Lattice Boltzmann for flow and transport phenomena

3. The lattice Boltzmann for multiphase flow

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3.1 Background

Multiphase fluid flows are ubiquitous in natural, scientific and engineering systems.

A phase refers to gas, liquid or solid state of matter. A multiphase flow is the flow of a mixture of phases such as gas (bubbles) in a liquid, or liquid (droplets) in a gas, and so on.

Same component:

Liquid water and water vapor system

Multiple components:

Small density: water and oil system

Large density ratio: water and air system

Crown
Simulation of multiphase flows is a problem encountered often in scientific and industrial applications, and has long been regarded as a challenging task.

**Microscopically**, the interfacial behaviors of multiphase flows result from the interactions among the constituent molecules.

**Conventional method:**
- Level set method
- Volume of Fluid (VOF)
- Additional techniques to track the phase interface

A need for alternative approaches to understand the connection between the macroscopic phenomena and the underlying microscopic dynamics at a much more fundamental level.
Molecular dynamics

Microscopic numerical methods such as molecular dynamics are suitable for capturing the microscopic interactions. However, they are often too computationally demanding to be applied to macroscopic scales of engineering interest.

The lattice Boltzmann method

The LBM can be considered as a mesoscopic method, occupying the middle ground between the microscopic molecular dynamics and macroscopic fluid dynamics. The main strength of the LBM is that it behaves like a solver for the conservation equations such as Navier-Stokes equation in the bulk flow yet its mesoscopic nature provides a viable way to include the microscopic dynamics at the fluid-fluid and fluid-solid interfaces.
3.2 Fundamental definitions

**Saturation**: ratio between the volume of one phase to the total volume of void space

\[ S_i = \frac{V_i}{V_{\text{void}}} \]

**Surface tension**: refers to the tensile force exists at the phase interface separating two fluids, arise due to a mutual attraction between molecules near the interface

*unit*: N/m  *Typical value*: water-air: 0.0725 N/m

**Contact angle**: is a measurement of the surface wettability. The angle of the triple-phase line. *Hydrophilic surface* with angle less than 90, liquid tends to spread. *Hydrophobic surface* with angle higher than 90, liquid tends to form droplet. *Neutral surface* with angle as 90.
Contact angle

\[ \theta > 90 \quad \theta = 90 \quad \theta < 90 \]

Capillary pressure: pressure difference across a phase interface

\[ P_c = P_1 - P_2 = \frac{\sigma \cos \theta}{r} \]
3.3 LB multiphase flow models

Due to its kinetic nature, LB does not need to track the phase interface, where the interface can be maintained automatically. Interface is a post-result of density field.

Several LB multiphase models:

1. **Color-gradient model by Gunstense et al. 1991**

2. **Pseudopotential model by Shan and Chen 1993**

3. **Free energy model by Swift 1995**
The pseudo-potential LB model

Proposed by Shan and Chen in 1993.


To the best of the authors’ knowledge, this model is the most widely used LB multiphase model due to its simplicity and versatility.
The basic idea is to represent the microscopic molecular interactions at the mesoscopic scale using a pseudo-potential (also often called effective mass) depending on the local density.

With such interactions, a single component system or a multicomponent system will spontaneously segregates into two phases if the interaction is properly calculated.

*: Such automatic phase separation is an attractive characteristic of the pseudopotential model, as the phase interface is no longer a mathematical boundary and no explicit interface-tracking or interface-capturing technique is needed.

The phase interface is a post-processed quantity that can be characterized through monitoring the variation of the fluid densities.
3.3.1 The single component pseudopotential LBM

\[ f_{\sigma,\alpha}(x + ce_{\alpha} \Delta t, t + \Delta t) - f_{\sigma,\alpha}(x, t) = -\frac{1}{\tau_{\sigma,\nu}}(f_{\sigma,\alpha}(x, t) - f_{\sigma,\alpha}^{eq}(x, t)) + F_{\sigma,\alpha}(x, t) \]

\[ f_{\sigma,\alpha}^{eq} = \omega_{\alpha} \rho_{\sigma} \left[ 1 + \frac{3}{c^2} (e_{\alpha} \cdot u) + \frac{9}{2c^4} (e_{\alpha} \cdot u)^2 - \frac{3}{2c^2} u^2 \right] \]

\[ \rho_{\sigma} = \sum_{\alpha} f_{\sigma,\alpha} \]

\[ \rho_{\sigma} u_{\sigma} = \sum_{\alpha} f_{\sigma,\alpha} e_{\alpha} \]

\[ \nu_{\sigma} = c_s^2 (\tau_{\sigma,\nu} - 0.5) \Delta t \]

