Numerical Heat Transfer

(数值传热学)

Chapter 11 Grid Generation Techniques

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11.1 Treatments of Irregular Domain in FDM,FVM

11.2 Introduction to Body-Fitted Coordinates

11.3 Algebraic Methods for Generating Body-Fitted Coordinates

11.4 PDE Method for Generating Body-Fitted Coordinates

11.5 Control of Grid Distribution

11.6 Transformation and Discretization of Governing Eq. and Boundary Conditions

11.7 SIMPLE Algorithm in Computational Plane

11.8 Post-Process and Examples

11.1 Treatments of Irregular Domain in FDM,FVM

11.1.1 Conventional orthogonal coordinates can not deal with variety of complicated geometries

11.1.2 Methods in FDM,FVM to deal with complicated geometries

1. Structured grid (结构化网格)

1) Domain extension method

2) Special orthogonal coordinates

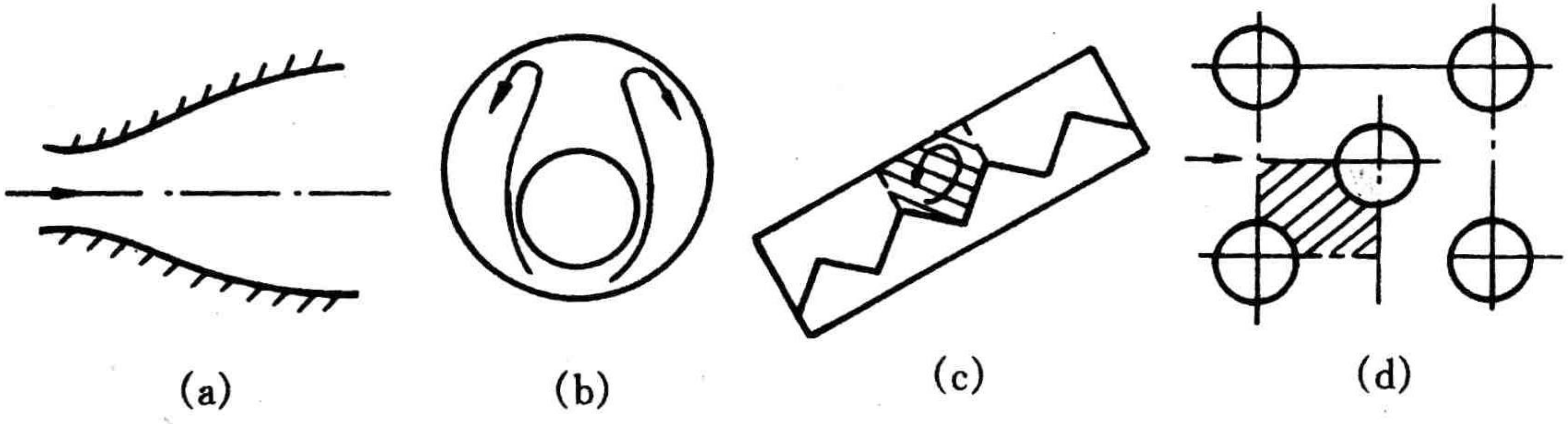
3) Composite grid (组合网格)

4) Body-fitted coordinate (适体坐标系)

2. Unstructured grid (非结构化)

11.1 Treatments of Irregular Domain in FDM,FVM

11.1.1 Conventional orthogonal (正交)coordinates can not deal with variety of complicated geometries



Plane nozzle

Eccentric annulus (偏心圆环)

Solar collector

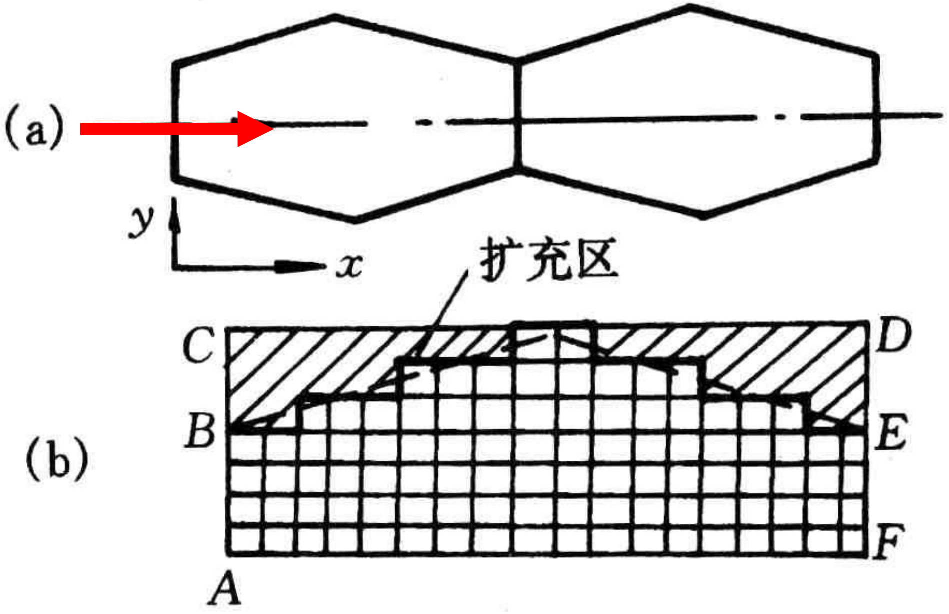
Tube bank

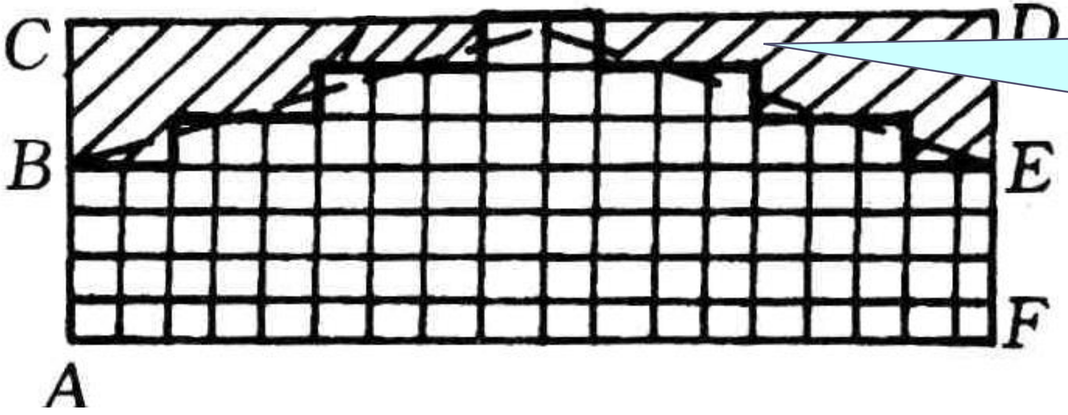
11.1.2 Methods in FDM,FVM to deal with complicated geometries

1. Structured grid (结构化网格)

1) Domain extension method (区域扩充法)

An irregular domain is extended to a regular one, the irregular boundary is replaced by a **step-wise approximation,** and simulation is performed in a conventional coordinate within the extended one.





Extended region

(1) Flow field simulation

(a) Set zero velocity at the boundaries of extended region at B-C-D-E: u=v=0;

(b) Set a very large viscosity in the extended region

$$\eta = 10^{25} \sim 10^{30};$$

(c) Set interface diffusivity by harmonic mean

(2) Temperature field prediction

(a) First kind boundary condition with uniform temperature: The same as for velocity： in the extended region the thermal conductivity is set to e very large, $\lambda = 10^{25} \sim 10^{30}$ and boundary temperatures are given；

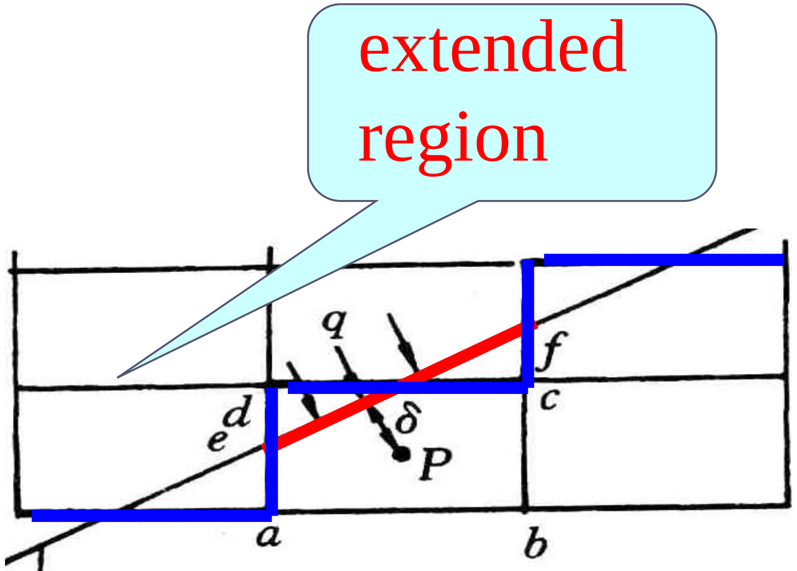
(b) Second kind boundary conditions by ASTM

Specified boundary heat flux distribution (not necessary uniform)

For CV. P adding additional source term:

$$S_{c,ad} = \frac{q \bullet \overline{ef}}{\Delta V_P}$$

And setting zero conductivity for the extended region to avoid heat transfers outward.

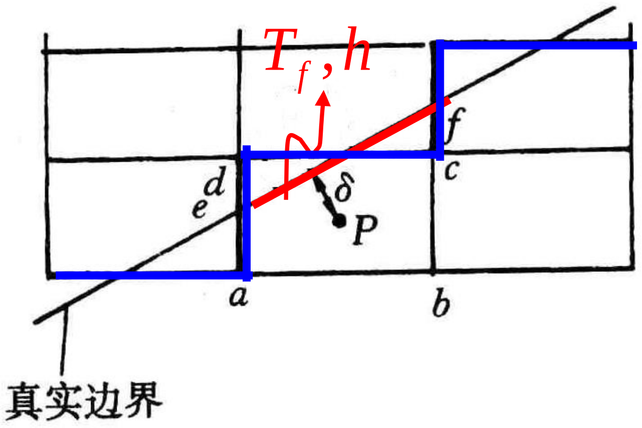


True boundary

(c) Third kind boundary conditions by ASTM

Specified external convective heat transfer coefficient and temperature, h and T_f,

For CV. P following source term is added



$$S_{C,ad} = \frac{\overline{ef}}{\Delta V_P} \frac{T_f}{1/h + \delta/\lambda};$$

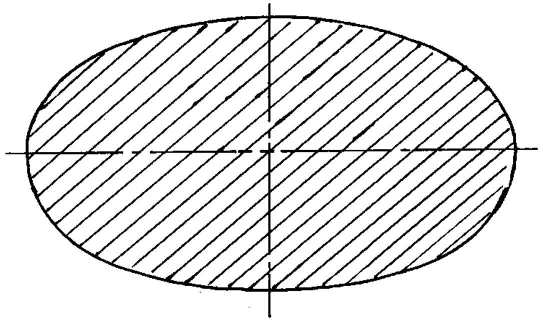
$$S_{P,ad} = -\frac{\overline{ef}}{\Delta V_P} \frac{1}{1/h + \delta/\lambda};$$

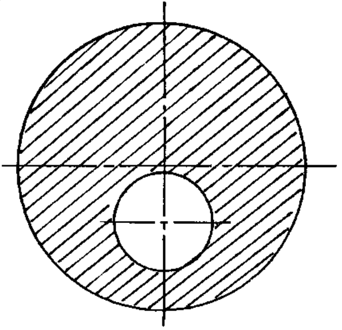
And setting zero conductivity ($\lambda = 0$) for the extended region to avoid heat transfers outward.

For not very complicated geometries, is is a convenient method.

2) Special orthogonal (正交的) coordinates

There are 14 orthogonal coordinates, and they can be used to deal with some irregular regions





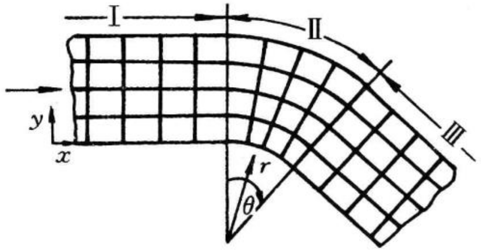
Elliptical coordinate can be used to simulate flow in elliptic tube

Bi-polar coordinate (双极坐标） can be used for flow in a biased annulus(偏心环)

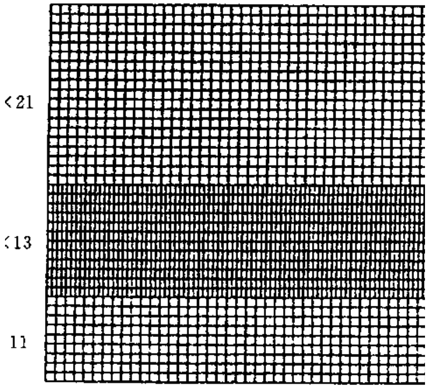
3) Composite coordinate (组合坐标)(block structured)

The entire domain is composed of several blocks, for each block individual coordinate is adopted and solutions are exchanged at the interfaces between different blocks.

