND, TREF, RHOREF,
     1   RHOT, FLOWIN, FL, FACTOR
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
     SUBROUTINE USER
C*****************************************************************
     USE START_L
     USE USER_L
     IMPLICIT NONE
C*****************************************************************
C---------------------------PROBLEM SIX-------------------------------
C     Laminar fluid flow and heat transfer in a two-dimensional sudden
C                                    expansion
C*****************************************************************
```

!Difference in section number and problem number:

!Section No. of the lecture-5;

!Prob. No. of the original code-6

C

```
ENTRY GRID
LAST=60
TITLE(1)=‘  .VEL  U.'
TITLE(2)=‘  .VEL  V.'
TITLE(3)=‘ .STR FN.'
TITLE(4)=‘  . TEMP .'
TITLE(11)=‘PRESSURE'
TITLE(12)=‘ DENSITY'
RELAX(1)=0.8 !  Underrelaxation of velocity is organized in the solution process.
RELAX(2)=0.8
LSOLVE(1)=.TRUE.  !  For SIMPLER set .TRUE. For NF＝1 is enough
LSOLVE(4)=.TRUE.
LPRINT(1)=.TRUE.
LPRINT(2)=.TRUE.
LPRINT(3)=.TRUE.
LPRINT(4)=.TRUE.
LPRINT(11)=.TRUE.
LPRINT(12)=.TRUE.
XL=0.5
YL=2.
L1=7
M1=12
CALL UGRID
RETURN
```

' VEL_U'
‘ VEL_V’
‘ STR_FN’
‘ TEMP. '
‘PRESSURE’
‘ DENSITY'

Titles for print out

```
      ENTRY START
      TIN=500
      TW=300.
      VIN=100.
      VOUT=VIN*XCV(L2)/X(L1)*TW/TIN
      DO 100 J=1,M1
      DO 101 I=1,L1
      U(I,J)=0
      V(I,J)=VOUT
      V(I,2)=0
      V(1,J)=0.
      T(I,J)=TW
 101  ENDDO
 100  ENDDO
      V(L2,2)=VIN
      T(L2,1)=TIN
       PR=.7
      AMU=1.
      AMUP=AMU*CPCON/PR
      TREF=300.
      RHOREF=1.
      RHOT=RHOREF*TREF
      RETURN
```

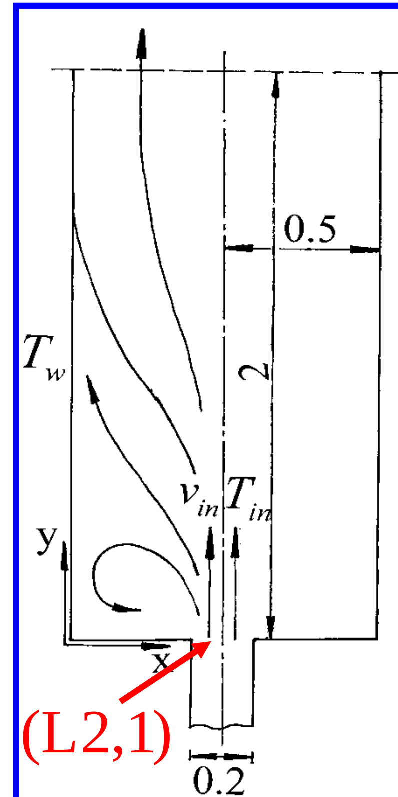
! Estimation of outlet normal velocity

! Initial field, including some boundary conditions.

! At the same location , different i, and j for V and T

$$\Pr = \mu c_p / \lambda,$$

$$\lambda = \mu c_p / \Pr$$

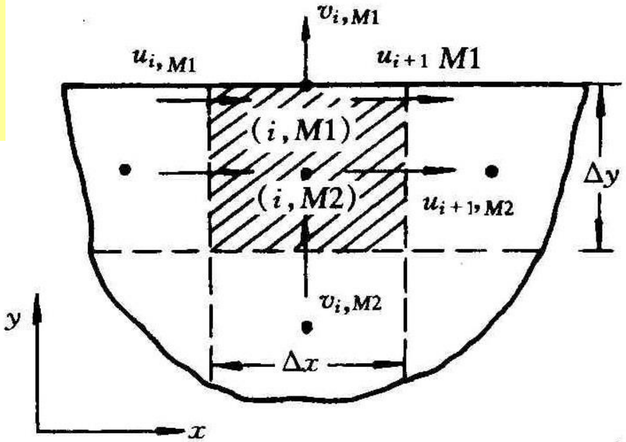


Total mass conservation for case of outlet without recirculation

- Assuming that relative changes of outlet normal velocity =constant

$$\frac{v_{i,M1}-v_{i,m2}}{v_{i,M2}}=k=const$$

$$v_{i,M1}=v_{i,M2}(1+k)=f\,v_{i,M2}$$



f is determined according to total mass conservation :

$$\sum_{i=2}^{L2}\rho_{i,M1}v_{i,M1}\Delta x_i=\sum_{i=2}^{L2}\rho_{i,M1}f\,v_{i,M2}\Delta x_i=FLOWIN$$

$$f=\frac{FLOWIN}{\sum_{2}^{L2}\rho_{i,M1}v_{i,M2}\Delta x_i}$$

$$\boxed{v_{i,M1}=f\bullet v^{*}_{i,M2}}$$

It is regarded as the boundary condition for next iteration.

(2) Assuming that the 1st derivatives at outlet =constant

$$\frac{v_{i,M1}-v_{i,M2}}{\Delta y}=k=const \rightarrow v_{i,M1}=v_{i,M2}+k\Delta y=v_{i,M2}+C$$

C is determined according to total mass conservation

$$\sum_{i=2}^{L2}\rho_{i,M1}(v_{i,M2}+C)\Delta x_i=FLOWIN \rightarrow$$

$$C=\frac{FLOWIN-\sum\rho_{i,M1}v_{i,M2}\Delta x_i}{\sum\rho_{i,M1}\Delta x_i}$$

$\underline{v_{i,M1}=v^*_{i,M2}+C}$ is taking as boundary condition for next iteration.

When fully developed at outlet,: f=1,C=0;
Otherwise there is some differences between the two treatments. **In this example FACTOR method will be used**

```
    ENTRY DENSE
     DO 200 J=1,M1
     DO 201 I=1,L1
     RHO(I,J)=RHOT/T(I,J)
201 ENDDO
200 ENDDO
     RETURN
*
     ENTRY BOUND
     IF(ITER= =0)  FLOWIN=RHO(L2,1)*V(L2,2)*XCV(L2)
     FL=0.
     DO 301 I=2,L2
     FL=FL+RHO(I,M1)*V(I,M2)*XCV(I)
301 ENDDO
     FACTOR=FLOWIN/FL
     DO 302 I=2,L2
     V(I,M1)=V(I,M2)*FACTOR
     T(I,M1)=T(I,M2)
302 ENDDO
     RETURN
```

! Variable density

! RHOT=RHOREF*TREF

!Inlet flow rate calculation

!Outlet flow rate calculation

$$\text{Factor} = \frac{\text{FLOWIN}}{\sum_{i=2}^{L2} \rho_{i,M1} * V_{i,M2} * XCV(i)}$$

Only for print out purpose—decoration! It can be executed after getting converged solution.

```
   ENTRY OUTPUT
   IF(ITER= =0)  THEN
   WRITE(8,401)
401 FORMAT(1X,'   ITER',7X,'SMAX',11X,'SSUM',10X,'V(4,7)',
  1  9X,'T(4,7)')
    ELSE
    PRINT 403, ITER, SMAX,  SSUM, V(4,7), T(4,7)
    WRITE(8,403) ITER, SMAX, SSUM, V(4,7), T(4,7)
403 FORMAT(1X,I6,1P4E15.3)
    ENDIF
    IF (ITER= =LAST) CALL PRINT
    RETURN
*
```

Print out **SMAX,SSUM** for observing the convergence of the iteration