Equation of State

\[ p = \rho c_s^2 \]

Ideal gas!
In order to introduce nonlocal interaction among particles, Shan and Chen defined the force experienced by the particles of component $\sigma$ at $x$ from the particles $\sigma'$ at $x'$ as the following form:

$$F(x, x') = -G(|x - x'|)\psi_\sigma(x)\psi_\sigma(x')(x' - x)$$

$\Psi$ is the effective mass, or the pseudopotential.
\[ F(x, x') = -G(|x - x'|)\psi_\sigma(x)\psi_\sigma(x')(x' - x) \]

\[ F(x', x) = -G(|x' - x|)\psi_\sigma(x)\psi_\sigma(x')(x - x') \]

The most general one that meets the Newton’s third law and conserves momentum globally

Such interaction force does not conserve the local momentum during the collision process because the force behaves as an external force acting on the site. The total momentum of the system is conserved and no net momentum is introduced into the system.
\[ F(x, x') = -G(|x - x'|)\psi_\sigma(x)\psi_\sigma(x')(x' - x) \]

Total force
\[ F(x) = -\psi(x) \sum G(|x - x'|)\psi(x')(x' - x) \]

\[ F(x) = -g\psi(x)c_s^2 \sum_{\alpha=1}^{N} w(|e_\alpha|^2)\psi(x + e_\alpha)e_\alpha \]

Interaction strength
Distance weight
\[ F(x) = -g\psi(x)c_s^2 \sum_{\alpha=1}^{N} w(|e_\alpha|^2)\psi(x + e_\alpha)e_\alpha \]

Three important elements

- The interaction strength, \( g \)
  \[ w(|e_\alpha|^2) = 1/3, \quad |e_\alpha|^2 = 1 \]
- The effective mass, \( \Psi \)
  \[ w(|e_\alpha|^2) = 1/12, \quad |e_\alpha|^2 = 2 \]
- Determination of the neighbor sites, \( N \) and \( w \)

\( \Psi \) in the original work of Shan and Chen is as follows

\[ \psi = \rho_0 (1 - \exp(-\frac{\rho}{\rho_0})) \]

\( \rho \to \infty, \quad \Psi \to 1 \)
\[
F(x) = -g \psi(x) c_s^2 \sum_{\alpha=1}^{N} w(|e_\alpha|^2) \psi(x + e_\alpha) e_\alpha
\]

Taylor expansion

\[
F = -g \left( c_s^2 \psi \nabla \psi + \frac{c_s^4}{2} \psi \nabla (\Delta \psi) + \ldots \right)
\]

Two major ingredients of non-ideal fluids are captured: a non-ideal EOS relating to the first term and the surface tension

\[
p = \rho c_s^2 + \frac{g}{2} c_s^2 \psi^2
\]
Force implementation: velocity shift scheme proposed by Shan and Chen

\[ f_{\sigma,\alpha}(\mathbf{x} + c\mathbf{e}_\alpha \Delta t, t + \Delta t) - f_{\sigma,\alpha}(\mathbf{x}, t) = -\frac{1}{\tau_{\sigma,\nu}} (f_{\sigma,\alpha}(\mathbf{x}, t) - f_{\sigma,\alpha}^{\text{eq}}(\mathbf{x}, t)) \]

\[ \mathbf{u}^{\text{eq}} = \mathbf{u} + \frac{\tau}{\rho} \mathbf{F} \Delta t \]

\[ f_{\sigma,\alpha}^{\text{eq}} = \omega_{\alpha}\rho_{\sigma} \left[ 1 + \frac{3}{c^2} (\mathbf{e}_\alpha \cdot \mathbf{u}^{\text{eq}}) + \frac{9}{2c^4} (\mathbf{e}_\alpha \cdot \mathbf{u}^{\text{eq}})^2 - \frac{3}{2c^2} (\mathbf{u}^{\text{eq}})^2 \right] \]

\[ \mathbf{F}(\mathbf{x}) = -g\psi(\mathbf{x})c_s^2 \sum_{\alpha=1}^{N} w(|\mathbf{e}_\alpha|^2)\psi(\mathbf{x} + \mathbf{e}_\alpha)\mathbf{e}_\alpha \]

\[ \mathbf{u}_p = \mathbf{u} + \frac{1}{2\rho} \mathbf{F} \Delta t \]
Bubble test (Laplace law)

Laplace law

\[ \nabla p = p_{in} - p_{out} = \frac{\sigma}{r} \]
First phase transition

$t=0$, uniform density field

A small disturbance is added: $\delta \rho < 5\%$. 

