Numerical Heat Transfer

Chapter 13 Application examples of fluent for flow and heat transfer problem

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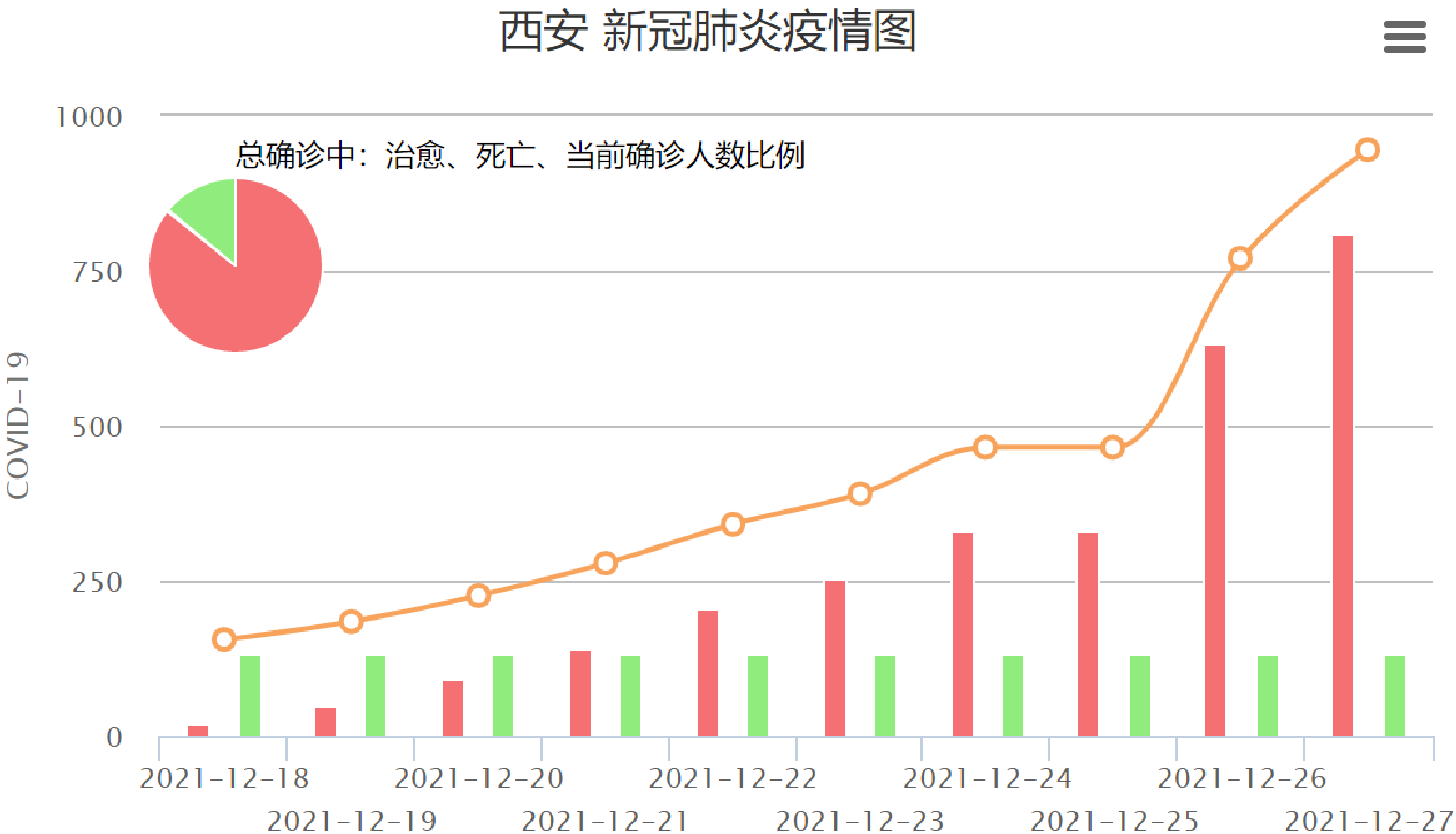
Xi'an, 2021-Dec.-28



Dec. 27th, Xi’ an: 175

“西”望你我，“安”然无恙

Hope Everyone Safe and Sound

13. 2 Flow and heat transfer in porous media

多孔介质流动换热问题

Focus: in this example, first the background of porous media is introduced, and then governing equations for fluid flow and heat transfer in porous media are discussed in detail.

13. 2 Flow and heat transfer in porous media

Known： **Steady state fluid flow and heat transfer of air in a channel filled with porous medium made of Aluminum (铝). The porosity (孔隙率) of the porous medium is 0.8. The permeability（渗透率）of the porous medium is 7. E-8 m^2. The computational domain is shown in Fig. 2. The boundary condition is as follows.**

- **Inlet: *u*---5m/s ; *T*---300K**
- **Pressure outlet: Gauge pressure（表压）: 0 Pa.**
- **Top and bottom boundary: 2^{rd} boundary condition**

Heat flux: q=10000 W/m^2

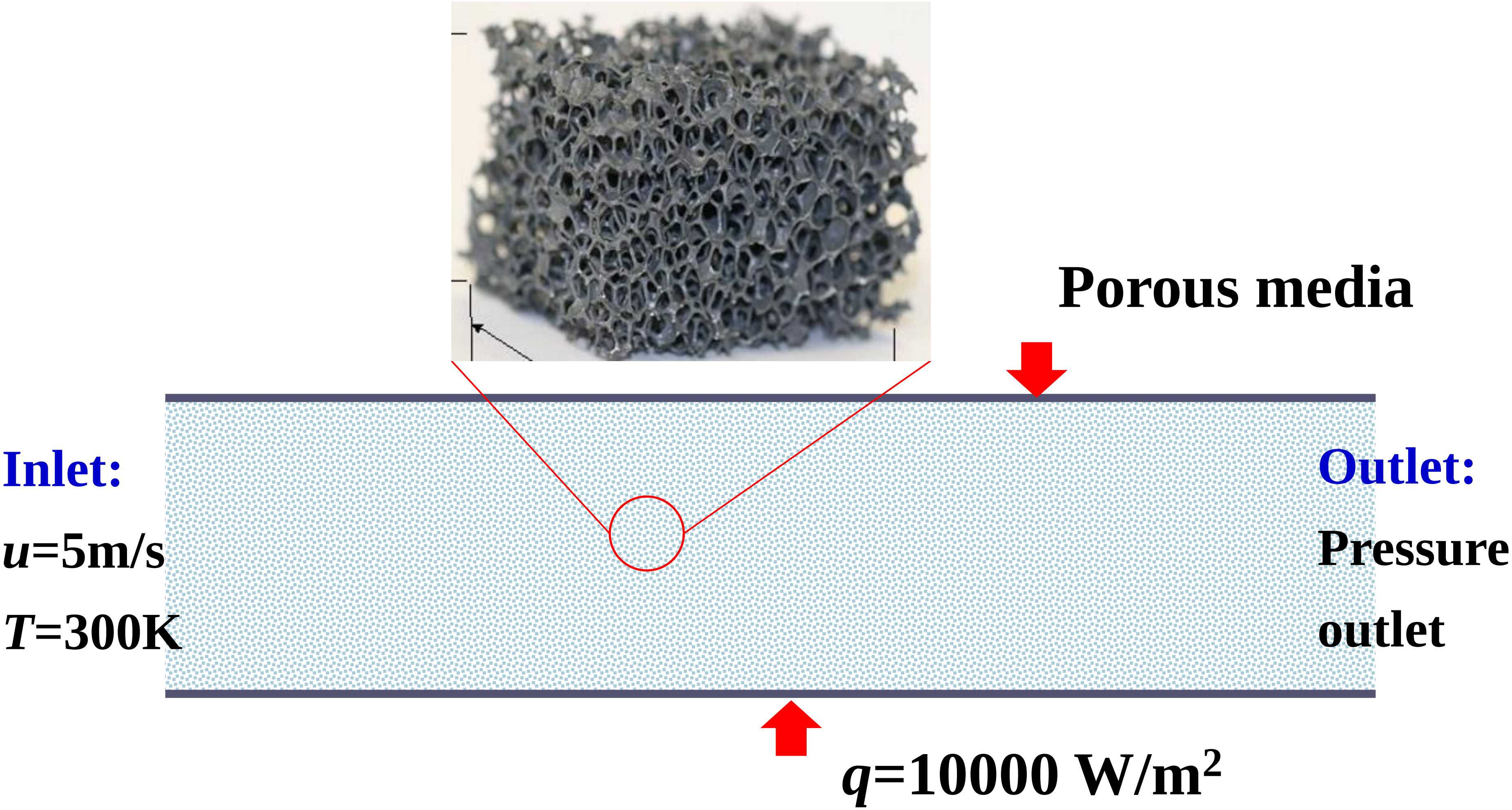


Fig. 2 Computational domain

Find: Temperature and velocity distribution in the domain

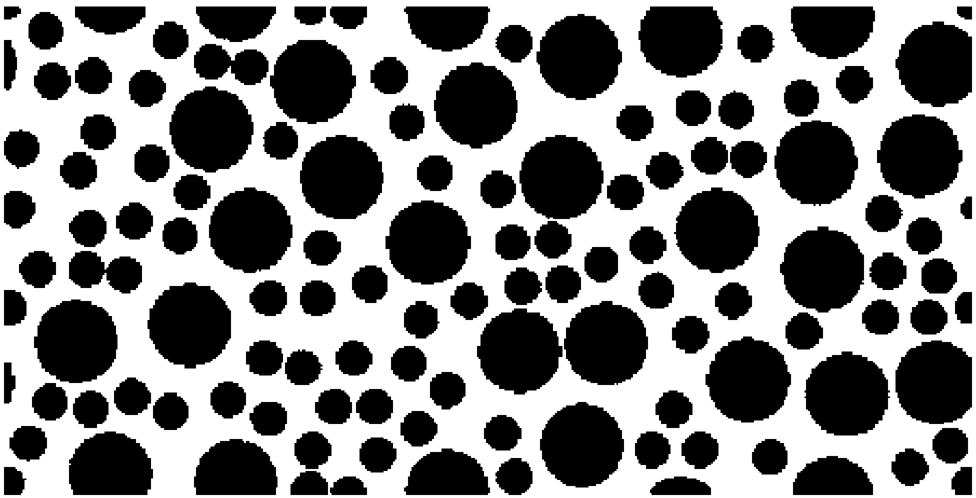
Solution:

Continuity, momentum and energy equation for porous media??

The governing equations for porous media are different from the original NS equation. Thus, we will study together background information of porous media and then derive the governing equations.

Introduction to Porous media

A porous medium is a material that contains plenty of pores (孔) between solid skeleton (骨架）.