```
    ENTRY GAMSOR
    DO 500 J=1,M1
    DO 501 I=1,L1
    GAM(I,J)=AMU
    IF(NF= =4) GAM(I,J)=AMUP  ! For solving temperature
    IF(NF/=1) GAM(L1,J)=0.    ! Except u others ---adiabatic
    GAM(I,M1)=0.  ! Local one way for both u and T, identical to
                    adiabatic.
501 ENDDO
500 ENDDO
    RETURN
    END
```

9-5-4 Results analysis

COMPUTATION IN CARTISIAN COORDINATES
****

ITER	SMAX	SSUM	V(4,7)	T(4,7)
0	0.000E+00	0.000E+00	1.200E+01	3.000E+02
1	2.366E+00	5.960E-08	1.269E+01	3.539E+02
2	1.068E+00	3.576E-07	1.526E+01	3.574E+02
3	1.059E+00	-2.980E-07	1.600E+01	3.609E+02
4	6.520E-01	-8.941E-08	1.609E+01	3.630E+02
5	1.605E-01	4.433E-07	1.618E+01	3.645E+02
6	1.039E-01	-8.754E-08	1.606E+01	3.655E+02
7	5.972E-02	-8.196E-08	1.594E+01	3.663E+02
8	3.817E-02	-3.101E-07	1.576E+01	3.668E+02
9	2.447E-02	-5.243E-07	1.559E+01	3.672E+02
10	1.535E-02	2.674E-07	1.543E+01	3.675E+02
11	9.663E-03	-8.473E-07	1.529E+01	3.677E+02
12	5.899E-03	4.657E-10	1.516E+01	3.678E+02

13	4.332E-03	-2.432E-07	1.506E+01	3.678E+02
14	3.456E-03	2.751E-07	1.498E+01	3.678E+02
15	2.698E-03	7.753E-08	1.491E+01	3.678E+02
16	2.052E-03	1.475E-07	1.486E+01	3.678E+02
17	1.539E-03	-5.428E-07	1.481E+01	3.678E+02
18	1.133E-03	2.519E-07	1.478E+01	3.677E+02
19	8.994E-04	2.108E-07	1.476E+01	3.677E+02
20	7.056E-04	5.479E-07	1.474E+01	3.677E+02
21	5.436E-04	2.256E-07	1.473E+01	3.677E+02
22	4.111E-04	9.380E-08	1.472E+01	3.676E+02
23	3.100E-04	1.485E-07	1.471E+01	3.676E+02
24	2.303E-04	2.160E-07	1.470E+01	3.676E+02
25	1.793E-04	4.192E-07	1.470E+01	3.676E+02
26	1.447E-04	-1.086E-08	1.470E+01	3.676E+02
27	1.149E-04	-9.684E-08	1.469E+01	3.676E+02
28	8.990E-05	1.732E-09	1.469E+01	3.676E+02
29	6.926E-05	-5.815E-07	1.469E+01	3.676E+02
30	5.170E-05	-3.065E-07	1.469E+01	3.676E+02
31	3.837E-05	-5.491E-07	1.469E+01	3.676E+02
32	3.084E-05	2.732E-07	1.469E+01	3.676E+02

33	2.032E-05	-9.269E-07	1.469E+01	3.676E+02
34	2.015E-05	3.659E-08	1.469E+01	3.676E+02
35	1.213E-05	4.555E-07	1.469E+01	3.676E+02
36	9.591E-06	-1.184E-07	1.469E+01	3.676E+02
37	6.249E-06	4.063E-07	1.469E+01	3.676E+02
38	4.888E-06	-2.038E-08	1.469E+01	3.676E+02
39	3.099E-06	1.491E-07	1.469E+01	3.676E+02
40	3.695E-06	4.564E-07	1.469E+01	3.676E+02
41	2.980E-06	-3.393E-07	1.469E+01	3.676E+02
42	2.923E-06	1.307E-06	1.469E+01	3.676E+02
43	3.150E-06	-3.455E-07	1.469E+01	3.676E+02
44	2.787E-06	5.100E-07	1.469E+01	3.676E+02
45	3.219E-06	-2.657E-07	1.469E+01	3.676E+02
46	2.980E-06	-8.977E-07	1.469E+01	3.676E+02
47	2.503E-06	-2.419E-07	1.469E+01	3.676E+02
48	2.205E-06	5.658E-08	1.469E+01	3.676E+02
49	3.517E-06	-9.167E-07	1.469E+01	3.676E+02
50	3.576E-06	-1.444E-07	1.469E+01	3.676E+02
51	3.278E-06	2.954E-07	1.469E+01	3.676E+02

52	2.772E-06	1.221E-08	1.469E+01	3.676E+02
53	2.146E-06	5.844E-07	1.469E+01	3.676E+02
54	2.104E-06	5.236E-07	1.469E+01	3.676E+02
55	2.921E-06	3.407E-07	1.469E+01	3.676E+02
56	2.712E-06	1.156E-07	1.469E+01	3.676E+02
57	2.801E-06	2.216E-07	1.469E+01	3.676E+02
58	3.005E-06	8.967E-08	1.469E+01	3.676E+02
59	2.886E-06	4.362E-07	1.469E+01	3.676E+02
60	2.623E-06	5.034E-07	1.469E+01	3.676E+02

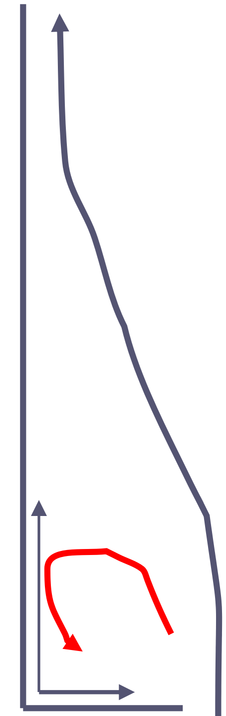
That SMAX reduces to a certain value can be regarded as an indicator of convergence

In the iteration process SSUM takes a very small value from beginning to the end. This can not be regarded as an indicator of convergence. Because it is resulted by our treatment of outflow boundary condition!

************************** .VEL U. **************************

J \ I =	2	3	4	5	6	7
12	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
11	0.00E+00	1.41E-02	3.39E-02	4.04E-02	2.71E-02	0.00E+00
10	0.00E+00	-6.73E-02	-1.96E-01	-2.78E-01	-2.11E-01	0.00E+00
9	0.00E+00	-1.55E-01	-4.33E-01	-5.97E-01	-4.48E-01	0.00E+00
8	0.00E+00	-3.26E-01	-8.75E-01	-1.19E+0 0	-8.95E-01	0.00E+00
7	0.00E+00	-6.17E-01	-1.61E+00	- 2.16E+00	-1.65E+00	0.00E+00
6	0.00E+00	-1.03E+00	-2.62E+00	-3.53E+00	-2.75E+00	0.00E+00
5	0.00E+00	-1.42E+00	-3.67E+00	5.06E+00	- 4.10E+00	0.00E+00
4	0.00E+00	-1.35E+00	-3.91E+00	.02E+00	-5.42E+00	0.00E+00
3	0.00E+00	1.37E-01	-1.24E+0	.69E+00	-6.33E+00	0.00E+00
2	0.00E+0	2.64E+00	6.16E+0	03E+00	-7.70E+00	0.00E+00
1	0.00E+	.00E+00	0.00E+0	00E+00	0.00E+00	0.00E+00

No decoration

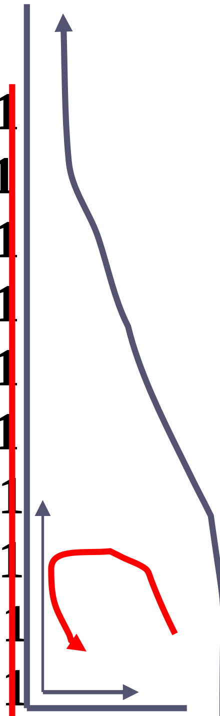


u larger than 0

u less than 0

**************************** .VEL V. ****************************

J \ I =	1	2	3	4	5	6	7
12	0.00E+00	3.73E+00	9.97E+00	1.50E+01	1.87E+01	2.07E+01	1.20E+01
11	0.00E+00	3.76E+00	1.01E+01	1.52E+01	1.89E+01	2.09E+01	1.20E+01
10	0.00E+00	3.65E+00	9.94E+00	1.53E+01	1.95E+01	2.19E+01	1.20E+01
9	0.00E+00	3.37E+00	9.57E+00	1.54E+01	2.04E+01	2.35E+01	1.20E+01
8	0.00E+00	2.76E+00	8.70E+00	1.52E+01	2.17E+01	2.61E+01	1.20E+01
7	0.00E+00	1.59E+00	7.02E+00	1.47E+01	2.35E+01	3.03E+01	1.20E+01
6	0.00E+00	-3.65E-01	4.21E+00	1.36E+01	2.60E+01	3.70E+01	1.20E+01
5	0.00E+00	-3.06E+00	1.81E-01	1.15E+01	2.89E+01	4.66E+01	1.20E+01
4	0.00E+00	-5.60E+00	-4.41E+[illegible]	8.01E+00	3.09E+01	5.93E+01	1.20E+01
3	0.00E+00	-5.24E+00	-6.77E[illegible]	1.43E+00	2.77E+01	7.51E+01	1.20E+01
2	0.00E+00	0.00E+00	0.00E[illegible]	0.00E+00	0.00E+00	1.00E+02	0.00E+00



v less than 0

V larger than 0

Inlet V

No decoration

***************************** .STR FN. ********************************

I = / J	2	3	4	5	6	7
12	0.00E+00	-3.63E-01	-1.29E+00	-2.63E+00	-4.24E+00	-6.00E+00
11	0.00E+00	-3.66E-01	-1.29E+00	-2.63E+00	-4.25E+00	-6.00E+00
10	0.00E+00	-3.53E-01	-1.26E+00	- 2.58E+00	-4.21E+00	-6.00E+00
9	0.00E+00	-3.24E-01	-1.18E+00	-2.48E+00	-4.14E+00	-6.00E+00
8	0.00E+00	-2.64E-01	-1.03E+00	-2.29E+00	-4.00E+00	-6.00E+00
7	0.00E+00	-1.51E-01	-7.61E-01	-1.95E+00	-3.74E+00	-6.00E+00
6	0.00E+00	3.46E-02	-3.26E-01	-1.40E+00	-3.34E+00	-6.00E+00
5	0.00E+00	2.89E-01	2.74E-01	-6.28E-01	-2.74E+00	-6.00E+00
4	0.00E+00	5.31E-01	9.10E-01	2.79E-01	-1.97E+00	-6.00E+00
3	0.00E+00	5.06E-01	1.12E+00	9.96E-01	-1.09E+00	-6.00E+00
2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	-6.00E+00

Stream function =0 at the wall

Total flow rate

*********************** . TEMP . *************************

Decoration has been made：

T(I,M1)=T(I,M2)

J \ I =	1	2	3	4	5	6	7
12	3.00E+02	3.08E+02	3.23E+02	3.37E+02	3.48E+02	3.53E+02	3.00E+02
11	3.00E+02	3.08E+02	3.23E+02	3.37E+02	3.48E+02	3.53E+02	3.00E+02
10	3.00E+02	3.09E+02	3.27E+02	3.43E+02	3.55E+02	3.62E+02	3.00E+02
9	3.00E+02	3.11E+02	3.31E+02	3.50E+02	3.65E+02	3.73E+02	3.00E+02
8	3.00E+02	3.12E+02	3.37E+02	3.59E+02	3.75E+02	3.84E+02	3.00E+02
7	3.00E+02	3.14E+02	3.43E+02	3.68E+02	3.87E+02	3.97E+02	3.00E+02
6	3.00E+02	3.16E+02	3.48E+02	3.76E+02	3.98E+02	4.10E+02	3.00E+02
5	3.00E+02	3.18E+02	3.53E+02	3.83E+02	4.07E+02	4.23E+02	3.00E+02