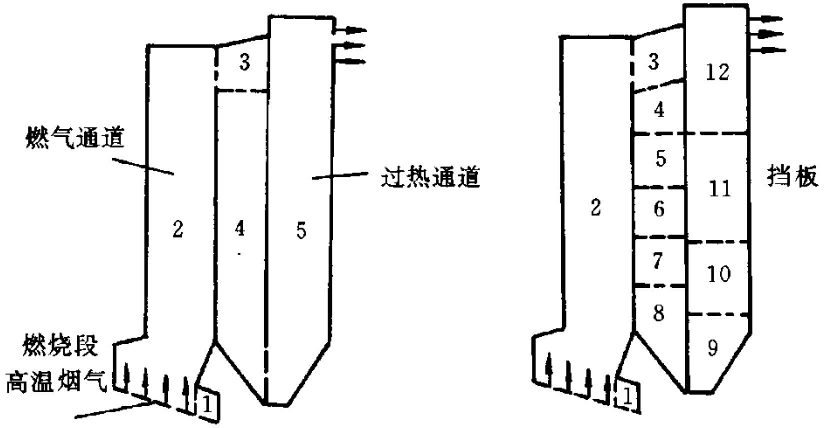
Mathematically it is called domain decomposition method (区域分解法）.



Grid lines are continuous. The entire domain can be solved by ADI.

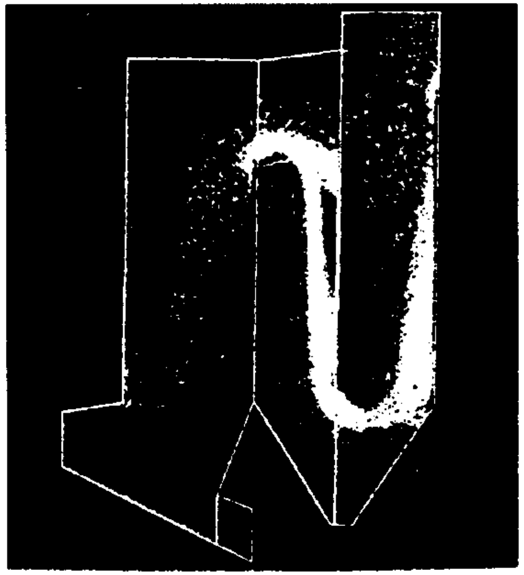


Grid lines are discontinuous



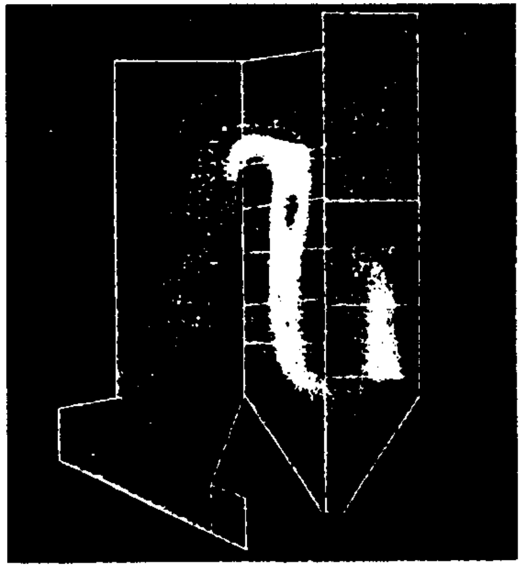
（a）原设计　　（b）改型设计

Application example



（a）原设计

Original design



（b）改型设计

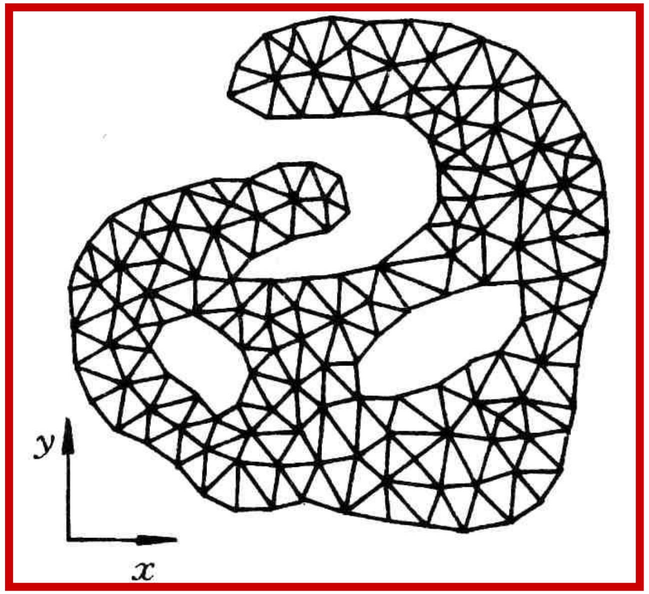
Improved design

4) Body-fitted coordinates (适体坐标)

In such coordinates the coordinates are fitted with(适应) the domain boundaries; The generation of such coordinates by numerical methods is the major concern of this chapter. It was proposed by TTM in Colorado Uni.

2. Unstructured grid (非结构化网格)

There are no fixed rules for the relationship between different nodes, and such relationship should be specially stored for each node. Computationally very expensive. Suitable for very complicated geometries.



11.2 Introduction to Body-Fitted Coordinates

11.2.1 Basic idea for solving physical problems by BFC

11.2.2 Why domain can be simplified by BFC

11.2.3 Methods for generation of BFC

11.2.4 Requirements for grid system constructed by BFC

11.2.5 Basic solution procedure by BFC

11.2 Introduction to Body-Fitted Coordinates

11.2.1 Basic idea for solving physical problems by BFC

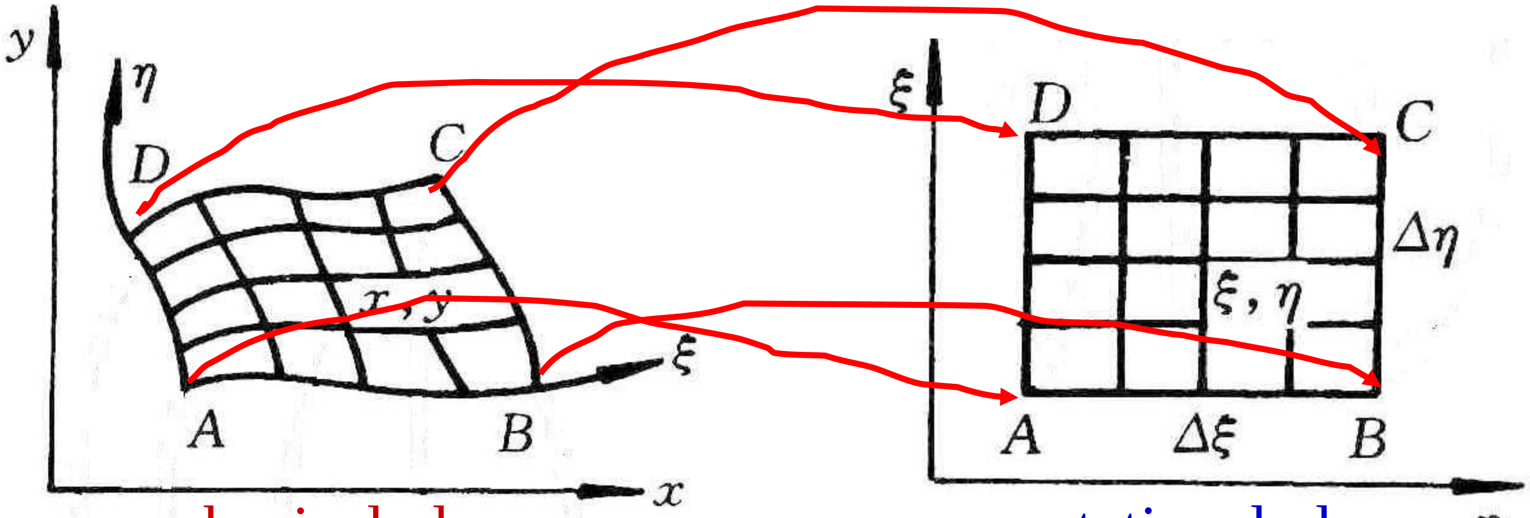
1.In the numerical simulation of physical problems the most ideal coordinate is the one which fits with the boundaries of the studied problem, called body-fitted coordinates(适体坐标系): Cartesian coordinate is the body-fitted one for rectangles, polar coordinate is the one for annular spaces.

2.The existing orthogonal coordinates can not deal with variety of complicated geometries in different fields ; Thus artificially constructed body-fitted coordinates are necessary to meet the different practical requirements.

11.2.2 Why domain can be simplified by BFC

1.Assuming that a BFC has been constructed in Cartesian coordinate *x*-*y*, denoted by $\xi-\eta$;

2.Regarding ξ and η as the two coordinates of a Cartesian coordinate in a imaginery computational plane, then the irregular geometry in physical plane transforms to a rectangle in the computational plane.

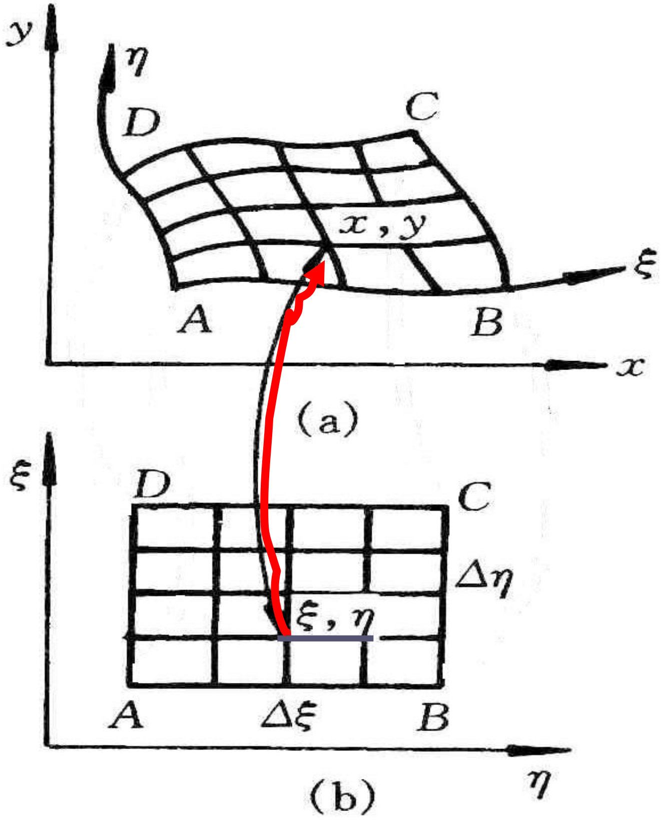


physical plane computational plane

3.The grids in computational plane are always uniformly distributed, thus once grid number is given, the grid system in computational plane can be constructed with ease.

4.Simulation is first conducted in the computational plane , then the converged solution is transferred from the computational plane to physical plane.

In such a way the simulation domain is greatly simplified.



5.In order to transfer solutions from computational domain to physical domain, it is necessary to obtain the corresponding relations of nodes between the two planes.

The so-called grid generation technique herafter refers to the methods by which from (ξ,η) in the computational plane the corresponding (x,y) in the physical Cartesian coordinate can be obtained.

11.2.3 Methods for generation of BFC

1. Conforming mapping (保角变换法)

2. Algebraic method (代数法)

The correspondent relations between grids of two planes are represented by algebraic equations.

3. PDE method(微分方程法)

The relations are obtained through solving a PDE. Three kinds of PDE, hyperbolic, parabolic and elliptic, all can be used to provide such relations.