Black: solid

White: pores

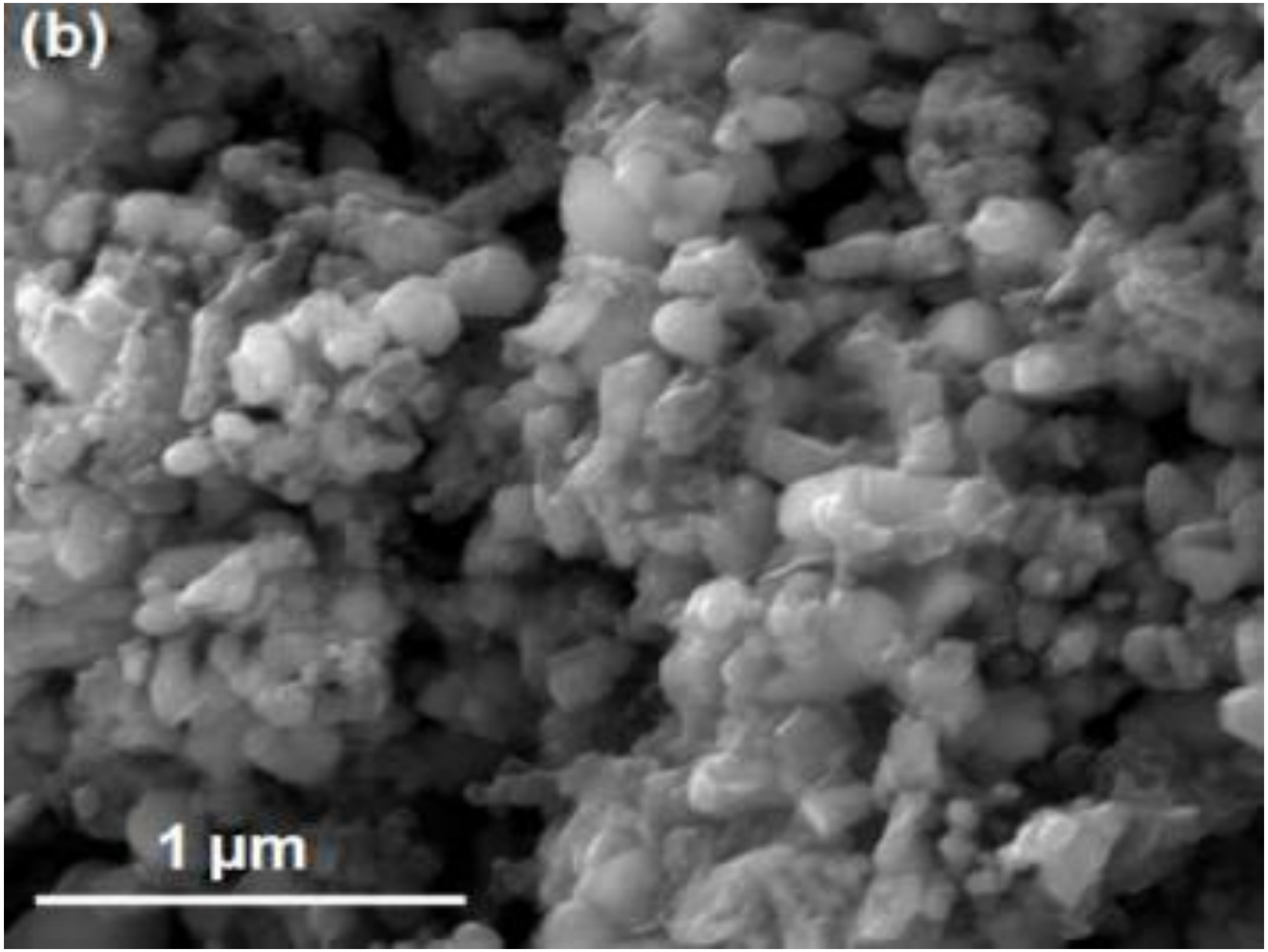
Two necessary elements in a porous medium:

Skeleton ：maintaining the shape of a porous medium

Pores: providing pathway for fluid transport.

Metal foam (金属泡沫）

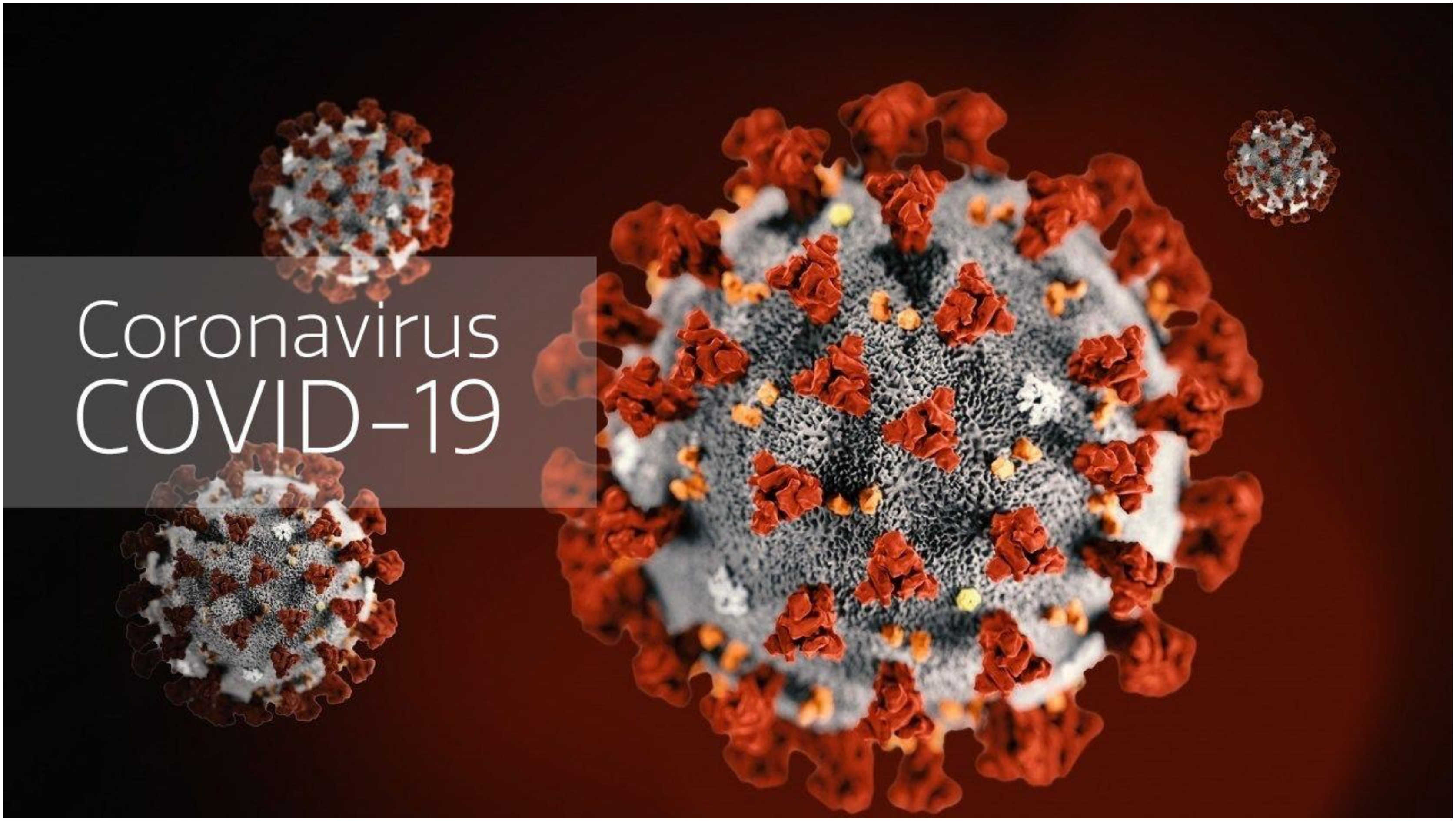
Carbon fiber （碳纤维）



Catalyst

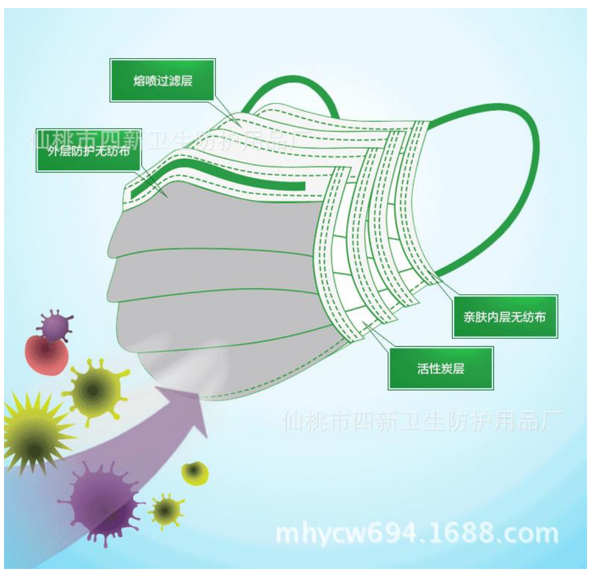
Stone

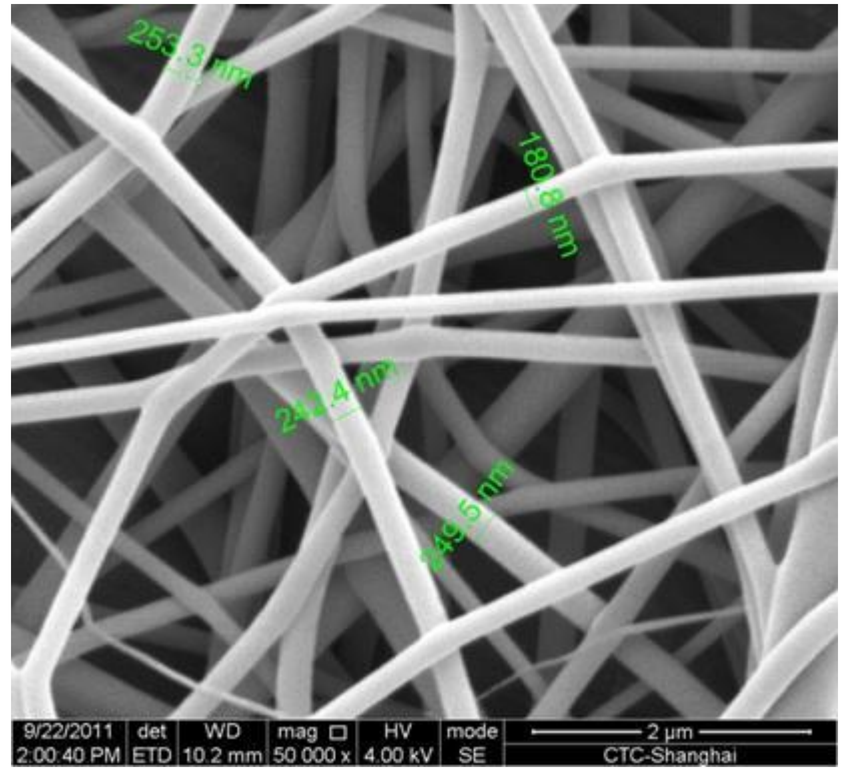
Bread



Covid-19


戴口罩 勤洗手 抗击肺炎


熔喷过滤层
外层防护无纺布
亲肤内层无纺布
活性炭层
仙桃市四新卫生防护用品厂
mhycw694.1688.com


253.3 nm
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Mask

Structure Characterization （结构表征）

Porosity (孔隙率)

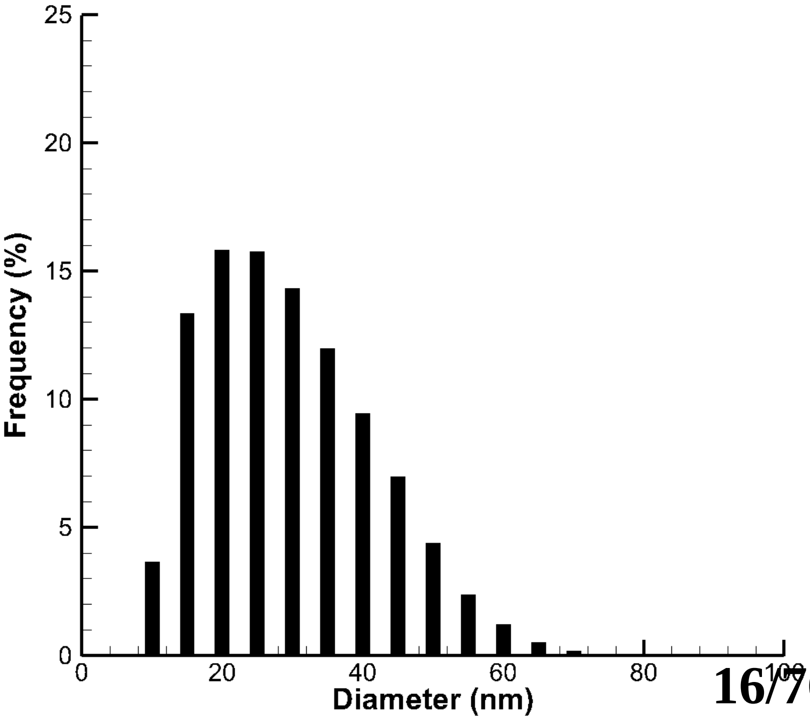
The volume ratio between pore volume and total volume

$$\varepsilon = \frac{V_{\text{pore}}}{V_{\text{total}}}$$

In the range of 0~1.