4	3.00E+02	3.18E+02	3.53E+02	3.85E+02	4.12E+02	4.35E+02	3.00E+02
3	3.00E+02	3.15E+02	3.45E+02	3.76E+02	4.10E+02	4.49E+02	3.00E+02
2	3.00E+02	3.06E+02	3.21E+02	3.42E+02	3.88E+02	4.69E+02	3.00E+02
1	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	5.00E+02	3.00E+02

Given wall temperature

Inlet temp.

No decoration

************************** PRESSURE **************************

J \ I =	1	2	3	4	5	6	7
12	8.40E+02	8.40E+02	8.39E+02	8.38E+02	8.34E+02	8.31E+02	8.30E+02
11	8.52E+02	8.52E+02	8.52E+02	8.50E+02	8.48E+02	8.45E+02	8.44E+02
10	8.77E+02	8.77E+02	8.76E+02	8.76E+02	8.75E+02	8.74E+02	8.73E+02
9	8.99E+02	8.98E+02	8.97E+02	8.95E+02	8.94E+02	8.92E+02	8.91E+02
8	9.12E+02	9.10E+02	9.08E+02	9.06E+02	9.05E+02	9.02E+02	9.00E+02
7	9.06E+02	9.04E+02	9.01E+02	8.99E+02	8.99E+02	8.96E+02	8.94E+02
6	8.63E+02	8.61E+02	8.56E+02	8.56E+02	8.62E+02	8.59E+02	8.58E+02
5	7.55E+02	7.52E+02	7.46E+02	7.50E+02	7.66E+02	7.69E+02	7.70E+02
4	5.57E+02	5.53E+02	5.45E+02	5.50E+02	5.85E+02	6.02E+02	6.11E+02
3	2.91E+02	2.84E+02	2.72E+02	2.55E+02	3.32E+02	3.56E+02	3.68E+02
2	9.85E+01	8.74E+01	6.54E+01	-3.27E+01	-2.08E+02	9.08E+01	2.40E+02
1	0.00E+00	-1.10E+01	-3.79E+01	-1.77E+02	-4.78E+02	-4.18E+01	1.07E+02

Maximum pressure caused by reattachment of flow

Low pressure region caused by high inlet velocity

From interpolation

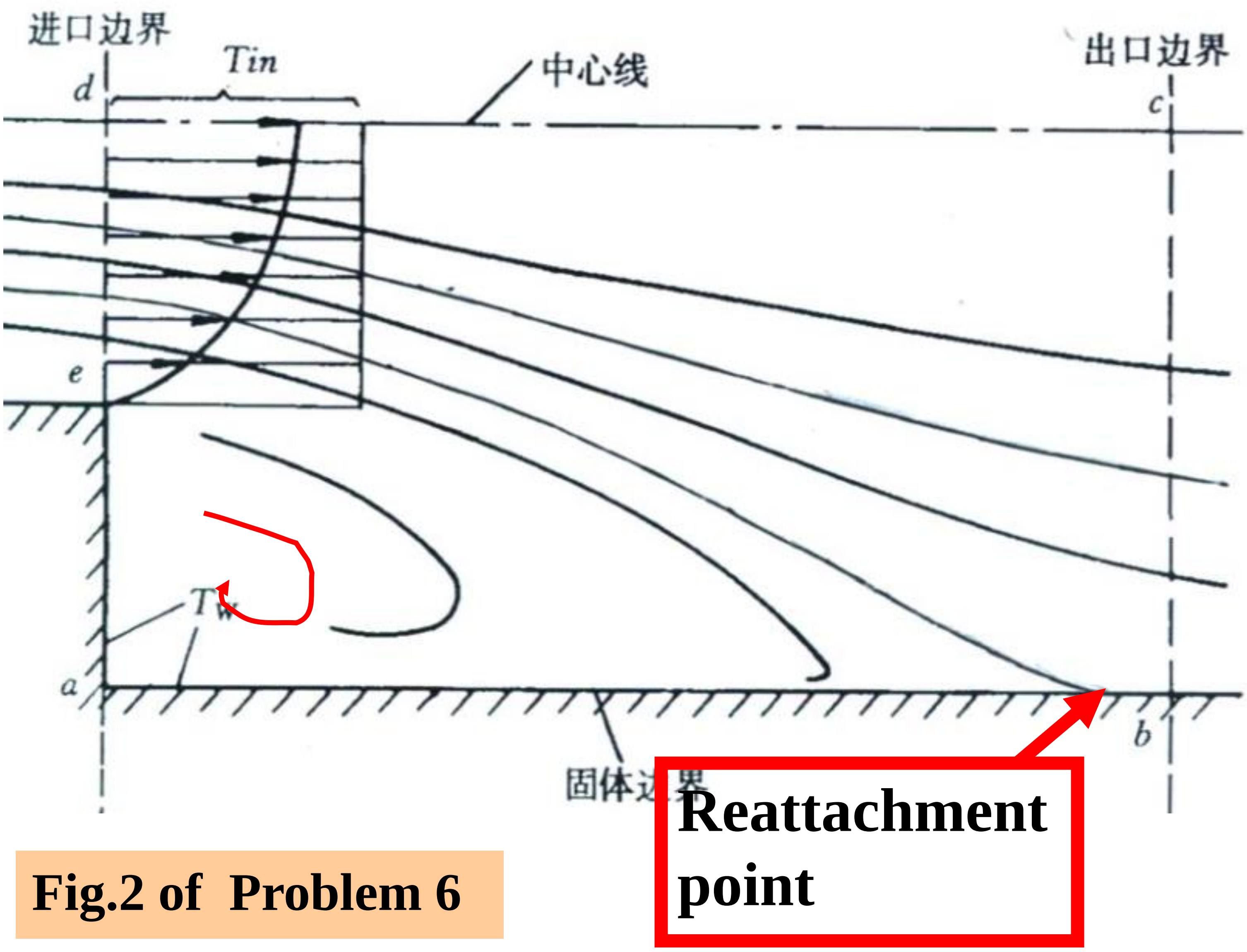


Fig.2 of Problem 6

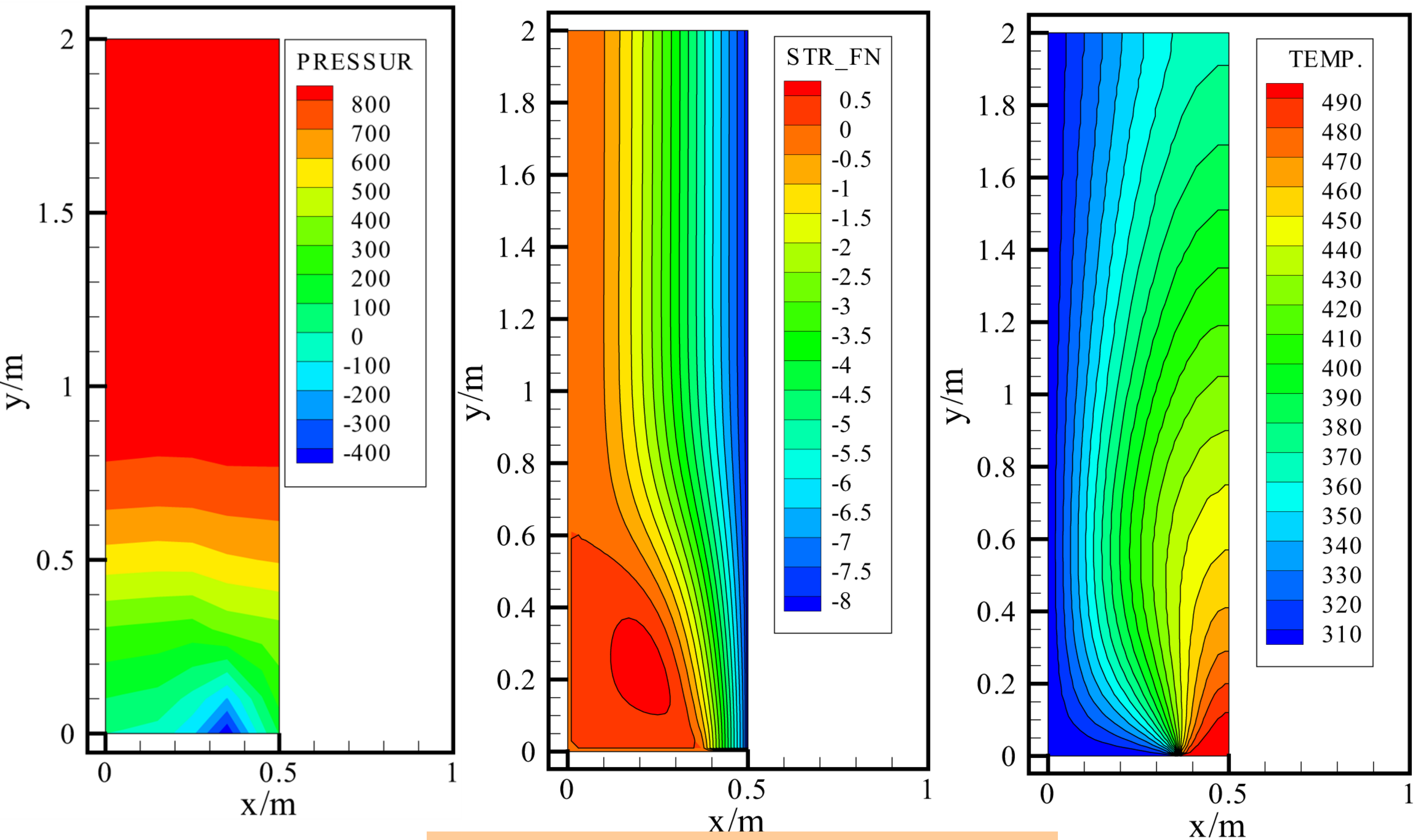


Fig. 3 Results of Problem 6

9-6 Complicated fully developed fluid flow and heat transfer in square duct

---Velocity is regarded as a ϕ variable

9-6-1 Physical problem and its math formulation

Known: Fully developed heat transfer in a square duct shown in Fig. 1. The effect of gravitation is taken into account by Boussinesq assumption. Duct top and bottom walls are adiabatic , while left and right walls are kept at constant and uniform tempertures: $T_1=T_c=0$, $T_2=T_h=1$; $Pr=0.7$, $AMU=1.0$, $dp/dz=-3000$, and $\rho g \beta = 10^4$.

Find: Cross sectional distributions of u,v, and w, temperature distribution and fRe.

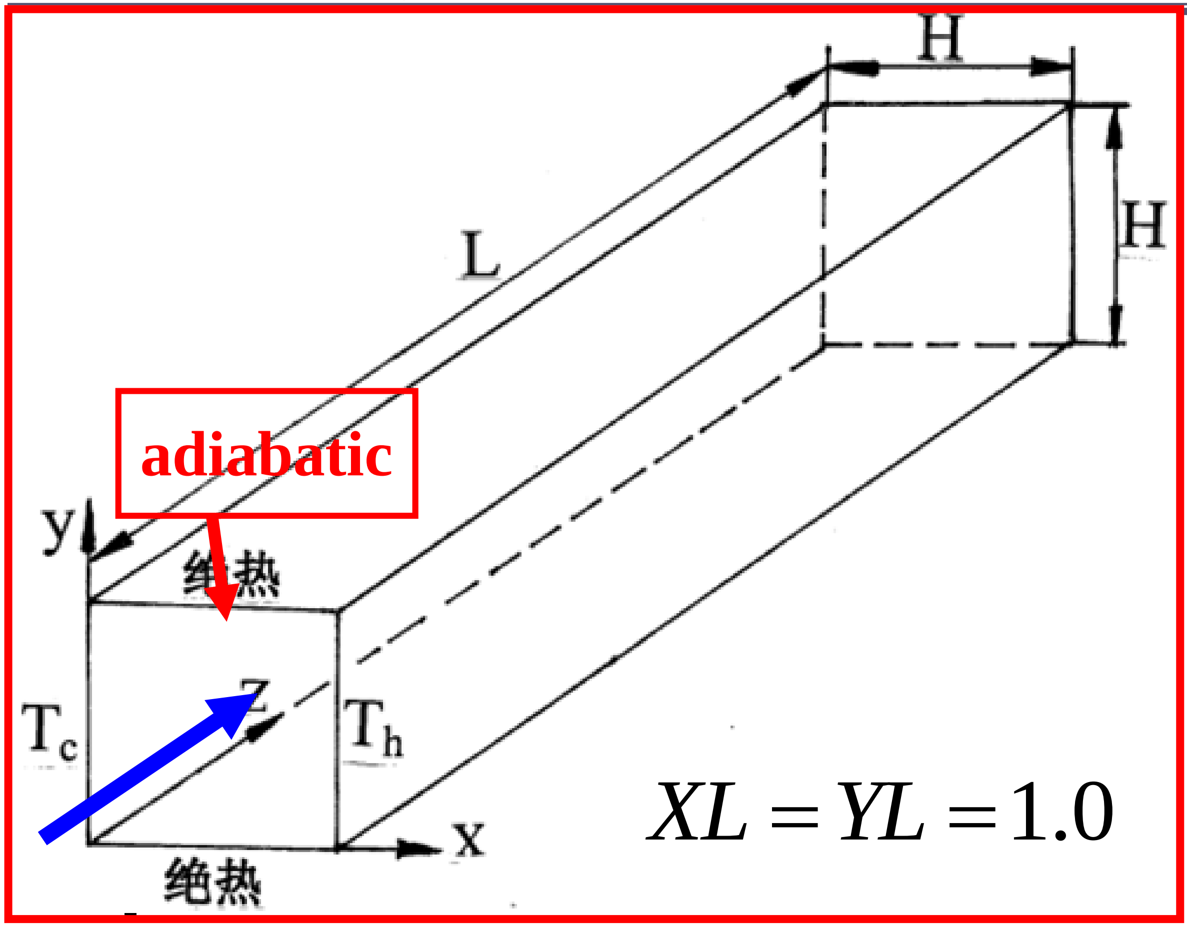


Fig. 1 Physical model of Problem 6

For the case studied, when heat transfer goes into the fully developed region， the heat leaves the hot wall goes into the cold wall, i.e., the heat transfer rate is determined by the flow at the cross-section, and the axial flow does not make any contribution to this heat transfer.

Analysis of the governing eq.:

According to the fully developed condition

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=-\frac{\partial p}{\partial x}+\eta(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2})$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z})=-\frac{\partial p}{\partial y}+\eta(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2})-\rho g$$

$$\rho(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z})=-\frac{\partial p}{\partial z}+\eta(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2})$$

Because heat leaving right wall transfers to left wall:

$$\rho(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z})=\frac{\lambda}{c_p}(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2})$$

Analysis for the computational domain:

This problem looks like Problem 3 where we take ¼ of the cross section as the computational domain. Can we still take such practice for this case?

No! Because of the cross sectional natural convection the entire region must be taken as the computational domain.

Boundary conditions:

At x=0, $T=T_c$: x=L, $T=T_h$ At y=0 and y=L: adiabatic

At four walls: u=v=w=0.

Major features of the problem

(1) There are three velocity components: *u, v, w*; However *u, v* are not coupled with *w*;

(2) For the coordinate adopted, temperature is coupled only with v-component.

9-6-2 Numerical methods

(1) How to use 2-D code for solving three velocity components? Using the partially coupled feature!

***u, v, T* are not coupled with *w*, while w is couled with u and v; Thus *w* can be regarded as a scalar variable: *u, v, T* are solved first, then *w* is solved;**

(2) The problem studied can be resolved into two sub-problems:

(a) Natural convection in a 2-D square cavity: *u, v, T* are solved;

(b) Fully developed axial flow for solving *w*，with a prespecified source term of $-dp/dz$ **.**

(3) Boussinesq assumption is adopted for *v* -equation:

Treatment of pressure gradient and gravitation term for *v*-equation

$$-\frac{\partial p}{\partial y}-\rho g=-\frac{\partial p}{\partial y}-\rho_{ref}[1-\beta(T-T_{ref})]g$$

$$=-\frac{\partial p}{\partial y}-\rho_{ref}(1+\beta T_{ref})g+g\rho_{ref}\beta T$$

$$=-\frac{\partial}{\partial y}[\underline{p+\rho_{ref}(1+\beta T_{ref})gy}]+g\rho_{ref}\beta T$$

$$=-\frac{\partial p_{eff}}{\partial y}+g\rho_{ref}\beta T$$

Governing equations of the problem studied:

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p_{eff}}{\partial x}+\eta(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p_{eff}}{\partial y}+\eta(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})+\rho g\beta T$$

$$\rho c_p(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})=\lambda(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})$$

Solved first to get u,v and T

$$\rho(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y})=-\frac{dp}{dz}+\eta(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2})$$

Solved 2nd with known u, v and specified pressure gradient!

dp/dz **(<0) can be assumed and is specified as -3000.**

9-6-3 Program reading

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      MODULE USER_L
C*********************************************************************
      INTEGER*4 I,J
      REAL*8  GBR, DPDZ, PR, AMU, FRE, WBAR, TM
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
     SUBROUTINE USER
C**********************************************************************
     USE START_L
     USE USER_L
     IMPLICIT NONE
C**********************************************************************
C------------------------------PROBLEM SEVEN---------------------------------
C        Complex fully developed laminar fluid flow and heat transfer in a
C                                        horizontal square duct
C**********************************************************************
```

```
ENTRY GRID
TITLE(1)=' .VEL  U.'
TITLE(2)=' .VEL  V.'
TITLE(3)=' .STR FN.'
TITLE(4)=' . TEMP .'
TITLE(5)=' .W/WBAR.'
TITLE(11)='PRESSURE'
RELAX(1)=0.8
RELAX(2)=0.8
LSOLVE(1)=.TRUE.
LSOLVE(4)=.TRUE.
LPRINT(1)=.TRUE.
LPRINT(2)=.TRUE.
LPRINT(3)=.TRUE.
LPRINT(4)=.TRUE.
LPRINT(5)=.TRUE.
LPRINT(11)=.TRUE.
LAST=25
XL=1.
YL=1.
L1=7
M1=7
CALL UGRID
RETURN
```