11.2.4 Requirements for grid system constructed by BFC

1. The nodes in two planes should be one to one correspondent（一一对应）.

2. Grid lines in **physical plane** should be normal to the boundary .

3. The grid spacing in the **physical plane** can be controlled easily.

11.2.5 Procedure of solving problem by BFC

1. Generating grid： find the one to one correspondence between $(\xi,\eta)\leftrightarrow(x,y)$;

2. Transforming governing eqs. and boundary conditions from physical plane to computational plane；

3. Discretizing gov. eq. and solving the ABEqs. in computational plane.

4. Transferring solutions from the computational plane to the physical plane.

11.3 Algebraic Methods for Generating Body-Fitted Coordinates

11.3.1 Boundary normalization（边界规范化）

1. 2-D nozzle

2. Trapezoid enclosure（梯形封闭空腔）

3. Eccentric annular space（偏心圆环）

4. Plane duct with one irregular boundary

11.3.2 Two-boundary method（双边界法）

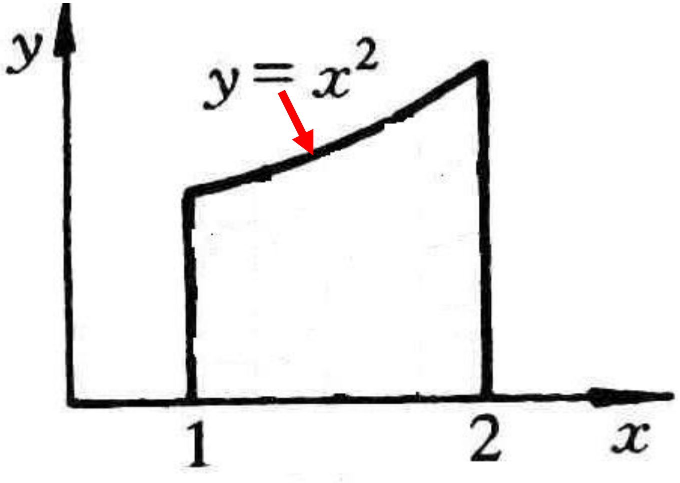
11.3 Algebraic Methods for Generating Body-Fitted Coordinates

11.3.1 Boundary normalization (边界规范化)

1. 2-D nozzle

A plane nozzle is given by following profile

$$y = x^2$$

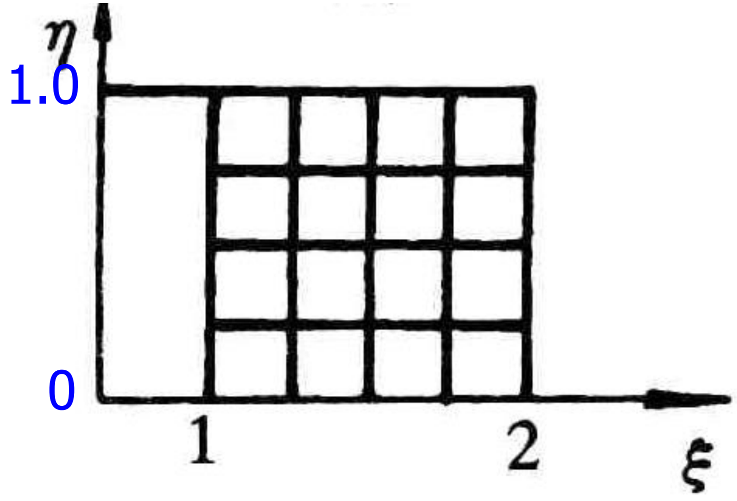


$$\boxed{\xi = x}$$

$$\boxed{\eta = y / y_{\max}}$$

normalization

$$y_{\max} = x^2$$

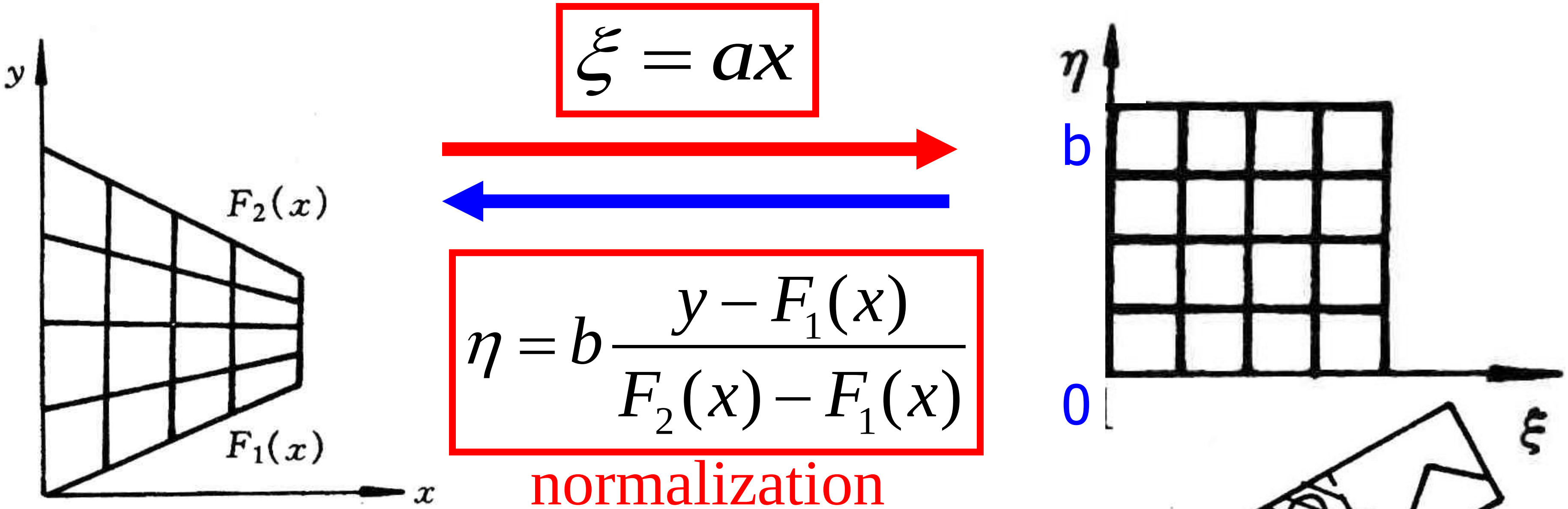


2. Trapezoid (梯形) enclosure

Functions of two tilted boundaries are given by：

$$F_1(x), F_2(x)$$

The grid in the trapezoid enclosure is generated.

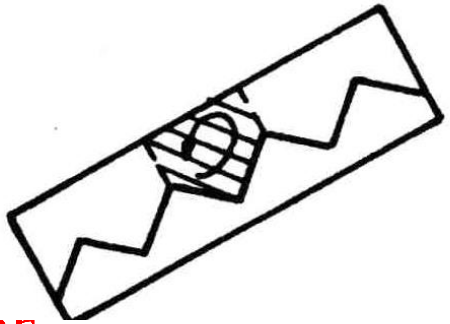


$$\boxed{\xi = ax}$$

$$\boxed{\eta = b\frac{y - F_1(x)}{F_2(x) - F_1(x)}}$$

normalization

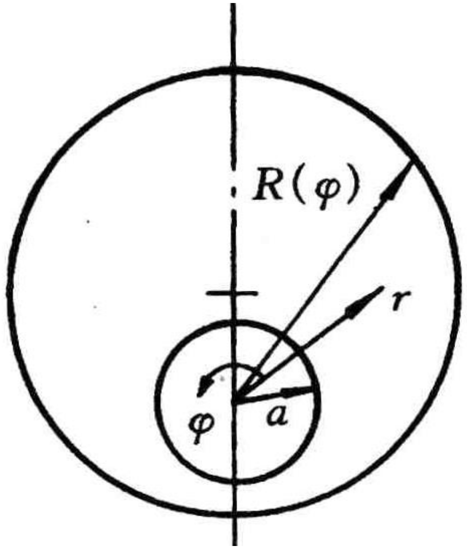
Normalized by the distance between top and bottom



Solar collector

3. Eccentric annular space

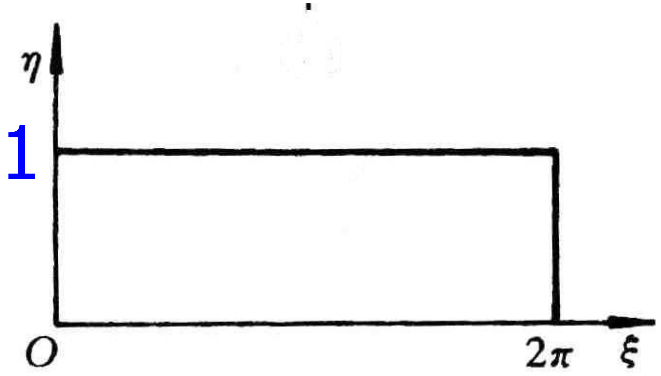
Given two radiuses (R,*a*) and the eccentric distance



$$\boxed{\xi = \varphi}$$

$$\boxed{\eta = \frac{r-a}{R(\varphi)-a}}$$

normalization

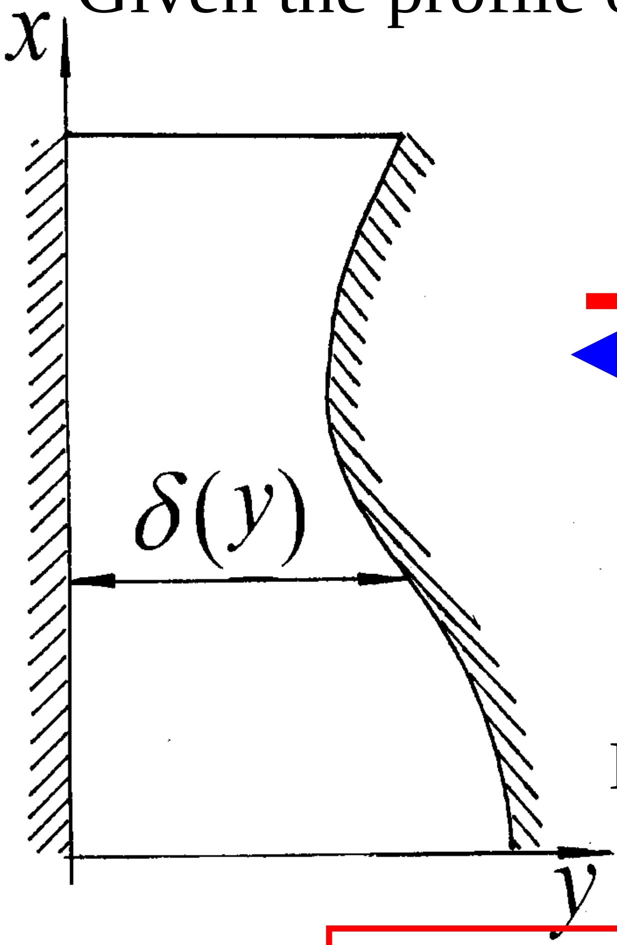


Normalized by the distance between outer and inner circles.

Prusa,Yao, ASME J H T, 1983， 105:105-116

4. Plane duct with one irregular boundary

Given the profile of the irregular boundary $\delta(y)$

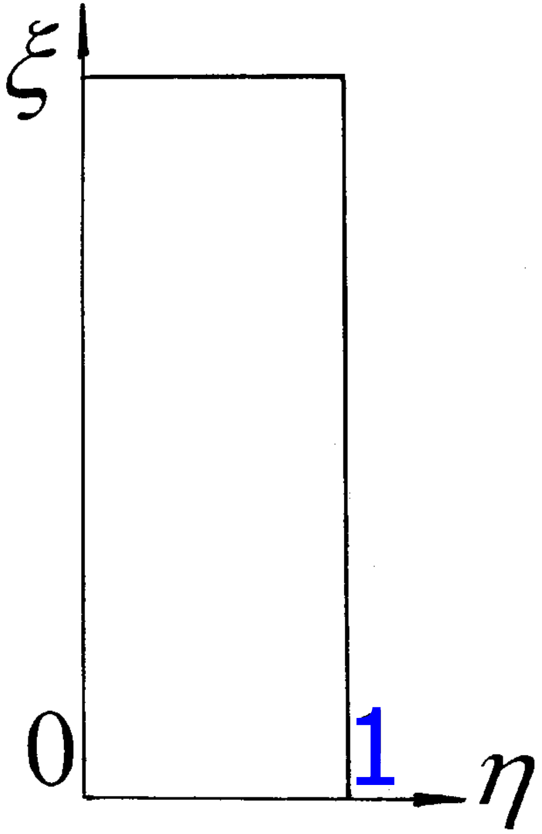


$$\boxed{\xi = x}$$

$$\boxed{\eta = \frac{y}{\delta(x)}}$$

normalization

Normalized by the distance between left and right boundaries



Sparrow-Faghri-Asako, p.479 of Textbook

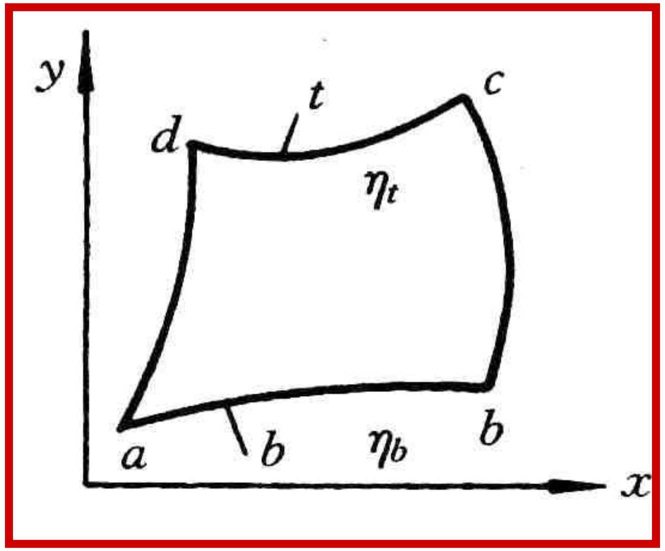
11.3.2 Two-boundary method

1. Method for transforming an irregular quadrilateral (四边形）in physical plane to a rectangle in computational plane.

Implementing procedure:

1) Setting values of η for two opposite (相对的) boundaries:

say: $\eta)_{ab} = \eta_b = 0;\ \eta)_{cd} = \eta_t = 1$



2) Setting the rules of how x,y vary with ξ on the two boundaries:

$$x_b = x_b(\xi), y_b = y_b(\xi)$$

$$x_t = x_t(\xi), y_t = y_t(\xi)$$

3) For any pair of (x,y) and (ξ,η) within the domain taking following interpolations

$$x(\xi,\eta)=x_b(\xi,0)\,[1-f_1(\eta)]+f_1(\eta)\,x_t(\xi,1)$$

$$y(\xi,\eta)=y_b(\xi,0)[1-f_1(\eta)]+f_1(\eta)y_t(\xi,1)$$

where $f_1(\eta)$ must satisfy following two conditions:

$$\eta=0, x(\xi,\eta)=x_b(\xi), y(\xi,\eta)=y_b(\xi)$$

$$\eta=1, x(\xi,\eta)=x_t(\xi), y(\xi,\eta)=y_t(\xi)$$

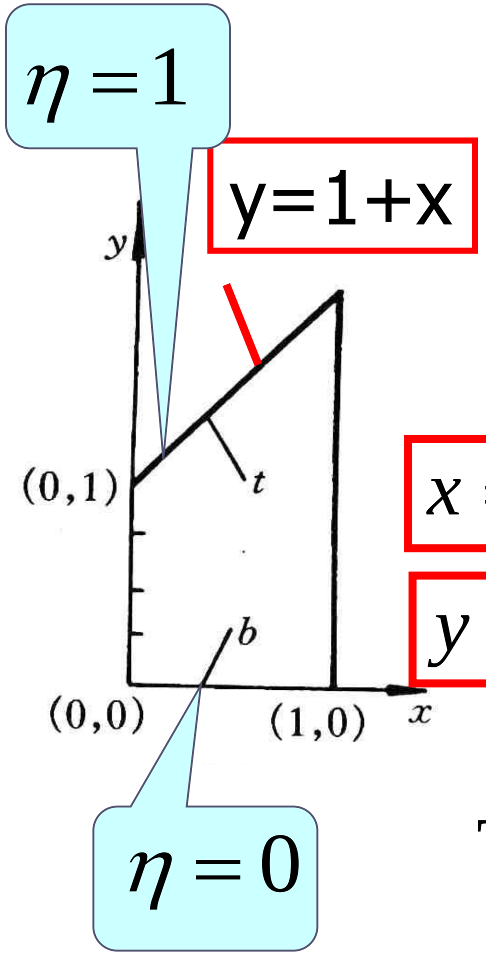
The most simple interpolation which satisfies such conditions is

$$f_1(\eta)=\eta$$

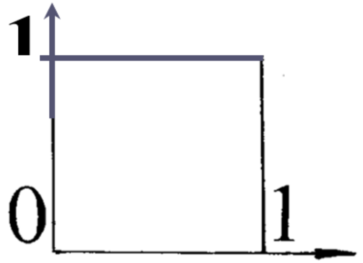
2. Example of two-boundary method

$$x(\xi,\eta)=x_b\bullet(1-\eta)+x_t\bullet\eta$$

$$y(\xi,\eta)=y_b\bullet(1-\eta)+y_t\bullet\eta$$



$$\boxed{x_b=\xi,\ y_b=0;\ x_t=\xi,\ y_t=1+\xi}$$



$$\boxed{x=\xi\bullet(1-\eta)+\xi\bullet\eta=\xi}\Rightarrow\boxed{x=\xi}$$

$$\boxed{y=0\bullet(1-\eta)+(1+\xi)\bullet\eta}\Rightarrow\boxed{y=\eta(1+\xi)}$$

That is: $\begin{cases}\boxed{\xi=x}\\ \boxed{\eta=\dfrac{y}{1+x}}\end{cases}$

The same as that by boundary normalization method.

11.4 PDE Method for Generating Body-Fitted Coordinates

11.4.1 Known conditions and task of grid generation by PDE

11.4.2 Problem set up of grid generation by PDE

1. **Starting from physical plane**

2. **Starting from computational plane**

11.4.3 Procedure of grid generation by solving an Elliptic-PDE

11.4.4 The metric identity should be satisfied

11.4 PDE Method for Generating Body-Fitted Coordinates

11.4.1 Known conditions and task of grid generation by PDE

1. The grid distribution in computational plane is given;

2. The grid arrangement on the physical boundary is given.

Find： the one to one correspondence between (x, y) ,(ξ, η)

i,e: $(x, y) \leftrightarrow (\xi, \eta)$

11.4.2 Problem set up of grid generation by PDE (用微分方程生成网格时问题的提法）

1. Starting from physical plane

Regarding (ξ,η) as two dependent variables to be solved in the physical plane; then above given conditions are equivalent to： Given boundary values of the two dependent variables:

$$\xi_B = f^{\xi}(x_B, y_B), \eta_B = f^{\eta}(x_B, y_B)$$

Find values of (ξ,η) for any inner point (x, y) within the solution region in physical plane.

This is a **boundary value problem (边值问题)** in physical plane. The most simple governing equation is Laplace eq.：

$$\nabla^2\xi = 0;\ \nabla^2\eta = 0 \quad \text{or} \quad \xi_{xx} + \xi_{yy} = 0,\ \eta_{xx} + \eta_{yy} = 0$$

ξ_B, η_B given（i.e., ξ, η of boundary nodes are known)

However, this problem should be solved for a domain in physical plane, which is irregular!Thus we have the same difficulty as for the original problem!

2. Starting from computational plane

Now we regard (x, y) as the dependent variables in computational domain, the above conditions are equivalent to solve a **boundary value problem** in computational domain: with given boundary values of x and y:

$$x_B = f^x(\xi_B,\eta_B), y_B = f^y(\xi_B,\eta_B)$$

it is required to find (x, y) for any inner point (ξ,η) in the computational plane.

This is a boundary value problem in a regular computational domain. This treatment greatly simplify the problem because in computational plane the solution region is either a rectangle or a square.

It should be noted that the boundary value problem in computational domain can not be simply expressed as:

$$x_{\xi\xi} + x_{\eta\eta} = 0; \quad y_{\xi\xi} + y_{\eta\eta} = 0$$

According to mathematical rules the correspondent expressions are:

$$\alpha x_{\xi\xi} - 2\beta x_{\xi\eta} + \gamma x_{\eta\eta} = 0; \qquad \alpha y_{\xi\xi} - 2\beta y_{\xi\eta} + \gamma y_{\eta\eta} = 0$$

$$\alpha = x_{\eta}^{2} + y_{\eta}^{2}; \quad \beta = x_{\xi}x_{\eta} + y_{\xi}y_{\eta}; \quad \gamma = x_{\xi}^{2} + y_{\xi}^{2}$$

where subscript stands for derivative and parameter β represents the orthogonality (正交性) of grid lines in physical plane：its value of two orthogonal lines is zero .

The above two equations are non-orthogonal and non-isotropic diffusion equations.

Thus the essence (本质) of grid generation by PDE is to solve two boundary value diffusion problems in computational domain! The boundary value problems are set up by elliptic partial differential equations.

11.4.3 Procedure of grid generation by solving an elliptic-PDE

1. Determining the number of nodes in physical plane and constructing grid network in computational plane;

2. Setting boundary nodes in physical plane according to given conditions;

3. Solving two boundary value problems in computational plane, by regarding them as non-isotropic and nonlinear diffusion problems with source term.

4. Calculating $x_\xi, x_\eta, y_\xi, y_\eta$ after getting the correspondence between (ξ,η) and (x,y) .

11.4.4 The metric identity should be satisfied

In the transformation of govern. eq. from physical plane to computational plane such kind of derivatives will be introduced.

$$\frac{\partial \phi}{\partial x}=\frac{\partial \phi}{\partial \xi}(\frac{\partial \xi}{\partial x})+\frac{\partial \phi}{\partial \eta}(\frac{\partial \eta}{\partial x})=\phi_\xi \xi_x+\phi_\eta \eta_x=\frac{1}{J}[(\phi y_\eta)_\xi-(\phi y_\xi)_\eta]$$

where: $J=x_\xi y_\eta-x_\eta y_\xi$, called Jakobi factor.

When ϕ is uniform $\frac{\partial \phi}{\partial x}=0,$ thus: $(\phi y_\eta)_\xi=(\phi y_\xi)_\eta$

That is for uniform field: $\boxed{y_{\eta\xi}=y_{\xi\eta}}$

This equation is called metric identity(度规恒等式). In the procedure of grid generation this identity should be satisfied. Otherwise artificial source will be introduced.

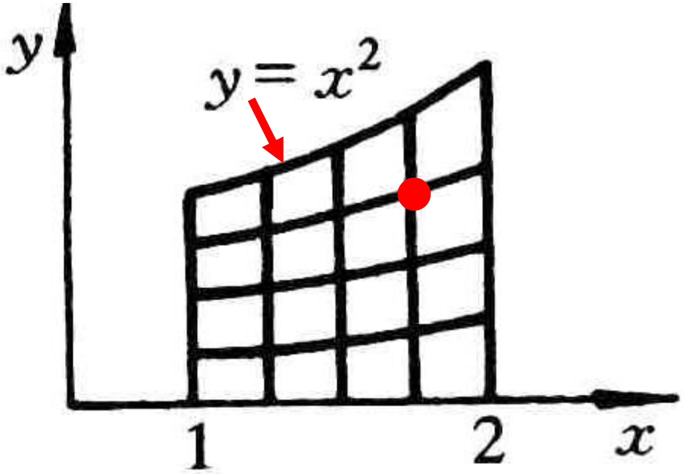
In order to guarantee the satisfaction of metric identity Thompson et al. (TTM) proposed following conditions:

(1) All derivatives with respect to geometric position must be determined by discretized form；

(2) Any such kind of derivative must be computed directly, no interpolation can be used.

Example

[Find] y_ξ, y_η for the position of x=1.75, y=2.2969 in the 2D nozzle problem.



