

Numerical Heat Transfer (数值传热学)

Chapter 10 Numerical Simulation for Turbulent Flow and Heat Transfer



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10.1 Introduction to turbulence

10.1.1 Present understanding of turbulence

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10.1.3 Reynolds time-averages and their characteristics

10.1 Introduction to turbulence

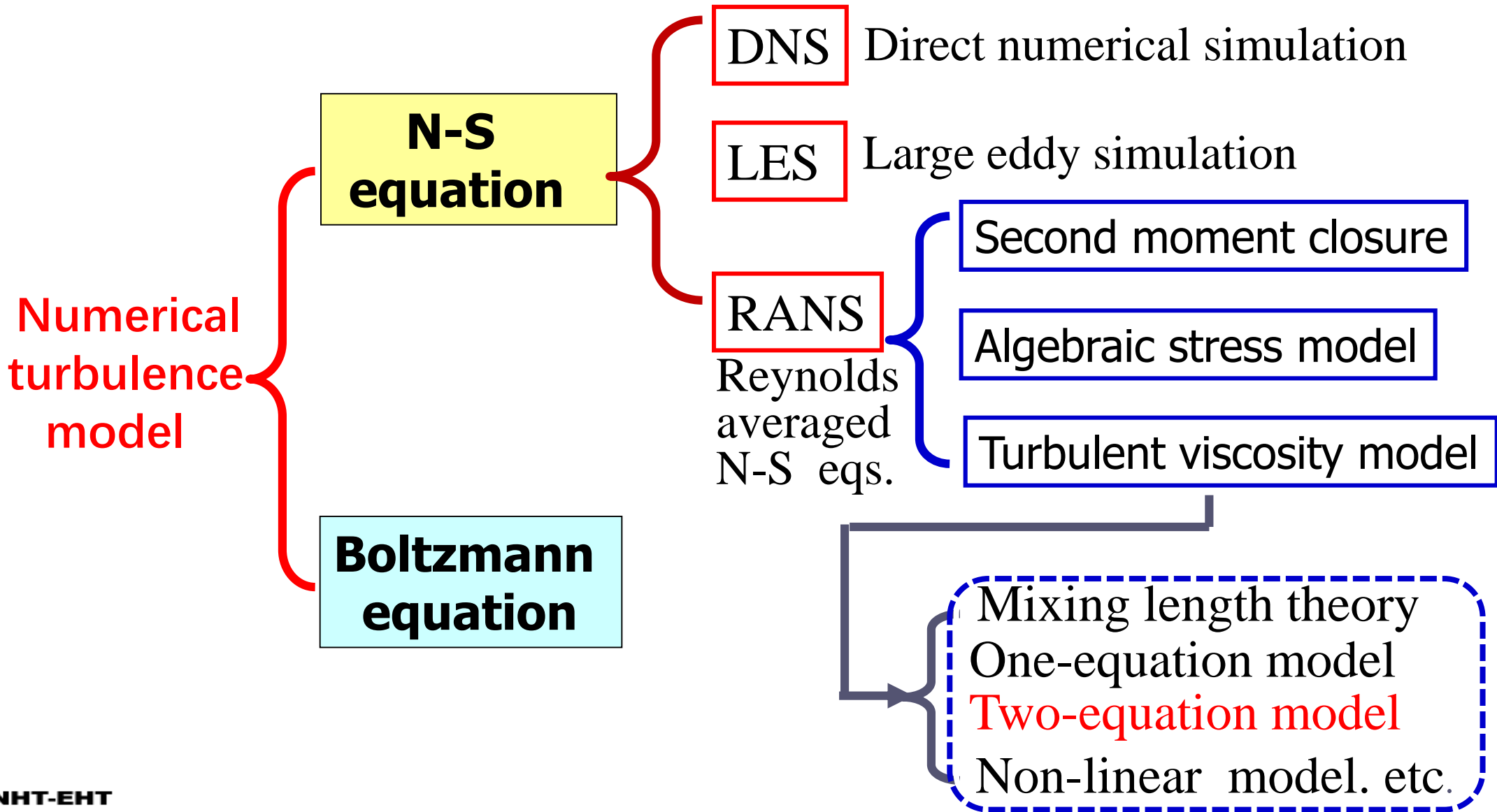
10.1.1 Present understanding of turbulence

1. Turbulence is a highly complicated unsteady flow, within which all kinds of physical quantities are randomly varying with both time and space;
2. Transient Navier-Stokes are valid for turbulent flows;
3. Turbulent flow field can be regarded as a collection of eddies (涡旋) with different geometric scales .

Eddy vs. vortex (漩涡): Eddy is characterized by turbulent flow with randomness, and it covers a wide range of geometric scales;

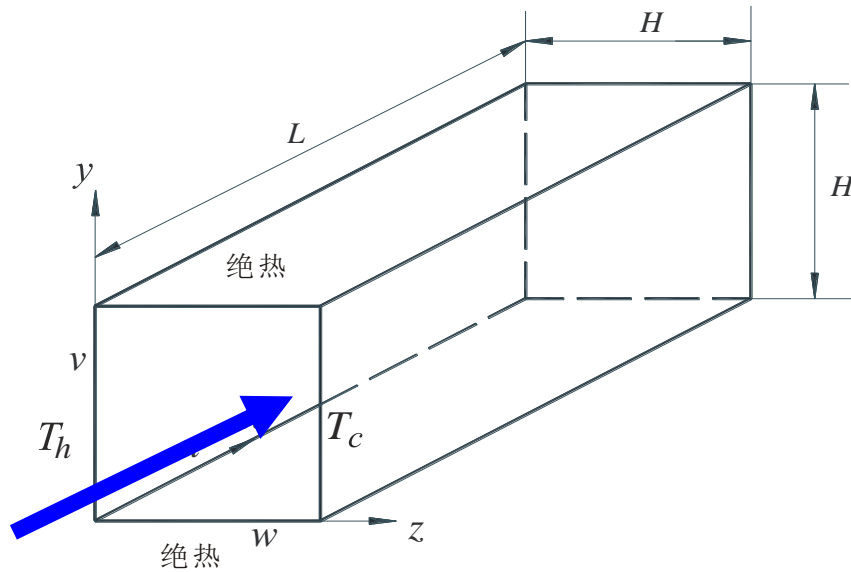
Vortex is caused by a flow phenomenon characterized by recirculation. Such vortex flow can be laminar or turbulent.

10.1.2 Classifications of turbulence simulation methods



1.DNS

In DNS very small time step and space step are needed to reveal the evolutions (演化) of eddies with different scales. Required computer resource is very high. Often high-performance computers are needed.



For a fully developed mixed convection in a square duct ($L=6.4H$), when $Re=6400$, $Gr=10^4 \sim 10^7$ DNS is conducted with 4.194×10^6 nodes ($=256 \times 128 \times 128$), and 8×10^5 time steps are needed for statistical average.

2. LES

Basic idea: Turbulent fluctuations are mainly generated by large scale eddies, which are non-isotropic(各向异性) and vary with flow situation; Small scale eddies dissipate(耗散) kinetic energy (from mechanic to thermal energy), and are almost isotropic. (各向同性)The N-S eqs. are used to simulate the large scale eddies, and the behavior of small scale eddies is simulated by simplified model.

LES requires less computer resource than that of DNS, even though still quite high, and has been used for some engineering problems

For the above problem when simulated by LES only $128 \times 80 \times 80 = 819200$ grids are needed (compared with 4.194×10^6).

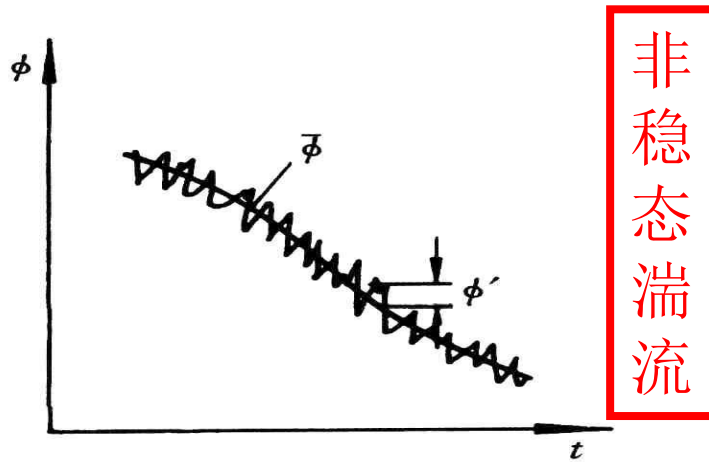
3. Reynolds time average N-S Eqs. methods

Expressing a transient term as the sum of average term and fluctuation(脉动) term. Time average is conducted for the transient N-S equations, and the time average terms of the fluctuations is expressed via some function of the average terms.

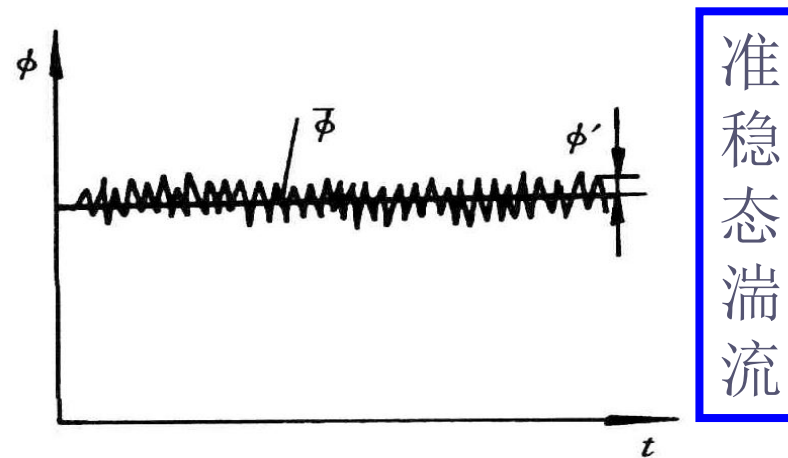
10.1.3 Reynolds time averages and their characteristics

$$\phi = \bar{\phi} + \phi' \quad \bar{\phi} = \frac{1}{\Delta t} \int_t^{t+\Delta t} \phi(t) dt$$

Δt is the time step, which should be large enough relative to the fluctuation but small enough with respect to the period of time average quantity.



(a)
Unsteady



(b)
Quasi-steady

Characteristics of time averages

1. $\overline{\phi'} \equiv 0$; 2. $\overline{\overline{\phi}} = \overline{\phi}$; 3. $\overline{\overline{\phi + \phi'}} = \overline{\phi}$; 4. $\overline{\overline{\phi\phi'}} = \overline{\phi\phi'} = 0$

5. $\overline{\phi f} = \overline{(\overline{\phi} + \phi')(\overline{f} + f')} = \overline{\overline{\phi} \overline{f}} + \overline{\phi' f'}$ 6. $\frac{\partial \overline{\phi}}{\partial x} = \frac{\partial \overline{\phi}}{\partial x}$;

7. $\frac{\partial \overline{\phi'}}{\partial x} = \frac{\partial \overline{\phi'}}{\partial x} = 0$

8. $\frac{\partial \overline{(\phi f)}}{\partial x} = \frac{\partial \overline{(\overline{\phi} \overline{f})}}{\partial x} + \frac{\partial \overline{(\phi' f')}}{\partial x}$

10.2 Time-averaged governing equation for incompressible convective heat transfer

10.2.1 Time average governing equation

10.2.2 Ways for determining additional terms

10.2.3 Governing equations with turbulent viscosity

10.2 Time-averaged governing equation for incompressible convective heat transfer

10.2.1 Time average governing equation

1. Continuity equation

$$\frac{\partial(\bar{u} + \bar{u}')}{\partial x} + \frac{\partial(\bar{v} + \bar{v}')}{\partial y} + \frac{\partial(\bar{w} + \bar{w}')}{\partial z} = \underbrace{\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z}}_{=0} + \underbrace{\frac{\partial \bar{u}'}{\partial x} + \frac{\partial \bar{v}'}{\partial y} + \frac{\partial \bar{w}'}{\partial z}}_{=0} = 0$$

Both time average velocity and time average fluctuation velocity satisfy continuity condition.

2. Momentum equation

Taking x -direction as an example:

$$\frac{\partial(\bar{u} + u')}{\partial t} + \frac{\partial(\bar{u} + u')^2}{\partial x} + \frac{\partial(\bar{u} + u')(\bar{v} + v')}{\partial y} + \frac{\partial(\bar{u} + u')(\bar{w} + w')}{\partial z} = -\frac{1}{\rho} \frac{\partial(\bar{p} + p')}{\partial x} + \nu \left[\frac{\partial^2(\bar{u} + u')}{\partial x^2} + \frac{\partial^2(\bar{u} + u')}{\partial y^2} + \frac{\partial^2(\bar{u} + u')}{\partial z^2} \right]$$

According to the above characteristics, yielding

$$\frac{\partial \bar{u}}{\partial t} + \frac{\partial(\bar{u}^2)}{\partial x} + \frac{\partial \bar{u}\bar{v}}{\partial y} + \frac{\partial \bar{u}\bar{w}}{\partial z} + \frac{\partial(u')^2}{\partial x} + \frac{\partial(u'v')}{\partial y} + \frac{\partial(u'w')}{\partial z} =$$

$$= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + \nu \left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial^2 \bar{u}}{\partial z^2} \right)$$

Moved to right hand side and combined with the corresponding viscous term

$$\frac{\partial \bar{u}}{\partial t} + \frac{\partial(\bar{u}^2)}{\partial x} + \frac{\partial \bar{u}\bar{v}}{\partial y} + \frac{\partial \bar{u}\bar{w}}{\partial z} =$$

$$-\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + \frac{\partial}{\partial x} \left[\nu \frac{\partial \bar{u}}{\partial x} - \overline{(u')^2} \right] + \frac{\partial}{\partial y} \left[\nu \frac{\partial \bar{u}}{\partial y} - \overline{(u'v')} \right] + \frac{\partial}{\partial z} \left[\nu \frac{\partial \bar{u}}{\partial z} - \overline{(u'w')} \right]$$

Rewritten in a tensor form in Cartesian coordinate:

$$\frac{\partial(\rho\bar{u})}{\partial t} + \frac{\partial(\rho\bar{u}_i\bar{u}_j)}{\partial x_j} = -\frac{\partial\bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\eta \frac{\partial\bar{u}_i}{\partial x_j} - \overline{\rho u'_i u'_j} \right) \quad (i = 1, 2, 3)$$

3. Other scalar (标量) variables

$$\frac{\partial(\rho\bar{\phi})}{\partial t} + \frac{\partial(\rho\bar{u}_j\bar{\phi})}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\Gamma \frac{\partial\bar{\phi}}{\partial x_j} - \overline{\rho u'_j \phi'} \right) + S$$

4. Discussion on the time averaged quantity

(1) Linear term remains unchanged during time average, while **product** term (乘积项) generates **product of fluctuations**, representing the additional transport caused by fluctuation.

(2) Equations are not closed: for 3-D problem, there are five equations, with 14 unknown variables:

Five time average variables —

$$\bar{u}, \bar{v}, \bar{w}, \bar{p}, \bar{\phi},$$

Nine products of fluctuations

$$\overline{u'_i u'_j} (i, j = 1, 2, 3) \text{ (6 terms)}; \quad \overline{u'_i \phi'} (i = 1, 2, 3) \text{ (3 terms)}$$

In order to close the above equations, additional relations must be added. **Such additional relations are called turbulence model, or closure model (封闭模型)。**

10.2.2 Ways of determining additional terms

1.Reynolds stress method

For the nine additional variables deriving their own governing equations.

However, in the derivation process new additional terms of higher order (product of three variables, four variables, etc...) are introduced.; If we still go along this direction then equations for much higher order products should be derived.,,,,,,. Thus we have to terminate such process at certain level. **Historically some complicated models with more than 20 equations have been derived.**

In the Reynolds stress models, the second moment model is quite famous and has been applied in some engineering problems. In the second moment model, for the product terms with two fluctuations their equations are derived, while for the terms with three or more fluctuations models are used to relate such terms with time average variables.

Prof. L X Zhou (周力行) in Tsinghua university contributed a lot in this regard.

2. Turbulent viscosity method

The product of fluctuations of two velocities is expressed via **turbulent viscosity**.

(1) Definition of turbulent viscosity

In 1877 Boussinesq introduced following equation, by mimicking(比拟) the **constitution equation (本构方程)** of laminar fluid flow:

$$(\tau_{i,j})_t = -\overline{\rho u'_i u'_j} = (-p_t \delta_{i,j}) + \underline{\eta}_t \left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right) - \frac{2}{3} \underline{\eta}_t \delta_{i,j} \text{div} \overline{\mathbf{U}} \quad 0$$

$$p_t = \frac{1}{3} \rho [(\overline{u'})^2 + (\overline{v'})^2 + (\overline{w'})^2] = \frac{2}{3} \rho k \quad k = \frac{1}{2} [(\overline{u'})^2 + (\overline{v'})^2 + (\overline{w'})^2]$$

(2) Definition of turbulent diffusivity of other scalar variables

$$-\overline{\rho u'_i \phi'} = \Gamma_t \frac{\partial \overline{\phi}}{\partial x_i} \quad \Gamma_t = \frac{\eta_t}{\text{Pr}_t}$$

Pr_t --- turbulent Prandtl number, usually treated as a constant.

For laminar heat transfer we have

$$\Gamma_l = \lambda = \frac{\lambda}{c_p} \frac{\eta_l}{\eta_l} c_p = \left(\frac{\lambda}{c_p \eta_l} \right) \eta_l c_p = \frac{\eta_l c_p}{\left(\frac{c_p \eta_l}{\lambda} \right)} = \frac{\eta_l c_p}{\text{Pr}_l}$$

Similarly: $\Gamma_t = \lambda_t = \eta_t c_p / \text{Pr}_t$

Therefore for turbulent viscosity model its major task is to find η_t, Pr_t .