$19/62$
Droplet coalesce

\[
d
\]
3.3.2 The limitation of SC original model

The original SC model suffers from some limitations.

Some of the limitations are inherent to this particular model and others are common problems shared by other multiphase flow models.

- Relatively large spurious current
- Thermodynamic inconsistency
- Low density and viscosity ratio
- Coupling between EOS and surface tension force
- Dependence of surface tension and density ratio on the viscosity.
Spurious current

The spurious current denotes the non-zero vortex-like fluid velocity in the vicinity of phase interface.

Physically, the droplet and its surrounding gas are at rest and the pressure difference inside and outside the droplet should be balanced by the surface tension force.

Due to the limited discretization in calculating the corresponding gradient, the pressure difference and surface tension are not exactly balanced, leading to the artificial vortex-like velocity.
Thermodynamic inconsistency

Maxwell construction

\[ \int_{v_1}^{v_v} p \, dv = p_s (v_v - v_1) \]

\[ \int_{v_1}^{v_v} (p - p_s) \, dv = 0 \]
\[ p = \rho RT \frac{1 + b \rho/4 + (b \rho/4)^2 - (b \rho/4)^3}{(1 - b \rho/4)^3} - a \rho^2 \]

Maxwell construction

\[ \int_{v_1}^{v_v} p \, dv = p_s (v_v - v_1) \]

\[ \int_{v_1}^{v_v} (p - p_s) \, dv = 0 \]

\[ \int_{\rho_v}^{\rho_1} \left( p_s - \rho c_s^2 - \frac{g}{2} c_s^2 \psi^2 \right) \frac{1}{\rho^2} \, d\rho = 0 \]

\[ \int_{\rho_v}^{\rho_1} \left( p_s - \rho c_s^2 - \frac{g}{2} c_s^2 \psi^2 \right) \frac{\psi'}{\psi} \, d\rho = 0 \]

\[ \psi \propto \exp(-1/\rho) \]

\[ a = 0.4963 (RT_c)^2/p_c, \quad b = 0.1873 RT_c/p_c \]
Thermodynamic inconsistency

\[ \psi \propto \exp(-1/\rho) \]

Any other choices of effective mass will violate the thermodynamic consistency.

\[ \rho_v = \rho_l, \text{临界点} \]
Limited density and viscosity ratio

Original SC model, density ratio \(~10\), viscosity ratio \(~1\)

Coupling between some properties

Density and surface tension cannot be adjusted separately. Dependence of density and surface tension on viscosity.

Remark 1: in real world, the various aspects of a multiphase system, e.g., the strength of the surface tension, the thickness of the interface, the EOS and its associated properties, are all manifestations of a single microscopic origin, i.e., the interaction potential, and therefore are “coupled together” naturally.

Remark 2: however, as a practical numerical tool for simulating engineering problems, it is desirable for those macroscopic properties to be adjustable separately
3.3.3 The improvement of pseudopotential model

Several techniques/schemes/methods have been developed to alleviate the above limitations of the original pseudo-potential model and to improve its performance.

- Incorporating realistic EOS into the model
- Increasing the isotropy order of the interaction force
- Modifying the interaction force
- Improving the force scheme to incorporate the interaction force
Incorporating realistic EOS

Equations of state in a lattice Boltzmann model

P Yuan, L Schaefer - Physics of Fluids, 2006 - aip.scitation.org

In this paper we consider the incorporation of various equations of state into the single-component multiphase lattice Boltzmann model. Several cubic equations of state, including the van der Waals, Redlich-Kwong, and Peng-Robinson, as well as a noncubic equation of state (Carnahan-Starling), are incorporated into the lattice Boltzmann model. The details of phase separation in these nonideal single-component systems are presented by comparing the numerical simulation results in terms of density ratios, spurious currents, and ...