! w is treated as fifth variable!

! Not for *T,w*; With known *u,v* , *T and w* eqs are linear.

!u,v,p,T are solved first

! In SIMPLER code when the 1st variable is set to be solved, the 2nd and 3rd ones (v and p) are automatically regarded as variables to be solved.

! Computation for the entire region

```
      ENTRY START
      GBR=1.E4        ! ρgβ
      DPDZ=-3000.
      DO 100 J=1,M1
      DO 101 I=1,L1
      U(I,J)=0.
      V(I,J)=0.
      T(I,J)=0.
      T(L1,J)=1. !Initial temperature and right side boundary condition
      F(I,J,5)=100.   !Initial field for axial velocity
      IF (I= =1.OR.I= =L1)  F(I,J,5)=0.
      IF (J= =1.OR.J= =M1)  F(I,J,5)=0. !Boundary cond. at four walls w=0
 101 ENDDO
 100 ENDDO
      PR=0.7                       !CPCON=1,default value
      AMU=1.
      AMUP=AMU*CPCON/PR            !GAMA for temperature
      RETURN
*
      ENTRY DENSE
      RETURN
```

$!\Pr = \mu c_p / \lambda; \quad \lambda = \mu c_p / \Pr$

With Bossinesq assumption, density is constant

```
      ENTRY BOUND
      FRE=0.
      IF(ITER<20) RETURN            !  w is not solved when ITER<20
      IF(.NOT.LSOLVE(5)) THEN       !  Switch of the solved variables,
      LSOLVE(1)=.FALSE.             !  only executed once
      LSOLVE(5)=.TRUE.
      ENDIF
                                    !  Once 5th variable is solved these
                                    !  two statements are not needed to
                                    !  executed any more.
      WBAR=0.
      DO 302 J=2,M2
      DO 303 I=2,L2
      WBAR=WBAR+F(I,J,5)*XCV(I)*YCV(J)
  303 ENDDO
  302 ENDDO
                                    !  Computing (fRe) according to definition;
                                    !  Shown in the next page.
      FRE=-DPDZ*2.*4.*(XL*YL)**3/(XL+YL)**2/(WBAR*AMU)
      RETURN
```

FRE=-DPDZ*2.*4.*(XL*YL)3/(XL+YL)**2/(WBAR*AMU)**

$$f\,\mathrm{Re} = -[(dp/dz)D_h / \frac{1}{2}\rho w_m^2]\frac{\rho w_m D_h}{\mu}$$

$$f\,\mathrm{Re} = -2[(dp/dz)D_h^2 / w_m \mu] = -\frac{2dp/dz}{\mu(\sum w_{i,j}\Delta A_{i,j} / A)} \bullet (\frac{4A}{P})^2$$

$$= \frac{-2dp/dz}{\mu \sum w_{i,j}\Delta A_{i,j}} \bullet (\frac{4A}{P})^2 A$$

$$= \frac{-2dp/dz}{\mu \sum w_{i,j}\Delta A_{i,j}} \bullet (\frac{4XL*YL}{2(XL+YL)})^2 \bullet XL*YL$$

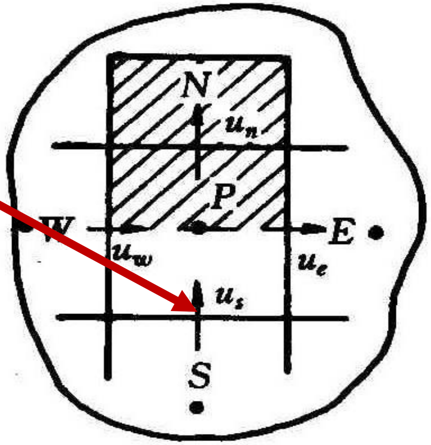
$$= \frac{-2dp/dz}{\mu \sum w_{i,j}\Delta A_{i,j}} \bullet \frac{4(XL*YL)^3}{(XL+YL)^2}$$

```
      ENTRY OUTPUT
      IF(ITER= =0) THEN
      PRINT401
      WRITE(8,401)
401 FORMAT(1X,'  ITER',6X,'SMAX',8X,'SSUM',7X,'V(6,4)',
     &  6X,'T(2,6)',6X,'F.RE')
       ELSE
       PRINT 403, ITER, SMAX, SSUM, V(6,4), T(2,6), FRE
       WRITE(8,403) ITER,SMAX,SSUM,V(6,4),T(2,6),FRE
403 FORMAT(1X,I6,1P5E12.3)
       ENDIF
      IF(ITER/=LAST) RETURN
      DO 410 J=1,M1
      DO 411 I=1,L1
      F(I,J,5)=F(I,J,5)/WBAR    !Dimensionless output for w
411 ENDDO
410 ENDDO
      CALL PRINT
      RETURN
```

```
    ENTRY GAMSOR
    DO 500 J=1,M1
    DO 501 I=1,L1
    GAM(I,J)=AMU
    IF(NF= = 4) THEN          ! GAMA for temp.
    GAM(I,J)=COND
    GAM(I,1)=0.               ! Adiabatic for south and north boundaries
    GAM(I,M1)=0.
    ENDIF
501ENDDO
500ENDDO
    DO 510 J=2,M2
    DO 511 I=2,L2
    IF(NF= =2) THEN
    IF(J/=2) THEN
    TM=(T(I,J)+T(I,J-1))*0.5
    CON(I,J)=TM*GBR           ! Source term of V-eq.
    ENDIF
    ENDIF
    IF(NF= =5)  CON(I,J)=-DPDZ
511 ENDDO                     ! Source term of W-eq.
510 ENDDO
    RETURN
    END
```

$$GBR = g\rho_{ref}\beta T$$

TM



9-6-4 Results analysis

COMPUTATION IN CARTESIAN COORDINATES

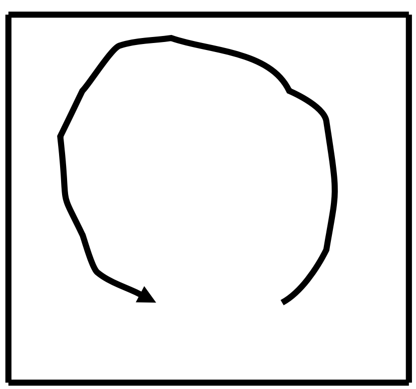
****

ITER	SMAX	SSUM	V(6,4)	T(2,6)	F.RE
0	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
1	0.000E+00	0.000E+00	0.000E+00	1.000E-01	0.000E+00
2	1.273E+01	-1.907E-06	1.016E+01	2.848E-01	0.000E+00
3	6.308E+00	1.073E-06	1.926E+01	3.445E-01	0.000E+00
4	2.978E+00	7.153E-07	2.076E+01	3.826E-01	0.000E+00
5	1.237E+00	-5.960E-07	2.284E+01	3.854E-01	0.000E+00
6	6.454E-01	-4.768E-07	2.304E+01	3.889E-01	0.000E+00
7	2.911E-01	7.153E-07	2.342E+01	3.894E-01	0.000E+00
8	1.338E-01	-3.278E-07	2.346E+01	3.900E-01	0.000E+00
9	6.046E-02	-5.364E-07	2.352E+01	3.900E-01	0.000E+00
10	2.868E-02	-5.364E-07	2.352E+01	3.900E-01	0.000E+00
11	1.286E-02	-4.321E-07	2.353E+01	3.900E-01	0.000E+00

12	6.224E-03	2.850E-07	2.353E+01	3.901E-01	0.000E+00
13	3.349E-03	-3.660E-07	2.353E+01	3.901E-01	0.000E+00
14	1.544E-03	1.974E-07	2.353E+01	3.901E-01	0.000E+00
15	8.407E-04	-2.626E-07	2.353E+01	3.901E-01	0.000E+00
16	3.686E-04	-1.118E-08	2.353E+01	3.901E-01	0.000E+00
17	1.961E-04	1.043E-07	2.353E+01	3.901E-01	0.000E+00
18	7.963E-05	2.775E-07	2.353E+01	3.901E-01	0.000E+00
19	4.327E-05	3.166E-08	2.353E+01	3.901E-01	0.000E+00
20	2.098E-05	-1.825E-07	2.353E+01	3.901E-01	6.000E+01
21	2.098E-05	-1.825E-07	2.353E+01	3.901E-01	5.323E+01
22	2.098E-05	-1.825E-07	2.353E+01	3.901E-01	5.238E+01
23	2.098E-05	-1.825E-07	2.353E+01	3.901E-01	5.236E+01
24	2.098E-05	-1.825E-07	2.353E+01	3.901E-01	5.236E+01
25	2.098E-05	-1.825E-07	2.353E+01	3.901E-01	5.236E+01

************************** .VEL U. **************************

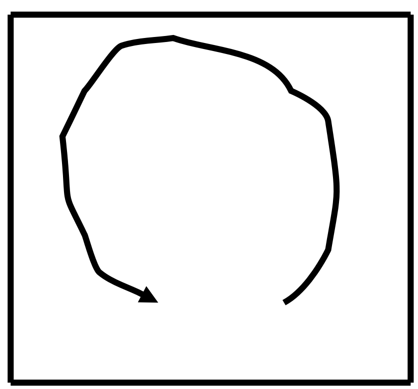
J \ I =	2	3	4	5	6	7
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
6	0.00E+00	-1.52E+01	-1.78E+01	-1.77E+01	-1.31E+01	0.00E+00
5	0.00E+00	-8.36E+00	-1.40E+01	-1.40E+01	-9.70E+00	0.00E+00
4	0.00E+00	7.76E-01	8.31E-02	-8.31E-0 2	-7.76E-01	0.00E+00
3	0.00E+00	9.70E+00	1.40E+01	1.40E+01	8.36E+00	0.00E+00
2	0.00E+00	1.31E+01	1.77E+01	1.78E+01	1.52E+01	0.00E+00
1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00