The name of engineering turbulence models comes from the number of PDEqs. included in the model to determine turbulence viscosity.

10.2.3 Governing equations of viscosity models

1. Governing equations

For simplicity of presentation, the symbol of time average “bar” is omitted hereafter:

$$\left\{ \begin{array}{l} \frac{\partial u_k}{\partial x_k} = 0 \\ \frac{\partial u_i}{\partial t} + \frac{\partial(\rho u_k u_i)}{\partial x_k} = -\frac{\partial p_{eff}}{\partial x_i} + \frac{\partial}{\partial x_k} \left[\underbrace{(\eta_l + \eta_t)}_{\eta_{eff}} \frac{\partial u_i}{\partial x_k} \right] + S_i \ ; \ p_{eff} = p + p_t \\ \frac{\partial(\rho^* \phi)}{\partial t} + \frac{\partial(\rho^* u_k \phi)}{\partial x_k} = \frac{\partial}{\partial x_k} \left[\underbrace{(\Gamma_l + \Gamma_t)}_{\Gamma_{eff}} \frac{\partial \phi}{\partial x_k} \right] + S_\phi \end{array} \right.$$

2. Differences from laminar governing equations:

- (1) u_i, p, ϕ -Time average; (2) Replacing Γ by $\Gamma_{eff} = \Gamma + \Gamma_t$
 (3) Replacing p by p_{eff} (4) In the source term S_i of u_i
 the additional terms caused by time averaging are included.

In the Cartesian coordinates, the source terms of the three components are:

$$u: S = \frac{\partial}{\partial x} \left(\eta_{\text{eff}} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(\eta_{\text{eff}} \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial z} \left(\eta_{\text{eff}} \frac{\partial w}{\partial x} \right)$$

$$v: S = \frac{\partial}{\partial x} \left(\eta_{\text{eff}} \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial y} \left(\eta_{\text{eff}} \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left(\eta_{\text{eff}} \frac{\partial w}{\partial y} \right)$$

$$w: S = \frac{\partial}{\partial x} \left(\eta_{\text{eff}} \frac{\partial u}{\partial z} \right) + \frac{\partial}{\partial y} \left(\eta_{\text{eff}} \frac{\partial v}{\partial z} \right) + \frac{\partial}{\partial z} \left(\eta_{\text{eff}} \frac{\partial w}{\partial z} \right)$$

In laminar flow of constant properties, all source terms are zero, but for turbulent flow they are not zero .

Thus the above governing eqs. for flow and heat transfer are not closed. Eq. for determining η_{eff} should be added.

3. Turbulent Prandtl number

Its value varies within a certain range, usually is taken as a constant, and $\Gamma_t = c_p \eta_t / \text{Pr}_t$

10.3 Zero equation model and one equation model

10.3.1 Zero equation model

1. Turbulent additional stress of zero equation model
2. Equations for mixing length
3. Application range of zero eq. model

10.3.2 One equation model

1. Turbulent fluctuation kinetic energy as dependent variable
2. Prandtl-Kolmogorov equation
3. Governing equation of turbulent fluctuation kinetic energy
4. Boundary condition

10.3 Zero Equation Model and One Equation Model

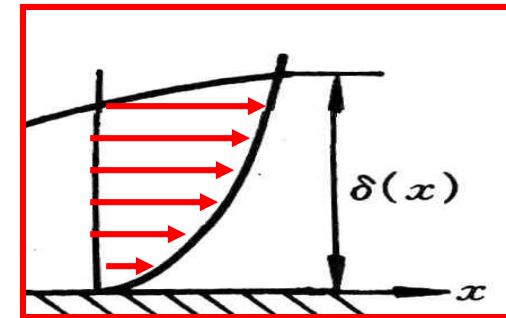
10.3.1 Zero equation model

1. Zero equation model for turbulent additional stress

In zero eq. model no PDE is involved to determine turbulent viscosity. The turbulent stress is expressed as:

Turbulent kinetic viscosity

$$\tau_t = -\rho \overline{u_i' u_j'} = \rho \overline{u' v'} = \rho \nu_t \left(\frac{du}{dy} \right) = \rho l_m^2 \left| \frac{du}{dy} \right| \left(\frac{du}{dy} \right)$$



From dimensionality consideration

Cause of momentum exchange

From Newton shear stress eq.

where l_m is called mixing length, whose determination is the key of zero-eq. model.

2. Equations for mixing length

(1) Flow and HT over a plate function (斜坡函数) :

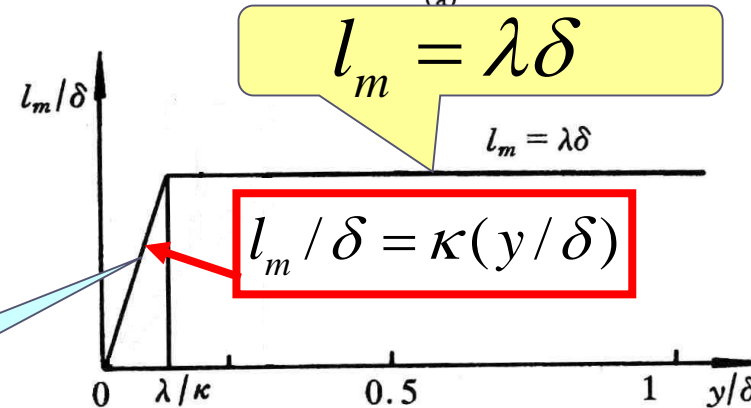
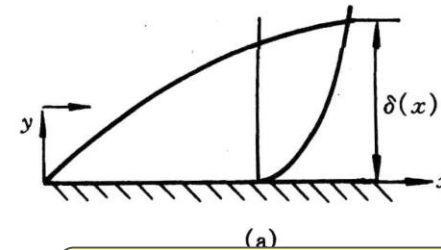
At $y / \delta < \lambda / \kappa$, $l_m = \kappa y$

At $y / \delta \geq \lambda / \kappa$, $l_m = \lambda \delta$

Authors	κ	λ
Cebeci	0.41	0.08
P-S	0.435	0.09

$$l_m = \kappa y$$

l_m / δ vs. y / δ is a slope

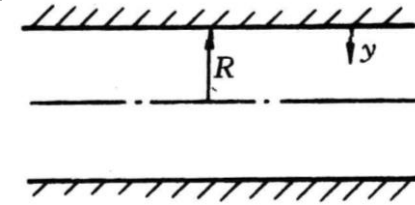


δ thickness of B.L.

(2) Turbulent HT in a circular tube--- Nikurads eq.

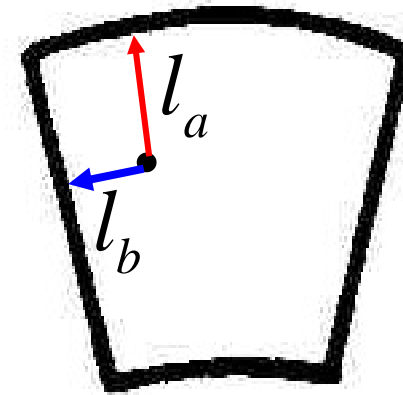
$$l_m / R = 0.14 - 0.08(1 - y/R)^2 - 0.06(1 - y/R)^4$$

Application range: $Re = 1.1 \times 10^5 \sim 3.2 \times 10^6$



(3) Fluid in a duct corner

$$\frac{1}{l_m} = \frac{1}{l_a} + \frac{1}{l_b}; \quad l_a, l_b \text{ from above eqs.}$$



(4) Modification caused by molecular viscosity — van Driest eq.

$$l_m = \kappa y \left[1 - \exp\left(-\frac{y(\tau_m / \rho)^{1/2}}{A\nu}\right) \right] = \kappa y \left[1 - \exp\left(-\frac{y^+}{A}\right) \right], \quad A = 26$$

Correction caused by molecular viscosity

For $\frac{y^+}{A} = 6$, its value = 0.997

3. Application range of zero eq. model

- (1) Boundary layer flow & HT (Flow over a wing before separation)
- (2) FF & HT in straight ducts;
- (3) Boundary layer type flow with weak recirculation.

Drawbacks of zero eq. model:

- (1) At duct center line velocity gradient equals zero but turbulent viscosity still exists.
- (2) Effects of oncoming flow turbulence is not considered.
- (3) Effects of turbulent flow itself is not considered.

Li ZY, Hung TC, Tao WQ. Numerical simulation of fully developed turbulent flow and heat transfer in annular-sector ducts. *Heat Mass Transfer*, 2002, 38 (4-5): 369-377

10.3.2 One-equation model

1. Turbulent fluctuation kinetic energy is taken as a dependent variable to be solved by a PDE.

The most important feature of turbulence is fluctuation. **Fluctuation kinetic energy** k is an appropriate quantity to indicate fluctuation intensity (**脉动强度**). It is taken as a dependent variable for reflecting the effects of turbulence itself.

2. Prandtl-Kolmogorov equation

Mimicking (**模仿**) the molecular viscosity caused by the random motion of molecules, which is:

Molecular viscosity $\eta_l \propto \rho \bar{u} \bar{\lambda}$

Then the viscosity caused by turbulent fluctuation (turbulent viscosity) can be expressed by

$$\eta_t \propto \rho k^{1/2} l \longrightarrow \eta_t = C'_\mu \rho k^{1/2} l$$

where l is the fluctuation scale, usually different from mixing length;

— Prandtl-Kolmogorov equation

Coefficient C'_μ is within the range from 0.2 to 1.0;

In order to get the distribution of k a related PDE is required.

3. Governing equation of turbulent kinetic energy k

Starting from the definition of $k = 0.5(\overline{u_i' u_i'})$, conducting time-average operation for N-S equations, and introducing some assumptions, following governing equation for k can be obtained:

$$\underbrace{\rho \frac{\partial k}{\partial t}}_{\text{transient}} + \underbrace{\rho u_j \frac{\partial k}{\partial x_j}}_{\text{convection}} = \underbrace{\frac{\partial}{\partial x_j} \left[(\eta_l + \frac{\eta_t}{\sigma_k}) \frac{\partial k}{\partial x_j} \right]}_{\text{diffusion}} + \underbrace{\eta_t \frac{\partial u_j}{\partial x_i} \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right)}_{\text{production}} - \underbrace{\rho \left(C_D \frac{k^{3/2}}{l} \right)}_{\text{dissipation}}$$

source

where σ_k is called turbulent Prandtl number of k , and its introduction can increase the application range of the model.

4. Boundary condition treatment: wall function method

10.4 Two-Equation Model

10.4.1 Second variables related to l

10.4.2 $k - \varepsilon$ governing equations

10.4.3 General governing equation for $k - \varepsilon$ model

10.4.4 Remarks

10.4 Two-Equation model

10.4.1 Second variables related to l

1. There are several physical variables related to l

Z-variables	$k^{1/2}/l$	<u>$k^{3/2}/l$</u>	kl	<u>k/l^2</u>
Proposed by	Kolmogorov [32]	Chou (周培源)[19]	Rodi, Spalding [38]	Spalding[39]
Symbol	f	ε	kl	W
Physical meaning	Eddy frequency	Energy dissipation	Product of energy and length scale	Mean square root of vorticity fluctuation

The dissipation term is defined by

$$\varepsilon = C_D \frac{k^{3/2}}{l}$$

This is the modeling definition (模拟定义). It can be regarded as the dissipation rate of fluctuation kinetic energy of unit mass; C_D is a dimensionless constant.

2. Two definitions of dissipation rate

(1) Strict definition

$$\varepsilon = \nu_l \overline{\left(\frac{\partial u'_i}{\partial x_k} \right) \left(\frac{\partial u'_i}{\partial x_k} \right)}$$

It represents dissipation rate of isotropic small eddies, and is used in the derivation of its governing equation.

(2) Modeling definition $\varepsilon = C_D k^{3/2} / l$

Understanding of its meaning: energy transit rate from large eddies to small eddies for unit volume is proportional to ρk , and $1/t$, where the transit time t is proportional to $l / k^{1/2}$, thus

$$\rho \varepsilon \sim \rho k / \left(\frac{l}{k^{1/2}} \right) \sim \rho \frac{k^{3/2}}{l} = C_D \rho \frac{k^{3/2}}{l}$$

This definition is used in the derivation process for simplifying treatment of some complicated terms.

10.4.2 $k - \varepsilon$ governing equations

(1) ε equation

Starting from strict definition, $\varepsilon = \overline{\nu_l \left(\frac{\partial u'_i}{\partial x_k} \right) \left(\frac{\partial u'_i}{\partial x_k} \right)}$ conducting time average operation for N-S equation, and adopting some assumptions (including modeling definition), yielding

$$\underbrace{\frac{\partial(\rho\varepsilon)}{\partial t}}_{\text{transient}} + \underbrace{\frac{\partial(\rho u_j \varepsilon)}{\partial x_j}}_{\text{convection}} = \underbrace{\frac{\partial}{\partial x_j} \left[\left(\eta_l + \frac{\eta_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right]}_{\text{diffusion}} - \underbrace{C_1 \frac{\varepsilon}{k} \eta_t \frac{\partial u_j}{\partial x_i} \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right)}_{\text{source}} - \underbrace{C_2 \rho \frac{\varepsilon^2}{k}}_{\text{source}}$$

σ_ε Prandtl number of ε ; C_1, C_2 are empirical coefficients

(2) k equation After introducing ε

k equation can be re-written as

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_j k)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\eta_l + \frac{\eta_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + \underbrace{\eta_t \frac{\partial u_j}{\partial x_i} \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right)}_{\text{Source term}} - \rho \varepsilon$$

$$-\rho \left(C_D \frac{k^{3/2}}{l} \right)$$

↓

Introducing: $G = \frac{\eta_t}{\rho} \frac{\partial u_i}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ called as unit mass production function

The source term of k eq. $\eta_t \frac{\partial u_j}{\partial x_i} \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right) - \rho \varepsilon \Rightarrow \rho G - \rho \varepsilon$

(3) Determination of turbulent viscosity of $k - \varepsilon$ model

$$\eta_t = C'_\mu \rho k^{1/2} l = \underline{C'_\mu C_D} \rho k^{1/2+3/2} \frac{l}{C_D k^{3/2}} = C_\mu \rho k^2 / \varepsilon$$

$$C'_\mu C_D \rightarrow C_\mu \quad \varepsilon = C_D \frac{k^{3/2}}{l}$$

10.4.3 General gov. eq. of $k - \varepsilon$ model

$$\frac{\partial(\rho^* \phi)}{\partial t} + \text{div}(\rho^* \vec{u} \phi) = \text{div}(\Gamma_\phi \text{grad} \phi) + S_\phi$$

ϕ represents: $u, v, w, T, k, \varepsilon$

Most widely accepted values of model constants

C_1	C_2	C_μ	σ_k	σ_ε	σ_T
1.44	1.92	0.09	1.0	1.3	0.9-1.0

Γ_ϕ, S_ϕ depend on variable and coordinate:

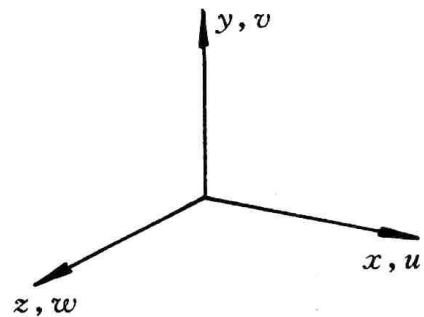
u, v, w, T, k, ϵ

For Cartesian Coordinate:

Text book, Page 350

But in our new G.Eqs. for temp.:

$$\Gamma_t = \lambda_t = \eta_t c_p / Pr_t$$

控制方程	$\frac{\partial(\rho u \phi)}{\partial x} + \frac{\partial(\rho v \phi)}{\partial y} + \frac{\partial(\rho w \phi)}{\partial z} = \frac{\partial}{\partial x}(\Gamma \frac{\partial \phi}{\partial x}) + \frac{\partial}{\partial y}(\Gamma \frac{\partial \phi}{\partial y}) + \frac{\partial}{\partial z}(\Gamma \frac{\partial \phi}{\partial z}) + S$ <p>对 u, v, w, k, ϵ, T 广义扩散系数 Γ 为:</p> <p>$u, v, w: \Gamma = \eta_{\text{eff}} = \eta + \eta_t$</p> <p>$k: \Gamma = \eta + \frac{\eta_t}{\sigma_k}$</p> <p>$\epsilon: \Gamma = \eta + \frac{\eta_t}{\sigma_\epsilon}$</p> <p>$T: \Gamma = \frac{\eta}{Pr} + \frac{\eta_t}{\sigma_T}$</p> <p style="color: red; font-size: 1.2em;">} Diffusion coefficients</p> 
源项	<p style="color: blue; font-size: 1.2em;">} Source term</p> <p>$u: S = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}(\eta_{\text{eff}} \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(\eta_{\text{eff}} \frac{\partial v}{\partial x}) + \frac{\partial}{\partial z}(\eta_{\text{eff}} \frac{\partial w}{\partial x})$</p> <p>$v: S = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}(\eta_{\text{eff}} \frac{\partial u}{\partial y}) + \frac{\partial}{\partial y}(\eta_{\text{eff}} \frac{\partial v}{\partial y}) + \frac{\partial}{\partial z}(\eta_{\text{eff}} \frac{\partial w}{\partial y})$</p> <p>$w: S = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x}(\eta_{\text{eff}} \frac{\partial u}{\partial z}) + \frac{\partial}{\partial y}(\eta_{\text{eff}} \frac{\partial v}{\partial z}) + \frac{\partial}{\partial z}(\eta_{\text{eff}} \frac{\partial w}{\partial z})$</p> <p>$k: S = \rho G_k - \rho \epsilon$</p> <p>$\epsilon: S = \frac{\epsilon}{k} (c_1 \rho G_k - c_2 \rho \epsilon)$</p> <p>$G_k = \frac{\eta_t}{\rho} \left\{ 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 \right\}$</p> <p>$T: S$ 按实际问题而定</p>

For the old general governing eq.