Pore size （孔径分布）

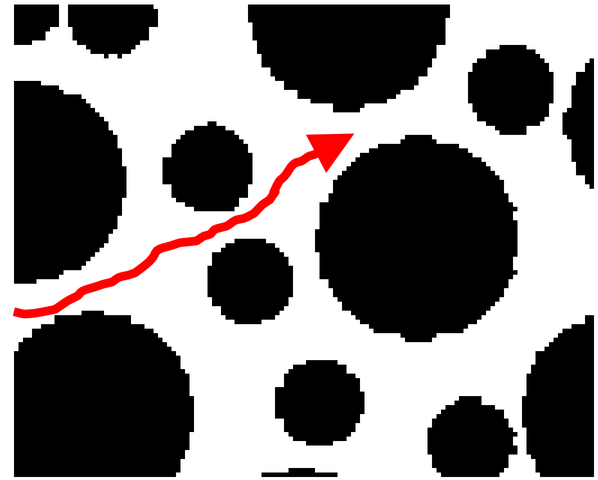
The size of pores. Use **pore size distribution （PSD)** to character (表征） size of pores in a porous medium.



Two velocity definition in a porous medium:

$$V_{\text{superficial}} = \varepsilon V_{\text{physical}}$$

Porosity



V_{physical}（真实速度）： the actual flow velocity in the pores.

$V_{\text{superficial}}$（表观速度)：the averaged velocity in the entire domain. Consider the domain as black box.

$$V_{\text{superficial}} < V_{\text{physical}}$$

Fluent uses superficial velocity as the default velocity.

Original continuity and momentum equation

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\mathbf{u})=0$$

$$\frac{\partial(\rho\mathbf{u})}{\partial t}+(\mathbf{u}\cdot\nabla)(\rho\mathbf{u})=-\nabla p+\eta\nabla^2\mathbf{u}$$

Mass conservation equation for porous media:

As the total mass of fluid is $\rho V_f = \rho\varepsilon V_{total} = \rho\varepsilon\Delta x\Delta y\Delta z$

$$\frac{\partial(\varepsilon\rho)}{\partial t}+\nabla\cdot(\varepsilon\rho\mathbf{u}_{\text{physical}})=0$$

Fluent uses superficial velocity as the default velocity.

$$\frac{\partial(\varepsilon\rho)}{\partial t}+\nabla\cdot(\rho\mathbf{u}_{\text{superficial}})=0$$

Momentum conservation equation for porous media:

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + (\mathbf{u}\cdot\nabla)(\rho\mathbf{u}) = -\nabla p + \eta\nabla^2\mathbf{u}$$

$$\rho u_p V_f = \rho u_p \varepsilon V_{total} = \rho u_p \varepsilon \Delta x \Delta y \Delta z$$

$$\frac{\partial(\varepsilon\rho\mathbf{u}_{\text{physical}})}{\partial t} + (\mathbf{u}_{\text{physical}}\cdot\nabla)(\varepsilon\rho\mathbf{u}_{\text{physical}}) = -\varepsilon\nabla(p) + \eta\varepsilon\nabla^2\mathbf{u}_{\text{physical}} + \mathbf{F}$$

Force due to porous media

$$\frac{\partial(\rho\mathbf{u}_{\text{superficial}})}{\partial t} + \left(\frac{\mathbf{u}_{\text{superficial}}}{\varepsilon}\cdot\nabla\right)(\rho\mathbf{u}_{\text{superficial}}) = -\varepsilon\nabla(p) + \varepsilon\eta\nabla^2\left(\frac{\mathbf{u}_{\text{superficial}}}{\varepsilon}\right) + \mathbf{F}$$

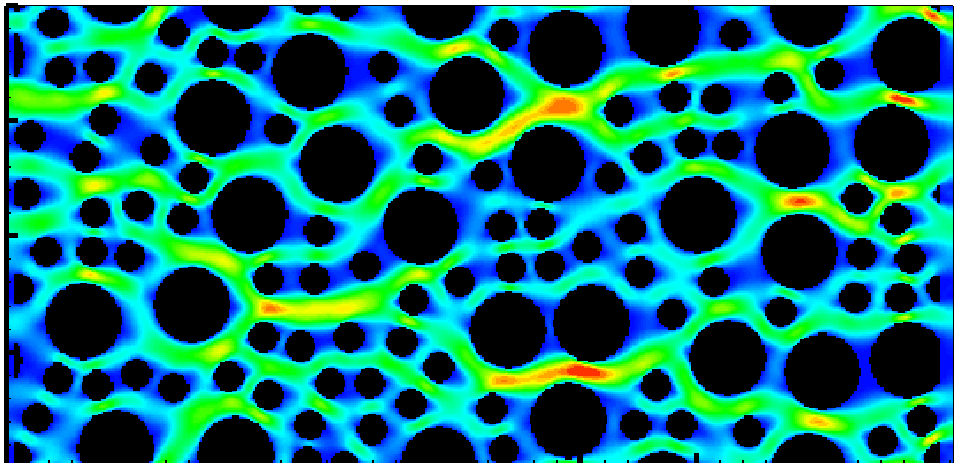
For incompressible steady state problem:

$$\nabla\cdot\mathbf{u}_{\text{superficial}} = 0$$

$$\left(\frac{\mathbf{u}_{\text{superficial}}}{\varepsilon}\cdot\nabla\right)(\mathbf{u}_{\text{superficial}}) = -\frac{1}{\rho}\varepsilon\nabla(p) + \nu\nabla^2(\mathbf{u}_{\text{superficial}}) + \frac{1}{\rho}\mathbf{F}$$

$$\frac{\partial(\varepsilon\rho\mathbf{u}_{\text{physical}})}{\partial t}+(\mathbf{u}_{\text{physical}}\cdot\nabla)(\varepsilon\rho\mathbf{u}_{\text{physical}})=-\varepsilon\nabla(p)+\eta\varepsilon\nabla^2\mathbf{u}_{\text{physical}}+\mathbf{F}$$

The fluid-solid interaction is strong in porous media. Porous media are modeled by adding a momentum source term:



$$\mathbf{F}=-\frac{\varepsilon\upsilon\rho}{k}\mathbf{u}_{\text{superficial}}-\frac{\varepsilon\rho F_\varepsilon}{\sqrt{k}}|\mathbf{u}_{\text{superficial}}|\mathbf{u}_{\text{superficial}}$$

$$\mathbf{F} = -\frac{\varepsilon \upsilon \rho}{k}\mathbf{u}_{\text{superficial}} - \frac{\varepsilon F_{\varepsilon} \rho}{\sqrt{k}} |\mathbf{u}_{\text{superficial}}| \mathbf{u}_{\text{superficial}}$$

The first term is the viscous loss term （黏性项） or the Darcy term, with unit of N/m^3

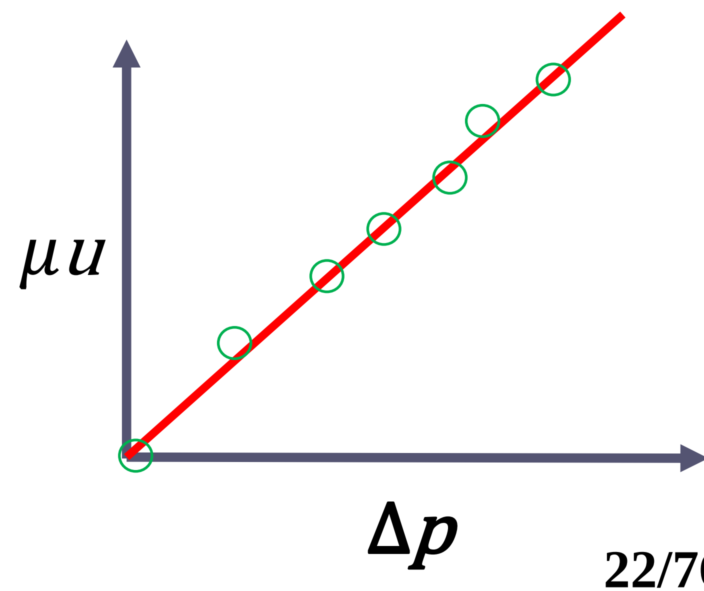
The second term is inertial loss term (惯性项） or the Forchheimer term, with unit of N/m^3

***k* is the permeability (渗透率) of a porous media, one of the most important parameter of a porous medium.**

Permeability (渗透率)

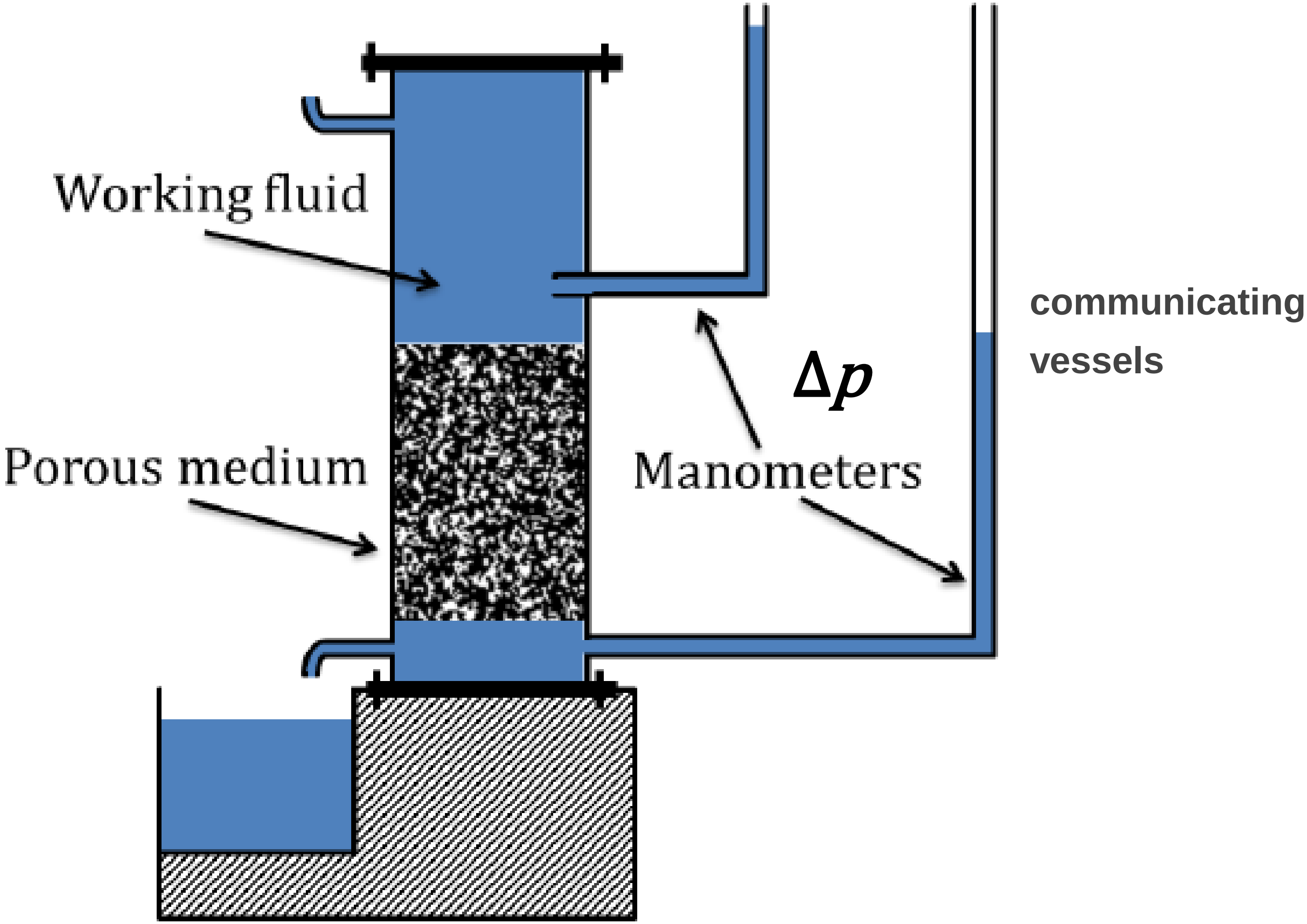
In 1856, Darcy (法国工程师） noted that for laminar flow through porous media, the flow rate <*u*> is linearly proportional to the applied pressure gradient Δ*p*, thus he introduced **permeability to describe the conductivity of the porous media**. The Darcy' law is as follows