Equations of state in a lattice Boltzmann model

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(Received 4 August 2005; accepted 16 February 2006; published online 3 April 2006)

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[DOI: 10.1063/1.2187070]
vdW EOS

\[ p = \rho c_s^2 + \frac{g}{2} c_s^2 \psi^2 \]

R-K EOS

\[ p = \frac{\rho RT}{1 - b \rho} - a \rho^2 \]

\[ p = \frac{\rho RT}{1 - b \rho} - \frac{a \rho^2}{\sqrt{T(1 + b \rho)}} \]

P-R EOS

\[ p = \frac{\rho RT}{1 - b \rho} - \frac{a \alpha(T) \rho^2}{1 + 2b \rho - b^2 \rho^2} \]

C-S EOS

\[ p = \rho RT \frac{1 + b \rho/4 + (b \rho/4)^2 - (b \rho/4)^3}{(1 - b \rho/4)^3} - a \rho^2 \]
\[ p = \rho c_s^2 + \frac{g}{2} c_s^2 \psi^2 \]

\[ \psi = \sqrt{\frac{2}{g c_s^2}} (p - \rho c_s^2) \]

vdW EOS

\[ p = \frac{\rho RT}{1 - b\rho} - a\rho^2 \]

Pseudo-potential

\[ \psi = \sqrt{\frac{2}{g c_s^2}} \left( \frac{\rho RT}{1 - b\rho} - a\rho^2 - \rho c_s^2 \right) \]
Yuan and Schaefer assessed different EOS in terms of four criteria:

- The maximum density ratio
- The lowest spurious current
- The widest temperature range
- The thermal consistency

\[
\psi = \sqrt{\frac{2}{gc_s^2}} (p - \rho c_s^2)
\]

\[
\psi \propto \exp\left(-\frac{1}{\rho}\right)
\]
Spurious current
Density ratio, lowest temperature and thermal consistency
Density ratio, lowest temperature and thermal consistency
Density ratio, lowest temperature and thermal consistency
High order

Analysis and reduction of the spurious current in a class of multiphase lattice Boltzmann models

X Shan - Physical Review E, 2006 - APS

We show that the spurious current present near a curved interface in a class of multiphase lattice Boltzmann (LB) models is due to the insufficient isotropy of the discrete gradient operator. A method of obtaining highly isotropic gradient operators on a lattice is given. Numerical simulations show that both the magnitude and the spatial extent of the spurious current are significantly reduced as gradient operators of increasingly higher order of isotropy is adopted in multiphase LB models.

PHYSICAL REVIEW E 73, 047701 (2006)

Analysis and reduction of the spurious current in a class of multiphase lattice Boltzmann models

Xiaowen Shan*

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(Received 20 December 2005; published 6 April 2006)

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\[
F(x) = -g \psi(x)c^2_s \sum_{\alpha=1}^{N} w(|e_\alpha|^2)\psi(x + e_\alpha)e_\alpha
\]

\[
w(|e_\alpha|^2) = 1/3, \quad |e_\alpha|^2 = 1
\]

\[
w(|e_\alpha|^2) = 1/12, \quad |e_\alpha|^2 = 2
\]

\[
F_x = -g\psi(i, j)c^2_s \left[ \frac{1}{3} (\psi(i+1, j) - \psi(i-1, j)) + \frac{1}{12} (\psi(i+1, j+1) - \psi(i-1, j+1)) + \frac{1}{12} (\psi(i+1, j-1) - \psi(i-1, j-1)) \right]
\]
\[
F_x = -g \psi(i, j)c_s^2 \left[ \frac{1}{3} (\psi(i+1, j) - \psi(i-1, j)) + \frac{1}{12} (\psi(i+1, j+1) - \psi(i-1, j+1)) + \frac{1}{12} (\psi(i+1, j-1) - \psi(i-1, j-1)) \right]
\]

\[
F_y = -g \psi(i, j)c_s^2 \left[ \frac{1}{3} (\psi(i, j+1) - \psi(i, j-1)) + \frac{1}{12} (\psi(i+1, j+1) - \psi(i+1, j-1)) + \frac{1}{12} (\psi(i-1, j+1) - \psi(i-1, j-1)) \right]
\]
\[ F(x) = -g \psi(x) c_s^2 \sum_{\alpha=1}^{N} w(|e_\alpha|^2) \psi(x + e_\alpha) e_\alpha \]

\[ F = -g \psi(x) c_s^2 \sum_{n=0}^{\infty} \frac{1}{n!} \nabla^{(n)} \psi(x) : E^{(n+1)} \]

\[ E_{i_1 i_2 \ldots i_n}^{(n)} = \sum_{\alpha=1}^{N} w(|e_\alpha|^2)(e_\alpha)_{i_1} \ldots (e_\alpha)_{i_n} \]