10.4.4 Remarks

(1) Expansion of G term for 2D case

$$G = \frac{\eta_t}{\rho} \frac{\partial u_i}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) = \frac{\eta_t}{\rho} \left(\frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right) =$$

$$\frac{\eta_t}{\rho} \left[\left(\frac{\partial u}{\partial x} \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \frac{\partial v}{\partial y} \right) + \left(\frac{\partial u}{\partial x} \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} + \frac{\partial v}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \frac{\partial v}{\partial y} \right) \right]$$

$$G = \frac{\eta_t}{\rho} \left\{ 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right\}$$

There are 18 terms for 3D case.

(2) The above model is called **standard** $k - \varepsilon$ model. It can be applied to vigorously developed (旺盛发展) turbulent flow , also called as high-Re $k - \varepsilon$ model.

10.5 Wall Function Method

10.5.1 Two ways for grid settlement near wall in turbulence simulation

10.5.2 Fundamentals of wall function method

10.5.3 Boundary conditions of k, ε for standard $k - \varepsilon$ model

10.5.4 Cautions in implementing wall function method

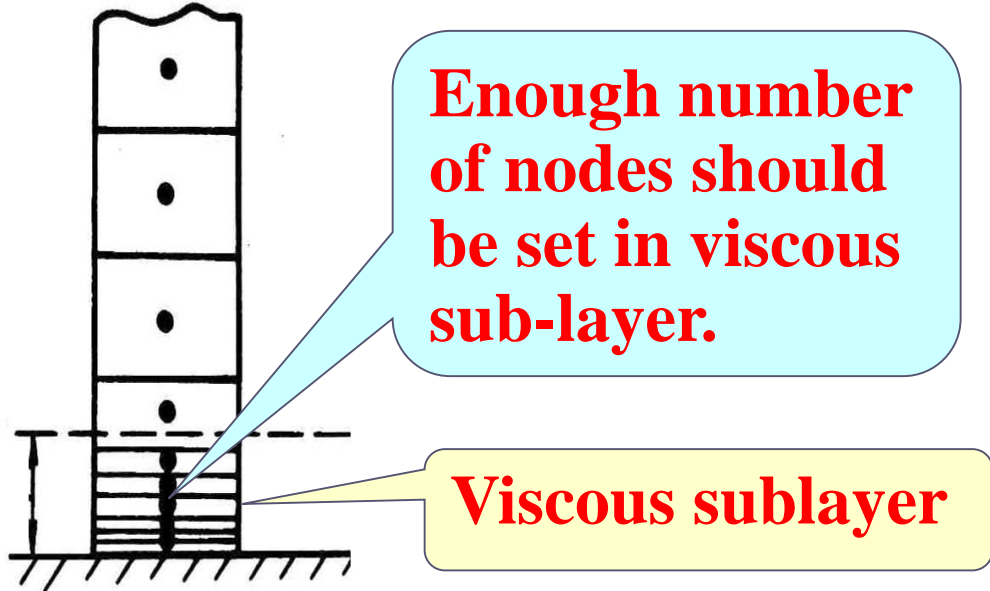
10.5 Wall Function Method

10.5.1 Two ways for grid settlement (节点设置) near wall in turbulence simulation

1. Setting enough number of grids in viscous sublayer (>10 grids)

For this treatment k equation can be used from vigorous turbulent flow to wall, and $k_w=0$ for its boundary condition.

This treatment will be used in low Reynolds number $k - \epsilon$ model.

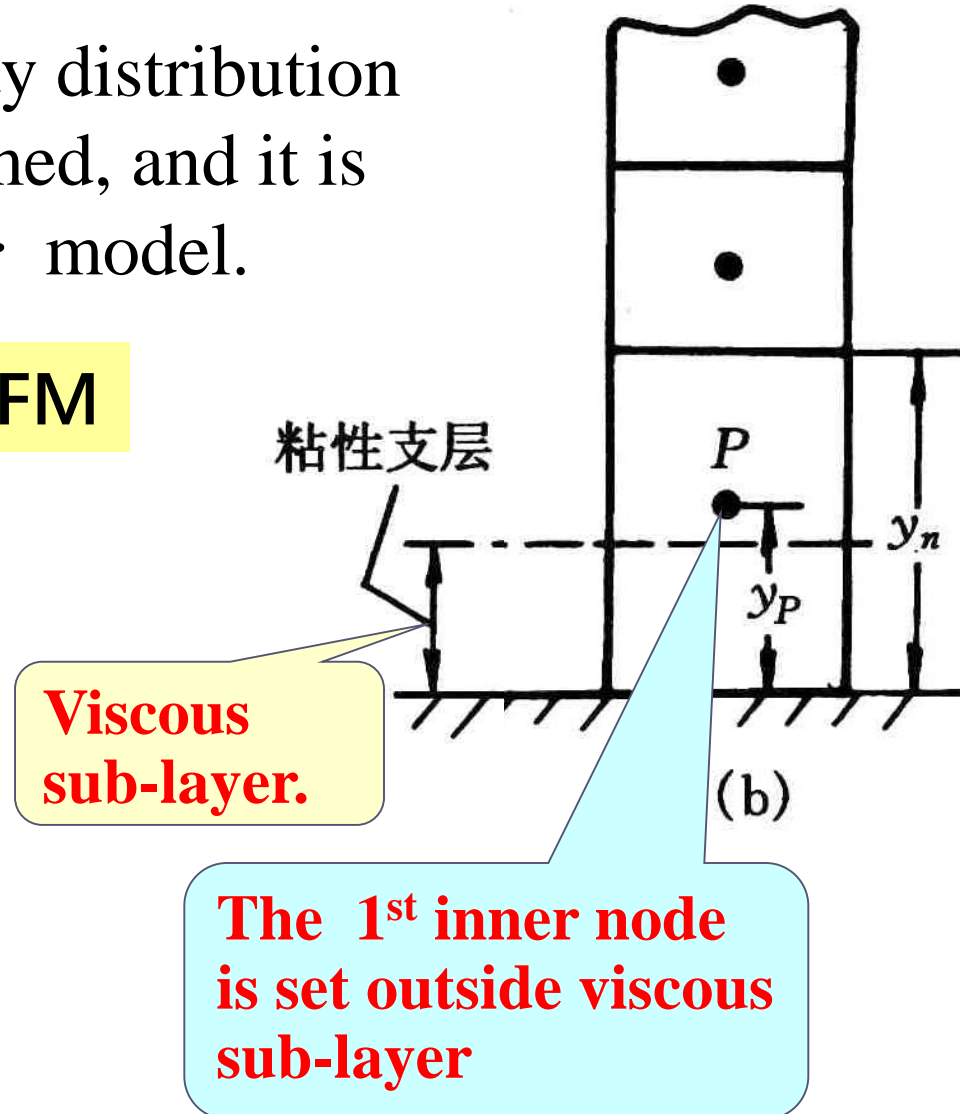


2. Set the 1st inner node outside the viscous sublayer

In this treatment velocity distribution near the wall should be assumed, and it is adopted in the high Re $k - \varepsilon$ model.

10.5.2 Fundamentals of WFM

1) Assuming that the dimensionless velocity and temp. distributions outside the viscous sub-layer are of logarithmic law(对数律) type.



(1) Logarithmic law in fluid mechanics

$$u^+ = \frac{u}{v^*} = \frac{1}{\kappa} \ln\left(\frac{yv^*}{\nu}\right) + B = \frac{1}{\kappa} \ln(y^+) + B = \frac{1}{\kappa} \ln(Ey^+)$$

$$v^* = \sqrt{\tau_w / \rho}, \quad \kappa = 0.4 \sim 0.42, \quad B = 5.0 \sim 5.5$$

(2) New definitions of y^+, u^+

In order that the logarithmic law can reflect some characteristics of turbulence the law is reformed in two aspects
 As follows:

Replacing v^* by $C_\mu^{1/4} k^{1/2}$
 to define y^+ :

$$y^+ = \frac{y(C_\mu^{1/4} k^{1/2})}{\nu} \quad \leftarrow \frac{yv^*}{\nu}$$

Introducing $C_\mu^{1/4} k^{1/2}$
 into u^+ definition

$$u^+ = \frac{u}{v^*} = \frac{u}{v^*} \frac{C_\mu^{1/4} k^{1/2}}{C_\mu^{1/4} k^{1/2}} = \frac{u(C_\mu^{1/4} k^{1/2})}{\tau_w / \rho}$$

When dissipation and production of fluctuation kinetic energy are balanced, the above definitions are identical to conventional definition in fluid mechanics.

(3) Definition of dimensionless temperature: mimicking (模仿) definition of u^+ :

$$T^+ = \frac{(T - T_w)(C_\mu^{1/4} k^{1/2})}{(q_w / \rho c_p)}$$

Mimicking velocity

Mimicking stress

Required by dimension consistency

(4) Logarithmic laws of u & T in turbulence model

For $y^+ > 11.0$ following distributions are adopted:

$$u^+ = \frac{1}{K} \ln(Ey^+), \quad \frac{1}{K} \ln(E) = 5.0 \sim 5.5$$

$$T^+ = \frac{\sigma_t}{K} \ln(Ey^+) + P\sigma_t \quad P = 8.96 \left(\frac{\sigma_l}{\sigma_t} - 1 \right) \left(\frac{\sigma_l}{\sigma_t} \right)^{-1/4}$$

$$\sigma_l = \text{Pr}_l; \sigma_t = \text{Pr}_t \quad \text{If } \sigma_l = \sigma_t \text{ then } T^+ = u^+$$

Then this is Reynolds analogy (雷诺比拟) .

For $y^+ < 11.0$, it is regarded as laminar sublayer.

2) **Placing the 1st inner** node P outside the viscous sub-layer, where logarithmic law valid ($y_P^+ > 11.0$)

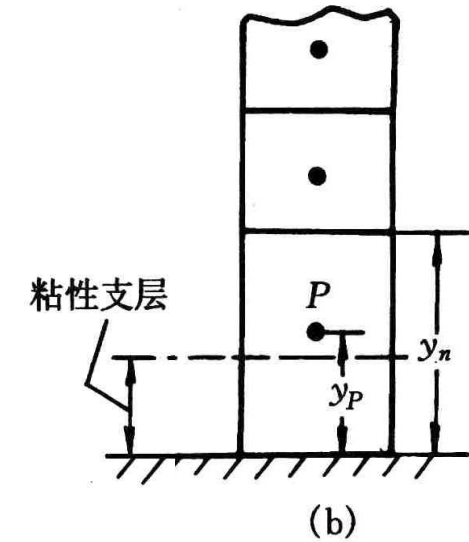
3) **The effective turbulent** viscosity and thermal conductivity between the 1st inner node and the wall should satisfy following equations:

$$\tau_w = \eta_B \frac{u_P - u_W}{y_P}, \quad q_w = \lambda_B \frac{T_P - T_W}{y_P}$$

The equations of effective viscosity and thermal conductivity between the 1st inner node and the wall can be derived as follows:

(1) Equation for η_B : At point P, u^+ satisfy :

$$\frac{u_P (C_\mu^{1/4} k_P^{1/2})}{\tau_w / \rho} = \frac{1}{K} \ln \left[E y_P \left(\frac{C_\mu^{1/4} k_P^{1/2}}{\nu} \right) \right]$$



This equation can be re-written as follows:

$$\tau_w = \frac{\rho u_P (C_\mu^{1/4} k_P^{1/2})}{\frac{1}{K} \ln \left[E y_P \frac{C_\mu^{1/4} k_P^{1/2}}{\nu} \right]} \xrightarrow{\text{According to Point 3}} = \eta_B \frac{u_P - \cancel{u_W}}{y_P}$$

0

η_B equation can be derived from this equation.

$$\frac{\cancel{\rho} u_P (C_\mu^{1/4} k_P^{1/2})}{\frac{1}{K} \ln[E y_P \frac{C_\mu^{1/4} k_P^{1/2}}{\cancel{\nu}}]} = \eta_B \frac{\cancel{u}_P}{y_P}$$

$$\eta_B = \left[\frac{y_P (C_\mu^{1/4} k_P^{1/2})}{\cancel{\nu}} \right] (\cancel{\rho} \cancel{\nu}) \frac{1}{\frac{1}{K} \ln(E y_P^+)} = \left(\frac{y_P^+}{u_P^+} \right) \eta_l$$

In the vigorous region , $y_P^+ \gg u_P^+$ above equation shows:

turbulent viscosity is y_P^+ / u_P^+ times of laminar viscosity.

For example $y_P^+ = 100$, $u_P^+ = \frac{1}{K} \ln(100) + B = \frac{1}{0.4} 4.605 + 5.0 = 16.5$

Then: $\eta_B = (100 / 16.5) \eta_l = 6.06 \eta_l$

(2) Equation for λ_B : At point P, T^+ satisfy :

$$\frac{(T_P - T_W)(C_\mu^{1/4} k_P^{1/2})}{\kappa} = \frac{\sigma_t}{\rho c_p} \ln(Ey_P^+) + \sigma_t P$$

From which: $q_w / \rho c_p$

$$q_w = \frac{\rho c_p (T_P - T_W)(C_\mu^{1/4} k_P^{1/2})}{\frac{\sigma_t}{\kappa} \ln(Ey_P^+) + \sigma_t P} \overset{\text{According to Point 3}}{=} \lambda_B \frac{(T_P - T_W)}{y_P}$$

$$\lambda_B = \frac{(C_\mu^{1/4} k_P^{1/2}) y_P \rho c_p \nu}{\frac{\sigma_t}{\kappa} \ln(Ey_P^+) + \sigma_t P} = \left(\frac{y_P^+}{T_P^+}\right) Pr_l \lambda_l$$

y_P^+

$$\frac{\eta c_p}{\lambda} = Pr_l$$

T_P^+

This is equivalent to magnify the molecular conductivity by $\left(\frac{y_P^+}{T_P^+}\right) Pr_l$ times.

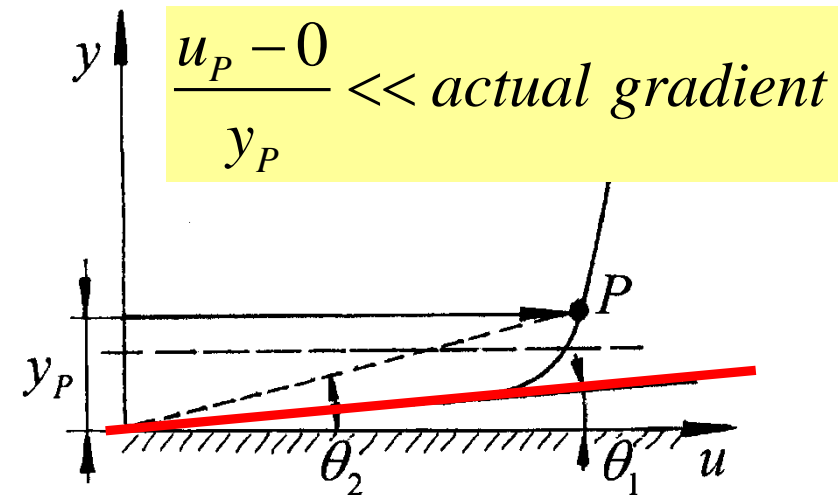
For $Pr_l = 5.0, Pr_t = 1.0, y_P^+ = 100,$

yielding $T_P^+ = 40.5, \frac{y_P^+}{T_P^+} Pr_l = \frac{100}{40.5} \times 5.0 = 12.3$

The molecular conductivity is magnified by 12.3 times!

Why wall viscosity and conductivity η_B, λ_B should be magnified? This is because the 1st inner node is far from wall, leading to reduced wall gradient determined by FD method.

In WFM the magnified transport properties compensate (弥补) the reduced gradients so that their products will be approximately close to the true values.



Wall functions refer to the expressions of η_B, λ_B

Fundamentals of Wall Function Method

1) **Assuming that the** dimensionless velocity and temp. distributions outside the viscous sub-layer are of logarithmic law(**对数律**) type.

$$\text{For } y^+ > 11.0 \quad \begin{cases} u^+ = \frac{1}{\kappa} \ln(Ey^+), & \frac{1}{\kappa} \ln(E) = 5.0 \sim 5.5 \\ T^+ = \frac{\sigma_t}{\kappa} \ln(Ey^+) + P\sigma_t; & P = 8.96 \left(\frac{\sigma_l}{\sigma_t} - 1\right) \left(\frac{\sigma_l}{\sigma_t}\right)^{-1/4} \end{cases}$$

2) **Placing the 1st inner** node P outside the viscous sub-layer, where logarithmic law valid ($y_P^+ > 11.0$)

3) **The effective turbulent** viscosity and thermal conductivity between the 1st inner node and the wall can be determined by following equations:

$$\eta_B = \left[\left(\frac{y_P^+}{u_P^+} \right) \right] \eta_l; \quad \lambda_B = \left[\left(\frac{y_P^+}{T_P^+} \right) \text{Pr}_l \right] \lambda_l; \quad y_P^+ = \frac{y_P (C_\mu^{1/4} k_P^{1/2})}{\nu}$$

4) The boundary condition of k equation: $\left. \frac{\partial k}{\partial n} \right|_w = 0$

Because outside the sublayer the production of fluctuation kinetic energy is much larger than diffusion towards wall, hence diffusion to the wall is approximately taken zero.

5) The dissipation of fluctuation kinetic energy at 1st inner node is determined by the model equation:

$$\varepsilon_P = \frac{C_D k^{3/2}}{l} = \frac{C_\mu^{3/4} k_P^{3/2}}{K y_P} \quad (\text{see page 355 of text book})$$

For the 1st inner node dissipation rate is specified by above equation, and computation is limited within the region surrounded by the 1st inner nodes.