$$< u > = -\frac{k}{\mu}\frac{\Delta p}{l}$$

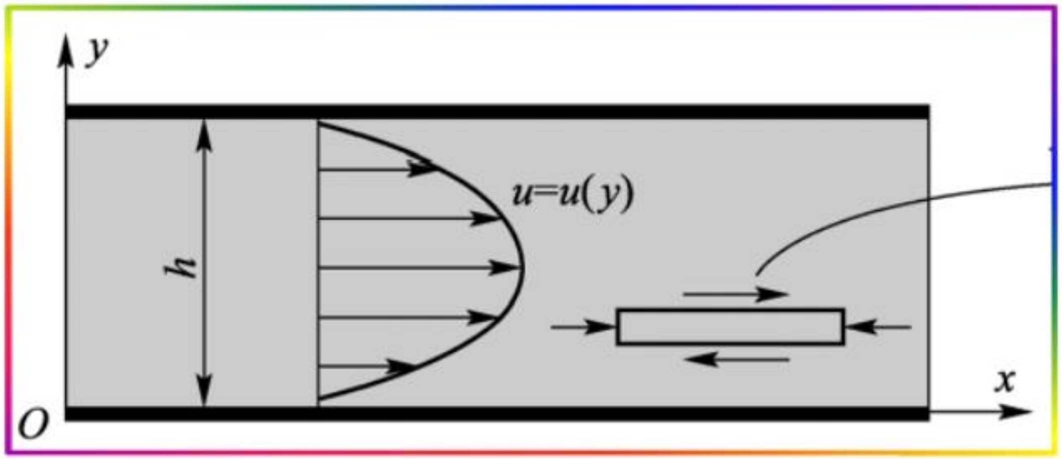


***k* is permeability** with unit of m^2

Schematic of Darcy’s experiment



Poiseuille flow



$$u = -\frac{h^2}{2\mu}\frac{\partial p}{\partial x}\frac{y}{h}\left(1-\frac{y}{h}\right)$$

$$\bar{V} = \frac{Q}{A} = \frac{h^2 \Delta p}{12\mu l}$$

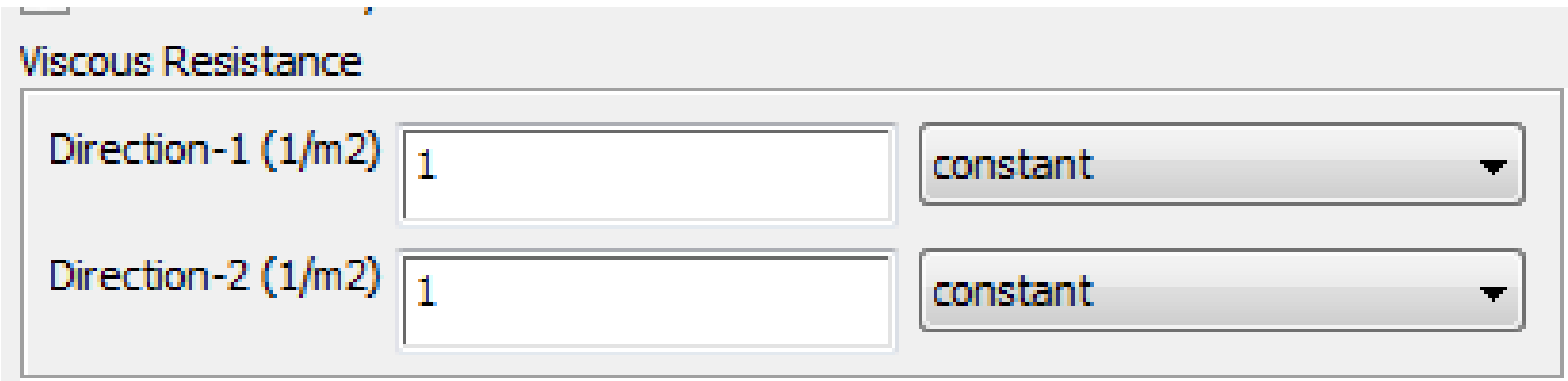
$$< u >= -\frac{k}{\mu}\frac{\Delta p}{l} \quad \Rightarrow \quad k = \frac{h^2}{12}$$

In Fluent, this force source term is expressed as

$$\mathbf{F} = -\frac{\mu}{k}\mathbf{u} - C_2 \frac{1}{2}\rho|\mathbf{u}|\mathbf{u}$$

***k*: permeability;** **C_2: inertial resistance factor (惯性阻力）**

Here, viscous resistance(黏性阻力） is 1/*k*!



Permeability is a transport property of a porous medium, and there are database of *k* for different porous materials.

$$\mathbf{F} = -\frac{\mu}{k}\mathbf{u} - C_2 \frac{1}{2}\rho|\mathbf{u}|\mathbf{u}$$

The second term can be canceled if the fluid flow is slow

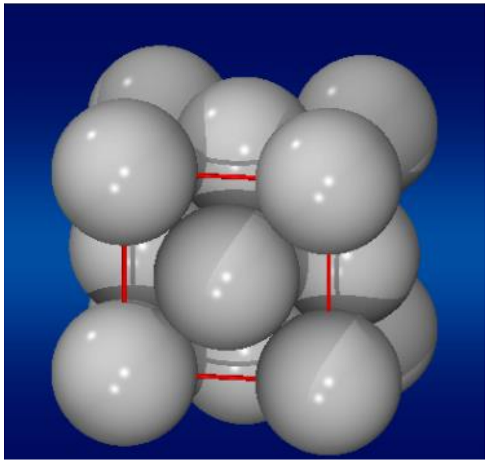
u **is small, <<1, thus u*u is smaller.**

Otherwise, this term should be considered.

There have been lots of experiments in the literature to determine the relationship between pressure drop and velocity of different kinds of porous media, and thus to determine permeability.

Ergun equation is one of the most adopted empirical equations (经验公式) for packed bed porous media.

$$\frac{\Delta P}{l}=\frac{150\mu}{D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}u+\frac{1.75\rho}{D_p}\frac{(1-\varepsilon)}{\varepsilon^3}u^2$$



Diameter of solid particle

$$\mathbf{F}=-\frac{\mu}{k}\mathbf{u}-C_2\frac{1}{2}\rho|\mathbf{u}|\mathbf{u}$$

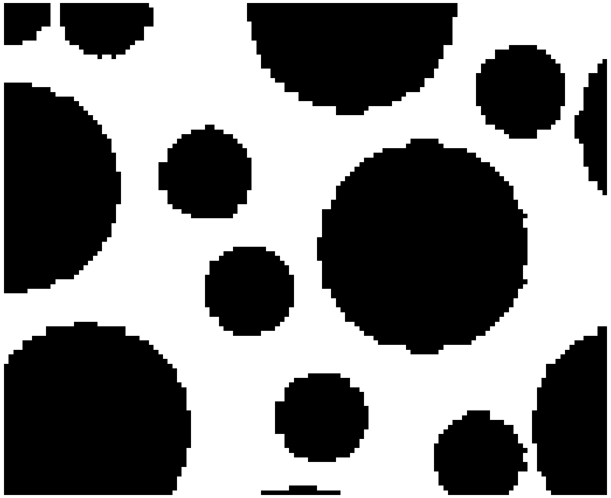
⬅ **Fluent**

Comparing the two equations, you can obtain *k* and C_2.

$$k=\frac{D_p^2}{150}\frac{\varepsilon^3}{(1-\varepsilon)^2}\qquad C_2=\frac{3.5}{D_p}\frac{(1-\varepsilon)}{\varepsilon^3}$$

Energy equation

$$\frac{\partial(\rho C_p T)}{\partial t} + (\mathbf{u}\cdot\nabla)(\rho C_p T) = \lambda\nabla^2\mathbf{T} + \mathbf{S}$$



For porous media:

Heat transfer in fluid phase as well as in solid phase.

There are two models for heat transfer:

Equilibrium thermal model (热平衡模型）

Non-Equilibrium thermal model （非热平衡模型）

Equilibrium thermal model （热平衡模型）

Assume solid phase and fluid phase are in thermal equilibrium. In other words, the temperature of fluid and solid in a mesh is the same.

Original

$$\frac{\partial(\rho C_p T)}{\partial t}+(\mathbf{u}\cdot\nabla)(\rho C_p T)=\nabla(\lambda\nabla T)+S$$

For the first term :

$$\rho C_p TV=(1-\varepsilon)V(\rho C_p)_{\text{solid}}T_{\text{solid}}+\varepsilon V(\rho C_p)_{\text{fluid}}T_{\text{fluid}}$$

$$=\left[(1-\varepsilon)(\rho C_p)_{\text{solid}}+\varepsilon(\rho C_p)_{\text{fluid}}\right]VT$$

$$\rho C_p T=\left[(1-\varepsilon)(\rho C_p)_{\text{solid}}+\varepsilon(\rho C_p)_{\text{fluid}}\right]T$$

For the second convection term:

$$(\mathbf{u}\cdot\nabla)(\varepsilon\rho C_p T)$$

As convective term is only for fluid phase!

For the diffusion term:

$$\nabla(\lambda\nabla T)V = \nabla(\lambda_s\nabla T_s)V(1-\varepsilon) + \nabla(\lambda_f\nabla T_f)V\varepsilon$$

$$= \nabla(\lambda_s(1-\varepsilon)\nabla T)V + \nabla(\lambda_f\varepsilon\nabla T)V$$

$$= V\nabla(\lambda_s(1-\varepsilon)\nabla T + \lambda_f\varepsilon\nabla T)$$

$$= V\nabla(\lambda_{\text{eff}}\nabla T)$$

$$\lambda\nabla^2 T = \left[(1-\varepsilon)\lambda_s + \varepsilon\lambda_f\right]\nabla^2 T$$

For the source term

$$SV = (1-\varepsilon)\mathrm{V}\,S_s + \varepsilon V S_f$$

$$\frac{\partial\left(\left[(1-\varepsilon)(\rho C_p)_{\text{solid}}+\varepsilon(\rho C_p)_{\text{fluid}}\right]T\right)}{\partial t}+(\mathbf{u}\cdot\nabla)(\varepsilon\rho C_p T)$$
$$=\left[(1-\varepsilon)\lambda_s+\varepsilon\lambda_f\right]\nabla^2 T+\left[(1-\varepsilon)S_s+\varepsilon S_f\right]$$

$$(\rho C_p)_{\text{eff}}=\left[(1-\varepsilon)(\rho C_p)_{\text{solid}}+\varepsilon(\rho C_p)_{\text{fluid}}\right]$$

$$\lambda_{\text{eff}}=(1-\varepsilon)\lambda_s+\varepsilon\lambda_f \qquad S_{\text{eff}}=(1-\varepsilon)S_s+\varepsilon S_f$$

The final energy equation for porous media

$$\frac{\partial((\rho C_p)_{\text{eff}}T)}{\partial t}+(\mathrm{u}_{\text{superficial}}\cdot\nabla)(\rho C_p T)=\lambda_{\text{eff}}\nabla^2 T+S_{\text{eff}}$$

No equilibrium thermal model (非平衡热模型)

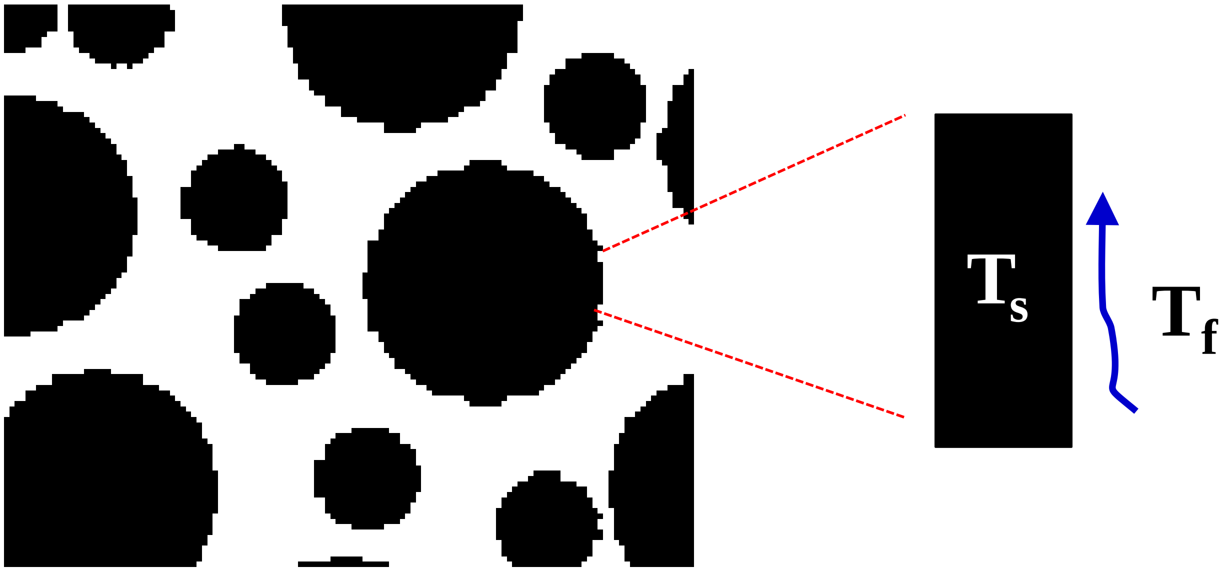
Solid phase and fluid phase are not in thermal equilibrium. The energy equation are solved for fluid and solid region respectively. At the fluid-solid phase, they are coupled by convective boundary condition.