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\[
F_x = -g\psi(i, j)c_s^2 \left[ \frac{4}{21}(\psi(i+1, j) - \psi(i-1, j)) + \frac{4}{45}(\psi(i+1, j+1) - \psi(i-1, j+1)) + \\
\frac{4}{45}(\psi(i+1, j-1) - \psi(i-1, j-1)) + \frac{1}{60}(2\psi(i+2, j) - 2\psi(i-2, j)) + \\
\frac{2}{315}(2\psi(i+2, j+1) - 2\psi(i-2, j+1)) + \frac{2}{315}(2\psi(i+2, j-1) - 2\psi(i-2, j-1)) + \\
\frac{2}{315}(\psi(i+1, j+2) - \psi(i-1, j+2)) + \frac{2}{315}(\psi(i+1, j-2) - \psi(i-1, j-2)) + \\
\frac{1}{5040}(2\psi(i+2, j+2) - 2\psi(i-2, j+2)) + \frac{1}{5040}(2\psi(i+2, j-2) - 2\psi(i-2, j-2)) \right]
\]

\[
F_y = -g\psi(i, j)c_s^2 \left[ \frac{4}{21}(\psi(i, j+1) - \psi(i, j-1)) + \frac{4}{45}(\psi(i+1, j+1) - \psi(i+1, j-1)) + \\
\frac{4}{45}(\psi(i-1, j+1) - \psi(i-1, j-1)) + \frac{1}{60}(2\psi(i, j+2) - 2\psi(i, j-2)) + \\
\frac{2}{315}(\psi(i+2, j+1) - \psi(i+2, j-1)) + \frac{2}{315}(\psi(i-2, j+1) - \psi(i-2, j-1)) + \\
\frac{2}{315}(2\psi(i+1, j+2) - 2\psi(i+1, j-2)) + \frac{2}{315}(2\psi(i-1, j+2) - 2\psi(i-1, j-2)) + \\
\frac{1}{5040}(2\psi(i+2, j+2) - 2\psi(i+2, j-2)) + \frac{1}{5040}(2\psi(i-2, j+2) - 2\psi(i-2, j-2)) \right]
\]
Force scheme

\[ f_{\sigma,\alpha}(x + ce_{\alpha}\Delta t, t + \Delta t) - f_{\sigma,\alpha}(x, t) = -\frac{1}{\tau_{\sigma,\nu}}(f_{\sigma,\alpha}(x, t) - f_{\sigma,\alpha}^{eq}(x, t)) + F_{\sigma,\alpha}(x, t) \]

\[ f_{\sigma,\alpha}^{eq} = \omega_{\alpha} \rho_{\sigma} \left[ 1 + \frac{3}{c^2} (e_{\alpha} \cdot u_{eq}) + \frac{9}{2c^4} (e_{\alpha} \cdot u_{eq})^2 - \frac{3}{2c^2} (u_{eq})^2 \right] \]

Velocity shifting scheme

\[ u_{eq} = u + \frac{\tau}{\rho} F \Delta t \]

Exact difference method

\[ F_{\alpha} = f_{\alpha}^{eq}(\rho, u + F \Delta t / \rho) - f_{\alpha}^{eq}(\rho, u) \]

\[ F_{\alpha} = (1 - \frac{1}{2\tau}) \omega_{\alpha} \left( \frac{e_{\alpha} - u_{eq}}{c_s^2} + \frac{e_{\alpha} u_{eq}}{c_s^4} \right) \cdot F \Delta t \]

Guo’s force scheme

\[ u_p = u + \frac{1}{2\rho} F \Delta t \]

\[ u_p = u + \frac{1}{2\rho} F \Delta t \]

\[ u_p = u_{eq} = u + \frac{1}{2\rho} F \Delta t \]
1. For the dependence of the surface tension and density on the kinematic viscosity, i.e., the relaxation time, both the EDM and Guo’s force scheme lead to independence, while in the velocity shift scheme the surface force and density change with the kinetic viscosity;

2. For the stable temperature range, Guo’s force scheme leads to a relatively narrow temperature range. For example, at $\tau=1$ for C-S EOS, the minimum $T_{\text{min}}$ achieved is only $T_{\text{min}} = 0.777T_c$. The velocity shifting scheme and the EDM scheme are demonstrated to perform almost the same.