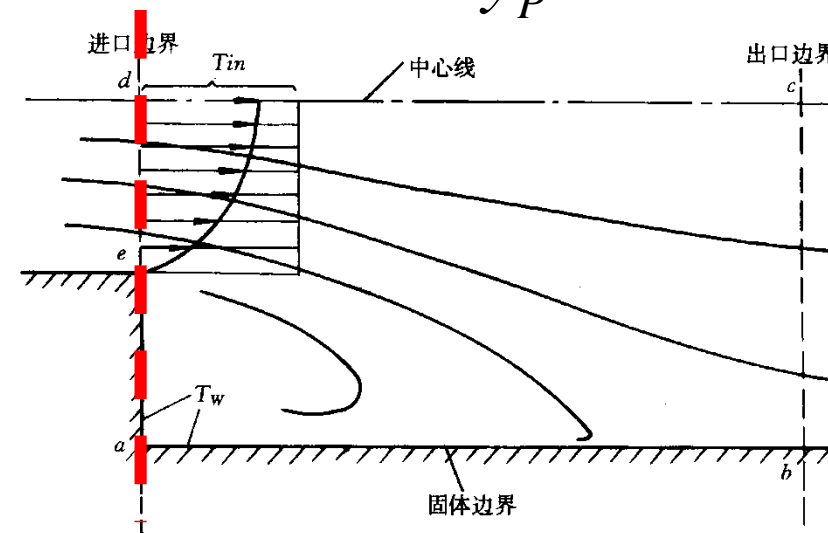
10.5.3 Boundary conditions of k, ε for standard $k - \varepsilon$ model

1. Inlet boundary

1) k : (1) Adopting test data; (2) Taking a percentage of kinetic energy of oncoming flow. For fully developed flow in ducts: 0.5~1.5%;

2) ε : (1) Using model equation:
$$\varepsilon = \frac{C_{\mu}^{3/4} k^{3/2}}{K y_P}$$

(2) Using $\eta_t = C_{\mu} \rho k^2 / \varepsilon$
 assuming $\frac{\rho u L}{\eta_t} = 100 \sim 1000$
 yielding η_t with inlet u
 and L .



2. At central line: $\frac{\partial k}{\partial n} = \frac{\partial \varepsilon}{\partial n} = 0$

3. Outlet: Adopting local one way coordinate assumption

3. Solid wall: **Adopting wall function method**

(1) Velocity — Velocity normal to wall $\left(\frac{\partial \phi}{\partial n}\right)_w = 0;$

Velocity parallel to wall $\phi_w = 0,$

And wall viscosity determined by WFM.

Remarks: here velocity is the dependent variable to be solved, not the one in the nonlinear part of convection term, for which wall velocities always equal zero: $u=v=0$.

(2) k — Adopting $\frac{\partial k}{\partial n} = 0$ implemented via setting $\Gamma_B = 0$

(3) ε — Specifying the 1st inner node by
 Then cutting connection with boundary.

$$\varepsilon_P = \frac{C_\mu^{3/4} k^{3/2}}{\kappa y_P}$$

10.5.4 Cautions in implementing wall function method

1) Approximate range of y_P^+, x_P^+

$$\underline{11.5 \sim 30} \leq (y_P^+, x_P^+) \leq 200 \sim 400$$

Logarithmic law is valid in this range

2) Underrelaxation

In the iteration process η_t, k, ε must be under-relaxed.

And it is organized within the solution process.

3) ε_P should be specified by large coefficient method

4) Source term treatment of k, ε

$$S_k = \rho G - \rho \varepsilon = \underbrace{\rho G}_{S_C} - \underbrace{(\rho \varepsilon / k^*)}_{S_P} k$$

$$S_\varepsilon = \frac{C_1 \rho \varepsilon G}{k} - \frac{C_2 \rho \varepsilon^2}{k} = \frac{C_1 \rho \varepsilon G}{\underbrace{k}_{S_C}} - \frac{C_2 \rho \varepsilon^*}{\underbrace{k}_{S_P}} \varepsilon$$

5) Treatment of solid located within fluid region

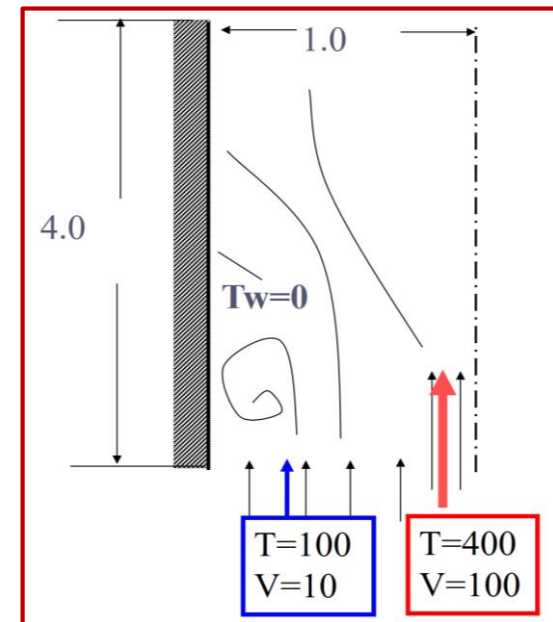
See pages 358—359 of textbook.

10-6 Turbulent flow and heat transfer in duct with a stepwise inlet velocity distribution ---k-epsilon turbulence model with WFM

10-6-1 Physical problem and its math formulation

Known: A stream with a central jet goes into a parallel channel; Flow is in turbulent state, $\text{AMU} = 10^{-6}$ and $\text{Pr}=0.7$.

Find: Adopt the standard k-Epsilon model and the wall function method to determine velocity and temperature fields in the channel.



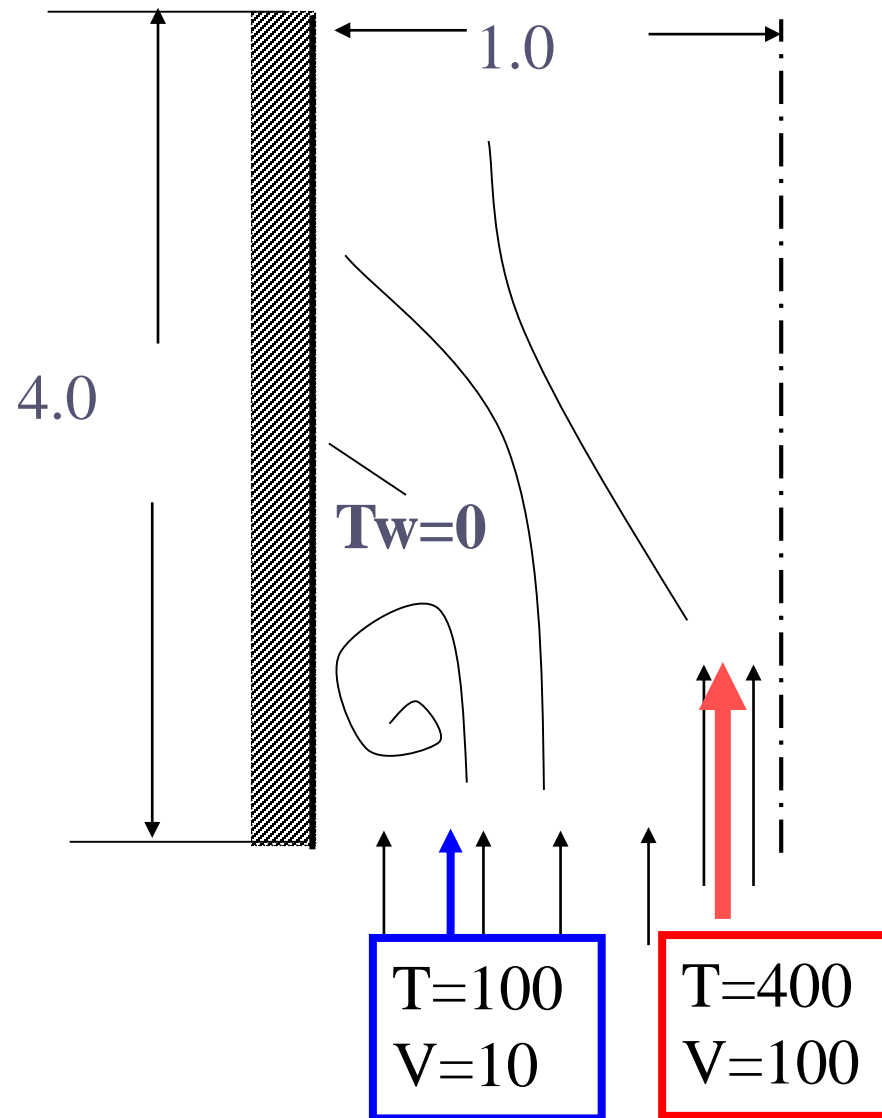


Fig. 1 of Example 8

Governing equation is:

$$\text{div}(\rho \vec{u} \phi) = \text{div}(\Gamma_{\phi} \text{grad} \phi) + S_{\phi}$$

where $\phi = u, v, T, k, \varepsilon, \rho, p$

The diffusion coefficients are:

NF=	1	2	3	4	5	6	7	8	11
Variable	U	V	PC	T	k	ε	AMUT	GEN	P
Γ_{ϕ}	η_t	η_t	/	$\frac{\eta_t c_p}{Pr_t}$	$\frac{\eta_t}{\sigma_k}$	$\frac{\eta_t}{\sigma_{\varepsilon}}$			
α	0.8	0.8		1.0	0.6	0.6			0.6

In our new governing equation for temperature:

$$\Gamma_t = \lambda_t = \eta_t c_p / Pr_t$$

$$S_u = \frac{\partial}{\partial x} \left(\eta_{\text{eff}} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(\eta_{\text{eff}} \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial z} \left(\eta_{\text{eff}} \frac{\partial w}{\partial x} \right)$$

$$S_v = \frac{\partial}{\partial x} \left(\eta_{\text{eff}} \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial y} \left(\eta_{\text{eff}} \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left(\eta_{\text{eff}} \frac{\partial w}{\partial y} \right)$$

$$k: S = \rho G_k - \rho \epsilon$$

$$\epsilon: S = \frac{\epsilon}{k} (c_1 \rho G_k - c_2 \rho \epsilon)$$

$$G_k = \frac{\eta_t}{\rho} \left\{ 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 \right\}$$

Boundary conditions are:

(1) Inlet:

k – taking 1 % of kinetic energy of oncoming flow;

Epsilon – determined by following eq.

$$\varepsilon = \frac{c_{\mu} \rho k^2}{\eta_t}$$

where η_t is determined by $Re_{eff} = \frac{\rho V (2L_{in})}{\eta_{eff}} = 100$

(2) Wall: adopting WFM;

(3) Outlet: taking local one –way;

(4) **At symmetric line:** normal velocity component =0, all others have their first order normal derivatives equal to zero!

10-6-2 Numerical method

(1) Source term treatment for $k - \varepsilon$

$$S_k = \eta_t G - \rho \varepsilon = \underbrace{\eta_t G}_{S_C} - \underbrace{\left(\frac{\rho \varepsilon}{k^*}\right) k}_{S_P}$$

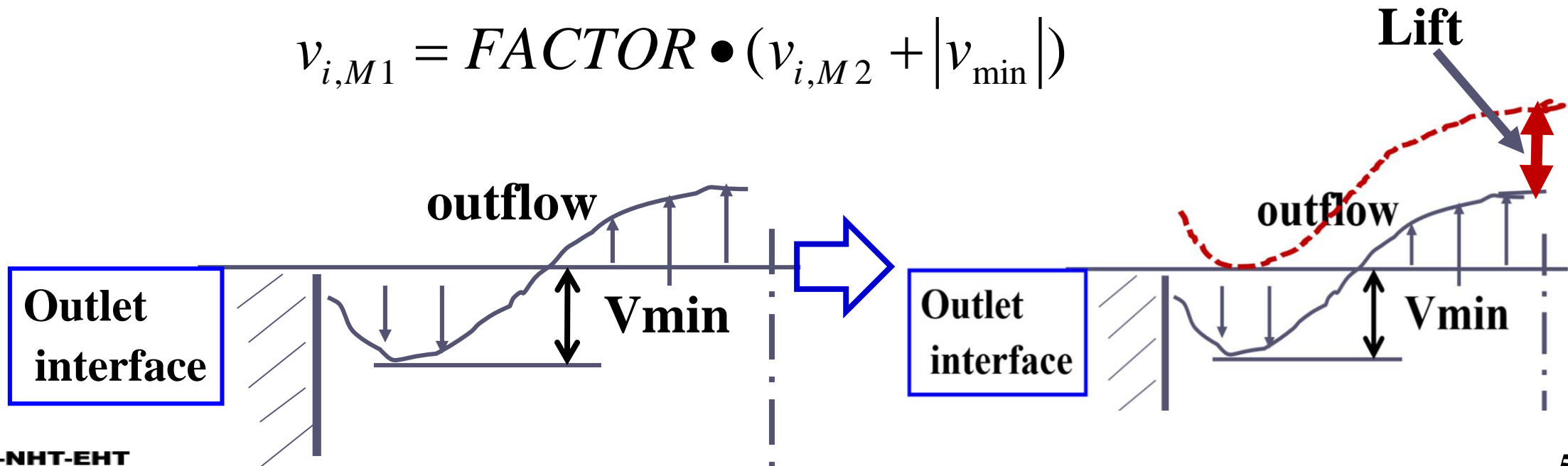
$$S_\varepsilon = \frac{c_1 \varepsilon \eta_t G}{k} - \frac{c_2 \rho \varepsilon^2}{k} = \underbrace{\frac{c_1 \varepsilon \eta_t G}{k}}_{S_C} - \underbrace{\left(\frac{c_2 \rho \varepsilon^*}{k}\right) \varepsilon}_{S_P}$$

(2) Lift (提升) of outlet velocity

In order to avoid negative outlet velocity during iteration, we may adopt method for lifting temporary (暂时的) outlet velocity:

$$FACTOR = \frac{FLOWIN}{\sum_{i=2}^{L2} [(V_{i,M2} + |V_{min}|) * RHO_{i,M1} * XCV(i)]}$$

$$v_{i,M1} = FACTOR \bullet (v_{i,M2} + |v_{min}|)$$



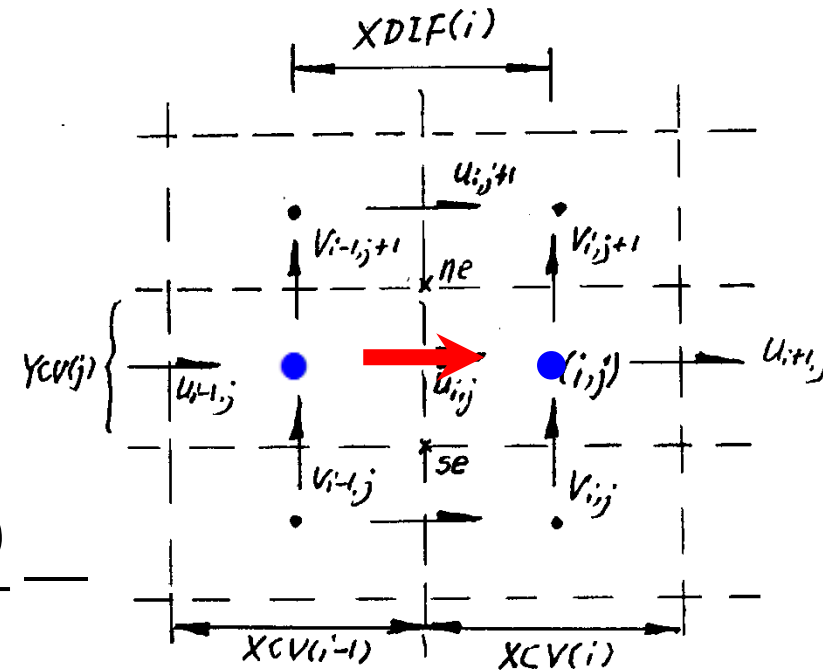
(3) Treatment of source term in u-momentum equation

$$S_u = \frac{\partial}{\partial x} \left(\mu_t \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu_t \frac{\partial v}{\partial x} \right)$$

$$\frac{\partial}{\partial x} \left(\mu_t \frac{\partial u}{\partial x} \right) = \frac{1}{XDIF(i)}$$

$$\left\{ GAM(i, j) \frac{u(i+1, j) - u(i, j)}{xcv(i)} - \right.$$

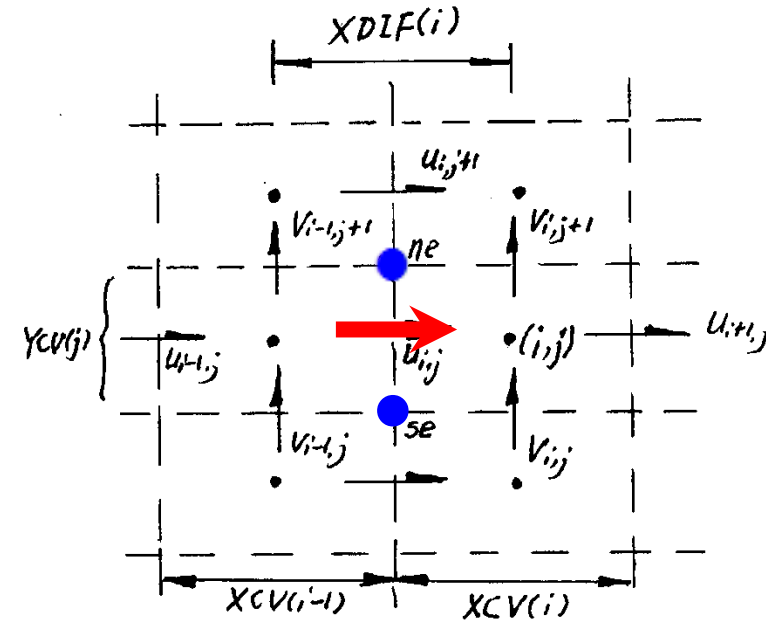
$$\left. GAM(i-1, j) \frac{u(i, j) - u(i-1, j)}{xcv(i-1)} \right\}$$



The above term is taken as S_c of u-equation!

$$\frac{\partial}{\partial y} \left(\mu_t \frac{\partial v}{\partial x} \right) = \frac{1}{YCV(j)}$$

$$\left\{ \mu_{t,ne} \frac{v(i, j+1) - v(i-1, j+1)}{XDIF(i)} - \mu_{t,se} \frac{v(i, j) - v(i-1, j)}{XDIF(i)} \right\}$$



Also, taken as Sc of u -equation!