[Calculation] (1) The position of this point (ξ, η) in computational plane is determined:

$$\xi = x = 1.75; \eta = y / y_{\max} = 2.2969 / 1.75^2 = 0.75$$

(2) According to definition:

$$y_\eta = \frac{\partial y}{\partial \eta})_{\xi = cons} = \frac{y(\xi, \eta + \Delta\eta) - y(\xi, \eta - \Delta\eta)}{2\Delta\eta} =$$

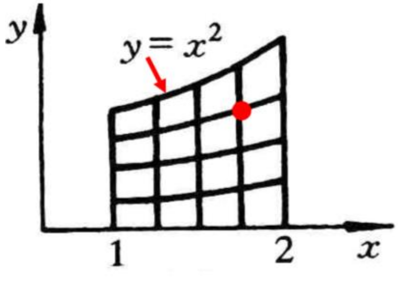
$$\frac{y[1.75,(0.75+0.25)] - y[1.75,(0.75-0.25)]}{2 \times 0.25} =$$

$$\frac{y(1.75,1.0) - y(1.75,0.5)}{0.5} \xrightarrow[\xi = x]{y = \eta x^2}$$

$$\frac{1 \times 1.75^2 - 0.5 \times 1.75^2}{0.5} = 3.0625$$

$$y_\xi = \frac{\partial y}{\partial \xi})_{\eta=cons} = \frac{y(\xi+\Delta\xi,\eta)-y(\xi-\Delta\xi,\eta)}{2\Delta\xi} =$$

$$\frac{y[(1.75+0.25),0.75]-y[(1.75-0.25),0.75]}{2\times 0.25} =$$



$$= \frac{y(2.0,0.75)-y(1.5,0.75)}{0.5} \xrightarrow[\xi = x]{y=\eta x^2}$$

$$= \frac{0.75\times 2.0^2 - 0.75\times 1.5^2}{0.5} = 2.6250$$

$$y_\eta = 3.0625; \quad y_\xi = 2.6250$$

11.5 Control of Grid Distribution

11.5.1 Major features of grid system generated by Laplace equation

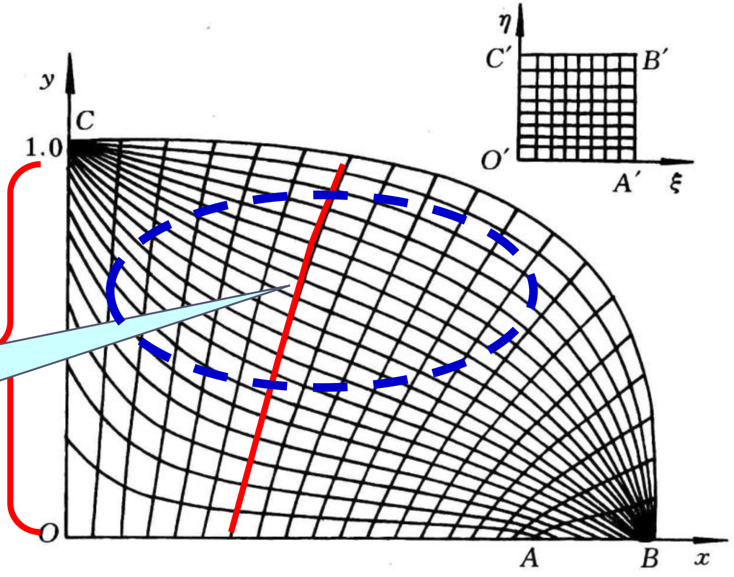
11.5.2 Grid system generated by Poisson equation

11.5.3 Thomas-Middlecoff method for determining P,Q function

11.5 Control of Grid Distribution

11.5.1 Major features of grid system generated by Laplace equation

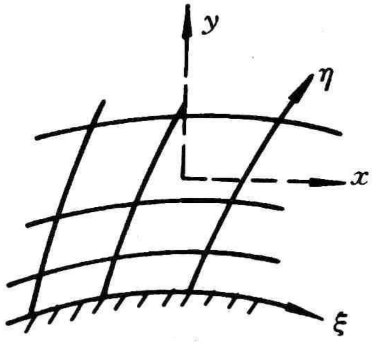
1.The grid distribution along the boundary in physical plane is automatically unified within the solution domain

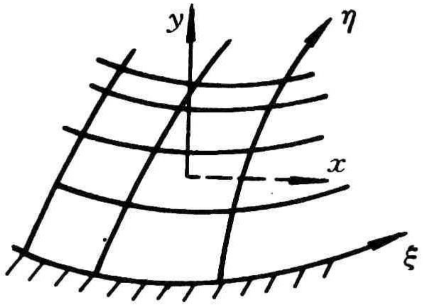


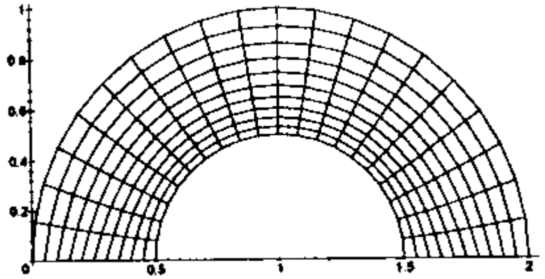
Strongly non-uniform distribution at left boundary

In the domain grid distribution has been unified.

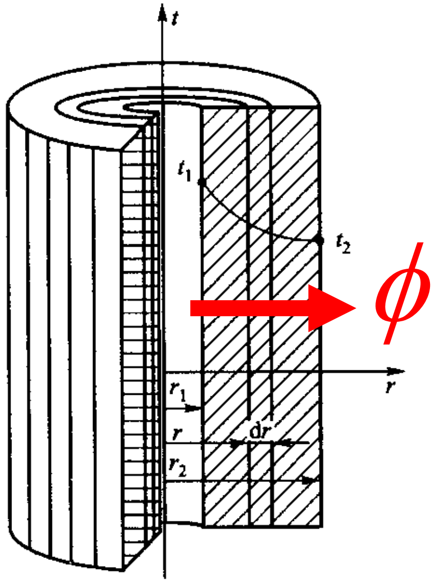
2.Along the normal to a curved wall spacing between grid lines changes automatically.







Such features are inherently related to diffusion process: For steady heat conduction through a cylindrical wall heat flux gradually deceases along radius and spacing between two isothermals increases.



Thus it is needed to develop techniques for controlling grid distribution: grid density and the orthogonality of gridline with boundary.

11.5.2 Grid generation by Poisson equation

1.Heat transfer theory shows that high heat flux leads to dense isothermal（等温线） distribution. If gridlines are regarded as isothermals， then their density can be controlled by heat source. Heat conduction with source term is governed by Poisson equation.

In physical plane Poisson equation is：

$$\nabla^2\xi = P(\xi,\eta)\,; \quad \nabla^2\eta = Q(\xi,\eta)$$

In computational plane, it becomes:

$$\alpha x_{\xi\xi} - 2\beta x_{\xi\eta} + \gamma x_{\eta\eta} = -J^2[P(\xi,\eta)x_\xi + Q(\xi,\eta)x_\eta]$$

$$\alpha y_{\xi\xi} - 2\beta y_{\xi\eta} + \gamma y_{\eta\eta} = -J^2[P(\xi,\eta)y_\xi + Q(\xi,\eta)y_\eta]$$

$$\alpha = x_\eta^2 + y_\eta^2; \quad \beta = x_\xi x_\eta + y_\xi y_\eta; \quad \gamma = x_\xi^2 + y_\xi^2$$

11.5.3 Thomas-Middlecoff method for P,Q

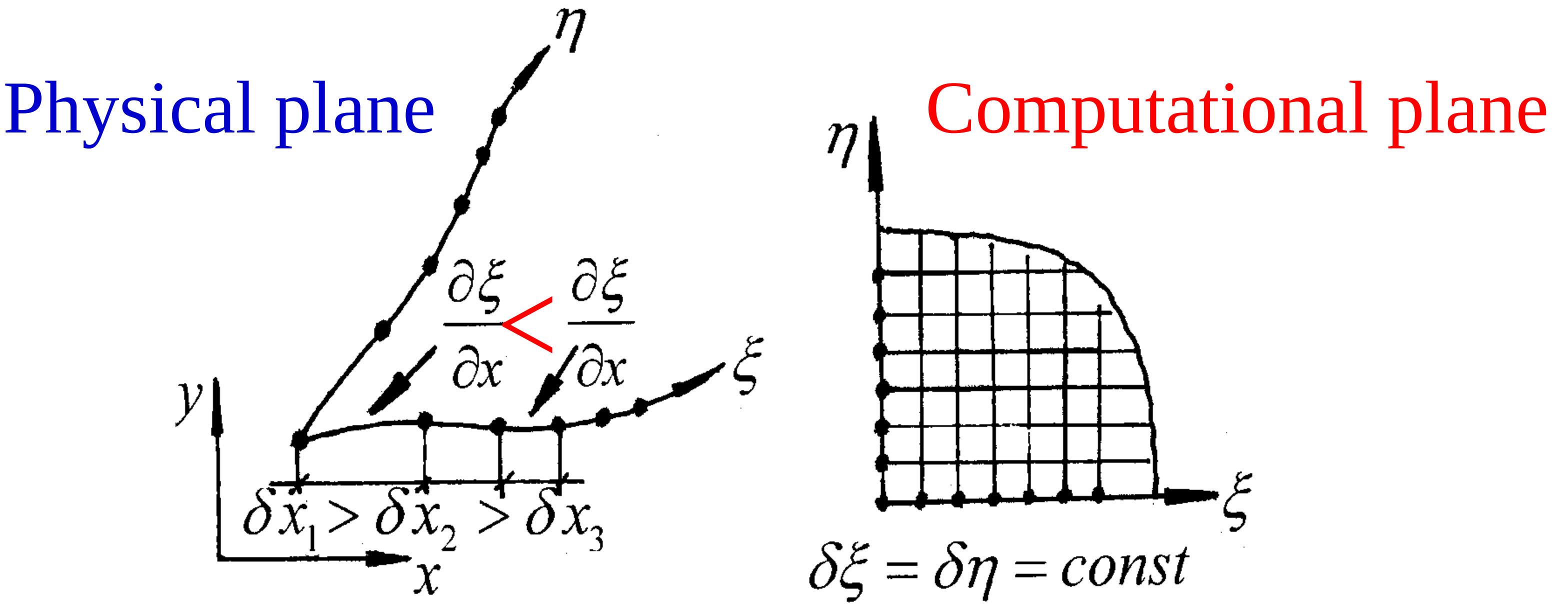
P,Q are source function for controlling density and orthogonality, and can be constructed by different methods. **Thomas－Middlecoff method** is very meaningful and easy to be implemented. Its implementation procedure is introduced as follows .

1.Assuming that

$$P(\xi,\eta)=\underline{\phi(\xi,\eta)}(\underline{\xi_x^2+\xi_y^2});Q(\xi,\eta)=\psi(\xi,\eta)(\eta_x^2+\eta_y^2)$$

Controlling the orthogonality of boundary grid line

Controlling grid density within domain---transmitting the specified density on the boundary to inner region


Physical plane
η
$\frac{\partial \xi}{\partial x} < \frac{\partial \xi}{\partial x}$
y
ξ
$\delta x_1 > \delta x_2 > \delta x_3$
x
Computational plane
η
ξ
$\delta \xi = \delta \eta = const$

Along the ξ coordinate: $\frac{\partial \xi}{\partial x})_1 < \frac{\partial \xi}{\partial x})_2 < \frac{\partial \xi}{\partial x})_3$

The first derivatives of ξ, η with respect to x, y , ξ_x, η_x , in the physical plane reflect the rate of changes. Thus $(\xi_x^2 + \xi_y^2)$ represents grid density distribution!

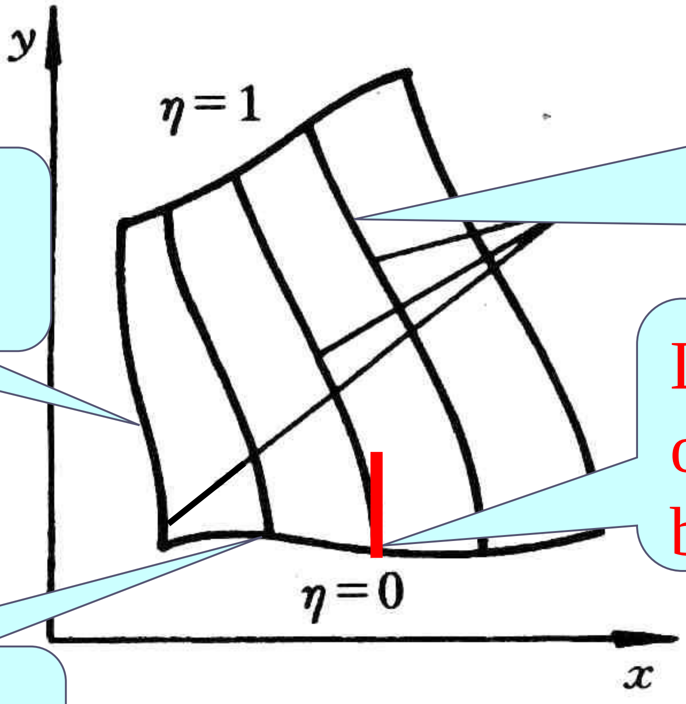
After grid generation, $\xi_x, \xi_y, \eta_x, \eta_y$ are known along the boundary; The key is to determine ϕ, ψ .

2. Ways for determining ϕ and ψ

1) ϕ is first determined for the bottom and top boundaries where η is constant; ψ is first determined for the left and right boundaries where ξ is constant.

The boundary values of ϕ and ψ should satisfy following conditions: the local gridlines are straight and normal to the relative boundary (局部网格线是直线且垂直边界).

2) On the constant ξ lines between bottom and top, the values of ϕ are linearly interpolated with respect to η ; On the constant η lines between left and right boundaries the values of ψ are interpolated linearly with respect to ξ .



ξ = C, **determining** ψ

η = C, **determining** ϕ

On the constant ξ line ϕ is linearly interpolated with respect to η

Locally straight and orthogonal to the boundary

Then our task is to determine ϕ for $\eta=0$ and $\eta=1$; and determine ψ for $\xi=0$ and $\xi=1$.