Natural convection in cross section

***************************** .VEL V. *****************************

I = / J	1	2	3	4	5	6	7
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
6	0.00E+00	-1.52E+01	-2.64E+00	1.01E-01	4.66E+00	1.31E+01	0.00E+00
5	0.00E+00	-2.35E+01	-8.26E+00	8.31E-02	8.96E+00	2.28E+01	0.00E+00
4	0.00E+00	-2.28E+01	-8.96E+00	-8.31E-02	8.26E+00	2.35E+01	0.00E+00
3	0.00E+00	-1.31E+01	-4.66E+00	-1.01E-01	2.64E+00	1.52E+01	0.00E+00
2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00



***************************   .STR FN.   ***************************

I =	2	3	4	5	6	7
J						
7	0.00E+00	-3.91E-07	2.60E-07	1.16E-07	1.26E-08	0.00E+00
6	0.00E+00	3.03E+00	3.56E+00	3.54E+00	2.61E+00	0.00E+00
5	0.00E+00	4.71E+00	6.36E+00	6.34E+00	4.55E+00	0.00E+00
4	0.00E+00	4.55E+00	6.34E+00	6.36E+00	4.71E+00	0.00E+00
3	0.00E+00	2.61E+00	3.54E+00	3.56E+00	3.03E+00	0.00E+00
2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

Stream functions of the four walls are zero

**************************** . TEMP . ****************************

No decoration

I =	1	2	3	4	5	6	7
J							
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E+00
6	0.00E+00	3.90E-01	7.20E-01	7.86E-01	8.33E-01	9.11E-01	1.00E+00
5	0.00E+00	3.25E-01	6.21E-01	6.77E-01	7.15E-01	8.43E-01	1.00E+00
4	0.00E+00	2.41E-01	4.58E-01	5.00E-01	5.42E-01	7.59E-01	1.00E+00
3	0.00E+00	1.57E-01	2.85E-0 1	3.23E-01	3.79E-01	6.75E-0 1	1.00E+00
2	0.00E+00	8.92E-02	1.67E-01	2.14E-01	2.80E-01	6.10E-0 1	1.00E+00
1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E+00

No decoration

**************************** .W/WBAR. ****************************

J \ I =	1	2	3	4	5	6	7
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
6	0.00E+00	4.96E-01	7.74E-01	7.73E-01	6.99E-01	4.72E-01	0.00E+00
5	0.00E+00	7.89E-01	1.50E+00	1.54E+00	1.34E+00	7.52E-01	0.00E+00
4	0.00E+00	8.21E-01	1.63E+00	1.85E+00	1.63E+00	8.21E-01	0.00E+00
3	0.00E+00	7.52E-01	1.34E+00	1.54E+00	1.50E+00	7.89E-01	0.00E+00
2	0.00E+00	4.72E-01	6.99E-01	7.73E-01	7.74E-01	4.96E-01	0.00E+00
1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

W velocity of the four walls are zero

*************************** PRESSURE ***************************

J \ I =	1	2	3	4	5	6	7
7	3.64E+03	3.73E+03	4.05E+03	4.33E+03	4.67E+03	4.89E+03	5.00E+03
6	3.09E+03	3.18E+03	3.36E+03	3.56E+03	3.84E+03	4.05E+03	4.16E+03
5	2.14E+03	2.09E+03	1.99E+03	2.02E+03	2.17E+03	2.36E+03	2.46E+03
4	1.10E+03	1.02E+03	8.42E+02	7.85E+02	8.42E+02	1.02E+03	1.10E+03
3	4.58E+02	3.63E+02	1.73E+02	2.45E+01	-7.31E+00	9.20E+01	1.42E+02
2	1.56E+02	5.04E+01	-1.61E+02	-4.37E+02	-6.35E+02	-8.17E+02	-9.08E+02
1	0.00E+00	-1.06E+02	-3.28E+02	-6.67E+02	-9.49E+02	-1.27E+03	-1.36E+03

Pmax (5.00E+03 at I = 7, J = 7)

Pmin (-1.36E+03 at I = 7, J = 1)

Pressure reference point (0.00E+00 at I = 1, J = 1)

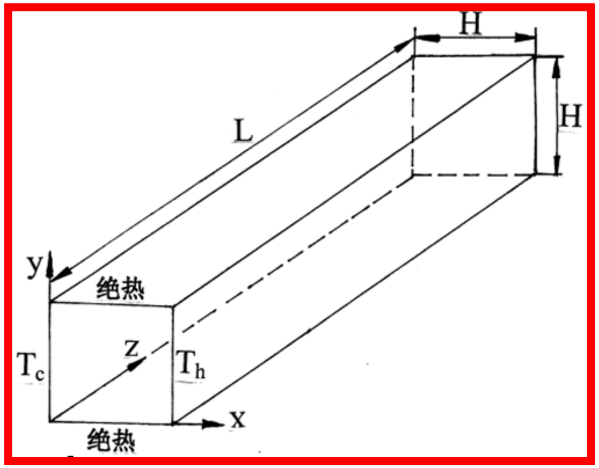
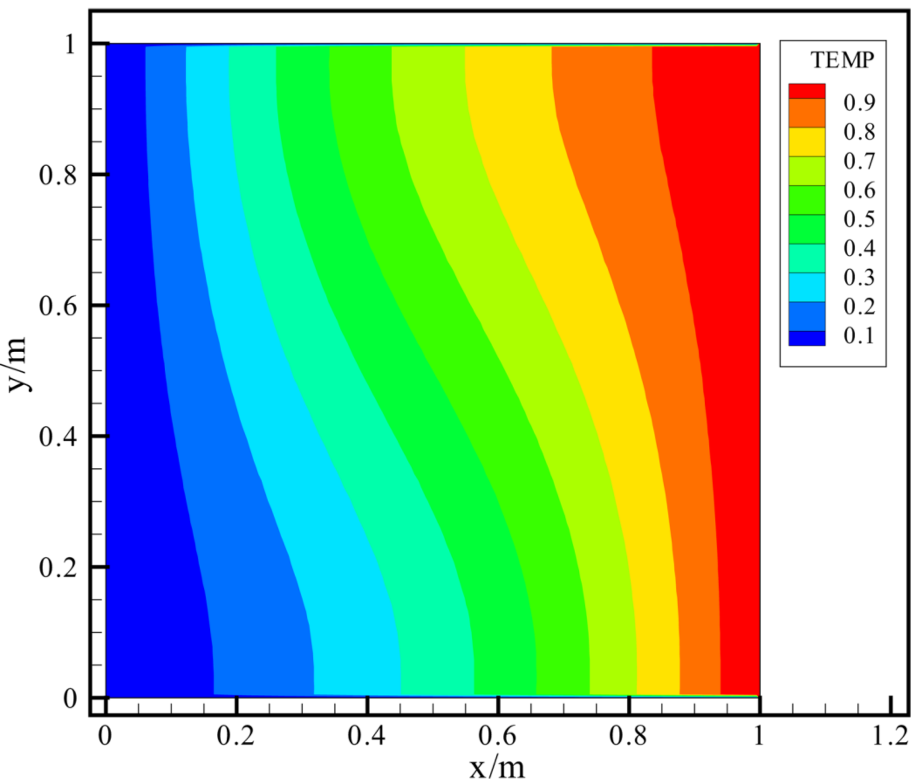
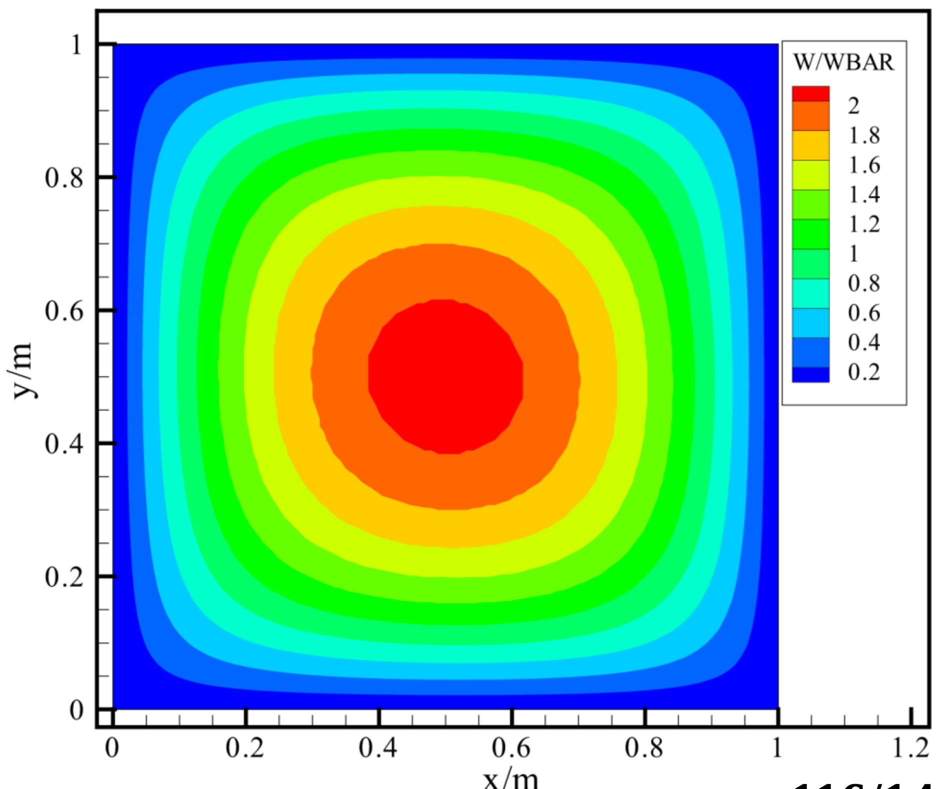


Fig. 2 Results of Problem 6



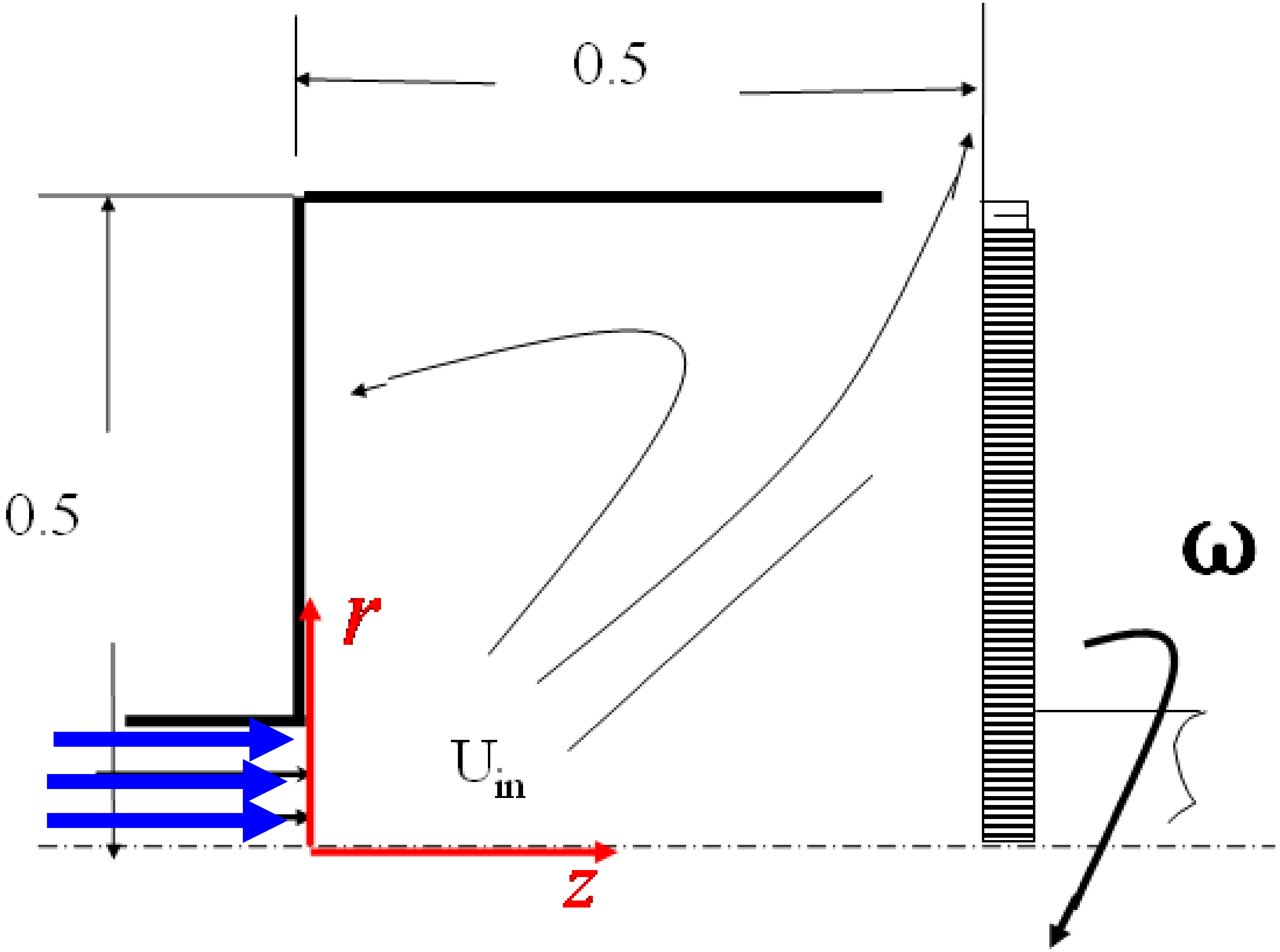


9-7 Impinging flow on a rotating disc
---Discretization of source term of momentum equation in cylindrical coordinate