(4) Flow field and temperature are solved separately

Because velocities are not coupled with temperature, the turbulent flow field can be solved first, then the fluid temperature.

10-6-3 Program reading

CC

MODULE USER_L

C*****

INTEGER*4 I,J

REAL*8 CMU, C1, C2, PRT, PRK, PRD, PRPRT, PFN, CMU4,

1 AFL, VMIN, REL, AMT, ALOG, GAP, GAMM, DUDX, DUDY, DVDX,

1 DVDY, DISS, AMU, PR, FLOWIN, FL, FACTOR

END MODULE

CC

SUBROUTINE USER

C*****

USE START_L

USE USER_L

IMPLICIT NONE

C*****

C-----**PROBLEM TEN**-----

C Turbulent fluid flow and heat transfer in a parallel duct with stepwise

C inlet velocity distribution

C*****

*

ENTRY GRID

TITLE(1)=' .VEL U.'

TITLE(2)=' .VEL V.'

TITLE(3)=' .STR FN.'

TITLE(4)=' .TEMP.'

TITLE(5)='KIN ENE'

TITLE(6)=' .DISIPA.'

TITLE(7)='TURB VI'

TITLE(11)='PRESSURE'

TITLE(12)=' DENSITY'

!All are titles for printing

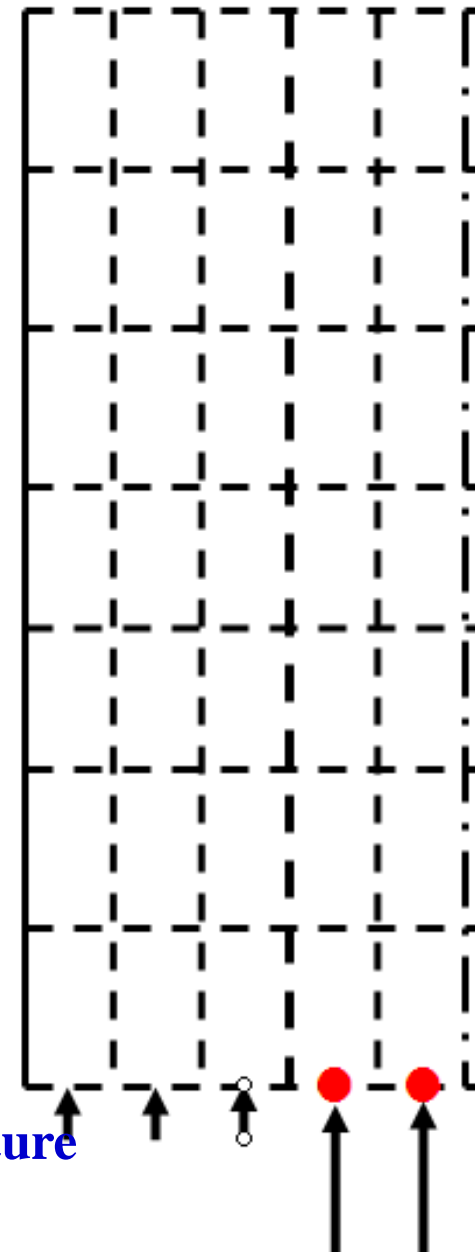
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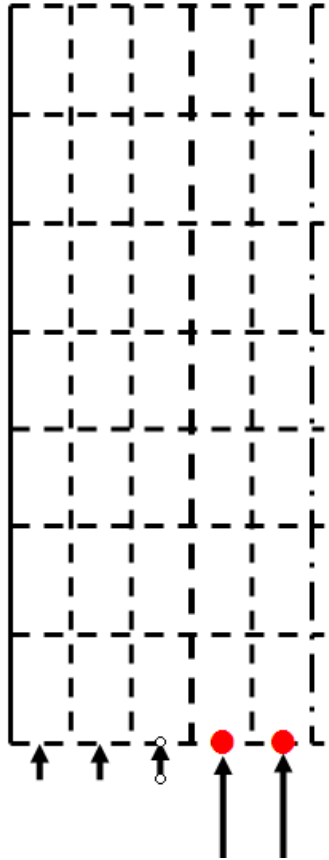
RELAX(1)=0.8
RELAX(2)=0.8
RELAX(5)=0.6
RELAX(6)=0.6
RELAX(13)=0.6 ! NGAM=13 for turbulent viscosity
LSOLVE(1)=.TRUE.
LSOLVE(5)=.TRUE.
LSOLVE(6)=.TRUE.
LPRINT(1)=.TRUE.
LPRINT(2)=.TRUE.
LPRINT(3)=.TRUE.
LPRINT(4)=.TRUE.
LPRINT(5)=.TRUE.
LPRINT(6)=.TRUE.
LPRINT(7)=.TRUE.
LPRINT(11)=.TRUE.
LAST=100
XL=1.
YL=4.
L1=7
M1=9
CPCON=1000. ! Cp in the Gama expression for temperature
CALL UGRID
RETURN
    
```

!All logical values for solving and printing

Regarding AMUT as the 7th element of F(i, j, NF)

$$\Gamma_t = \lambda_t = \eta_t c_p / Pr_t$$





ENTRY START

```

DO 100 J=1,M1
DO 101 I=1,L1
U(I,J)=0.
V(I,J)=10.
V(1,J)=0.
V(I,2)=10.
IF(I.GT.4) V(I,2)=100.
T(I,J)=100.
T(1,J)=0.
IF(I.GT.4) T(I,1)=400.
AKE(I,J)=0.005*V(I,2)**2
DIS(I,J)=0.1*AKE(I,J)**2
101 ENDDO
100 ENDDO
    
```

**1% of inlet kinetic energy,
Also initial value!**

η_t : determined form

$$Re_{eff} = \frac{\rho V (2L_{in})}{\eta_{eff}} = 100$$

$$100 = \frac{1 \times 100 \times 1.0}{\eta_t}, \eta_t = 1.0$$

$$\varepsilon = C_{\mu} \rho k^2 / \eta_t = 0.09 \times 1 \times k^2 \approx 0.1 k^2$$

Initial value!

AMU=1.E-6 ! Attention, very small values

CMU=0.09

C1=1.44

C2=1.92

PRT=0.9

PRK=1.0

PRD=1.3

PR=0.7

PRPRT=PR/PRT

PFN=9.*(PRPRT-1.)/PRPRT**.25

CMU4=CMU**.25

RETURN

ENTRY DENSE

RETURN

! Constants of Standard k-Epsilon

! P function of WFM for T

$$P = 9.0 \left(\frac{\sigma_l}{\sigma_t} - 1 \right) \left(\frac{\sigma_l}{\sigma_t} \right)^{-0.25}$$

e

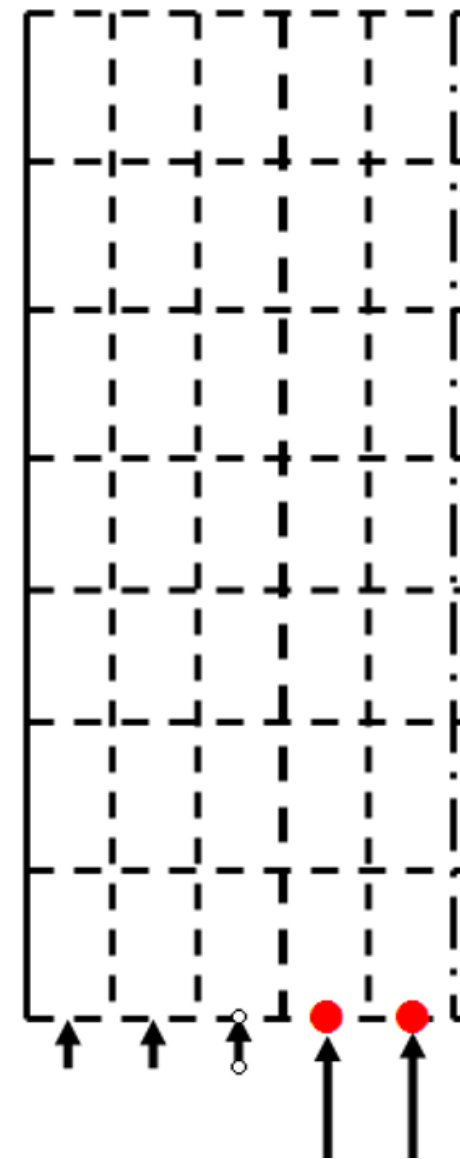
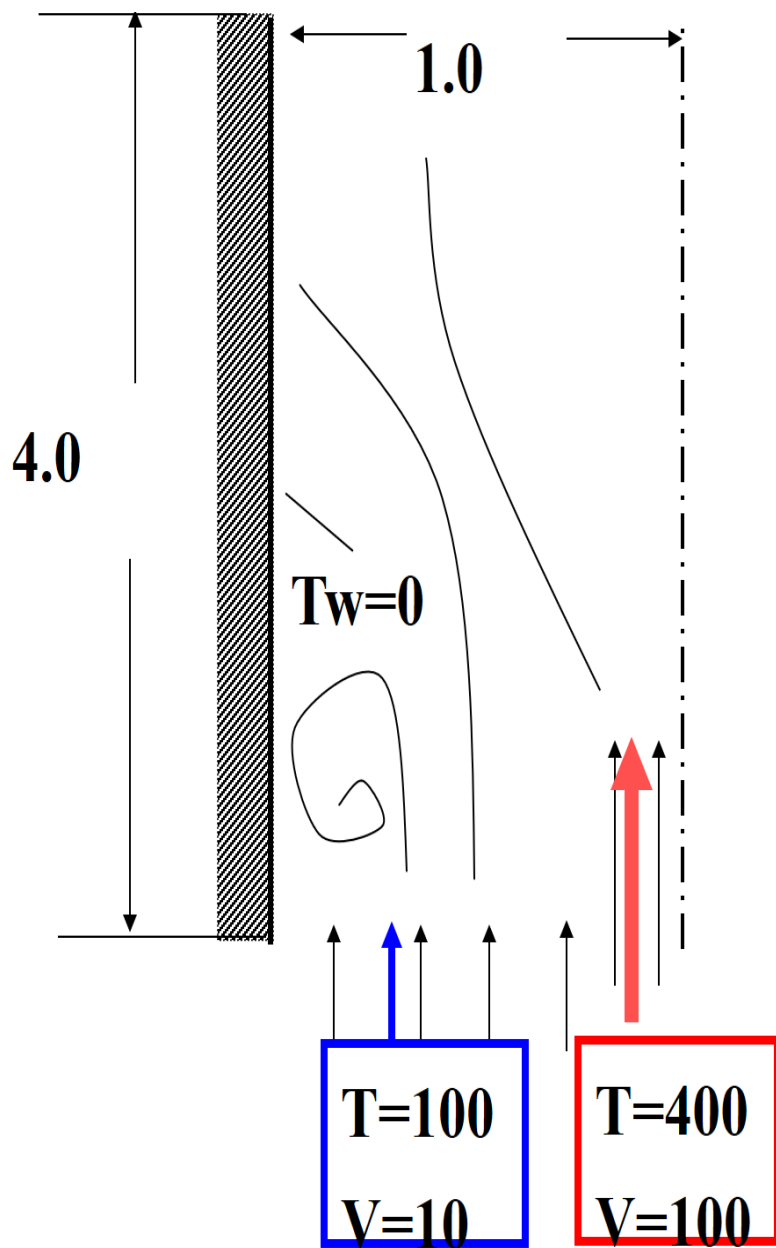
```

ENTRY BOUND
IF(ITER == 0) THEN
FLOWIN=0.
DO 310 I=2,L2
FLOWIN=FLOWIN+RHO(I,1)*V(I,2)*XCV(I)  ! Flow rate at inlet
310 ENDDO
ELSE
FL=0.
AFL=0.
VMIN=0.
ENDIF
DO 301 I=2,L2
IF(V(I,M2)< 0.) VMIN=DMAX1(VMIN,-V(I,M2))  ! Search for Vmin
AFL=AFL+RHO(I,M1)*XCV(I)  ! DMAX1 ( ) is
FL=FL+RHO(I,M1)*V(I,M2)*XCV(I)  ! more accurate than
FACTOR=FLOWIN/(FL+AFL*VMIN)  AMAX1( )
301 ENDDO
DO 302 I=2,L2
V(I,M1)=(V(I,M2)+VMIN)*FACTOR  !  $v_{i,M1} = FACTOR \cdot (v_{i,M2} + |v_{min}|)$ 
302 ENDDO
DO 303 J=2,M2
AKE(L1,J)=AKE(L2,J)  ! Equivalent to fully developed; also
DIS(L1,J)=DIS(L2,J)  ! decoration for print out
303 ENDDO
RETURN
    
```

$$FACTOR = \frac{FLOWIN}{\sum_{i=2}^{L2} [(V_{i,M2} + |V_{min}|) * RHO_{i,M1} * XCV(i)]}$$

```
ENTRY OUTPUT  
IF(ITER= =0) THEN  
PRINT 401  
WRITE(8,401)  
401 FORMAT(1X,' ITER',6X,'SMAX',6X,'SSUM',5X,'V(6,6)',  
1 4X,'T(5,6)',4X,'KE(5,6)')  
ELSE  
PRINT 403, ITER, SMAX, SSUM, V(6,6),T(5,6), AKE(5,6)  
WRITE(8,403) ITER,SMAX,SSUM,V(6,6),T(5,6),AKE(5,6) 403  
FORMAT(1X,I6,1P5E11.3)  
ENDIF  
IF(ITER>=55) THEN  
LSOLVE(4)=.TRUE.  
LSOLVE(1)=.FALSE.  
LSOLVE(5)=.FALSE.  
LSOLVE(6)=.FALSE.  
ENDIF  
IF (ITER==LAST) CALL PRINT  
RETURN
```

**! Switch off the solution valuables:
Flow is not coupled with
temperature ! After obtaining
converged flow field temperature
is solved**



ENTRY GAMSOR

IF(NF= = 3) RETURN

IF(NF= = 1) THEN

REL=1.-RELAX(NGAM) ! NGAM=13 for turbulent viscosity

DO 500 J=1,M1

DO 501 I=1,L1

AMT=CMU*RHO(I,J)*AKE(I,J)**2/(DIS(I,J)+1.E-30)

IF(ITER= =0) AMUT(I,J)=AMT ! Initial values

AMUT(I,J)=RELAX(NGAM)*AMT+REL*AMUT(I,J)

501 ENDDO

! Underrelaxation for turbulent viscosity

500 ENDDO

FACTOR=1.

ELSE

IF(NF== 4) FACTOR=CPCON/PRT

IF(NF== 5) FACTOR=1./PRK

IF(NF= = 6) FACTOR=1./PRD

DO 520 J=1,M1

DO 521 I=1,L1

GAM(I,J)=AMUT(I,J)*FACTOR

IF(NF/= 1) GAM(L1,J)=0. ! Symmetric line, u=0

GAM(I,M1)=0. ! Local one way for outlet

521 ENDDO

520 ENDDO

$$\varepsilon = \frac{c_{\mu} \rho k^2}{\eta_t}$$

$$\text{Pr} = \mu c_p / \lambda, \quad \lambda = \mu c_p / \text{Pr}$$

$$\left(\eta_l + \frac{\eta_t}{\sigma_k} \right) - \text{for } k; \quad \left(\eta_l + \frac{\eta_t}{\sigma_{\varepsilon}} \right) - \text{for } \varepsilon$$

! Laminar part is omitted.

WFM implementation!

W
F
M

I
m
p
L
e
m
e
n
t
a
t
i
o
n

DO 530 J=2,M2

SELECT CASE (NF) ! For u, p',k, epsilon

CASE (1,3,5,6)

GAM(1,J)=0

CASE (2) ! For velocity v and temp., WFM should be used!

GAM(1,J)=AMU ! First laminar viscosity is given for the left wall

XPLUS(J)=RHO(2,J)*SQRT(AKE(2,J))*CMU4*XDIF(2)/AMU

IF(XPLUS(J)>11.5) GAM(1,J)=AMU*XPLUS(J)/

1 (ALOG(9.*XPLUS(J))*2.5) ! Turbulence viscosity $\eta_B = \left(\frac{x_P^+}{u_P^+}\right)\eta_l$

CASE (4) ! For temperature, WFM for temperature

GAM(1,J)=AMU*CPCON/PR! First laminar thermal conductivity

IF(XPLUS(J)>11.5) GAM(1,J)=AMU/PRT*XPLUS(J)

1 / (2.5*ALOG(9.*XPLUS(J))+PFN) ! Turbulence thermal conductivity

ENDSELECT

530 ENDDO

$$x^+ = \frac{\rho x (C_\mu^{1/4} k^{1/2})}{\mu} \quad \lambda_B = \left(\frac{x_P^+}{T_P^+}\right) Pr_l \lambda_l$$

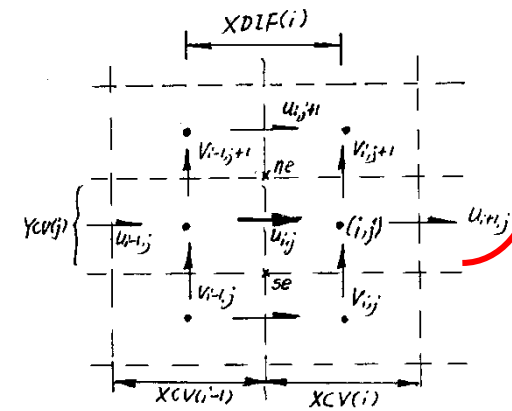
```