Fluid region

$$\frac{\partial(\left[\varepsilon(\rho C_p)_{\mathrm{fluid}}\right]T_f)}{\partial t}+(\mathbf{u}\cdot\nabla)(\varepsilon\rho C_p T_f)=\left[\varepsilon\lambda_f\right]\nabla^2 T_f+\left[\varepsilon S_f\right]+hA(T_f-T_s)$$

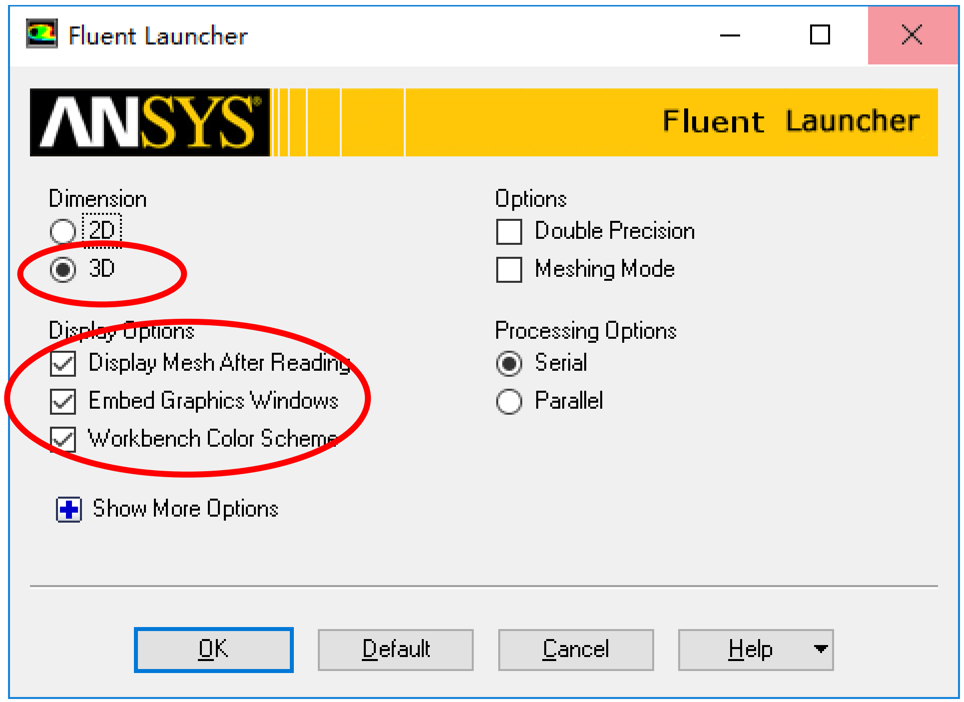
Solid region

$$\frac{\partial\left(\left[(1-\varepsilon)(\rho C_p)_{\text{solid}}\right]T\right)}{\partial t}=\left[(1-\varepsilon)\lambda_s\right]\nabla^2 T+\left[(1-\varepsilon)S_s\right]+hA(T_f-T_s)$$



Two equations are solved separately, and 3rd boundary condition is adopted to couple the two equations.

Start the Fluent software



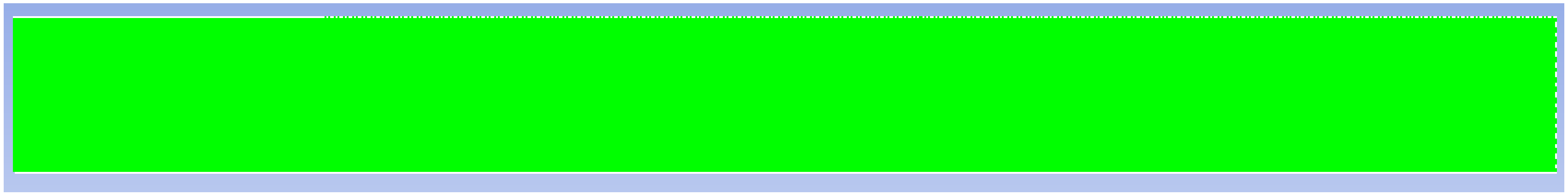
1. Choose 2-Dimension

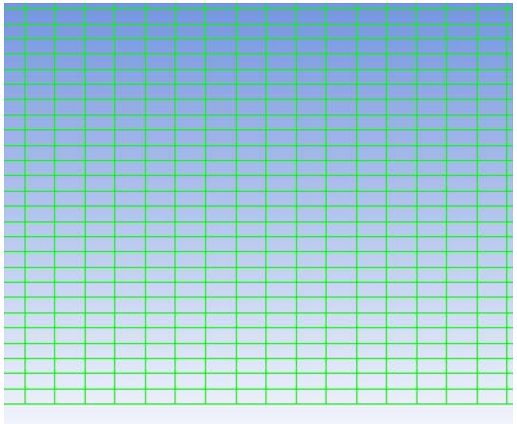
2. Choose display options

3. Choose Serial processing option

Step 1: Read and check the mesh

Read the mesh and check the quality and topological information of the mesh.



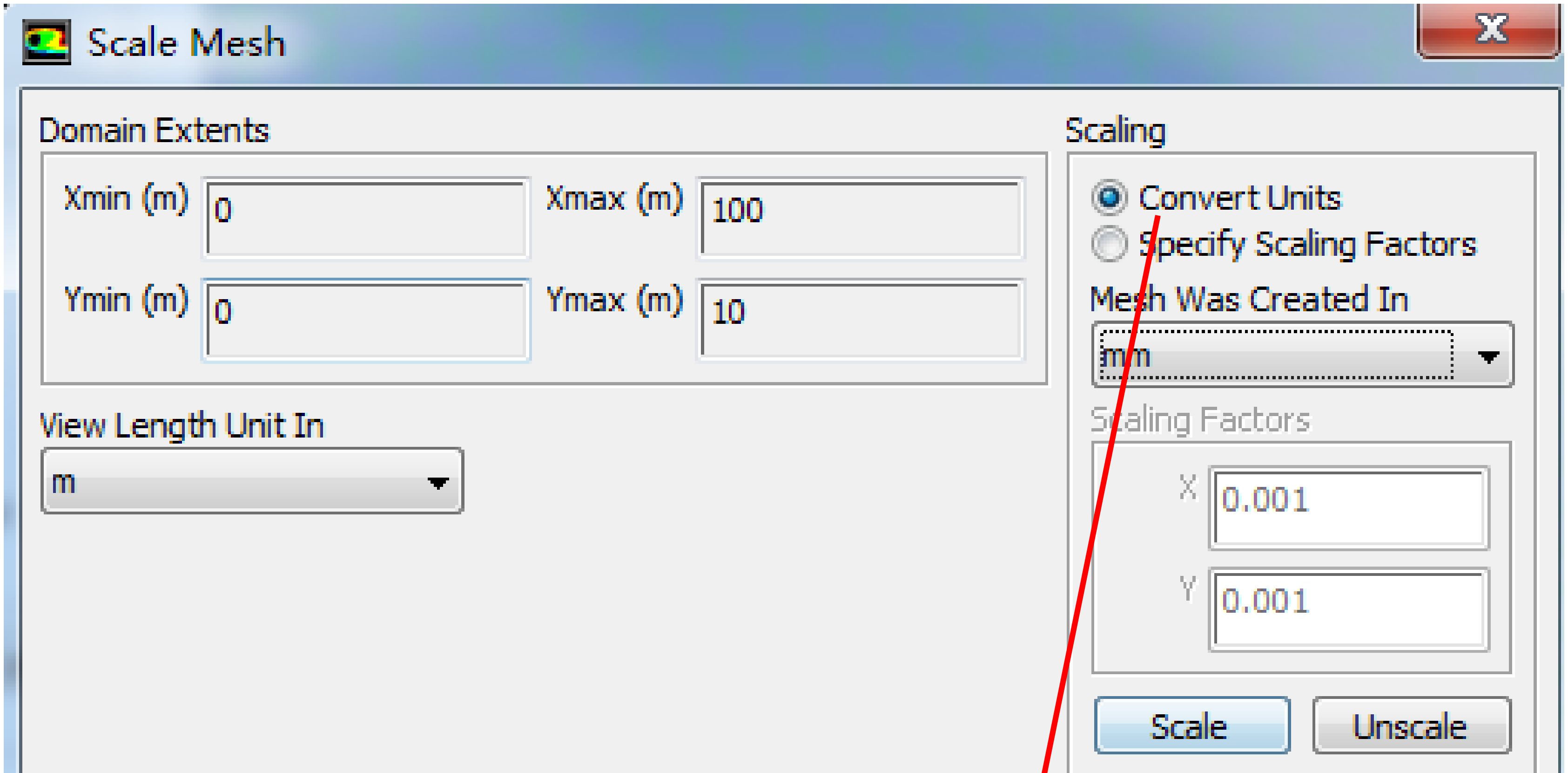


```
Done.

Preparing mesh for display...
Done.

 Domain Extents:
   x-coordinate: min (m) = 0.000000e+00, max (m) = 1.000000e+02
   y-coordinate: min (m) = 0.000000e+00, max (m) = 1.000000e+01
 Volume statistics:
   minimum volume (m3): 2.024164e-02
   maximum volume (m3): 2.024260e-02
     total volume (m3): 1.000000e+03
 Face area statistics:
   minimum face area (m2): 1.010094e-01
   maximum face area (m2): 2.004013e-01
 Checking mesh..........................
Done.
```