9-7-1 Physical problem and its math formulation

Known: **A rotating disc with ω=100 is partially covered by a shell（壳体）. Fluid flows into the shell through the central inlet of the shell with inlet velocity U_{in}＝100, impinges onto the disc and then leaves the disc (盘)through the gap between the shell and the disc. Fluid viscosity AMU＝1.**

Fig.1 Schematic diagram of Section 7



No change along the peripheral direction(圆周方向)

Find： **Velocity and pressure distribution in the cavity.**

Solution： **This is a fluid flow problem in three dimensional cylindrical coordinate. The rotating disc and the shell form a cavity. The fluid flow within the cavity is caused by the impingement of the inlet flow and the rotating effect of the disc. The circumferential velocity component is purely caused by the rotating disc. Thus there exists circumferential velocity component, but no circumferential pressure drop .**

Governing equations of the three velocities are:

$$z\ \text{direction}: \rho(v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z}) = -\frac{\partial p}{\partial z} + \eta(\frac{\partial^2 v_z}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_z}{\partial r}))$$

$$r\ \text{direction}: \rho(v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z}) = -\frac{\partial p}{\partial r} + \eta(\frac{\partial^2 v_r}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_r}{\partial r}))$$

$$+\rho\frac{v_\theta^2}{r} - \eta\frac{v_r}{r^2}$$

Source term

$$\theta\ \text{direction}: \quad \rho(v_r \frac{\partial v_\theta}{\partial r} + v_z \frac{\partial v_\theta}{\partial z}) = \eta(\frac{\partial^2 v_\theta}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_\theta}{\partial r}))$$

$$-\rho\frac{v_r v_\theta}{r} - \eta\frac{v_\theta}{r^2}$$

Source term

No pressure gradient!

There exists v_θ but no term with $\frac{\partial}{\partial \theta}$.

Since the circumferential flow is caused by shear stress, there is no pressure gradient in θ direction.

9-7-2 Numerical method

(1) There are three velocity components, but no terms contain $\partial/\partial\theta$, such as no terms with $\partial/\partial z$ in Example 6.

(2) v_θ is not in the convection terms of v_z, v_r, but it is included in the source term of v_r. v_θ can be viewed as a scalar variable (such as temperature) coupled with v_r, v_z; Thus it is 2-D cylingdrical case with MODE＝2.

(3) rv_θ will be taken as variable to be solved to enhance solution stability .

The original momentum eq. of θ direction :

$$\rho(v_r \frac{\partial v_\theta}{\partial r} + v_z \frac{\partial v_\theta}{\partial z}) = \eta(\frac{\partial^2 v_\theta}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_\theta}{\partial r}))$$

It is transformed to： $\rho(v_r \frac{\partial (rv_\theta)}{\partial r} + v_z \frac{\partial (rv_\theta)}{\partial z}) =$

$$\eta(\frac{\partial^2 (rv_\theta)}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial (rv_\theta)}{\partial r})) - \frac{2\eta}{r}\frac{\partial (rv_\theta)}{\partial r}$$

(4) The source term of v_r is transformed as follows:

$$S_{v_r} = \rho\frac{v_\theta^2}{r} - \eta\frac{v_r}{r^2} = \rho\frac{(rv_\theta)^2}{r^3} - \eta\frac{1}{r^2}v_r$$

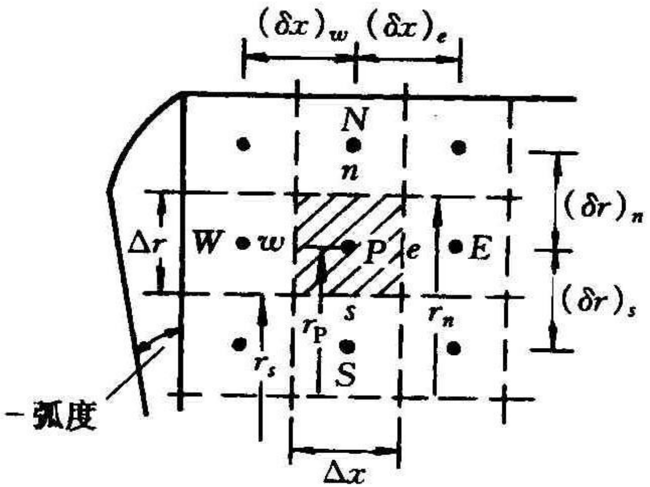
S_P

Numerical treatment of source term of rv_θ

$$S_{(rv_\theta)} = -\frac{2\eta}{r}\frac{\partial(rv_\theta)}{\partial r} = -\frac{2}{r_P}\frac{\eta}{YCV(j)}\frac{(rv_\theta)_n-(rv_\theta)_s}{1}$$

Idea of FUD

$$= -\frac{2}{r_P}\frac{\eta}{YCV(j)}[(rv_\theta)_P-(rv_\theta)_S]$$

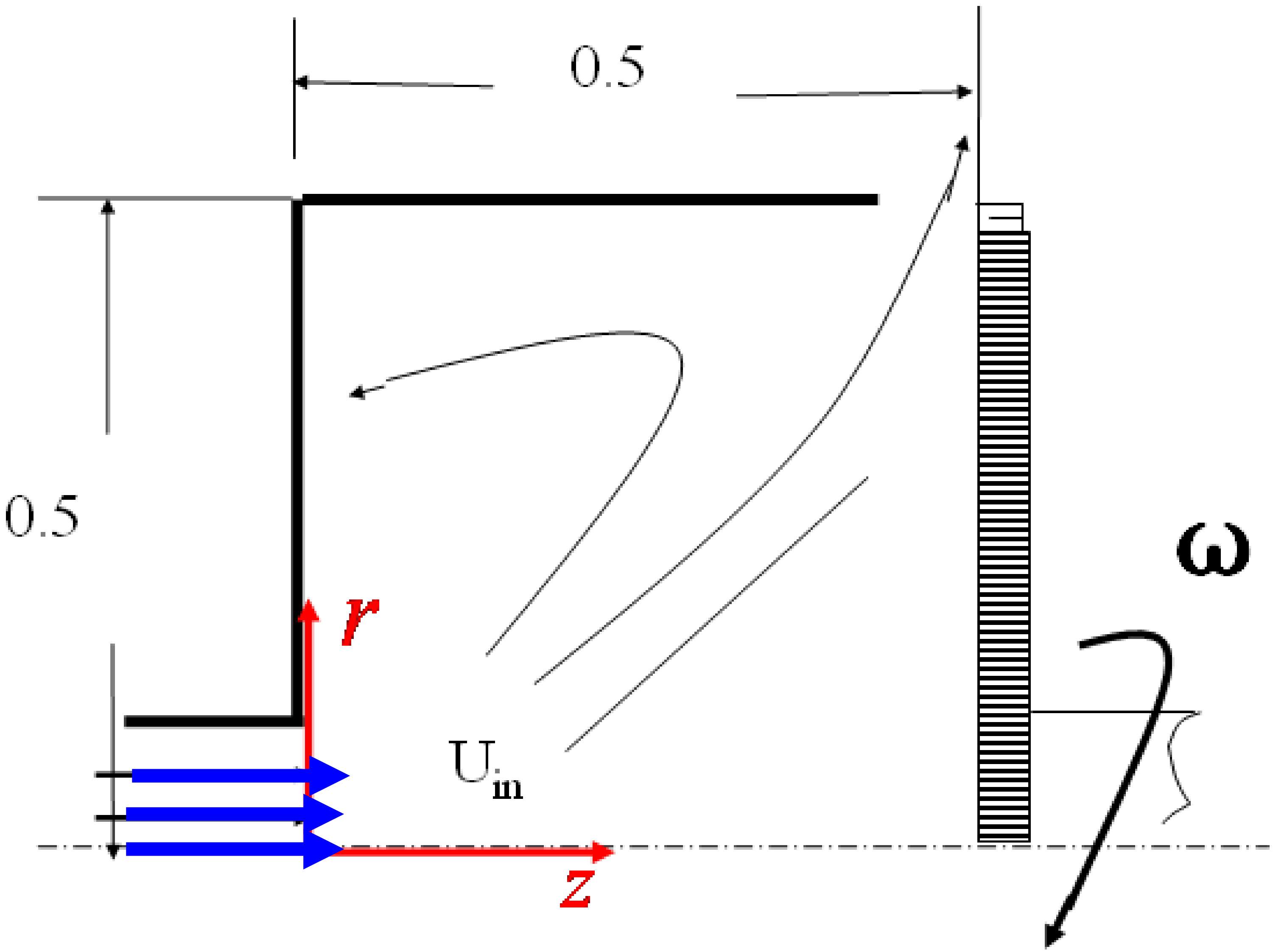


$$= \frac{2}{r_P}\frac{\eta(rv_\theta)_S}{YCV(j)} - \frac{2}{r_P}\frac{\eta}{YCV(j)}(rv_\theta)_P$$

S_C $\quad$ S_P

Introducing S_p to enhance solution stability.

Fig.1 Schematic diagram of Example 7



9-7-3 Program reading

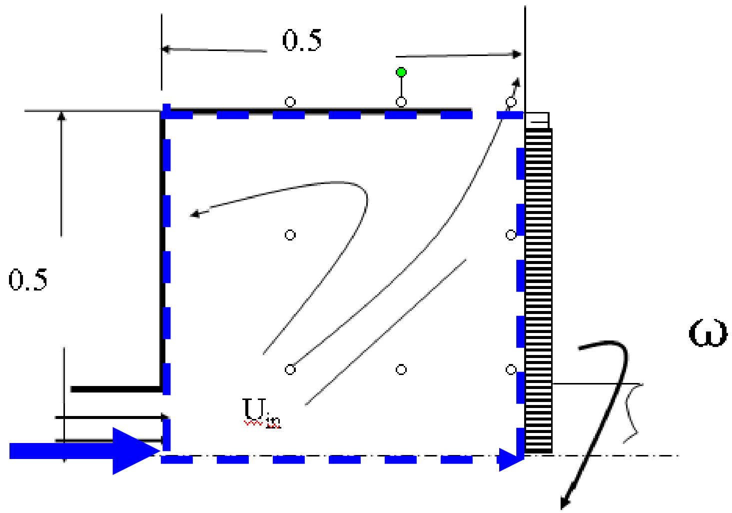
```
      MODULE USER_L
C*************************************************************
      INTEGER*4 I,J
      REAL*8 OMEGA, UIN, AMU, FLOWIN, AR, ADD, FL,
     1 RSWM, RHOM, FLT
      END MODULE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE USER
C******************************************************************
      USE START_L
      USE USER_L
      IMPLICIT NONE
C*******************************************************************
C--------------------------PROBLEM EIGHT----------------------------
C                     Laminar impinging flow over a rotating disk
C*******************************************************************
```

ENTRY GRID

```
TITLE(1)=' .VEL  U.'
TITLE(2)=' .VEL  V.'
TITLE(3)=' .STR FN.'
TITLE(5)=' .R.VTH.'
TITLE(11)='PRESSURE'
RELAX(1)=0.8
RELAX(2)=0.8
LSOLVE(1)=.TRUE.
LSOLVE(5)=.TRUE.
LPRINT(1)=.TRUE.
LPRINT(2)=.TRUE.
LPRINT(3)=.TRUE.
LPRINT(5)=.TRUE.
LPRINT(11)=.TRUE.
LAST=25
MODE=2
XL=0.5
YL=0.5
L1=7
M1=7
R(1)=0.
CALL UGRID
RETURN
```

Regarding (R.VTheta) as 5th variable

In SIMPLER code when the 1st variable is set to be solved, the 2nd and 3rd ones are automatically regarded as variables to be solved.