IF(NF= =1) THEN
DO 590 J=2,M2
DO 591 I=3,L2
CON(I,J)=(GAM(I,J)*(U(I+1,J)-U(I,J))/XCV(I)
1 -GAM(I-1,J)*(U(I,J)-U(I-1,J))/XCV(I-1))/XDIF(I)
GAMP=GAM(I,J+1)*GAM(I-1,J+1)/(GAM(I,J+1)+GAM(I-1,J+1)+1.E-30)
GAMP=GAMP+GAM(I,J)*GAM(I-1,J)/(GAM(I,J)+GAM(I-1,J)+1.E-30)
GAMM=GAM(I,J-1)*GAM(I-1,J-1)/(GAM(I,J-1)+GAM(I-1,J-1)+1.E-30)
GAMM=GAMM+GAM(I,J)*GAM(I-1,J)/(GAM(I,J)+GAM(I-1,J)+1.E-30)
CON(I,J)=CON(I,J)+(GAMP*(V(I,J+1)-V(I-1,J+1)
1 -GAMM*(V(I,J)-V(I-1,J)))/(YCV(J)*XDIF(I))
AP(I,J)=0.
591 ENDDO
590 ENDDO
RETURN
    
```

$$S_u = \frac{\partial}{\partial x} \left(\mu_t \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu_t \frac{\partial v}{\partial x} \right)$$

Source term calculation for u-eq.

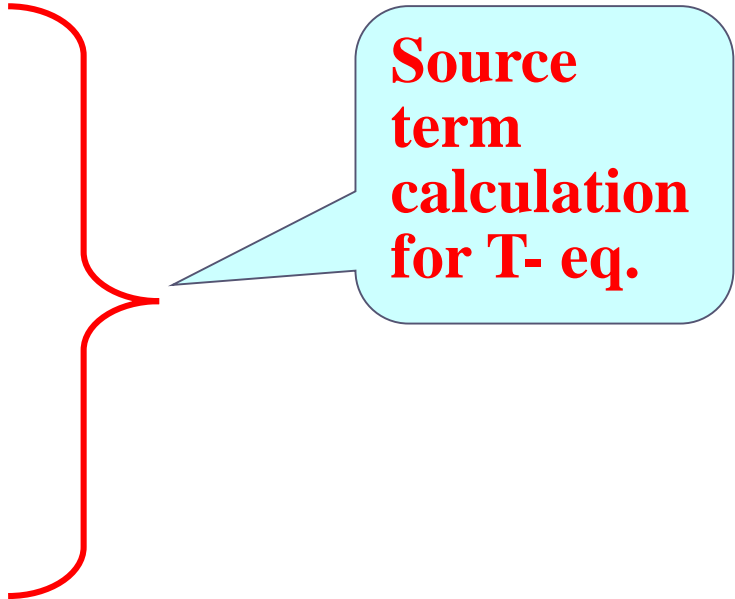
$$GAMM = D_{n-e} = \frac{(\delta x)_{e^-}}{(\delta y)_n} + \frac{(\delta x)_{e^+}}{(\delta y)_n}$$




```
509 IF(NF= =2) THEN
    DO 594 J=3,M2
    DO 595 I=2,L2
    CON(I,J)=(GAM(I,J)*(V(I,J+1)-V(I,J))/YCV(J)-
1 GAM(I,J-1)*(V(I,J)-V(I,J-1))/YCV(J-1))/(YDIF(J))
    GAMP=GAM(I+1,J)*GAM(I+1,J-1)/(GAM(I+1,J)+GAM(I+1,J-1)+1.E-30)
    GAMP=GAMP+GAM(I,J)*GAM(I,J-1)/(GAM(I,J)+GAM(I,J-1)+1.E-30)
    GAMM=GAM(I-1,J)*GAM(I-1,J-1)/(GAM(I-1,J)+GAM(I-1,J-1)+1.E-30)
    GAMM=GAMM+GAM(I,J)*GAM(I,J-1)/(GAM(I,J)+GAM(I,J-1)+1.E-30)
    CON(I,J)=CON(I,J)+(GAMP*(U(I+1,J)-U(I+1,J-1))
1 -GAMM*(U(I,J)-U(I,J-1)))/(XCV(I)*YDIF(J))
    AP(I,J)=0.
595 ENDDO
594 ENDDO
    RETURN
    ENDIF
```

Source term
calculation
for v- eq.

```
IF(NF= =4) THEN  
  DO 596 J=2,M2  
  DO 597 I=2,L2  
    CON(I,J)=0.  
    AP(I,J)=0.  
597 ENDDO  
586 ENDDO  
RETURN
```



**Source
term
calculation
for T- eq.**

! Following part is for the source term of k- eq.:

$$S_k = \eta_t G - \rho \varepsilon = \eta_t G - \left(\frac{\rho \varepsilon}{k^*} \right) k$$

! Most part of the code is for calculation of GEN term

```
ELSE IF(NF==5) THEN
DO 598 J=2,M2
DO 599 I=2,L2
DUDX=(U(I+1,J)-U(I,J))/XCV(I)
DVDY=(V(I,J+1)-V(I,J))/YCV(J)
IF(J= 2) DUDY=(0.5*(U(I,J+1)-U(I,J))+0.5*(U(I+1,J+1)-
C U(I+1,J)))/YDIF(J+1)
```

```

IF(J= =M2) DUDY=(0.5*(U(I,J)-U(I,J-1))+0.5*(U(I+1,J)-U(I+1,J-1))) /YDIF(J)
IF(J/=2.AND.J/=M2) DUDY=(0.5*(U(I,J+1)-U(I,J-1))+0.5*(U(I+1,J+1)-
1 U(I+1,J-1)))/(YDIF(J)+YDIF(J+1))
IF(I= =2) DVDX=(0.5*(V(I+1,J)-V(I-1,J))+0.5*(V(I+1,J+1)
1 -V(I-1,J+1)))/(XDIF(I)+XDIF(I+1))
IF(I= =L2) DVDX=(0.5*(V(I,J)-V(I-1,J))+0.5*(V(I,J+1)
1 -V(I-1,J+1)))/XDIF(I)
IF(I/=2.AND.I/=L2) DVDX=(0.5*(V(I+1,J)-V(I,J))+0.5*(V(I+1,J+1)
1 -V(I,J+1)))/XDIF(I+1))

```

GEN(I,J)=2.*(DUDX**2+DUDY**2)+(DUDY+DVDX)**2 ! GEN term

CON(I,J)=GEN(I,J)*AMUT(I,J)

AP(I,J)=-RHO(I,J)*DIS(I,J)/(AKE(I,J)+1.E-30)

598 ENDDO

599 ENDDO

RETURN

ENDIF

Sp of k-eq.

$$S_k = \eta_t G - \rho \varepsilon = \underline{\eta_t G} - \left(\frac{\rho \varepsilon}{k^*} \right) k$$

$$S_\varepsilon = \frac{c_1 \varepsilon \eta_t G}{k} - \frac{c_2 \rho \varepsilon^2}{k} = \frac{c_1 \varepsilon \eta_t G}{k} - \left(\frac{c_2 \rho \varepsilon^*}{k} \right) \varepsilon$$

$$S_c = A \phi_{given}, S_P = -A,$$

$$A = 10^{20} \sim 10^{30}$$

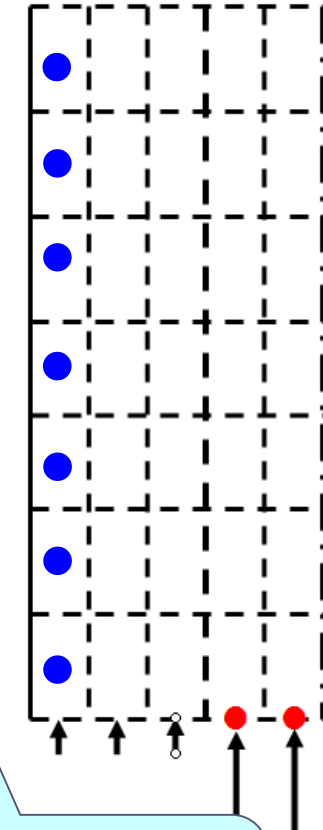
Large source term method

```

DO 600 J=2,M2
DO 601 I=2,L2
CON(I,J)=C1*GEN(I,J)*CMU*RHO(I,J)*AKE(I,J)
AP(I,J)=-C2*RHO(I,J)*DIS(I,J)/(AKE(I,J)+1.E-30)
601 ENDDO
600 ENDDO
DO 602 J=2,M2
DISS=CMU*AKE(2,J)**1.5/(0.4*CMU4*XDIF(2))
CON(2,J)=1.E30*DISS
AP(2,J)=-1.E30
602 ENDDO
RETURN
END
    