3. Way for determining ϕ on $\eta=0,\eta=1$

1) Substituting

$$P(\xi,\eta)=\phi(\xi,\eta)(\xi_x^2+\xi_y^2);Q(\xi,\eta)=\psi(\xi,\eta)(\eta_x^2+\eta_y^2)$$

into the Poisson equation in computational plane

$$\alpha x_{\xi\xi}-2\beta x_{\xi\eta}+\gamma x_{\eta\eta}=-J^2[\underline{P(\xi,\eta)}x_\xi+\underline{Q(\xi,\eta)}x_\eta]$$

$$\alpha y_{\xi\xi}-2\beta y_{\xi\eta}+\gamma y_{\eta\eta}=-J^2[\underline{P(\xi,\eta)}y_\xi+\underline{Q(\xi,\eta)}y_\eta]$$

Rewriting above equations in terms of ϕ,ψ , obtaining following two simultaneous equations:

$$\alpha(y_{\xi\xi}+\phi y_\xi)-2\beta y_{\xi\eta}+\gamma(y_{\eta\eta}+\psi y_\eta)=0$$

$$\alpha(x_{\xi\xi}+\phi x_\xi)-2\beta x_{\xi\eta}+\gamma(x_{\eta\eta}+\psi y_\eta)=0$$

2) **Eliminating** ψ from above two equations, obtaining equation of ϕ

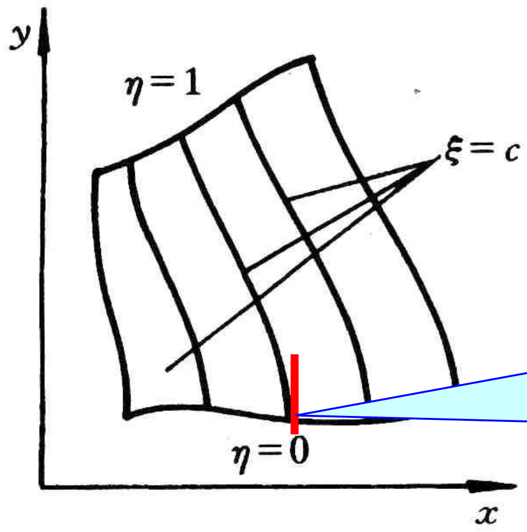
$$\alpha[y_\eta(x_{\xi\xi}+\phi x_\xi)-x_\eta(y_{\xi\xi}+\phi y_\xi)]=$$

$$y_\eta^2[2\beta(x_\eta/y_\eta)_\xi+\gamma\underbrace{(x_{\eta\eta}y_\eta-y_{\eta\eta}x_\eta)/y_\eta^2}_{=(x_\eta/y_\eta)_\eta}]$$

0

Straight and normal

$$=(x_\eta/y_\eta)_\eta$$



Locally straight and normal(局部平直正交）

On the local straight line, we have:

$$\frac{dy}{dx}=const \longrightarrow \frac{dx}{dy}=const \longrightarrow \frac{dx/d\eta}{dy/d\eta}=(x_\eta/y_\eta)=const$$

Thus $$(x_\eta/y_\eta)_\eta=\frac{d}{d\eta}(x_\eta/y_\eta)=\frac{d}{d\eta}(const)\equiv 0$$

3) Summarizing: Local orthogonality leads to $\beta=0$, local straight requires $(x_\eta/y_\eta)_\eta=0$.Thus the right hand side of the above equation equals zero:

$$\alpha[y_\eta(x_{\xi\xi}+\phi x_\xi)-x_\eta(y_{\xi\xi}+\phi y_\xi)]=0$$

Further: $$x_{\xi\xi}+\phi x_\xi=(\frac{x_\eta}{y_\eta})(y_{\xi\xi}+\phi y_\xi)$$

We are now working on the boundary with constant η.

Thus we have no way to calculate x_η / y_η； In order to determine this term following condition is utilized:

From $\beta = x_\xi x_\eta + y_\xi y_\eta = 0 \longrightarrow \dfrac{x_\eta}{y_\eta} = -\dfrac{y_\xi}{x_\xi}$

y_ξ / x_ξ can be computed on the line of $\eta = \text{cons tan t}$

Thus substituting into： $x_{\xi\xi} + \phi x_\xi = \left(\dfrac{x_\eta}{y_\eta}\right)(y_{\xi\xi} + \phi y_\xi) \longrightarrow$

$$x_{\xi\xi} + \phi x_\xi = -\left(\frac{y_\xi}{x_\xi}\right)(y_{\xi\xi} + \phi y_\xi) \longrightarrow$$

$$x_\xi (x_{\xi\xi} + \underline{\phi x_\xi}) = -y_\xi (y_{\xi\xi} + \underline{\phi y_\xi})$$

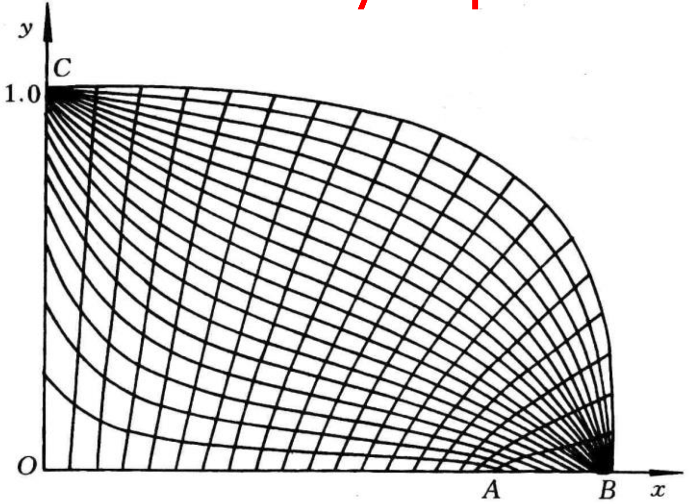
Finally：$\phi = -\dfrac{y_\xi y_{\xi\xi} + x_\xi x_{\xi\xi}}{x_\xi^2 + y_\xi^2}$ （on $\eta = 0, \eta = 1$ boundaries）

Similarly： $\psi=-\frac{y_\eta y_{\eta\eta}+x_\eta x_{\eta\eta}}{x_\eta^2+y_\eta^2}$ (On $\xi=0,\xi=1$ boundaries）

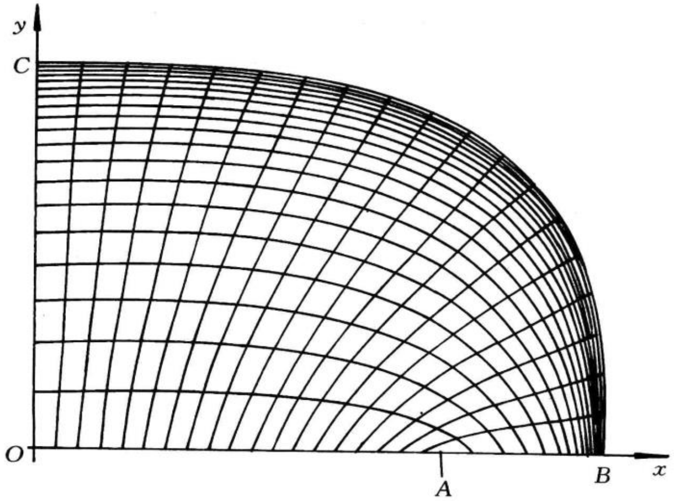
Application example of Thomas－Middlecoff method

Thomas－Middlecoff method for determining source functions of P,Q is a good example of creative numerical method proposed by non-mathematicians!

Generated by Laplace eq.



Poisson eq.＋T-M method



11.6 Transformation and Discretization of Governing Eq. and Boundary Conditions

11.6.1 Transformation of Governing Equation

11.6.2 Transformation of Boundary Conditions

11.6.3 Discretization in computational plane

11.6 Transformation and Discretization of Governing Eq. and Boundary Conditions

11.6.1 Transformation of Governing Equation

1.Mathematical tools used for transformation

1)Chain rule for composite function（复合函数链导法）

$$u(x,y)=u(x(\xi,\eta),y(\xi,\eta)) \quad v(x,y)=v(x(\xi,\eta),y(\xi,\eta))$$

$$\begin{vmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y} \\ \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} \dfrac{\partial u}{\partial \xi} & \dfrac{\partial u}{\partial \eta} \\ \dfrac{\partial v}{\partial \xi} & \dfrac{\partial v}{\partial \eta} \end{vmatrix} \times \begin{vmatrix} \dfrac{\partial \xi}{\partial x} & \dfrac{\partial \xi}{\partial y} \\ \dfrac{\partial \eta}{\partial x} & \dfrac{\partial \eta}{\partial y} \end{vmatrix}$$

yielding：$\dfrac{\partial u}{\partial x}=\dfrac{\partial u}{\partial \xi}\dfrac{\partial \xi}{\partial x}+\dfrac{\partial u}{\partial \eta}\dfrac{\partial \eta}{\partial x}$

2) Derivatives of function and its inverse function(反函数)

$\xi(x,y),\eta(x,y)$ are the inverse function of $x(\xi,\eta),y(\xi,\eta)$

Their derivatives have following relation：

$$\xi_x=\frac{1}{J}y_\eta;\ \eta_x=-\frac{1}{J}y_\xi;\xi_y=-\frac{1}{J}x_\eta;\eta_y=\frac{1}{J}x_\xi$$

2.Results of transformation of 2-D diffusion-convection equation in physical Cartesian coordinate

$$\frac{\partial(\rho u\phi)}{\partial x}+\frac{\partial(\rho v\phi)}{\partial y}=\frac{\partial}{\partial x}(\Gamma_\phi\frac{\partial\phi}{\partial x})+\frac{\partial}{\partial y}(\Gamma_\phi\frac{\partial\phi}{\partial y})+R_\phi(x,y)$$

Results：

$$\underline{\frac{1}{J}\frac{\partial}{\partial\xi}(\rho U\phi)+\frac{1}{J}\frac{\partial}{\partial\eta}(\rho V\phi)}=\underline{\frac{1}{J}\frac{\partial}{\partial\xi}[(\frac{\Gamma_\phi}{J}(\alpha\phi_\xi-\beta\phi_\eta)]}+$$

$$\underline{\frac{1}{J}\frac{\partial}{\partial\eta}[\frac{\Gamma_\phi}{J}(-\beta\phi_\xi+\gamma\phi_\eta)]}+\underline{S_\phi(\xi,\eta)}$$

3. Explanation for results

1) Velocity *U*, *V*: $U = uy_\eta - vx_\eta, V = vx_\xi - uy_\xi$

U, *V* are velocities in ξ, η direction respectively in comput. plane, called **contravariant velocity** (逆变速度);

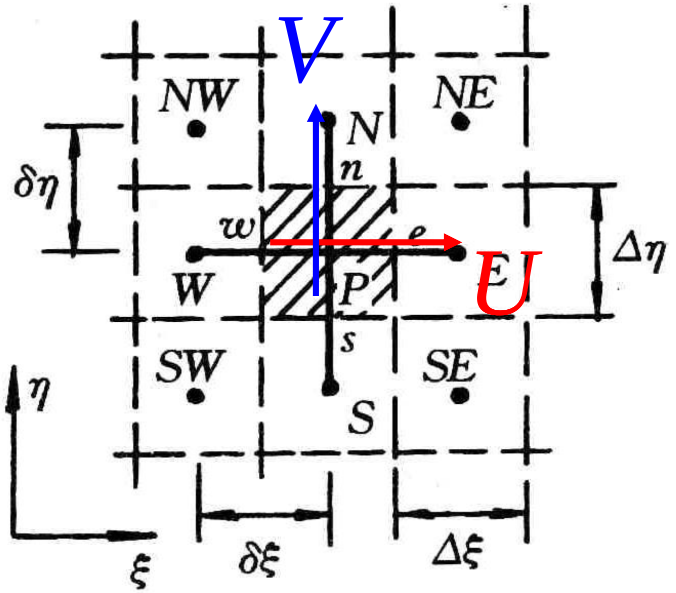
2) *J*: Jakobi factor, representing variation of volume during transformation:

$$\underbrace{dV}_{\text{Physical space volume}} = J \, \underline{d\xi d\eta d\varsigma}$$

Physical space volume

Computational. space volume

Factor of volume change: Larger than 1 means volume in computational space is reduced.