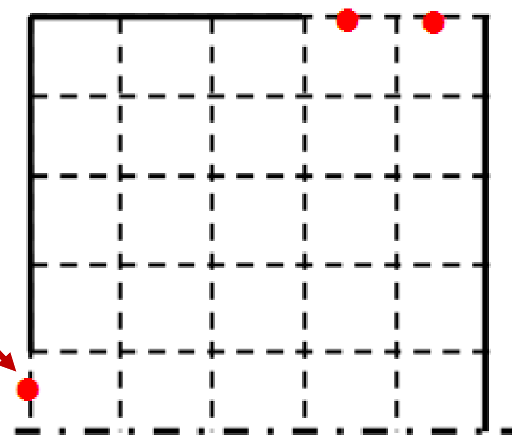



```
ENTRY START
OMEGA=100.
UIN=100.
DO 100 J=1,M1
DO 101 I=1,L1
U(I,J)=0.
V(I,J)=0.
F(I,J,5)=0.
F(L1,J,5)=R(J)**2*OMEGA
101 ENDDO
100 ENDDO
U(2,2)=UIN
AMU=1.
RETURN
*
ENTRY DENSE
RETURN
```

5th variable is R.VTheta

$$r \bullet v_{\theta} = r \cdot \omega r = \omega r^{2}$$

! Velocity on disc, causing circumferential movement



One way for obtaining outlet velocty of open system:

Assuming that the 1st derivatives at outlet =constant

$$\frac{v_{i,M1}-v_{i,M2}}{\Delta y}=k=const \longrightarrow v_{i,M1}=v_{i,M2}+k\Delta y=v_{i,M2}+C$$

C is determined according to total mass conservation

$$\sum_{i=2}^{L2}\rho_{i,M1}(v_{i,M2}+C)\Delta x_i=FLOWIN \longrightarrow$$

$$C=\frac{FLOWIN-\sum\rho_{i,M1}v_{i,M2}\Delta x_i}{\sum\rho_{i,M1}\Delta x_i}$$

$v_{i,M1}=v^{*}_{i,M2}+C$ **is taking as boundary condition for next iteration.**

In this example this method is used

```
ENTRY BOUND
IF(ITER.NE.0)   FLOWIN=RHO(1,2)*U(2,2)*YCVR(2)
FL=0.
AR=0.
DO 301 I=L3,L2
FLT=R(M1)*XCV(I)*RHO(I,M1)
AR=AR+FLT    ! Denominator
FL=FL+FLT*V(I,M2)
          ! Par of the Numerator
301 ENDDO
ADD=(FLOWIN-FL)/AR
DO 302 I=L3,L2      ! C---ADD
V(I,M1)=V(I,M2)+ADD
302 ENDDO
RETURN
```

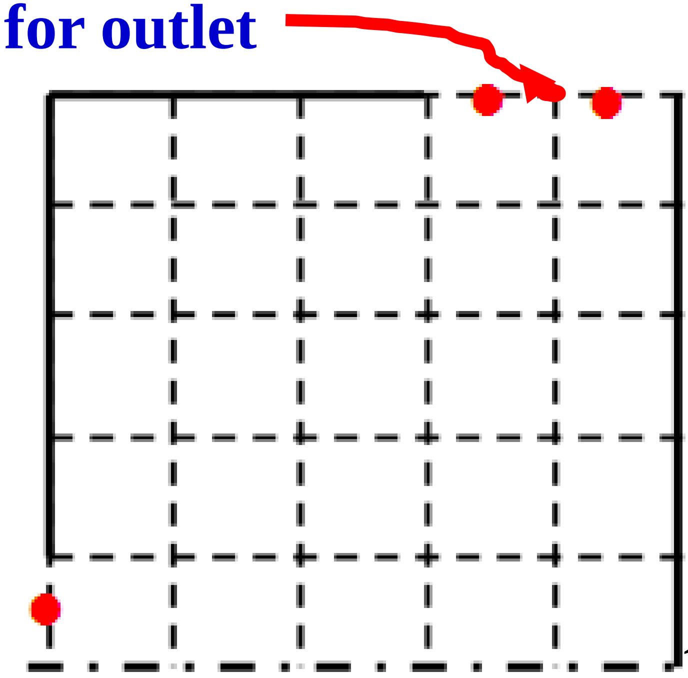
! FLOWIN = $\sum \rho(I,M1) \bullet XCV(I,M1) \bullet R(I,M1) \bullet (V(I,M2)+C)$

! $C = \dfrac{FLOWIN - \sum \rho(i,M1) \bullet XCV(i) \bullet R(M1) \bullet V(i,M2)}{\sum \rho(i,M1) \bullet XCV(i) \bullet R(M1)}$

!C-method is adopted to guarantee the total mass conservation condition

```
      ENTRY OUTPUT
      IF(ITER= =0) THEN
      PRINT 401
      WRITE(8,401)
401 FORMAT(1X,'  ITER',7X,'SMAX',11X,'SSUM',10X,'U(4,4)',
     & 9X,'V(4,4)')
       ELSE
       PRINT 403
       WRITE(8,403) ITER,SMAX,SSUM,U(4,4),V(4,4)
403 FORMAT(1X,I6,1P5E15.4)
       ENDIF
       IF(ITER= =LAST) CALL PRINT
       RETURN
```

```
ENTRY GAMSOR
    IF(ITER= = 0) THEN      ! Constant viscosity, calculation once is enough
    DO 500 J=1,M1
    DO 501 I=1,L1
    GAM(I,J)=AMU
501 ENDDO
502 ENDDO
    GAM(L3,M1)=0.           ! Local one-way for outlet
    GAM(L2,M1)=0.
    ENDIF
```



(E)

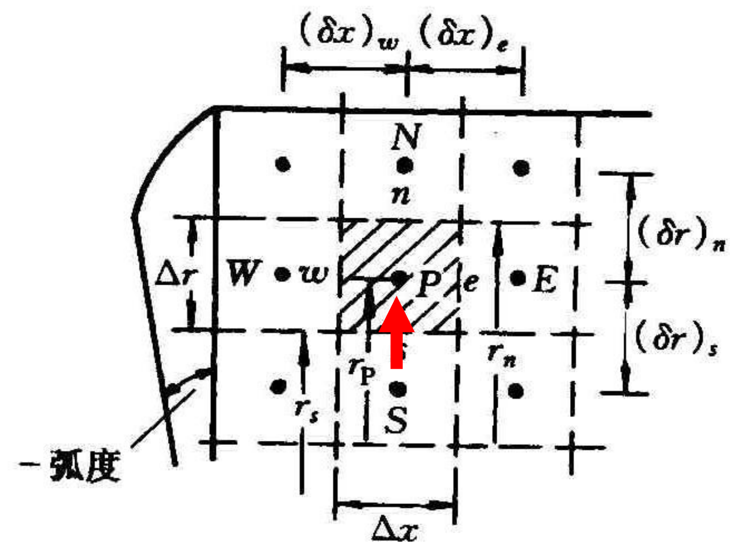
```
IF(NF= = 2) THEN
DO 502 J=3,M2
DO 503 I=2,L2
RSWM=FY(J)*F(I,J,4)+FYM(J)*F(I,J-1,4)
RHOM=FY(J)*RHO(I,J)+FYM(J)*RHO(I,J-1)
CON(I,J)=RHOM*RSWM**2/RMN(J)**3
AP(I,J)=-AMU/RMN(J)**2
503 ENDDO
502 ENDDO
ENDIF
```

! Source term of v –eq.

! rv_θ Interpolated from main nodes

! Interface density is interpolated from node density for the source term of Vr

$$S_{v_r} = \rho\frac{v_\theta^2}{r} - \eta\frac{v_r}{r^2} = \rho\frac{(rv_\theta)^2}{r^3} - \eta\frac{1}{r^2}v_r$$



```
510 IF(NF/=5) RETURN
    DO 512 J=2,M2   ! Source term of rv_θ is calculated at main node
    DO 513 I=2,L2
    AR=2.*AMU/YCVR(J)
    CON(I,J)=AR*F(I,J-1,5)
    AP(I,J)=-AR
512 ENDDO
513 ENDDO
    RETURN
    END
```

$$S_{(rv_\theta)} = \frac{2}{r_P}\frac{\eta (rv_\theta)_S}{YCV(j)} - \frac{2}{r_P}\frac{\eta}{YCV(j)}(rv_\theta)_P$$

$$= \underline{\frac{2\eta}{YCVR(j)}(rv_\theta)_S} - \underline{\frac{2\eta}{YCVR(j)}}(rv_\theta)_P$$

9-7-4 Results analysis

COMPUTATION FOR AXISYMMETRICAL SITUATION

****

ITER	SMAX	SSUM	U(4,4)	V(4,4)
0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
1	3.1852E-01	0.0000E+00	3.3742E+00	4.8158E+00
2	3.6224E-01	1.1921E-07	2.9314E+00	7.6065E+00
3	1.1265E-01	7.4506E-09	1.8755E+00	8.5863E+00
4	6.1974E-02	-3.7253E-08	1.5199E+00	8.8029E+00
5	3.2279E-02	-3.1665E-08	1.2971E+00	8.4019E+00
6	1.7869E-02	-4.0280E-08	1.2738E+00	7.6836E+00
7	1.2370E-02	5.1223E-09	1.3363E+00	6.8852E+00
8	1.0312E-02	-1.1176E-08	1.4400E+00	6.1421E+00
9	7.9294E-03	-2.9569E-08	1.5480E+00	5.5244E+00
10	5.9429E-03	4.8894E-08	1.6437E+00	5.0452E+00
11	4.6140E-03	-1.6531E-08	1.7207E+00	4.6926E+00
12	3.3741E-03	3.1199E-08	1.7787E+00	4.4432E+00
13	2.6291E-03	-5.1106E-08	1.8202E+00	4.2728E+00

14	1.9695E-03	-2.6543E-08	1.8486E+00	4.1597E+00
15	1.4364E-03	6.2981E-08	1.8674E+00	4.0867E+00
16	1.0142E-03	-4.5111E-08	1.8792E+00	4.0409E+00
17	6.9815E-04	8.9640E-09	1.8864E+00	4.0129E+00
18	4.6667E-04	3.8388E-08	1.8906E+00	3.9963E+00
19	3.0389E-04	3.3469E-09	1.8929E+00	3.9868E+00
20	1.9290E-04	-1.1176E-08	1.8941E+00	3.9816E+00
21	1.1830E-04	5.2169E-09	1.8946E+00	3.9790E+00
22	7.0846E-05	4.6941E-08	1.8947E+00	3.9778E+00
23	4.0823E-05	5.4388E-08	1.8947E+00	3.9773E+00
24	2.2590E-05	-8.0094E-08	1.8945E+00	3.9772E+00
25	1.1003E-05	-3.8743E-08	1.8944E+00	3.9773E+00