```

$$\frac{C_\mu^{3/4} k_P^{3/2}}{K y_P}$$

Adopt large source term method for 1st inner node where i=2; Page 26, Chapter 6 (3)



Source term calculation for Epsilon eq.

10.6.4 Results analysis

COMPUTATION IN CARTESIAN COORDINATES

ITER	SMAX	SSUM	V(6, 6)	T(5, 6)	KE(5, 6)
1	8.411E+00	1.421E-14	4.326E+01	1.000E+02	9.108E+00
2	2.675E+00	8.882E-15	4.354E+01	1.000E+02	2.939E+01
3	9.943E-01	-4.441E-15	4.409E+01	1.000E+02	5.808E+01
4	1.321E+00	6.661E-16	4.538E+01	1.000E+02	9.042E+01
5	1.147E+00	-1.998E-15	4.668E+01	1.000E+02	1.233E+02
6	7.209E-01	3.331E-16	4.747E+01	1.000E+02	1.550E+02
7	5.410E-01	2.109E-15	4.762E+01	1.000E+02	1.848E+02
8	4.211E-01	8.882E-16	4.725E+01	1.000E+02	2.119E+02
9	3.760E-01	3.886E-15	4.642E+01	1.000E+02	2.363E+02
10	3.451E-01	-2.776E-15	4.521E+01	1.000E+02	2.577E+02
11	3.723E-01	-5.773E-15	4.376E+01	1.000E+02	2.760E+02
12	3.797E-01	-4.441E-16	4.217E+01	1.000E+02	2.912E+02
13	3.811E-01	1.044E-14	4.054E+01	1.000E+02	3.031E+02
14	3.785E-01	-8.216E-15	3.899E+01	1.000E+02	3.120E+02
15	3.723E-01	-9.437E-15	3.757E+01	1.000E+02	3.183E+02
16	3.714E-01	-1.332E-15	3.633E+01	1.000E+02	3.226E+02
17	3.640E-01	-4.441E-16	3.529E+01	1.000E+02	3.254E+02
18	3.615E-01	1.776E-15	3.446E+01	1.000E+02	3.273E+02

19	3.499E-01	5.773E-15	3.380E+01	1.000E+02	3.285E+02
20	1.993E-01	0.000E+00	3.331E+01	1.000E+02	3.293E+02
21	1.916E-01	7.327E-15	3.294E+01	1.000E+02	3.298E+02
22	1.632E-01	-3.275E-15	3.267E+01	1.000E+02	3.299E+02
23	1.494E-01	-5.773E-15	3.248E+01	1.000E+02	3.299E+02
24	1.283E-01	-3.220E-15	3.234E+01	1.000E+02	3.295E+02
25	1.071E-01	-8.327E-16	3.224E+01	1.000E+02	3.290E+02
26	8.615E-02	-1.024E-14	3.218E+01	1.000E+02	3.282E+02
27	7.442E-02	5.301E-15	3.213E+01	1.000E+02	3.273E+02
28	7.219E-02	-3.969E-15	3.210E+01	1.000E+02	3.261E+02
29	6.907E-02	-1.638E-15	3.207E+01	1.000E+02	3.248E+02
30	6.246E-02	-5.704E-15	3.205E+01	1.000E+02	3.234E+02
31	5.292E-02	-6.689E-15	3.202E+01	1.000E+02	3.218E+02
32	4.163E-02	-3.039E-15	3.199E+01	1.000E+02	3.201E+02
33	3.782E-02	6.467E-15	3.196E+01	1.000E+02	3.183E+02
34	3.624E-02	1.332E-15	3.193E+01	1.000E+02	3.165E+02
35	3.316E-02	-7.938E-15	3.189E+01	1.000E+02	3.145E+02
36	2.901E-02	1.693E-15	3.185E+01	1.000E+02	3.126E+02
37	2.497E-02	-1.303E-14	3.181E+01	1.000E+02	3.105E+02
38	2.160E-02	-1.010E-14	3.177E+01	1.000E+02	3.085E+02
39	1.930E-02	1.041E-16	3.173E+01	1.000E+02	3.064E+02
40	1.730E-02	1.774E-14	3.168E+01	1.000E+02	3.043E+02
41	1.535E-02	-9.714E-16	3.164E+01	1.000E+02	3.022E+02
42	2.275E-02	5.967E-16	3.160E+01	1.000E+02	3.002E+02

			V	T	KE
43	4.093E-02	-4.635E-15	3.156E+01	1.000E+02	2.981E+02
44	4.235E-02	-1.457E-15	3.152E+01	1.000E+02	2.961E+02
45	3.395E-02	8.327E-16	3.148E+01	1.000E+02	2.941E+02
46	2.645E-02	1.388E-16	3.144E+01	1.000E+02	2.921E+02
47	2.060E-02	8.188E-16	3.140E+01	1.000E+02	2.901E+02
48	1.581E-02	4.718E-15	3.136E+01	1.000E+02	2.882E+02
49	1.193E-02	-6.939E-16	3.133E+01	1.000E+02	2.863E+02
50	8.833E-03	-2.772E-15	3.130E+01	1.000E+02	2.845E+02
51	6.423E-03	7.556E-15	3.127E+01	1.000E+02	2.827E+02
52	6.119E-03	-2.288E-15	3.124E+01	1.000E+02	2.810E+02
53	6.003E-03	-3.456E-15	3.121E+01	1.000E+02	2.793E+02
54	5.891E-03	-5.551E-15	3.118E+01	1.000E+02	2.776E+02
55	5.779E-03	-7.527E-15	3.116E+01	1.000E+02	2.760E+02
56	5.779E-03	-7.527E-15	3.116E+01	2.126E+02	2.760E+02
57	5.779E-03	-7.527E-15	3.116E+01	2.170E+02	2.760E+02
58	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
59	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
60	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
61	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
62	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
63	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
64	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
65	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
66	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02

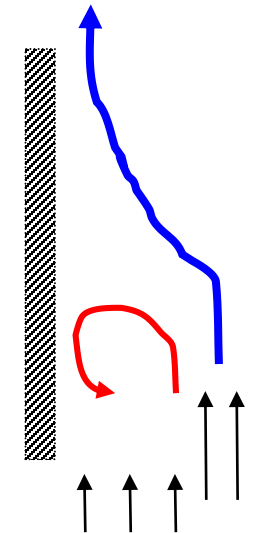
Changing solution variables

Seven iterations of T reach covered solution

67	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
68	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
69	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
70	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
71	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
72	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
73	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
74	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
75	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
76	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
77	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
78	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
79	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
80	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
81	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
82	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
83	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
84	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
85	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
86	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
87	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
88	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
89	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02
90	5.779E-03	-7.527E-15	3.116E+01	2.174E+02	2.760E+02

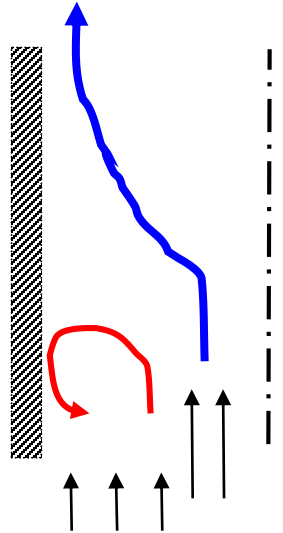
***** .VEL U. *****

I =	2	3	4	5	6	7
J						
9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
8	0.00E+00	1.56E-02	3.75E-02	3.84E-02	2.04E-02	0.00E+00
7	0.00E+00	-1.65E+00	-2.68E+00	-2.78E+00	-1.33E+00	0.00E+00
6	0.00E+00	-2.37E+00	-3.56E+00	-3.57E+00	-1.63E+00	0.00E+00
5	0.00E+00	-2.38E+00	-3.88E+00	-3.98E+00	-1.66E+00	0.00E+00
4	0.00E+00	-1.39E+00	-3.33E+00	-3.86E+00	-1.45E+00	0.00E+00
3	0.00E+00	3.74E+00	-3.47E-01	-2.75E+00	-8.62E-01	0.00E+00
2	0.00E+00	4.44E+00	6.55E+00	-2.87E+00	-6.77E-01	0.00E+00
1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00



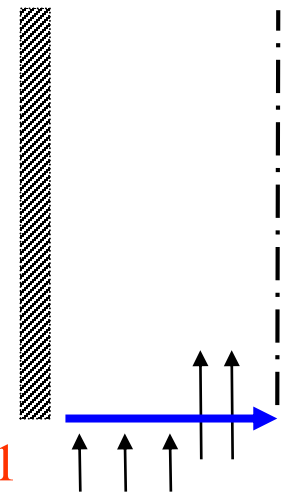
***** .VEL V. *****

I =	1	2	3	4	5	6	7
J							
9	0.00E+00	8.87E+00	3.18E+01	4.59E+01	6.52E+01	7.82E+01	1.00E+01
8	0.00E+00	8.87E+00	3.18E+01	4.59E+01	6.52E+01	7.82E+01	1.00E+01
7	0.00E+00	4.16E+00	2.89E+01	4.56E+01	6.93E+01	8.20E+01	1.00E+01
6	0.00E+00	-2.61E+00	2.55E+01	4.56E+01	7.48E+01	8.67E+01	1.00E+01
5	0.00E+00	-9.41E+00	2.12E+01	4.53E+01	8.15E+01	9.14E+01	1.00E+01
4	0.00E+00	-1.34E+01	1.56E+01	4.38E+01	8.83E+01	9.56E+01	1.00E+01
3	0.00E+00	-2.70E+00	3.98E+00	3.69E+01	9.37E+01	9.81E+01	1.00E+01
2	1.00E+01	1.00E+01	1.00E+01	1.00E+01	1.00E+02	1.00E+02	1.00E+02



*****.STR FN *****

I =	2	3	4	5	6	7
J						
9	0.00E+00	-1.77E+00	-8.12E+00	-1.73E+01	-3.03E+01	-4.60E+01
8	0.00E+00	-1.77E+00	-8.14E+00	-1.73E+01	-3.04E+01	-4.60E+01
7	0.00E+00	-8.31E-01	-6.61E+00	-1.57E+01	-2.96E+01	-4.60E+01
6	0.00E+00	5.21E-01	-4.58E+00	-1.37E+01	-2.87E+01	-4.60E+01
5	0.00E+00	1.88E+00	-2.36E+00	-1.14E+01	-2.77E+01	-4.60E+01
4	0.00E+00	2.68E+00	-4.55E-01	-9.22E+00	-2.69E+01	-4.60E+01
3	0.00E+00	5.39E-01	-2.57E-01	-7.64E+00	-2.64E+01	-4.60E+01
2	0.00E+00	-2.00E+00	-4.00E+00	-6.00E+00	-2.60E+01	-4.60E+01



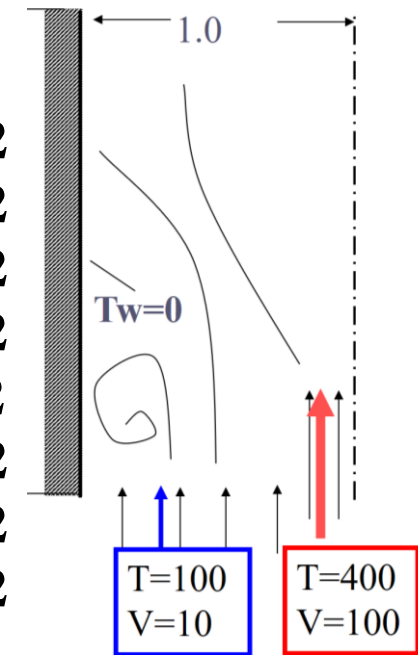
Stream function increase along this direction

***** . TEMP. *****

I =	1	2	3	4	5	6	7
J							
9	0.00E+00	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.00E+02
8	0.00E+00	3.01E+02	3.26E+02	3.39E+02	3.60E+02	3.80E+02	1.00E+02
7	0.00E+00	3.00E+02	3.21E+02	3.35E+02	3.63E+02	3.85E+02	1.00E+02
6	0.00E+00	2.93E+02	3.10E+02	3.26E+02	3.64E+02	3.89E+02	1.00E+02
5	0.00E+00	2.80E+02	2.93E+02	3.11E+02	3.67E+02	3.92E+02	1.00E+02
4	0.00E+00	2.65E+02	2.69E+02	2.88E+02	3.72E+02	3.95E+02	1.00E+02
3	0.00E+00	2.52E+02	2.36E+02	2.53E+02	3.79E+02	3.97E+02	1.00E+02
2	0.00E+00	1.29E+02	1.16E+02	2.01E+02	3.90E+02	3.99E+02	1.00E+02
1	0.00E+00	1.00E+02	1.00E+02	1.00E+02	4.00E+02	4.00E+02	4.00E+02

Given wall temp

Given inlet temp.



***** KIN ENE *****

I =	1	2	3	4	5	6	7
J	Initial values, No decoration!						
9	5.00E-01	5.00E-01	5.00E-01	5.00E-01	5.00E+01	5.00E+01	5.00E+01
8	5.00E-01	1.59E+02	4.93E+02	4.65E+02	3.53E+02	2.15E+02	2.15E+02
7	5.00E-01	1.90E+02	5.34E+02	4.85E+02	3.35E+02	1.74E+02	1.74E+02
6	5.00E-01	2.20E+02	5.83E+02	5.22E+02	3.20E+02	1.37E+02	1.37E+02
5	5.00E-01	2.39E+02	6.06E+02	5.46E+02	2.94E+02	1.06E+02	1.06E+02
4	5.00E-01	2.15E+02	5.40E+02	5.31E+02	2.54E+02	8.23E+01	8.23E+01
3	5.00E-01	1.15E+02	3.30E+02	4.69E+02	2.06E+02	6.62E+01	6.62E+01
2	5.00E-01	1.88E+01	1.03E+01	3.22E+02	1.46E+02	5.55E+01	5.55E+01
1	5.00E-01	5.00E-01	5.00E-01	5.00E-01	5.00E+01	5.00E+01	5.00E+01

**Initial values,
No decoration!**

*****.DISIPA.*****

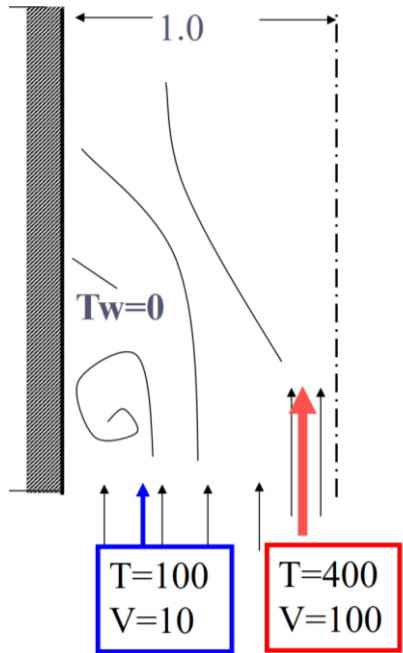
I =	1	2	3	4	5	6	7
J	Initial values, No decoration!						
9	2.50E-02	2.50E-02	2.50E-02	2.50E-02	2.50E+02	2.50E+02	2.50E+02
8	2.50E-02	8.18E+03	1.25E+04	1.13E+04	7.78E+03	3.60E+03	3.60E+03
7	2.50E-02	1.07E+04	1.44E+04	1.28E+04	7.79E+03	2.82E+03	2.82E+03
6	2.50E-02	1.34E+04	1.71E+04	1.53E+04	7.94E+03	2.12E+03	2.12E+03
5	2.50E-02	1.51E+04	1.93E+04	1.80E+04	7.66E+03	1.50E+03	1.50E+03
4	2.50E-02	1.29E+04	1.79E+04	1.98E+04	6.81E+03	1.01E+03	1.01E+03
3	2.50E-02	5.08E+03	1.04E+04	1.99E+04	5.46E+03	6.63E+02	6.63E+02
2	2.50E-02	3.34E+02	1.53E+02	1.52E+04	3.43E+03	4.02E+02	4.02E+02
1	2.50E-02	2.50E-02	2.50E-02	2.50E-02	2.50E+02	2.50E+02	2.50E+02

**Initial values,
No decoration!**

***** TURB VI *****

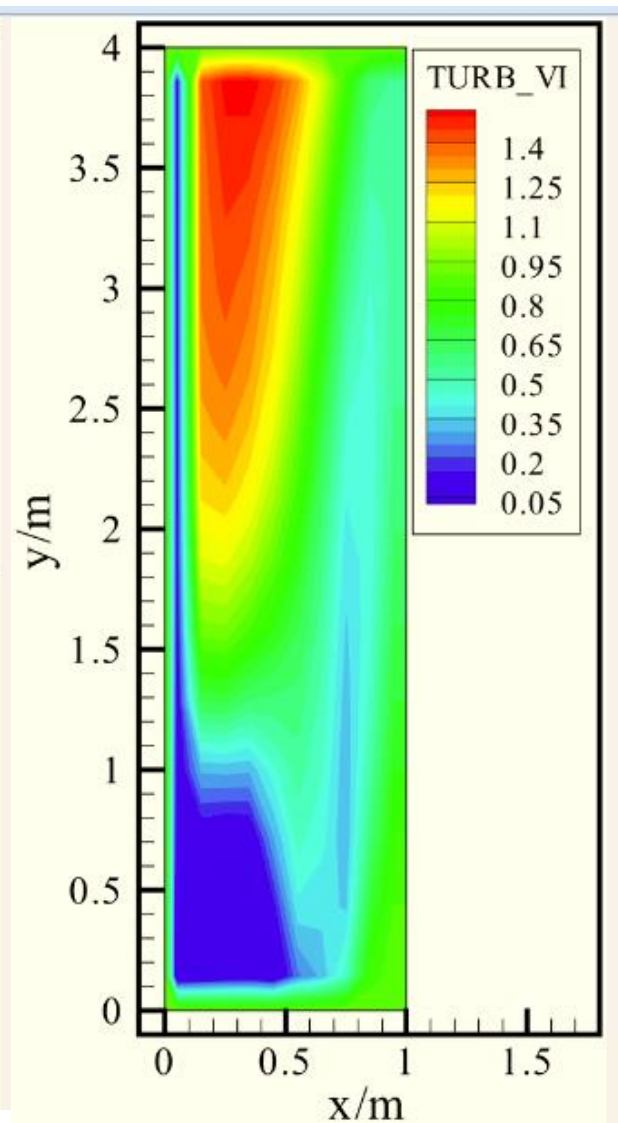
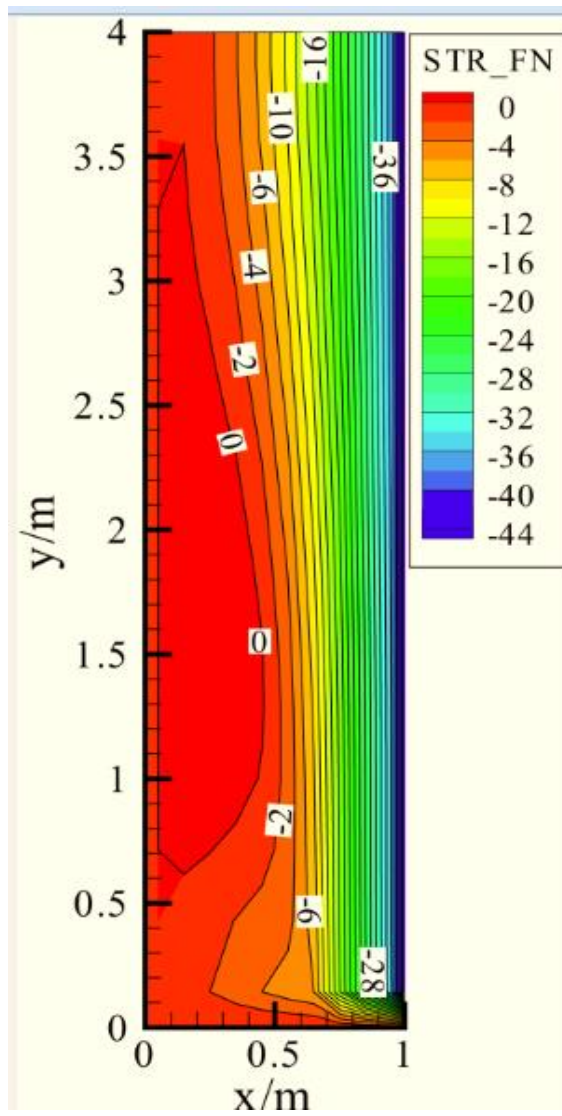
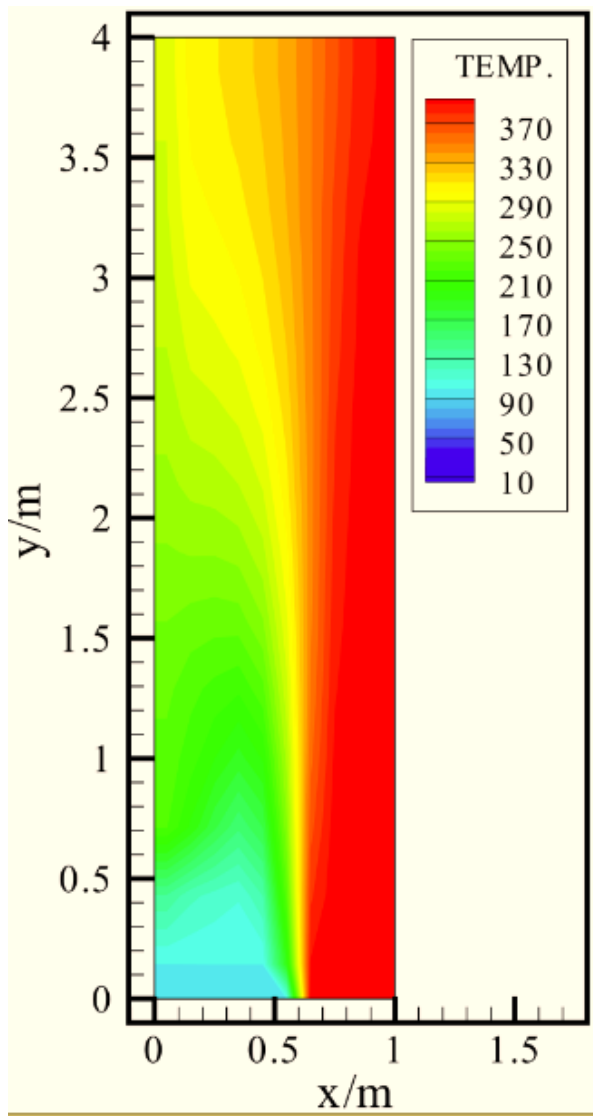
I =	1	2	3	4	5	6	7
J							
9	9.00E-01	9.00E-01	9.00E-01	9.00E-01	9.00E-01	9.00E-01	9.00E-01
8	9.00E-01	2.78E-01	1.72E+00	1.70E+00	1.42E+00	1.14E+00	1.14E+00
7	9.00E-01	3.04E-01	1.76E+00	1.65E+00	1.29E+00	9.59E-01	9.59E-01
6	9.00E-01	3.27E-01	1.77E+00	1.59E+00	1.16E+00	7.99E-01	7.99E-01
5	9.00E-01	3.40E-01	1.71E+00	1.48E+00	1.01E+00	6.75E-01	6.75E-01
4	9.00E-01	3.22E-01	1.46E+00	1.28E+00	8.54E-01	6.02E-01	6.02E-01
3	9.00E-01	2.36E-01	9.39E-01	9.99E-01	7.00E-01	5.94E-01	5.94E-01
2	9.00E-01	9.50E-02	6.24E-02	6.19E-01	5.58E-01	6.88E-01	6.88E-01
1	9.00E-01	9.00E-01	9.00E-01	9.00E-01	9.00E-01	9.00E-01	9.00E-01

Molecular viscosity $\mu_l \approx 10^{-6}$



***** PRESSURE *****

I =	1	2	3	4	5	6	7
J							
9	1.44E+03	1.43E+03	1.41E+03	1.33E+03	1.21E+03	1.14E+03	1.12E+03
8	1.36E+03	1.35E+03	1.33E+03	1.28E+03	1.20E+03	1.15E+03	1.13E+03
7	1.20E+03	1.19E+03	1.17E+03	1.17E+03	1.17E+03	1.16E+03	1.16E+03
6	9.40E+02	9.31E+02	9.11E+02	9.19E+02	9.28E+02	9.26E+02	9.25E+02
5	6.02E+02	5.92E+02	5.72E+02	5.96E+02	6.22E+02	6.25E+02	6.27E+02
4	2.24E+02	2.16E+02	1.99E+02	2.54E+02	3.08E+02	3.24E+02	3.32E+02
3	4.20E+01	3.16E+01	1.09E+01	1.03E+02	1.39E+02	1.44E+02	1.46E+02
2	1.31E+01	5.48E+00	-9.74E+00	-6.55E+01	2.53E+01	4.85E+01	6.02E+01
1	0.00E+00	-7.61E+00	-2.01E+01	-1.50E+02	-3.17E+01	1.07E+00	1.27E+01



10.7 Low Reynolds Number k -epsilon Model

10.7.1 Application range of standard $k - \varepsilon$ model

10.7.2 Jones – Launder low Re $k - \varepsilon$ model

10.7.3 Other low Re $k - \varepsilon$ models

10.7 Low Reynolds Number k-epsilon Model

10.7.1 Application range of standard $k - \varepsilon$ model

1. Near wall velocity distribution obeys logarithmic law
2. Shear stress is distributed uniformly from wall to 1st inner node;
3. Production and dissipation are nearly balanced for fluctuation kinetic energy.

Above assumptions are valid only when

$$Re_t = \frac{\rho k^2}{\eta \varepsilon} > 150$$

When this Re_t less than 150, the standard $k - \varepsilon$ model can not be used. When approaching wall this Reynolds number becomes smaller and smaller. In order that simulation can be conducted from vigorous part to the wall, model should be modified.

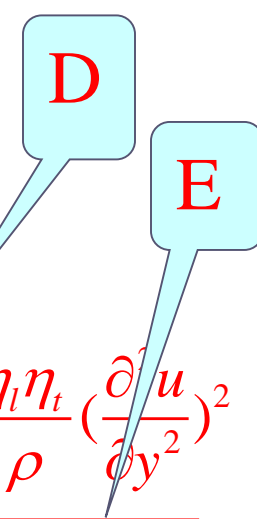
10.7.2 Jones – Launder low Re $k - \varepsilon$ model

1. Jones-Launder low Re model considerations(1972)

- (1) Both molecular and turbulent diffusions should be considered;
- (2) Effects of $Re_t = \frac{\rho k^2}{\eta \varepsilon}$ on coefficients should be considered;
- (3) Near a wall dissipation of fluctuation kinetic energy is not isotropic, and should be taken into account in k eq.

2. Jones-Launder low Reynolds $k - \varepsilon$ model

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_j k)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\eta_l + \frac{\eta_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + \rho G - \rho \varepsilon - \underline{2\eta \left(\frac{\partial k^{1/2}}{\partial y} \right)^2}$$