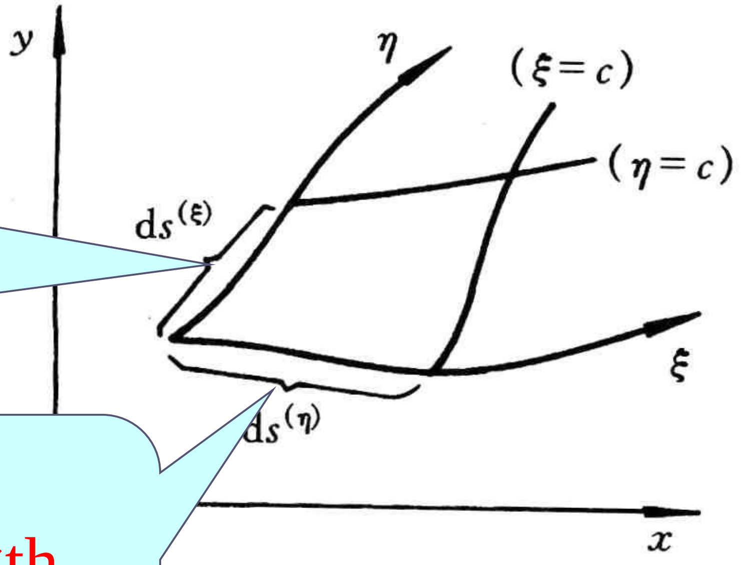


3) α,γ are metric (度规） coefficients in η,ξ direction

$\sqrt{\alpha},\sqrt{\gamma}$ are called Lame coefficient in η,ξ direction, respectively.

$$ds^{(\xi)} = \sqrt{\alpha}\,d\eta$$

is a differential arc length in curve with constant ξ



$$ds^{(\eta)} = \sqrt{\gamma}\,d\xi$$

is a differential arc length in curve with constant η

4) β represents local orthogonality

11.6.2 Transformation of boundary condition

1.Uniform expression of B.C. in physical plane

A,B,C are given constants：

A = 0: second kind

B = 0: first kind

$$A\phi + B\Gamma_{\phi}\frac{\partial \phi}{\partial n} = C$$

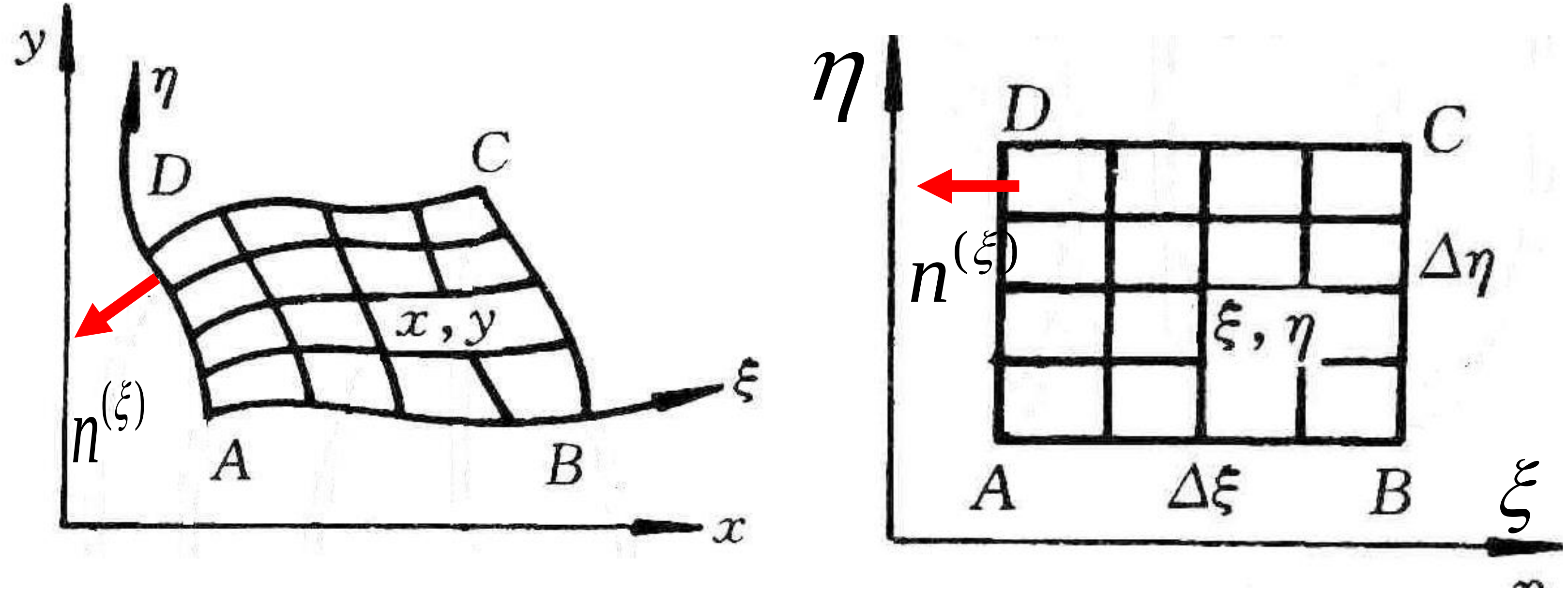
A，B are not zero: 3rd kind boundary condition

During the transformation from physical plane to computational plane

(1)The values of physical variables at correspondent positions remain unchanged

(2)Physical properties /constant remain unchanged.

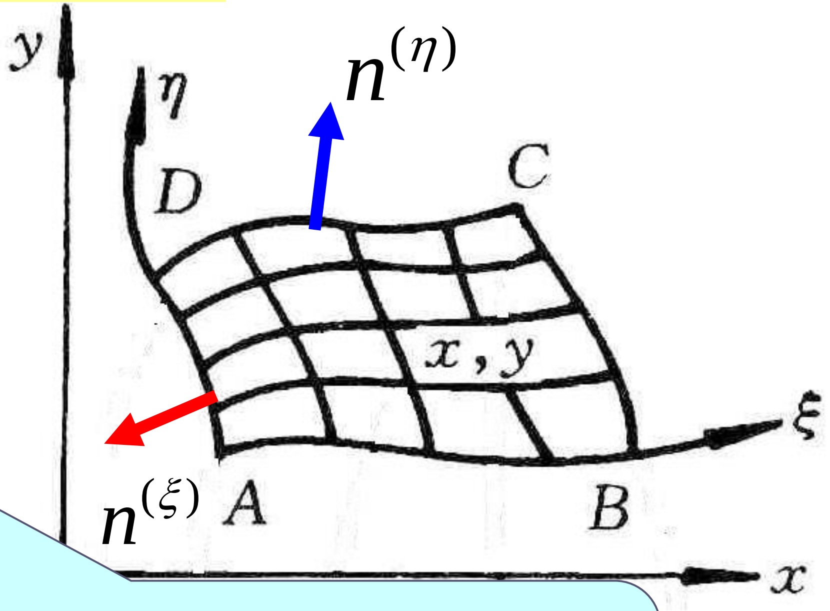
What different is the derivative normal to a boundary in physical plane and in computational plane:



$$\left[\frac{\partial\phi}{\partial n^{(\xi)}}\right]_{Phy} \neq \left[\frac{\partial\phi}{\partial n^{(\xi)}}\right]_{Comp}$$

It can be shown that

$$\begin{cases}\dfrac{\partial\phi}{\partial n^{(\xi)}} = \dfrac{\alpha\phi_\xi - \beta\phi_\eta}{J\sqrt{\alpha}}; \\ \dfrac{\partial\phi}{\partial n^{(\eta)}} = \dfrac{\gamma\phi_\eta - \beta\phi_\xi}{J\sqrt{\varphi}}\end{cases}$$



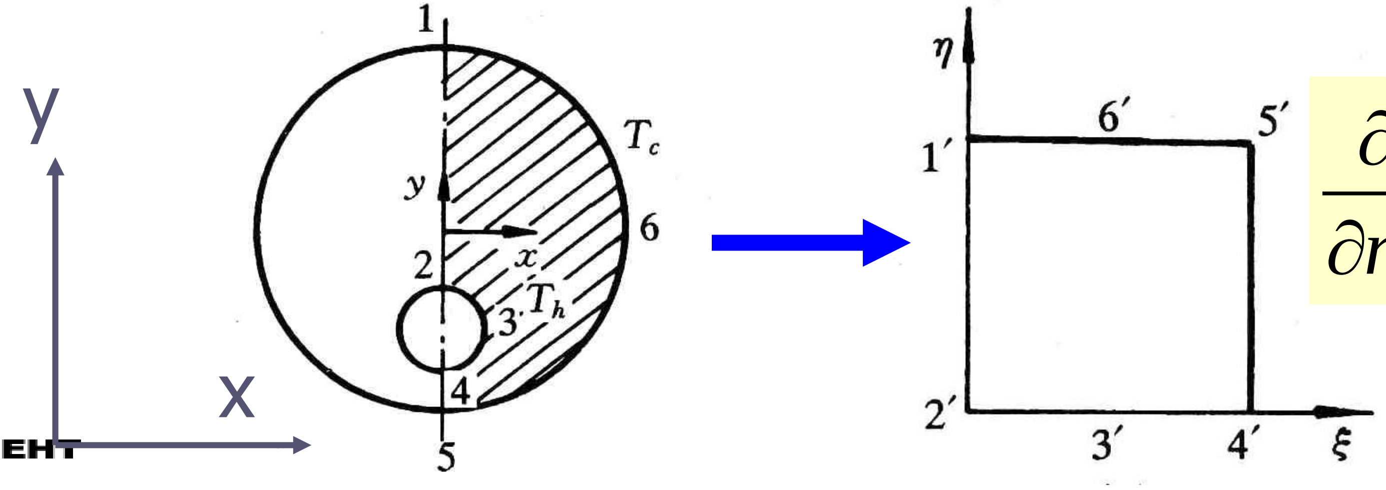
Boundary normal derivative in physical space

ϕ_ξ and ϕ_η are boundary normal derivatives in computational space

Boundary normal derivative in physical space is not equal to boundary normal derivative in computational space.

Example of boundary condition transformation

Boundary	Condition -Physical	Condition-Computational
1-2	$u=0,\dfrac{\partial v}{\partial x}=\dfrac{\partial T}{\partial x}=0$	$u=0;\alpha v_{\xi}-\beta v_{\eta}=\alpha T_{\xi}-\beta T_{\eta}=0$
2-3-4	$u=v=0,T=T_{h}$	$u=v=0,T=T_{h}$
4-5	$u=0,\dfrac{\partial v}{\partial x}=\dfrac{\partial T}{\partial x}=0$	$u=0;\alpha v_{\xi}-\beta v_{\eta}=\alpha T_{\xi}-\beta T_{\eta}=0$
5-6-1	$u=v=0,T=T_{c}$	$u=v=0,T=T_{c}$

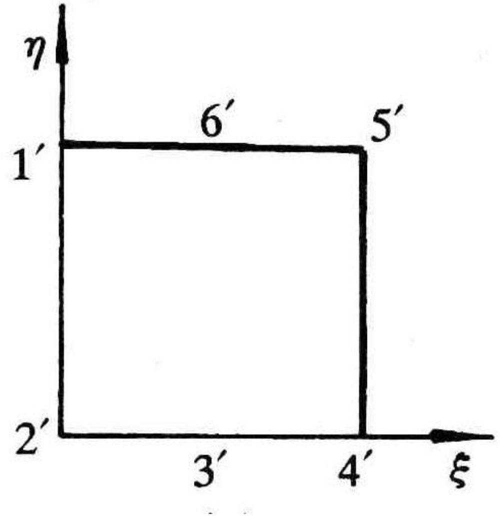


$$\frac{\partial \phi}{\partial n^{(\xi)}}=\frac{\alpha \phi_{\xi}-\beta \phi_{\eta}}{J\sqrt{\alpha}};$$

Implementation of boundary condition at 1’-2’

$$\alpha T_\xi - \beta T_\eta = 0 \longrightarrow T_\xi = \frac{\beta T_\eta}{\alpha}$$

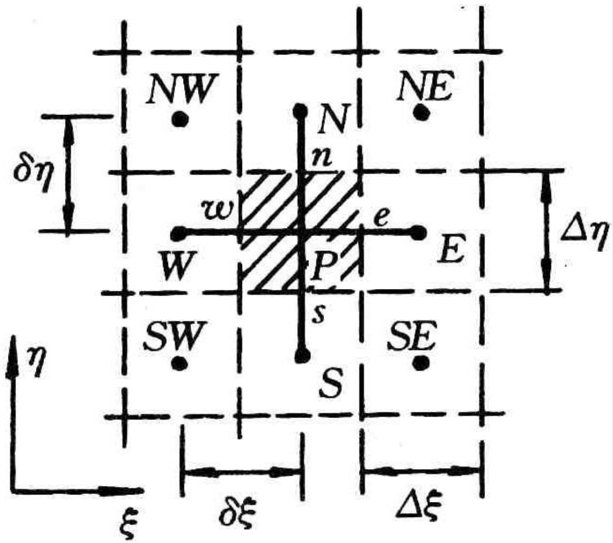
The discretization of   will be shown later.



11.6.3 Discretization in computational plane

1.Discretization of G.E.