************************** .VEL U. **************************

I =	2	3	4	5	6	7
J						
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
6	0.00E+00	-1.33E+00	-2.67E+00	-2.12E+00	-8.37E-01	0.00E+00
5	0.00E+00	-1.86E+00	-2.70E+00	-1.86E+00	-6.39E-01	0.00E+00
4	0.00E+00	-2.17E-01	1.89E+00	2.90E+00	1.65E+00	0.00E+00
3	0.00E+00	1.33E+01	1.97E+01	1.92E+01	1.04E+01	0.00E+00
2	1.00E+02	8.63E+01	7.43E+01	5.99E+01	3.27E+01	0.00E+00
1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

************************** .VEL V. **************************

I =	1	2	3	4	5	6	7
J							
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.99E+00	6.01E+00	0.00E+00
6	0.00E+00	-1.50E+00	-1.50E+00	6.18E-01	6.44E+00	8.45E+00	0.00E+00
5	0.00E+00	-4.17E+00	-2.98E+00	1.81E+00	1.00E+01	1.20E+01	0.00E+00
4	0.00E+00	-6.53E+00	-1.84E+00	3.98E+00	1.34E+01	1.60E+01	0.00E+00
3	0.00E+00	6.87E+00	5.96E+00	7.21E+00	1.36E+01	1.63E+01	0.00E+00
2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

**************************** .STR FN. ****************************

I =	2	3	4	5	6	7
J						
7	5.00E-01	5.00E-01	5.00E-01	5.00E-01	3.00E-01	0.00E+00
6	5.00E-01	5.60E-01	6.20E-01	5.95E-01	3.38E-01	0.00E+00
5	5.00E-01	6.25E-01	7.15E-01	6.60E-01	3.60E-01	0.00E+00
4	5.00E-01	6.31E-01	6.67E-01	5.88E-01	3.19E-01	0.00E+00
3	5.00E-01	4.31E-01	3.72E-01	3.00E-01	1.63E-01	0.00E+00
2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

At the shell flow rate is constant

Zero flow rate on disc

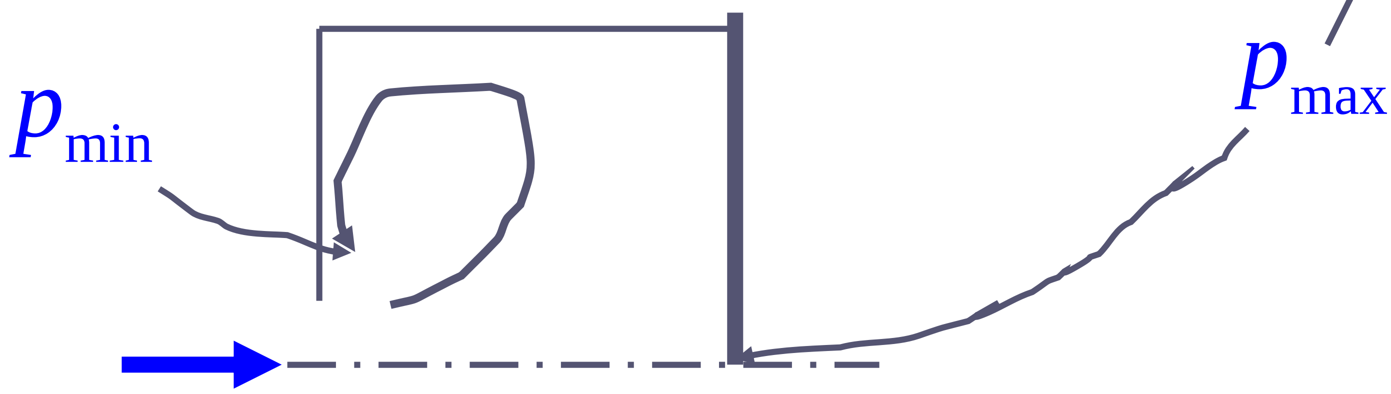
**************************** R. VTH ****************************

I =	1	2	3	4	5	6	7
J							
7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.50E+01
6	0.00E+00	1.24E-01	5.24E-01	1.64E+00	5.76E+00	1.26E+01	2.02E+01
5	0.00E+00	2.02E-01	7.28E-01	1.69E+00	3.66E+00	7.75E+00	1.23E+01
4	0.00E+00	1.40E-01	4.46E-01	8.49E-01	1.53E+00	3.54E+00	6.25E+00
3	0.00E+00	5.15E-02	1.49E-01	2.47E-01	3.84E-01	1.09E+00	2.25E+00
2	0.00E+00	4.66E-03	1.84E-02	3.72E-02	5.53E-02	1.55E-01	2.50E-01
1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

$\varpi * r^2$

********************************** PRESSURE **********************************

J \ I =	1	2	3	4	5	6	7
7	-4.93E+02	-4.81E+02	-4.57E+02	-3.68E+02	-3.47E+02	-3.61E+02	-3.61E+02
6	-5.08E+02	-4.96E+02	-4.72E+02	-3.94E+02	-3.61E+02	-3.61E+02	-3.61E+02
5	-5.38E+02	-5.26E+02	-5.02E+02	-4.46E+02	-3.89E+02	-3.61E+02	-3.47E+02
4	-6.85E+02	-6.47E+02	-5.72E+02	-4.92E+02	-3.60E+02	-2.41E+02	-1.81E+02
3	-1.15E+03	-9.63E+02	-5.97E+02	-4.57E+02	-1.85E+02	1.02E+02	2.46E+02
2	-3.01E+02	-3.62E+02	-4.84E+02	-3.04E+02	1.83E+02	6.20E+02	8.39E+02
1	0.00E+00	-6.11E+01	-4.27E+02	-2.28E+02	3.67E+02	8.79E+02	1.10E+03



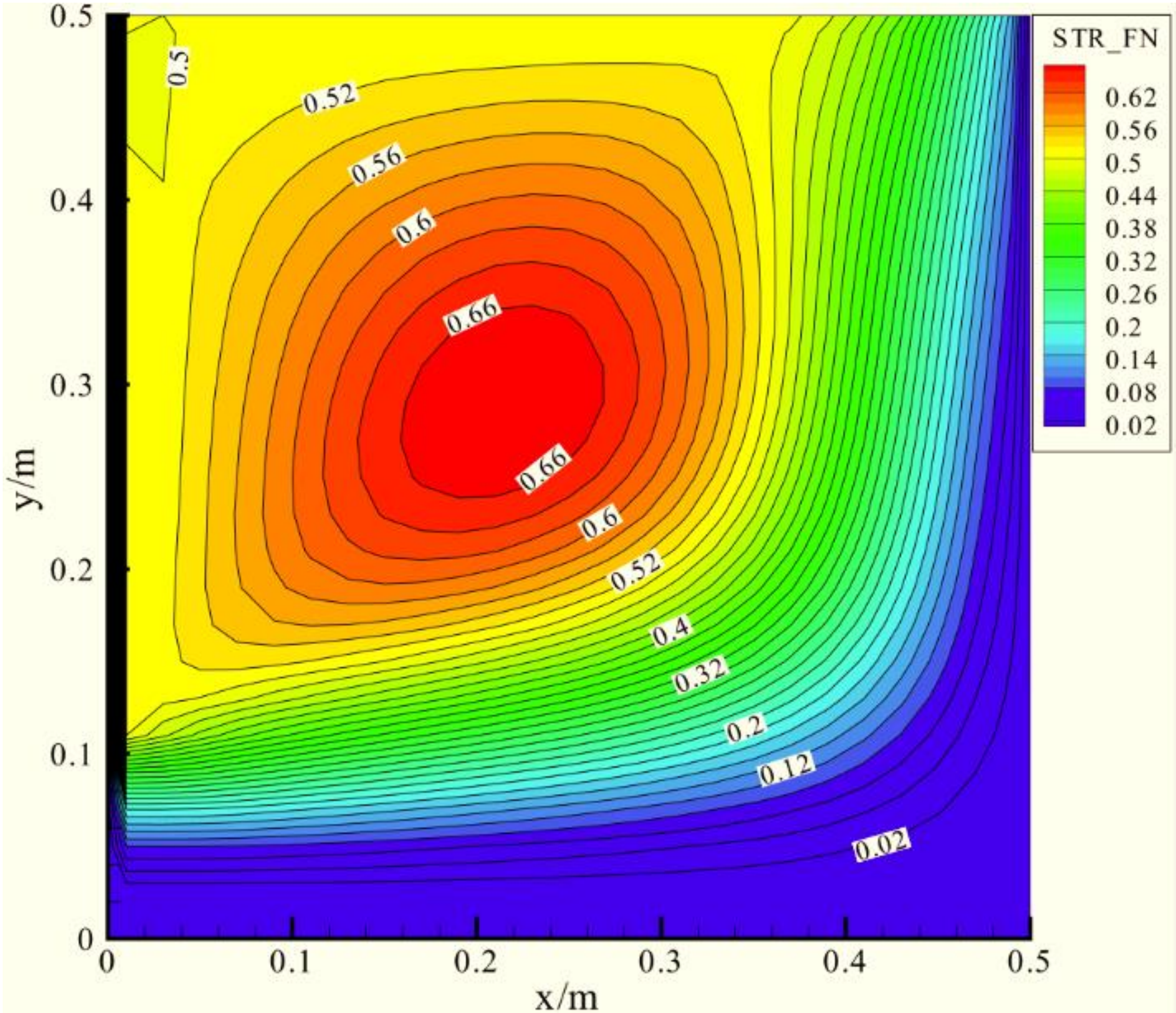
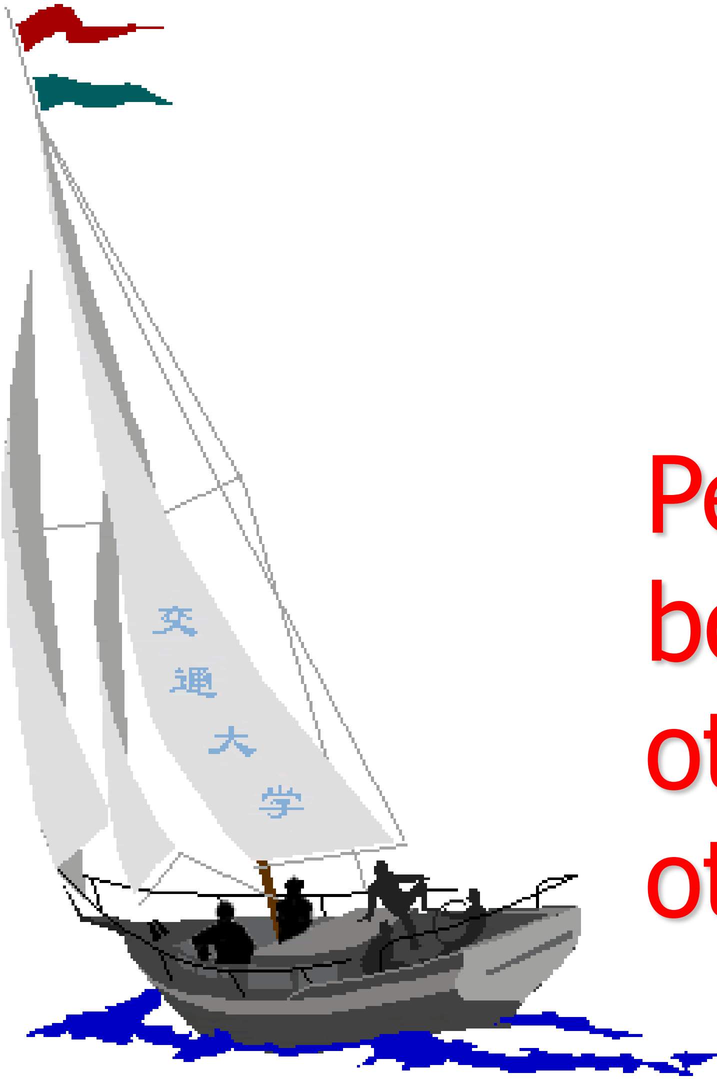


Fig.2 Schematic diagram of Section 7



同舟共济
渡彼岸！

People in the same boat help each other to cross to the other bank, where….