$$\frac{\partial(\rho \varepsilon)}{\partial t} + \frac{\partial(\rho u_j \varepsilon)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\eta_l + \frac{\eta_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + f_1 C_1 \frac{\rho G \varepsilon}{k} - f_2 C_2 \rho \frac{\varepsilon^2}{k} + \underline{2 \frac{\eta_l \eta_t}{\rho} \left(\frac{\partial u}{\partial y^2} \right)^2}$$


Callout D points to the term $2\eta \left(\frac{\partial k^{1/2}}{\partial y} \right)^2$ in the k equation.

Callout E points to the term $2 \frac{\eta_l \eta_t}{\rho} \left(\frac{\partial u}{\partial y^2} \right)^2$ in the ε equation.

$$\eta_t = C_\mu f_\mu \rho k^2 / \varepsilon$$

where $f_1 = 1.0$

$$f_2 = 1.0 - 0.3 \exp(-\text{Re}_t^2)$$

$$f_\mu = \exp(-2.5 / (1 + \text{Re}_t / 50))$$

$$\text{Re}_t = \frac{\rho k^2}{\eta \varepsilon}$$

Explanation: The vertical lines in Eqs. (9-47),(9-48) (page.363 of textbook) just show that the term is newly added, not the symbols of absolute value.

3. Explanations for additional terms

- (1) $D = -2\eta \left(\frac{\partial k^{1/2}}{\partial y} \right)^2$ (y is normal to wall), for considering that near a wall fluctuation kinetic energy is not isotropic, and with this term the condition of $\varepsilon_w = 0$ can be used;
- (2) The term **E** is for a better agreement with test data.

4. Boundary condition of J-L low Re model

$$k_w = \varepsilon_w = 0$$

10.7.3 Other low Re $k - \varepsilon$ models

Since the proposal of J-L low Re model in 1972, more than 20 variants (变体) have been proposed. The major differences between them are in four aspects:

(1) Different values of **the three modified coefficients**:

$$f_1, f_2, f_\mu$$

(2) Different expressions of additional **terms D and E** ;

(3) Different **wall boundary condition for ε**

$$\varepsilon = 0;$$

$$\frac{\partial \varepsilon}{\partial n} = 0$$

(4) **Different values of coefficients C_1, C_2, C_μ and constants $\sigma_k, \sigma_\varepsilon$**

Table 9-8 of Textbook

No	模型	简称	ε_w 条件	c_μ	c_1	c_2	σ_k	σ_ε	f_μ	f_1	f_2	D	E
1	高 Re 数	HR	壁面 函数法	0.09	1.44	1.92	1.0	1.3	1.0	1.0	1.0	0	0
2	Janes/Launder	JL	0	0.09	1.44	1.92	1.0	1.3	$\exp[-2.5/(1+Re_t/50)]$	1.0	$1-0.3\exp(-Re_t^2)$	$2\eta\left(\frac{\partial k^{1/2}}{\partial y}\right)^2$	$2\frac{\eta h}{\rho}\left(\frac{\partial^2 u}{\partial y^2}\right)^2$
3	Launder/Shar- ma[78]	LS	0(附注 1)	0.09	1.44	1.92	1.0	1.3	$\exp[-3.4/(1+Re_t/50)^2]$	1.0	$1-0.3\exp(-Re_t^2)$	$2\eta\left(\frac{\partial k^{1/2}}{\partial y}\right)^2$	$2\frac{\eta h}{\rho}\left(\frac{\partial^2 u}{\partial y^2}\right)^2$
4	Hassid/Porch [79]	HP	0	0.09	1.45	2.0	1.0	1.3	$1-\exp(-0.0015Re_t)$	1.0	$1-0.3\exp(-Re_t^2)$	$2\eta\frac{k}{y^2}$	$-2\eta\left(\frac{\partial \varepsilon^{1/2}}{\partial y}\right)^2$
5	Hoffman[80]	HO	0	0.09	1.81	2.0	2.0	3.0	$\exp[-1.75/(1+Re_t/50)]$	1.0	$1-0.3\exp(-Re_t^2)$	$\frac{\eta}{y}\frac{\partial k}{\partial y}$	0
6	Dutoya/ Michard[81]	DM	0	0.09	1.35	2.0	0.9	0.95	$1-0.86\exp[-(Re_t/600)^2]$	$1-0.04\exp\left[-\left(\frac{Re_t}{50}\right)^2\right]$	$1-0.3\exp\left[-\left(\frac{Re_t}{50}\right)^2\right]$	$2\eta\left(\frac{\partial k^{1/2}}{\partial y}\right)^2$	$-c_2f_2(\varepsilon D/k)^2$
7	Chien[82]	CH	0	0.09	1.35	1.8	1.0	1.3	$1-\exp(-0.0115y^+)$	1.0	$1-0.22\exp\left[-\left(\frac{Re_t}{6}\right)^2\right]$	$2\eta\frac{k}{y^2}$	$-2\eta(\varepsilon/y^2)\exp(-0.5y^+)$
8	Reynolds[83]	RE	$\nu\frac{\partial^2 k}{\partial y^2}$	0.084	1.0	1.83	1.69	1.3	$1-\exp(-0.0198Re_y)$ (附注 2)	1.0	$\left\{1-0.3\exp\left[-\left(\frac{Re_t}{6}\right)^2\right]\right\}f_\mu$	0	0
9	Lam/Bremhost [84](Dirich- let)	LB	$\nu\frac{\partial^2 k}{\partial y^2}$	0.09	1.44	1.92	1.0	1.3	$[1-\exp(-0.0165Re_y)]^2$ $\times\left(1+\frac{20.5}{Re_t}\right)$	$1+(0.05/f_\mu)^3$	$1-\exp(-Re_t^2)$	0	0
10	Lam/ Bremhost [84] (Neumann)	LB1	$\frac{\partial \varepsilon}{\partial y}=0$	0.09	1.44	1.92	1.0	1.3	同 LB	同 LB	同 LB	0	0

Table 9-8 in Textbook (Continued)

续表 9-8

No	模型	简称	ϵ_w 条件	c_μ	c_1	c_2	σ_k	σ_ϵ	f_μ	f_1	f_2	D	E
11	Nagano/Hishida[86]	NH	0	0.09	1.45	1.90	1.0	1.3	$[1 - \exp(-y^+ / 26.5)]^2$	1.0	$1 - 0.3 \exp(-Re_t^2)$	$2\eta \left(\frac{\partial k^{1/2}}{\partial y}\right)^2$	$\eta_\mu (1 - f_\mu) \left(\frac{\partial^2 u}{\partial y^2}\right)^2$
12	Myong/Kosagi[86]	MK	$\nu \frac{\partial^2 k}{\partial y^2}$ (附注 3)	0.09	1.40	1.80	1.4	1.3	$(1 + 3.45 Re_t^{1/2}) \times [1 - \exp(-y^+ / 70)]$	1.0	$\left[1 - \frac{2}{9} \exp\left(\frac{Re_t}{6}\right)^2\right] \times [1 - \exp(-y^+ / 5)]^2$	0	0
13	Abid[87]	AB	$\nu \frac{\partial^2 k}{\partial y^2}$	0.09	1.45	1.83	1.0	1.4	$\tanh(0.008 Re_y) \left(1 + \frac{4}{Re_t^{3/4}}\right)$	1.0	$1 - \frac{2}{9} \exp\left(1 - \frac{Re_t^2}{36}\right) \cdot [1 - \exp\left(\frac{-Re_y}{12}\right)]$	0	0
14	Abe \ Kondoh Nagano[88]	AKN	$2\nu \frac{R_p}{y_p^2}$	0.09	1.5	1.9	1.4	1.4	$\left\{1 + 5/Re_\tau^{3/4} \exp\left[1 - \left(\frac{Re_\tau}{200}\right)^2\right]\right\} [1 - \exp(-y^* / 14)]^2$ (附注 4)	1.0	$\left\{1 - 0.3 \exp\left[-\left(\frac{Re_t}{6.5}\right)^2\right]\right\} \cdot [1 - \exp(-y^* / 3.1)]^2$	0	0
15	Fan \ Barnett Lakshminarayana[89]	FBL	$\frac{\partial \epsilon}{\partial y} = 0$	0.09	1.4	1.8	1.0	1.3	$0.4 f_w / \sqrt{Re_t} + (1 - 0.4 f_w / \sqrt{Re_t}) \cdot [1 - \exp(Re_y / 42.63)]^3$ (附注 5)	1.0	$f_w^2 \left\{1 - 0.22 \exp\left[-\left(\frac{Re_t}{6}\right)^2\right]\right\}$	0	0
16	Cho/Goldstein[90]	CG	$\frac{\partial \epsilon}{\partial y} = 0$	0.09	1.44	1.92	1.0	1.3	$1 - 0.95 \exp(-5 \times 10^{-5} Re_t)$	1.0	$1 - 0.222 \exp\left(\frac{-Re_t^2}{36}\right)$	0	(附注 6)

For the low Re number $k - \varepsilon$ model

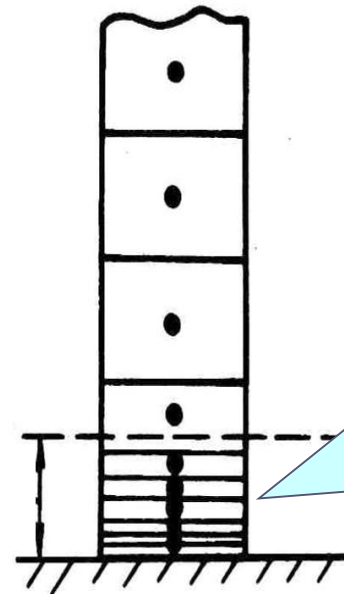
1. The Reynolds number refers to the turbulent Reynolds number, defined by

$$\text{Re}_t = \frac{\rho k^2}{\eta \varepsilon}$$

2. The grid number within the laminar sublayer should be larger than 10;

3. For J-L low Reynolds number model, the wall boundary condition of k and ε are:

$$k_w = \varepsilon_w = 0$$



Enough number of nodes should be set in viscous sub-layer.

10.8 Brief Introduction to Recent Developments

10.8.1 Developments in $k - \varepsilon$ two-equation model

10.8.2 Brief introduction to second moment model

10.8.3 Near wall region treatment of different models

10.8.4 Chen model for indoor air movement

10.8.5 $\overline{V^2} - f$ turbulence model for highly inhomogeneous turbulent flow

10.8 Brief Introduction to Recent Developments

10.8.1 Developments of $k - \varepsilon$ two-eq. model

1. Non-linear $k - \varepsilon$ model

Boussinesq's constitution eq.

$$(\tau_{i,j})_t = -\overline{\rho u_i' u_j'} = (-p_t \delta_{i,j}) + \eta_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)$$

In Boussinesq's constitution eq. every term is of 1st power (一次方)---linear leading to $\tau_{xx} = \tau_{yy}$ for fully developed turbulent flow in parallel plate duct, which does not agree with test results.

Speziale et al. proposed a non-linear model in 1987, see reference [95] of the textbook.

2. Multi-scale $k - \varepsilon$ model

In the standard $k - \varepsilon$ model only one geometric scale is used. Actually turbulent flow fluctuations cover a wide range of time scales and geometric scales. A simple improvement is adopting two geometric scales: big eddies for carrying kinetic energy(载能涡) and small eddies for dissipating energy(耗能涡). See reference [108] in book.

3. Renormalized group (重整化群) model

Starting from transient N-S eq. Yakhot-Orzag adopted spectral analysis (谱分析) method and derived k-epsilon equations with different coefficients and constants.

See Ref.[113] in the textbook.

3. Realizable $k - \varepsilon$ model (可实现)

In the standard k-epsilon model when fluid strain is very large the normal stress will be negative, which is not realizable; In order to establish all-cases realizable model the coefficient C_μ should be related with strain. (应变)
See ref. [115] in the textbook。

10.8.2 Brief introduction to second moment model(二阶矩模型)

For the products with two fluctuations, $-\overline{\rho u'_i u'_j}$, their governing eqs. are derived; for products with more than two fluctuations, say $u'_i u'_j u'_k$, models are introduced to close the model.

1. Original form of Reynolds stress equation

$$\frac{\overline{\partial u'_i u'_j}}{\partial t} + u_k \frac{\overline{\partial u'_i u'_j}}{\partial x_k} = P_{i,j} + \pi_{i,j} + D_{i,j} - \varepsilon_{i,j}$$

where $P_{i,j} = -\overline{(u'_i u'_k \frac{\partial u'_j}{\partial x_k} + u'_i u'_k \frac{\partial u'_i}{\partial x_k})}$ - **Production term**

$$\pi_{i,j} = \frac{\overline{p'}}{\rho} \left(\frac{\partial u'_j}{\partial x_i} + \frac{\partial u'_i}{\partial x_j} \right) -$$

Redistribution term

$$D_{i,j} = -\frac{\partial}{\partial x_k} \overline{(u'_i u'_j u'_k)} - \nu \frac{\partial \overline{(u'_i u'_j)}}{\partial x_k} + \delta_{i,k} \overline{\frac{u'_j p'}{\rho}} -$$

Diffusion term

The above three terms $P_{i,j}$, $D_{i,j}$, $\pi_{i,j}$ have to be simplified or modeled. Different treatments lead to different second moment models.

3. Eqs. and constants in 2nd moment closure for convective heat transfer

(1) 3-D time average governing eqs.---16:

5 time average eqs. for five variables: u, v, w, p, T

6 time average fluctuation stress eqs.

$$\overline{u'_i u'_j} (i, j = 1, 2, 3) \text{ (6 terms)};$$

3 eqs. for additional heat flux

$$\overline{u'_i \phi'} (i = 1, 2, 3) \text{ (3 terms)}$$

1 eq. for k , and

1 eq. for \mathcal{E}

(2) Nine empirical constants.

10.8.3 Near wall region treatment of different models

All the above improvements are **only for the vigorous part of turbulent flow**; for near wall region the molecular viscosity must be taken into account. At present following methods are used:

1. Adopting WFM;

2. Adopting two-layer model: several choices

- (1) With $Re_t=150$ as a **deviding line(分界线)**: **adopting one of the above model when it is larger than 150** ; if Re_t is less than 150 low Re k-epsilon model is used.
- (2) In near wall region k equation model is used, and in the vigorous part above model is adopted.

Emphasis should be paid for the near wall region.

10.8.4 Chen model for indoor air movement

Q Y Chen proposed following simple model for indoor air turbulent flow:

$$\eta_t = 0.03874 \rho v l$$

ρ – Air density

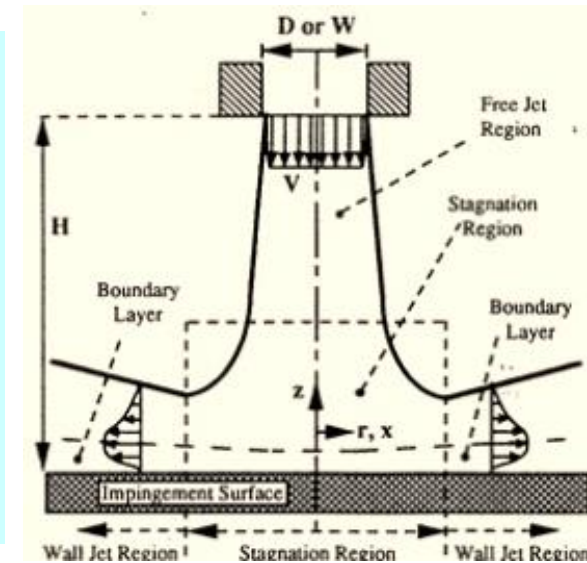
v – Local time average velocity

l – The shortest distance to the wall

Qingyang Chen, Weiran Xu. A zero-equation model for indoor airflow simulation. Energy and Building, 1998, 28, 137-144

10.8.5 $\overline{v^2} - f$ Turbulence model for highly inhomogeneous turbulent flow

For highly inhomogeneous flow and heat transfer, such as jet impingement flow, this $\overline{v^2} - f$ turbulence model may obtain reasonable simulation results.



[1] Durbin PA. Near wall turbulence closure modeling without damping functions. *Theoretical and Computational Fluid Dynamics*, 1991, 3:1-13

[2] Laurence D, Popovac M, and Uribe JC., and Utsyuzhinikov SV. A robust formulation of $\overline{v^2} - f$ model, *Flow, Turbulence and Combustion*, 2004, 73, 169-185

[3] Hanjalic K, Laurence D, Popovac M, and Uribe JC. $\overline{v^2} / k - f$ turbulence model and its applications to forced and natural convections, *Engineering Turbulence Modeling and Experiments*, 2005, 6: 67-86

Home work

9-1

9-2

9-4

9-5

Due on Dec. 8th

Home work

Problem # 9-1

Take the following data to estimate the difference between the fluid thermodynamic pressure and turbulent effective pressure; for the air flow through the wind tunnel, the pressure of the air is 1 bar, the average velocity is $u = 50$ m/s, the temperature of air is 20°C , and turbulence intensity $\sqrt{u'^2} / u = 5\%$, (which is a quite large value). Assumed that the turbulence is isotropic, i.e. various statistical values regardless of the direction of turbulence, here is $\overline{u'^2} = \overline{v'^2} = \overline{w'^2}$.

Problem#9-2

Try to write down the generation term of k-equation in 3-D Cartesian coordinates (See Eq. (9-21)).

Problem#9-4

In a 2-D boundary layer flow ,if the generation of turbulence kinetic energy and the dissipation are balanced each other, try to show:

$$\sqrt{\tau_w / \rho} = C_\mu^{1/4} k^{1/2}$$

Problem # 9-5

The definition of turbulent kinetic energy dissipation rate is $\varepsilon = \nu \overline{\left(\frac{\partial u'_i}{\partial x}\right)^2}$. Try to write the expression of ε in three dimension Cartesian coordinates, and identify its dimension and unit (SI). Then to analyze c_μ, c_1, c_2 are dimensionless numbers or not.

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same boat help
each other to
cross to the other
bank, where....