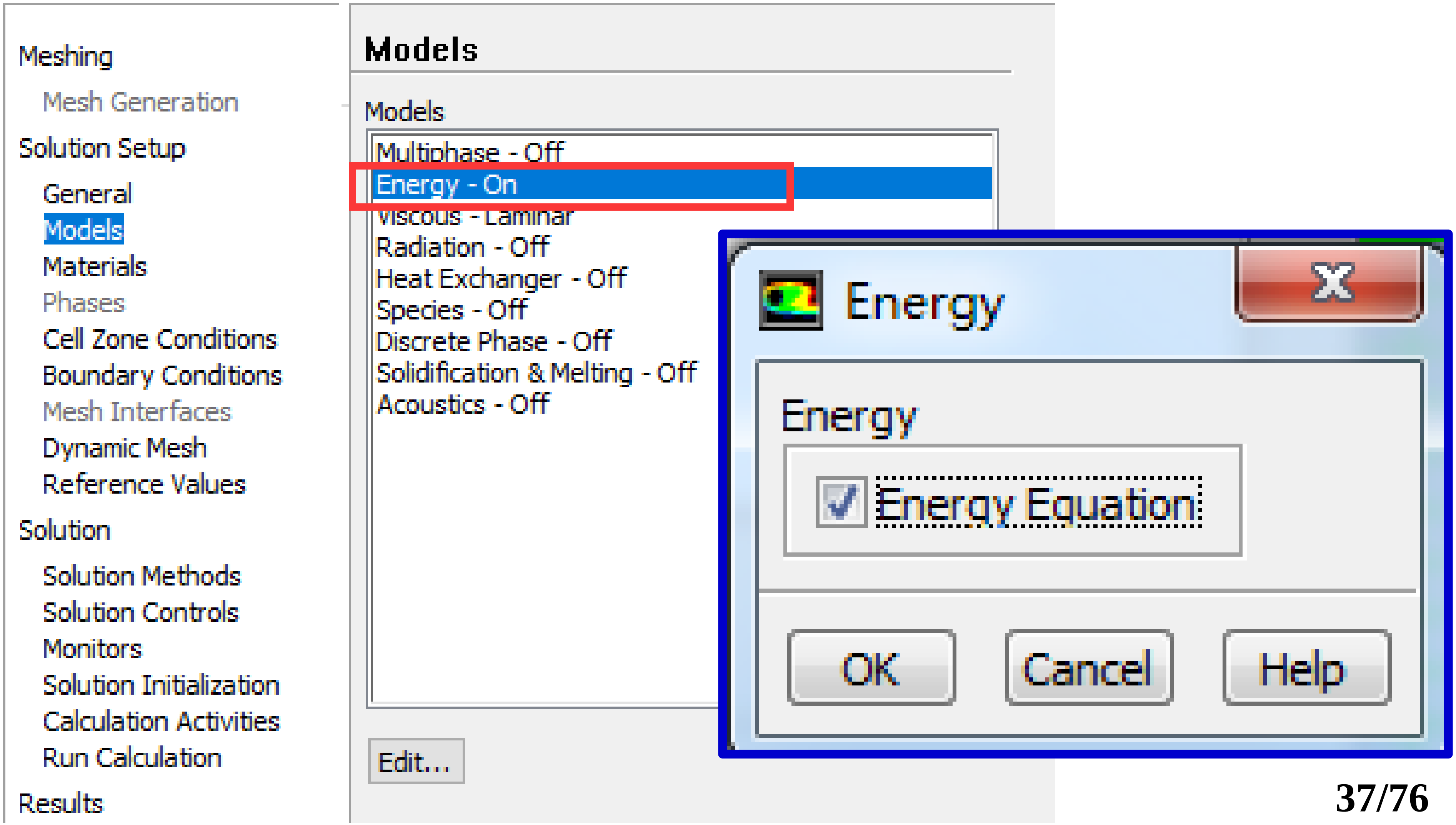
Step 2: Scale the domain size



The mesh is generated in ICEM using unit of mm. Fluent import it as unit of m. Thus, "Convert units" is used.

Step 3: Choose the physicochemical model

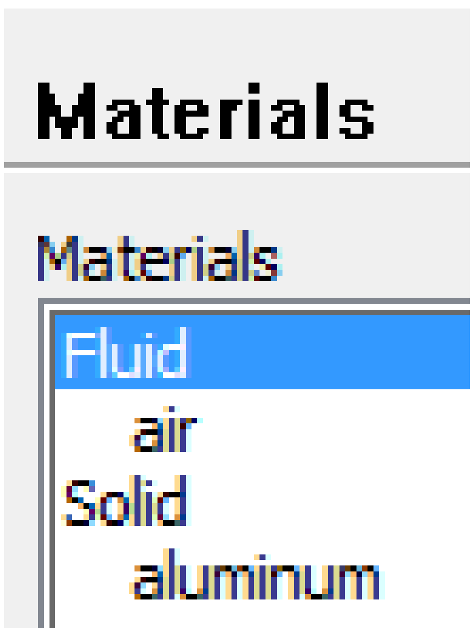
Activate fluid flow and energy model in Fluent.

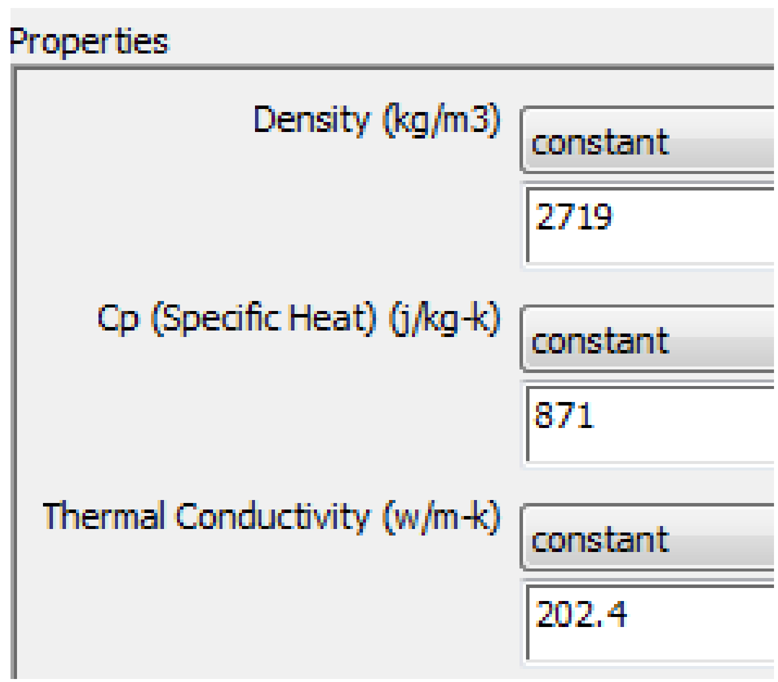


Step 4: Define the material

Define the materials and their properties required for modeling!

In Fluent, the default fluid is air and the default solid is Al. They are the materials we will use in Example A2.

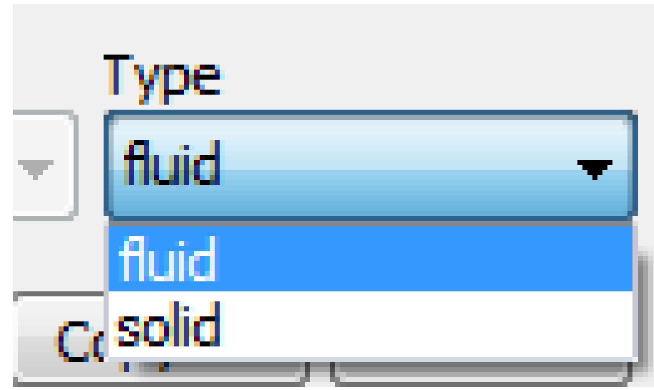




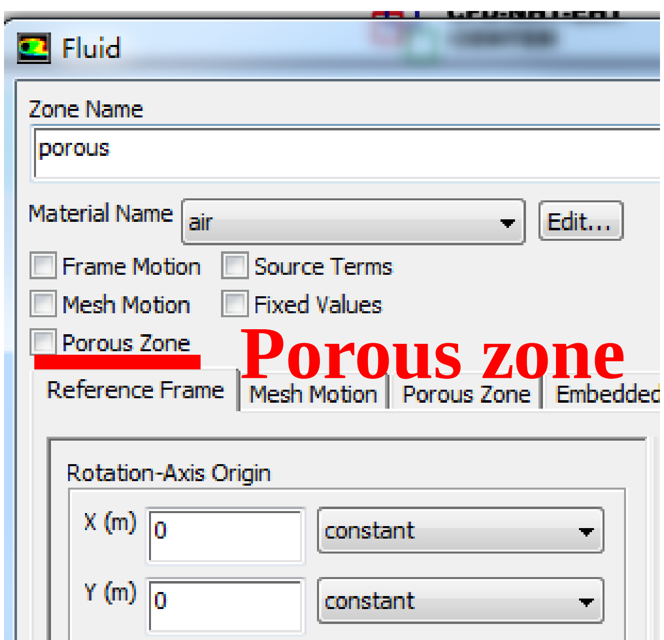
Step 5: Define zone condition

There are two options in Fluent for zone condition:

Fluid **Solid**

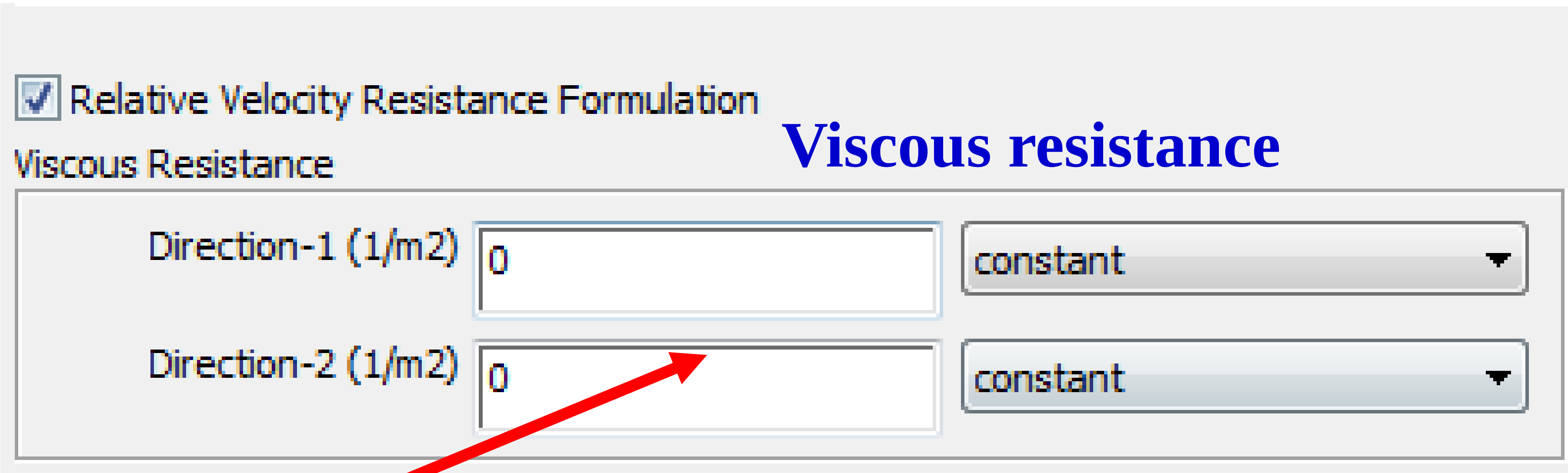


Porous media is treated as Fluid in Fluent.



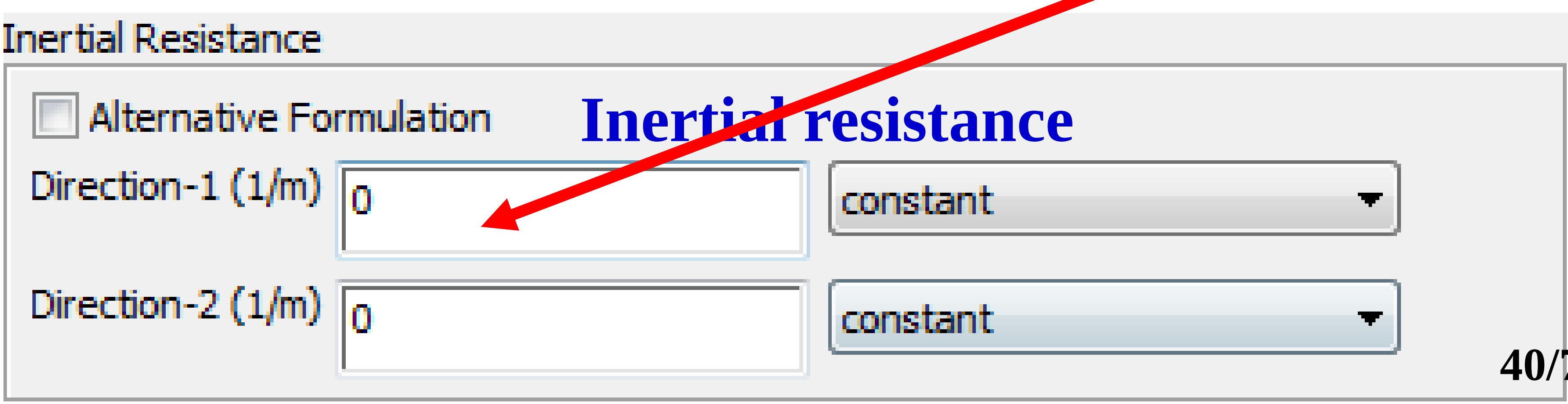
Here you can click "Porous zone" to define the porous media.

Then you can define related porosity and transport properties.