3. For the thermodynamic coexistence, Guo’s model gives the best agreement. Discrepancy of EDM increases as temperature decreases.
### 3.3.4 Fluid-solid interaction

\[ F_{\text{ads}}(x) = -g_w \psi(x) \sum_{\alpha=1}^{N} w(|e_\alpha|^2) \psi(\rho_w) s(x + e_\alpha) e_\alpha \]

where:
- \( F_{\text{ads}}(x) \) is the force acting on an adsorbed particle
- \( g_w \) is the gravitational constant
- \( \psi(x) \) is a potential function
- \( \rho_w \) is the density of the fluid
- \( s(x) \) is an indicator function that equals 1 for solid nodes and 0 for fluid nodes

\[ s(x) = \begin{cases} 1 & \text{if } x \text{ is a solid node} \\ 0 & \text{if } x \text{ is a fluid node} \end{cases} \]
3.3.5 Multicomponent multiphase model

\[ F_{\sigma\sigma}(x) = -g_{\sigma\sigma}\psi_{\sigma}(x)c_s^2 \sum_{\alpha=1}^{N} w(\| e_\alpha \|^2)\psi_{\sigma}(x + e_\alpha)e_\alpha \]

\[ F_{\sigma}(x) = -g_{\sigma\sigma}\psi_{\sigma}(x)c_s^2 \sum_{\alpha=1}^{N} w(\| e_\alpha \|^2)\psi_{\sigma}(x + e_\alpha)e_\alpha - g_{\sigma\sigma}\psi_{\sigma}(x)c_s^2 \sum_{\alpha=1}^{N} w(\| e_\alpha \|^2)\psi_{\sigma}(x + e_\alpha)e_\alpha \]

\[ F_{\sigma}(x) = -g_{\sigma\sigma}\psi_{\sigma}(x)c_s^2 \sum_{\alpha=1}^{N} w(\| e_\alpha \|^2)\psi_{\sigma}(x + e_\alpha)e_\alpha - g_{\sigma\sigma}\psi_{\sigma}(x)c_s^2 \sum_{\alpha=1}^{N} w(\| e_\alpha \|^2)\psi_{\sigma}(x + e_\alpha)e_\alpha \]
Water/air system
Laplace law
Contact angle
3.4 Application

LB pseudopotential model
Two-phase in PEM fuel cell

(a) Case 1 completely hydrophobic

(b) Case 2 completely hydrophilic

(c) Case 3 (1 denotes hydrophilic region, 2 denotes hydrophobic region)

Mixed wettability
Droplet behaviors under air flow

L Chen, YL He, WQ Tao, P Zelenay, R Mukundan, Q Kang, Pore-scale study of multiphase reactive transport in fibrous electrodes of vanadium redox flow batteries, Electrochimica Acta 248, 425-439;
Boiling

Density distribution

Case 1  Case 2  Case 3  Case 4

A critical review of the pseudopotential multiphase lattice Boltzmann model: Methods and applications

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Abstract

This article presents a critical review of the theory and applications of a multiphase model in the community of the lattice Boltzmann method (LBM). The pseudopotential model proposed by Shan and Chen (1993) [4], which has been successfully applied to a wide range of multiphase flow problems during the past two decades. The first part of the review begins with a description of the LBM and the original pseudopotential model. The distinct features and the limitations of the original model are described in detail. Then various enhancements necessary to improve the pseudopotential model in terms of decreasing the spurious currents, obtaining high density/velocity ratio, reducing thermodynamic inconsistency, unraveling the coupling between surface tension and equations of state (EOS), and unraveling the coupling between viscosity and surface tension, are reviewed. Then the fluid–solid interactions are presented and schemes to obtain different contact angles are discussed. The final section of this part focuses on the multi-component multiphase pseudopotential model. The second part of this review describes fruitful applications of this model to various multiphase flows. Coupling of this model with other models for more complicated multiple physical processes are also introduced in this part.

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