西安交通大学《Numerical Heat Transfer》课程大作业(1)

（20211208）

请使用本课程的教学程序研究以下非稳态热传导问题：

将金属铸件在型砂中的冷却过程简化成为如图 **1** 所示二维方形铸件的第一类边界条件下的非稳态导热问题，及图 **2** 所示型砂的非稳态导热问题，研究：

1. 在第一类边界条件下影响固体中温度变化快慢的物性参数是热扩散率：

影响固体导热过程的物性参数有：密度ρ、比热容 c、导热系数 λ。试运用数值方法验证，在第一类边界条件下，保证热扩散系数$a=\dfrac{\lambda}{\rho c}$不变，固体的非稳态导热过程的温度变化快慢与材料的导热系数无关。

2. 在砂型的内边界保持一个不变温度时，一定时间内，砂型的吸热量与$\sqrt{\rho c\lambda}$几乎成直线关系

在铸造工程中，把型砂的$\sqrt{\rho c\lambda}$称为其吸热系数，表征砂型的吸热能力，试运用教学程序验证上述论。

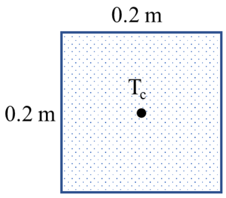

图 1

（铸件材料热物性自行选取）

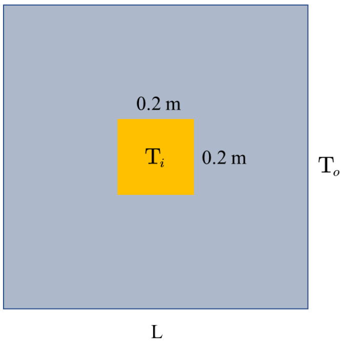

图 2

（型砂材料的热物性自行选取）

提示与要求：

1. 采用如下的有量纲的常物性导热控制方程进行离散，ρc_p及λ分别与非稳态项及扩散项结合

$$\rho c_p \frac{\partial T}{\partial t}=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

2. 要求第一类边界条件要通过第三类边界条件来实现，即按照第 **3** 类边界条件发展程序，练习附加源项法的实施方法；然后使换热系数 h 取一个很大的值来逼近第一类边界条件；

3. 建议定义中心温度降低到初始值的 **1/2** 所用的时间为热响应时间，记为τ_o，可通过比较热响应时间的大小来比较温度变化的快慢；

4. 对于第一个问题，可以设定一个较高的初始温度，t**=0** 时刻，固体边界温度突然降低到一个较低的温度并保持不变，通过比较保持 a（参考值在 **10^{-5}m^2/s** 的量级）不变而λ 改变时（取 **5-6** 个不同的数值，至少变化 **5** 倍）物体的热响应时间；需要进行网格独立性与时间步长独立性的考核；

5. 对于第二个问题，可采用图 **2** 所示计算区域（灰色部分）；设定外边界的温度为一定值，例如环境温度，内边界温度突然升身高到一定数值，在 **5-6** 个吸热系数下（至少变化 **2-3** 倍）计算在一定时间间隔内（例如前 **100** 秒时间内）计算区域所得到的热量；除了网格独立性及时间步长独立性考核外，还需要让砂型的尺寸足 **L** 够大，即再增加外边界尺寸对吸热量几乎没有影响了，以排除外边界的影响。

Project Assignment of Numerical Heat Transfer course of Xi'an Jiaotong University (1) (20211208)

Please use the teaching code of this course (available for download on http://nht.xjtu.edu.cn/) to calculate the following unsteady heat conduction problems:

The cooling process of a metal casting in molding sand is simplified by two sub-problems: the unsteady heat conduction problem of two-dimensional square castings under the first kind of boundary conditions as shown in Fig. 1 and the unsteady heat conduction problem of molding sand as shown in Fig. 2 by the grey part. Please verify by numerical simulation:

1. Physical property that affect the speed of temperature changes in solids under the first kind of boundary conditions is the thermal diffusivity:

The main physical parameters affecting the thermal conductivity of solids are density ρ, specific heat capacity c, and thermal conductivity λ. Please use the numerical method to verify that under the first kind of boundary conditions, when the thermal diffusion coefficient $a=\dfrac{\lambda}{\rho c}$ remains unchanged, the unsteady thermal conduction process of the solid is independent of the thermal conductivity λ of the material.

2. When the inner boundary of the sand mold maintains a constant temperature, the heat absorption within a certain time of the sand mold varies almost linearly with $\sqrt{\rho c \lambda}$.

In foundry engineering, the $\sqrt{\rho c \lambda}$ of molding sand is called its heat absorption coefficient to characterize the heat absorption capacity of sand mold. Try to verify the above statement numerically by using teaching code.

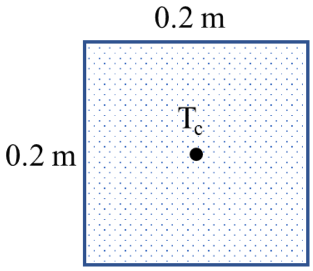

Fig. 1

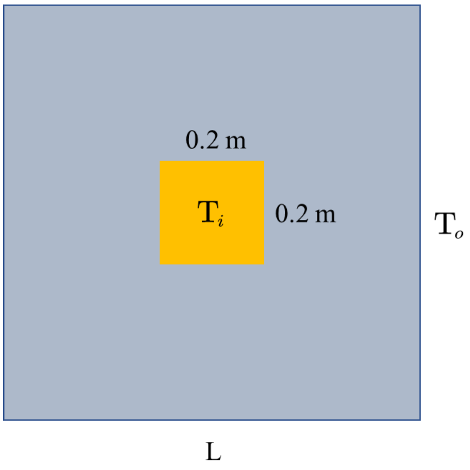

Fig. 2

(Select the thermophysical properties of casting materials and molding sand materials by yourself)

Tips and requirements:

1. The following governing equation of constant heat conduction should be adopted for discretization:

$$\rho c_p \frac{\partial T}{\partial t} = \lambda\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

in which ρc_p and λ are respectively attached with transient term and diffusion term.

2. It is required that the first kind of boundary conditions should be realized through the third kind of boundary conditions, that is, the program is developed according to the third kind of boundary conditions to practice the implementation of the additional source term method; Then make the heat transfer coefficient h take a large value to approximate the first kind of boundary conditions.

3. It is suggested to define the time taken for the central point temperature to decrease to 1 / 2 of the initial value as the thermal response time τ_0, and the speed of temperature change can be compared by comparing the thermal response time.

4. For the first problem, a higher initial temperature can be set. When t = 0, the solid boundary temperature suddenly decreases to a lower temperature and remains unchanged. It can be verified by calculating and comparing the thermal response time τ_0 with the thermal diffusivity a remaining unchanged (the reference value is in the order of $10^{-5}\mathrm{m}^2/\mathrm{s}$) and different λ taken (take 5-6 different values, the ratio of the maximum and minimum values at least 5). Grid independence and time step independence need to be assessed.

5. For the second problem, the calculation area shown in Figure 2 (gray part) can be used; Set the temperature of the outer boundary to a certain value, such as the ambient temperature, the temperature of the inner boundary suddenly rises to a certain value, and calculate the heat obtained in the calculation area within a certain time interval (such as the first 100 seconds) under 5-6 heat absorption coefficients (change at least 2-3 times). In addition to the assessment of grid independence and time step independence, it is also necessary to make the size of sand mold L large enough, that is, further increasing the size of outer boundary has little effect on heat absorption, so as to eliminate the influence of outer boundary.