Viscous resistance

$$\frac{1}{k} \Leftarrow \mathbf{F} = -\frac{\mu}{k}\mathbf{u} - C_2 \frac{1}{2}\rho|\mathbf{u}|\mathbf{u} \Rightarrow C_2$$



Inertial resistance

KC equation is adopted, another equation obtained from experiments

$$\frac{\Delta P}{l} = \frac{180\mu}{D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}u$$

$$\mathbf{F} = -\frac{\mu}{k}\mathbf{u} - C_2\frac{1}{2}\rho|\mathbf{u}|\mathbf{u}$$

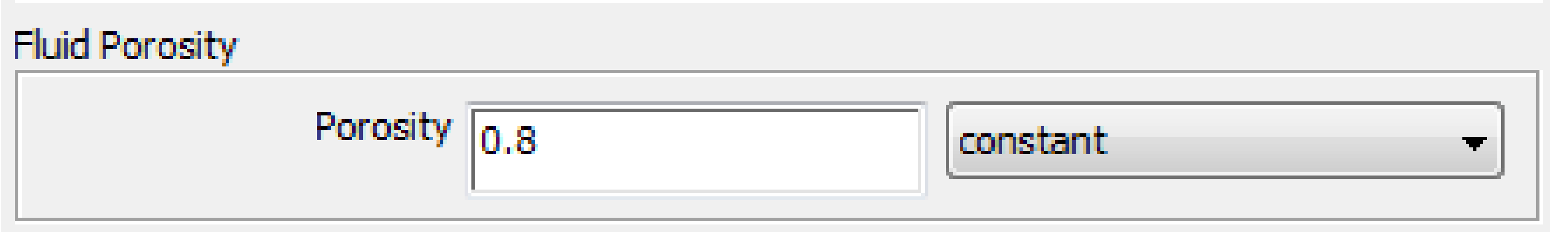
$$k = \frac{D_p^2}{180}\frac{\varepsilon^3}{(1-\varepsilon)^2}$$

$$C_2 = 0$$

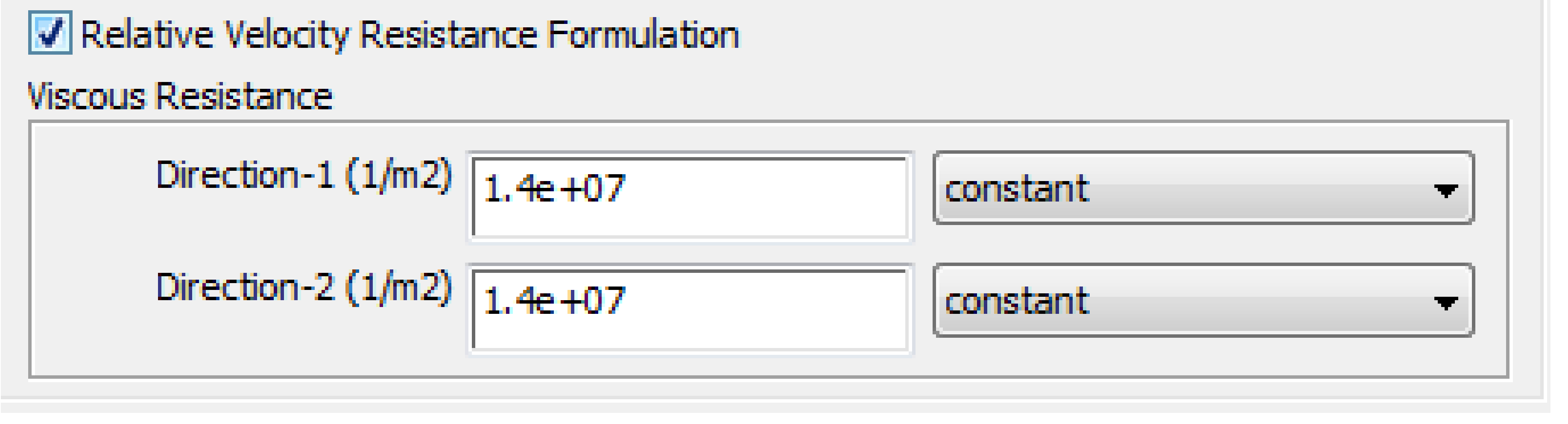
D_p=1mm ε=0.8

k=7.11E-8, 1/k=1.4E7 C_2=0

Porosity

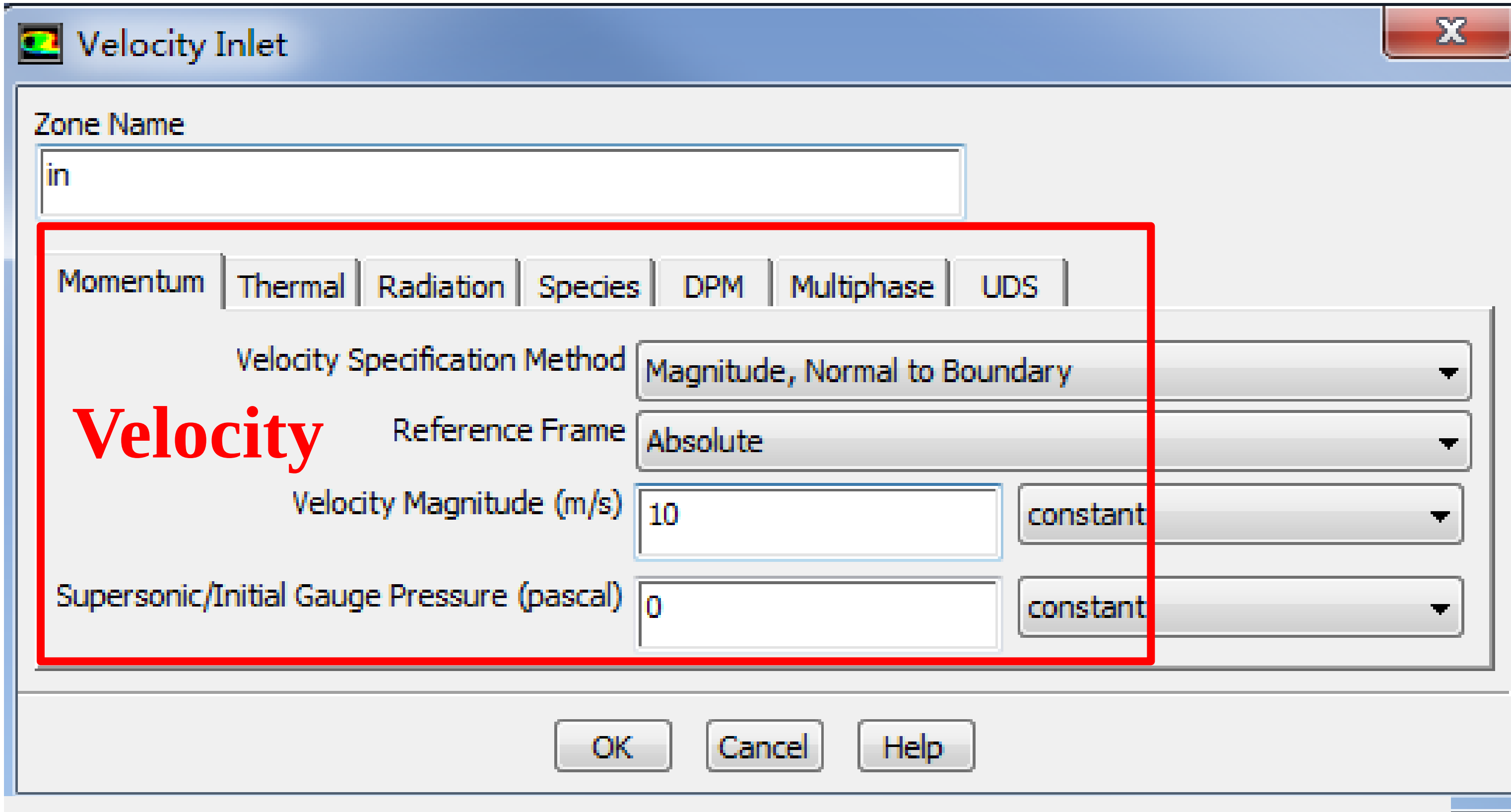


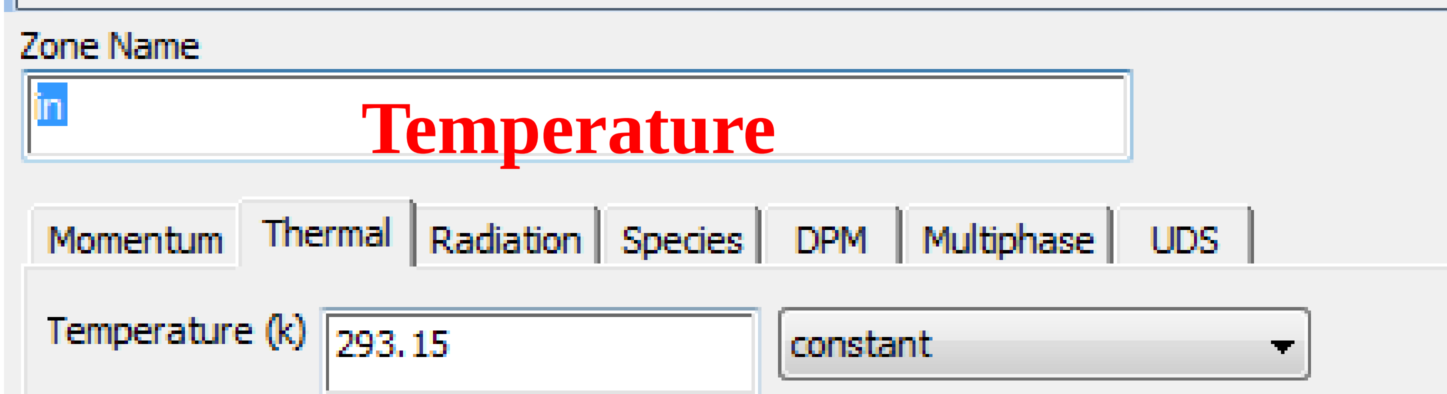
k=7.11E-8, 1/k=1.4E7



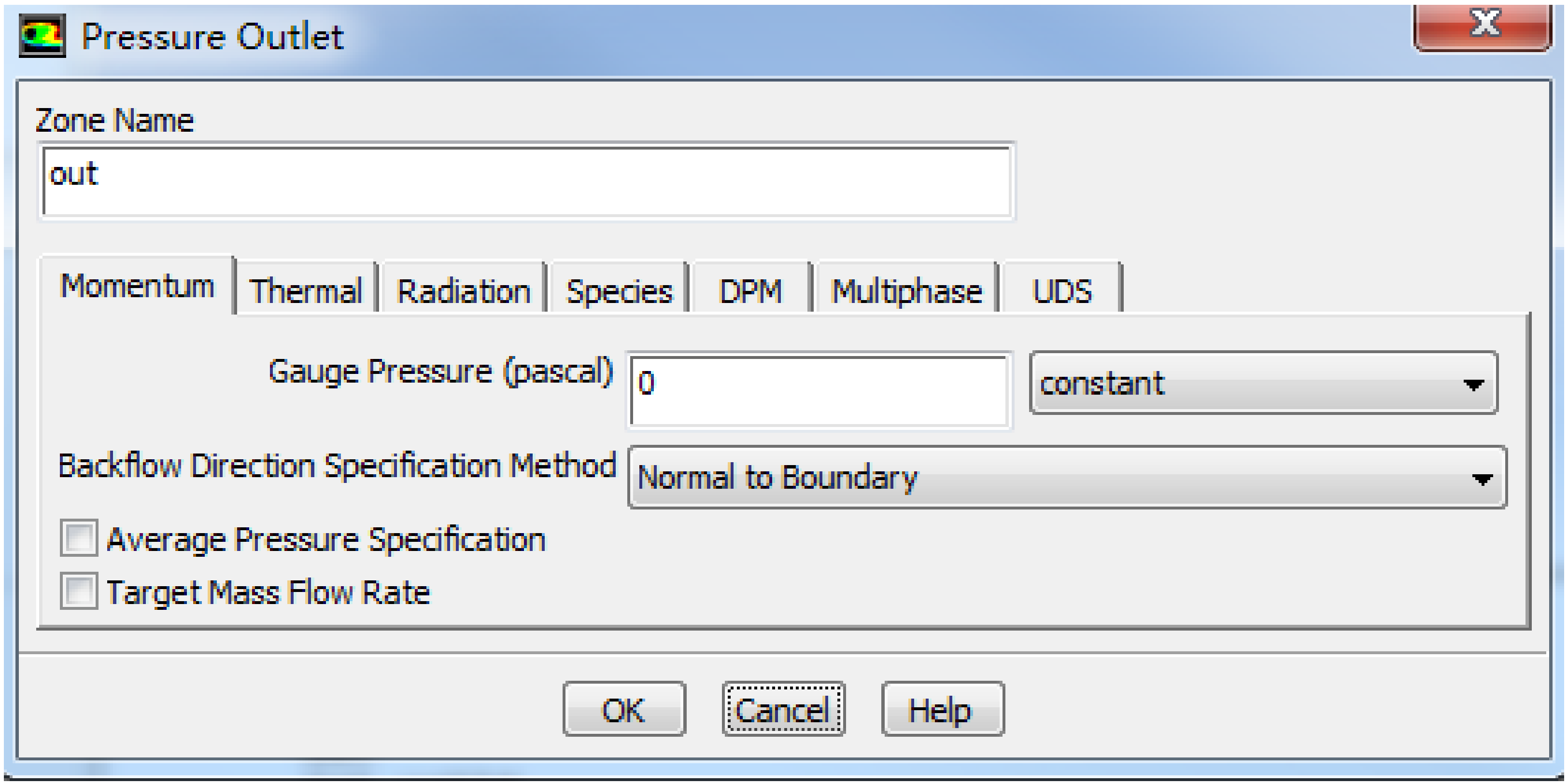
Step 6: Define the boundary condition

Inlet



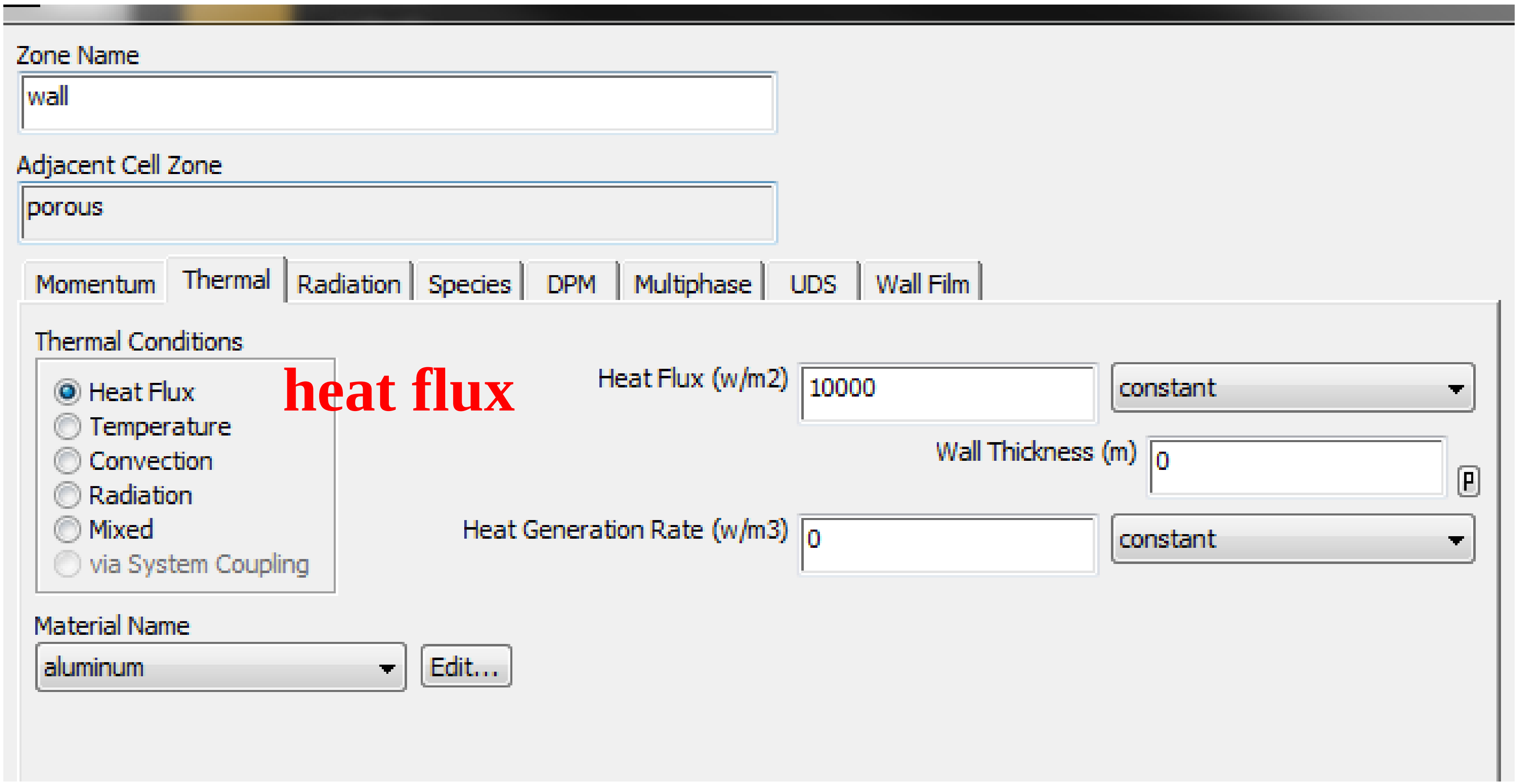


Outlet: pressure outlet



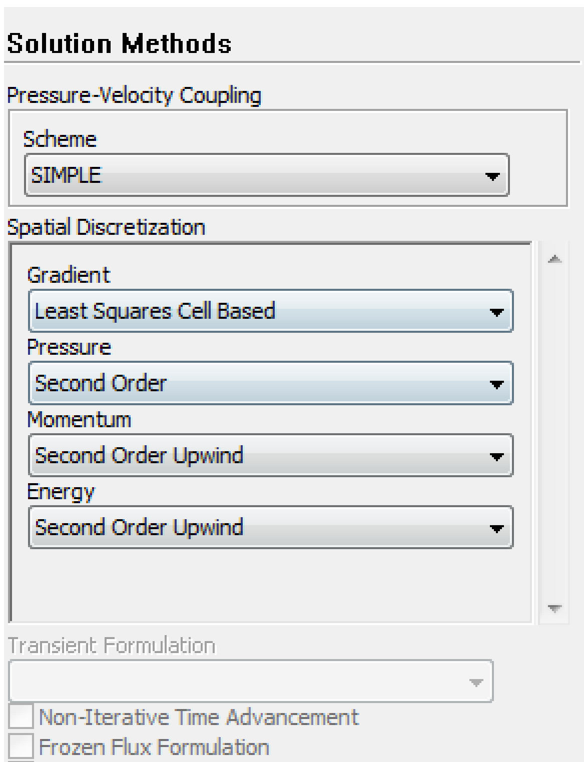
Gauge Pressure (表压)

Wall: heat flux



7st step: Define the solution

For algorithm and schemes, keep it as default. For more details of this step, one can refer to Example A1 of Chapter 13.



Algorithm: simple

Gradient: Least Square Cell Based

Pressure: second order

Momentum: second order upwind

Energy: second order Upwind

7st step: Define the solution

For under-relaxation factor, keep it default. For more details, refer to Example A1.

8st step: Initialization

Use the standard initialization, for more details of Hybrid initialization, refer to Example A1.

Step 9: Run the simulation

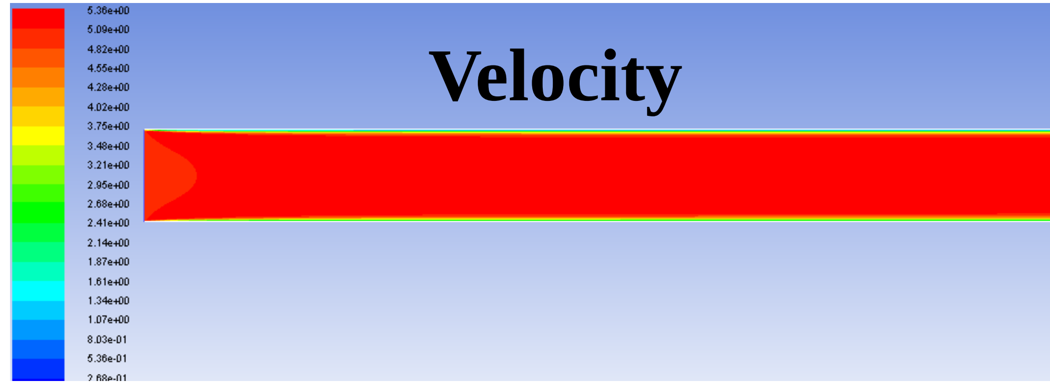
Step 10: Post-processing results

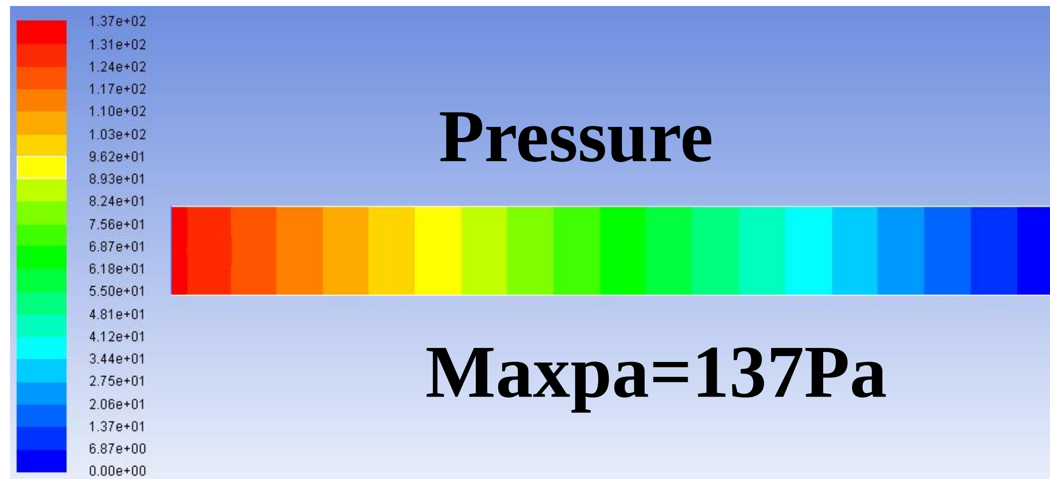
$1/k=1.4E7$

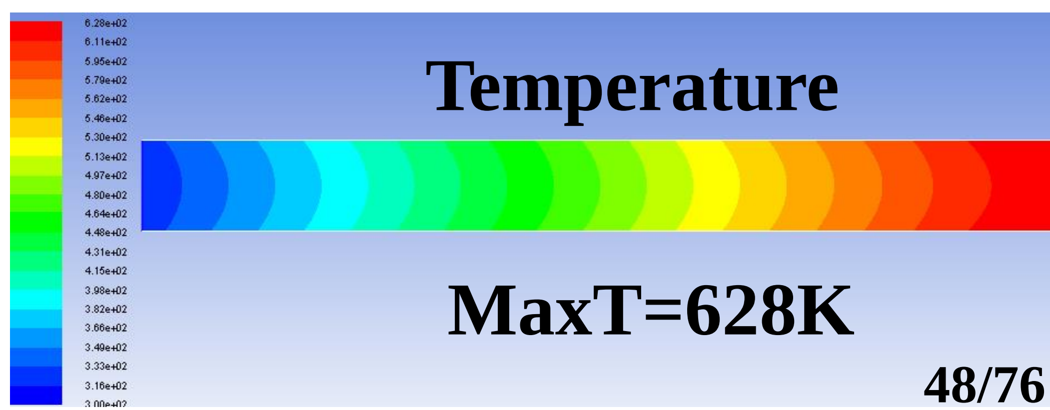
$C_2=0$

Porosity=0.8

$u=5$







2： Operating the Fluent software to simulate the example and post-process the results. （运行软件）

“西”望你我，“安”然无恙

Hope Everyone Safe and Sound

感谢各位同学!

同舟共济渡彼岸!

People in the same boat help each other to cross to the other bank, where....