Multiplying two sides of the Gov.Eqs. by J， and integrating it over a CV at staggered grid system:



$$[(\rho U\phi)_e-(\rho U\phi)_w]\Delta\eta+[(\rho V\phi)_n-(\rho V\phi)_s]\Delta\xi=$$

$$[\frac{\Gamma_\phi}{J}(\alpha\phi_\xi-\underline{\beta\phi_\eta})]_e\Delta\eta-[\frac{\Gamma_\phi}{J}(\alpha\phi_\xi-\beta\phi_\eta)]_w\Delta\eta+$$

$$[\frac{\Gamma_\phi}{J}(-\beta\phi_\xi+\gamma\phi_\eta)]_n\Delta\xi-[\frac{\Gamma_\phi}{J}(-\beta\phi_\xi+\gamma\phi_\eta)]_s\Delta\xi+S\bullet J\bullet\Delta\eta\bullet\Delta\xi$$

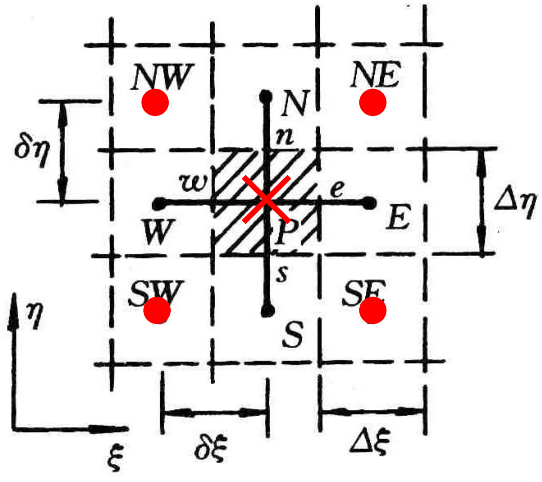
Note： Cross derivatives(交叉导数) occurs in diffusion terms.

2) Discretization of convective term –the same as in physical space.

3) Cross derivatives in diffusion term

Say: $$(\phi_\eta)_e=\frac{(\phi_N+\phi_{NE})-(\phi_S+\phi_{SE})}{4\Delta\eta}$$

leading to 9-point scheme of 2-D case.



Putting the cross derivatives into source term, obtaining following results:

$$a_P\phi_P = a_E\phi_E + a_W\phi_W + a_S\phi_S + a_N\phi_N + b$$

$$b = S_\phi J\Delta\eta\Delta\xi - [(\underline{\frac{\Gamma_\phi}{J}\beta\phi_\eta})_w^e\Delta\eta + (\underline{\frac{\Gamma_\phi}{J}\beta\phi_\xi})_s^n\Delta\xi]$$

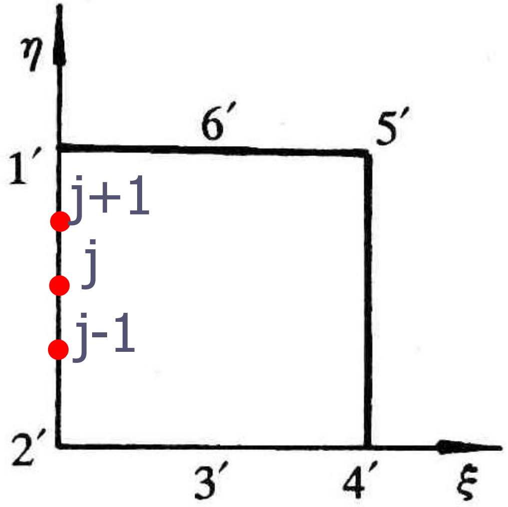
The pressure gradient term is temporary included in S_ϕ .

4.Discretization of boundary condition

The key is boundary derivative，

As shown in the above example:

$$T_\xi = \frac{\beta T_\eta}{\alpha} \longrightarrow (T_\xi)_j = \frac{\beta}{\alpha}\frac{T_{B(j+1)} - T_{B(j-1)}}{2\Delta\eta}$$



11.7 SIMPLE Algorithm in Computational Plane

11.7.1 Choice of velocity in computational space

11.7.2 Discretized momentum equation in computational plane

11.7.3 Velocity correction in computational plane

11.7.4 Pressure correction equation in computational plane

11.7.5 Solution procedure of SIMPLE in computational plane

11.7 SIMPLE Algorithm in Computational Plane

11.7.1 Choice of velocity in computational space

1. Three kinds of velocity

1) Components in physical plane (u, v)

2) Contravariant velocity (U, V) (逆变分量)

$$U = uy_\eta - vx_\eta, \quad V = vx_\xi - uy_\xi$$

3) Covariant velocity $(\overline{U}, \overline{V})$ (协变分量)

$$\overline{U} = ux_\xi + vy_\xi, \quad \overline{V} = ux_\eta + vy_\eta$$

All the three kinds of velocity were adopted in refs.

According to W. Shyy（史维）： following combination can satisfy the conservation condition the best: taking u, v as solution variables and U, V as the velocity in computational plane. We will take this practice.

11.7.2 Discretized momentum equation in computational plane

1.Separating pressure gradient from source term

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial p}{\partial \eta}\frac{\partial \eta}{\partial x} = \frac{1}{J}\left(\frac{\partial p}{\partial \xi} y_\eta - \frac{\partial p}{\partial \eta} y_\xi\right) = \frac{1}{J}(p_\xi y_\eta - p_\eta y_\xi)$$

Note： cross derivatives occur.

2. Discretized momentum equation in physical plane

$$a_e u_e = \sum a_{nb} u_{nb} + b - \Delta y \bullet \underline{\delta x} \left(\frac{p_E - p_P}{\underline{\delta x}} \right) = \sum a_{nb} u_{nb} + b - \Delta y \bullet \delta x \bullet \underline{p_x}$$

$$u_e = \sum \left(\frac{a_{nb}}{\underline{a_e}} \right) u_{nb} + \left(\frac{-\Delta y \bullet \delta x}{\underline{a_e}} \right) p_x + \left(\frac{b}{\underline{a_e}} \right)$$

Subscript here denotes derivative

3. Discretized u,v equations in computational plane

Mimicking the above form for *u*,*v* in physical plane for computational plane following form is taken:

$$u_P = \sum A_{nb}^u u_{nb} + (B^u p_\xi + C^u p_\eta) + D^u$$

$$v_P = \sum A_{nb}^v v_{nb} + (B^v p_\xi + C^v p_\eta) + D^v$$

1) (u_P, v_P) are the velocities at respective locations of staggered grid.

2) A,B,C,D are coefficients and constants generated during discretization.

11.7.3 Velocity correction in computational plane

1. u’,v’ equations in computational plane

From assumed p^*, yielding u^*, v^*:

$$u_P^* = \sum A_{nb}^u u_{nb}^* + (B^u p_\xi^* + C^u p_\eta^*) + D^u$$

$$v_P^* = \sum A_{nb}^v v_{nb}^* + (B^v p_\xi^* + C^v p_\eta^*) + D^v$$

The correspondent U^*,V^* may not satisfy mass conservation, and improvement of pressure is needed.

Denoting pressure correction by p', and the correspondent velocity corrections by u',v';

According to the SIMPLE practice，(p*+p’), (u*+u’), and (v*+v’) also satisfy momentum equation：

$$(u_P^* + u_P^{'}) = \sum A_{nb}(u_{nb}^* + u_{nb}^{'}) + [B^u(p_\xi^* + p_\xi^{'}) + C^u(p_\eta^* + p_\eta^{'})] + D^u$$

$$u_P^* = \sum A_{nb}^u u_{nb}^* + (B^u p_\xi^* + C^u p_\eta^*) + D^u$$

Subtraction of the two equations：

$$u_P^{'} = \sum \cancelto{0}{A_{nb}^u u_{nb}^{'}} + B^u p_\xi^{'} + C^u p_\eta^{'}$$

Similarly $$v_P^{'} = \sum \cancelto{0}{A_{nb}^v v_{nb}^{'}} + B^v p_\xi^{'} + C^v p_\eta^{'}$$

Omitting the effects of neighboring nodes：

yielding velocity correction： $$\left\{\begin{array}{l} u_P^{'} = B^u p_\xi^{'} + C^u p_\eta^{'} \\ v_P^{'} = B^v p_\xi^{'} + C^v p_\eta^{'} \end{array}\right.$$

2. U’,V’ equations in computational plane（20191126）

By definition： $U = uy_\eta - vx_\eta, \quad V = vx_\xi - uy_\xi$

Thus $U' = u'y_\eta - v'x_\eta = y_\eta(\underline{B^u p'_\xi} + \underline{C^u p'_\eta}) - x_\eta(\underline{B^v p'_\xi} + \underline{C^v p'_\eta})$

$$U'_P = \underline{p'_\xi}(B^u y_\eta - B^v x_\eta) + \underline{p'_\eta}\overbrace{(C^u y_\eta - C^v x_\eta)}^{0}$$

New assumption ：cross derivatives in contravariant velocity are neglected

Thus： $U'_P = p'_\xi\underline{(B^u y_\eta - B^v x_\eta)} = (Bp'_\xi)_{U_P} \quad , B = B^u y_\eta - B^v x_\eta$

Similarly： $V'_P = p'_\eta(C^v x_\xi - C^u y_\xi) = (Cp'_\eta)_{V_P}$

At location of V_P

At location of U_P

11.7.4 Pressure correction equation in computational plane

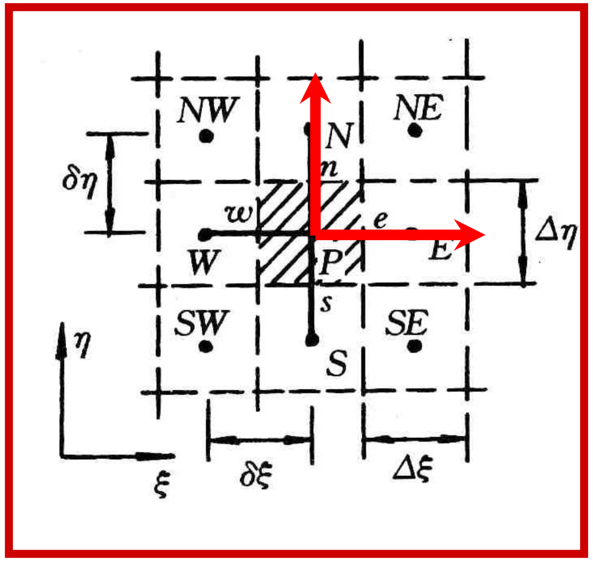
1. Discretized mass conservation in computational plane

From mass conservation in physical plane:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Its correspondent form in computational plane can be obtained:

$$\frac{\partial U}{\partial \xi}+\frac{\partial V}{\partial \eta}=0$$



Integrating over control volume P

$$(\rho U\Delta\eta)_e-(\rho U\Delta\eta)_w+(\rho V\Delta\xi)_n-(\rho V\Delta\xi)_s=0$$

2. Pressure correction equation in computational plane

Substituting $(U^*+U^{'}),(V^*+V^{'}),U^{'}=Bp_{\xi}^{'},V^{'}=Bp_{\eta}^{'}$

into mass conservation eq., and re-writing in terms of *p*’：

$$A_P p_P^{'} = A_E p_E^{'} + A_W p_W^{'} + A_N p_N^{'} + A_S p_S^{'} + b$$

$$b = (\rho U^* \Delta\eta)_e - (\rho U^* \Delta\eta)_w + (\rho V^* \Delta\xi)_n - (\rho V^* \Delta\xi)_s$$

$$A_E = (\rho B \frac{\Delta\eta}{\delta\xi})_e, \quad A_W = (\rho B \frac{\Delta\eta}{\delta\xi})_w, A_N = (\rho C \frac{\Delta\xi}{\delta\eta})_n, \quad A_S = (\rho C \frac{\Delta\xi}{\delta\eta})_s$$

3. Boundary condition of pressure correction equation

Homogeneous Neumann condition：

boundary coefficient = 0

11.7.5 Solution procedure of SIMPLE in computational plane

1. Assuming velocity field of u,v ,calculating *U*,*V* by definition and discretization coefficients；

2. Assuming pressure field *p** and solving for (u_P^*, v_P^*)

3. From u^*, v^* calculating (U_P^*, V_P^*) by definition；

4. Solving pressure correction eq., yielding *p*’；

5. Determining revised velocities

$$u_P = u_P^* + \underline{(B^u p'_\xi + C^u p'_\eta)} \longrightarrow \boxed{u'_P = B^u p'_\xi + C^u p'_\eta}$$

$$v_P = v_P^* + \underline{(B^v p'_\xi + C^v p'_\eta)} \longrightarrow \boxed{v'_P = B^v p'_\xi + C^v p'_\eta}$$

$$U_P = U_P^* + \underline{(B^u y_\eta + C^u x_\eta) p'_\xi} \longrightarrow \boxed{U'_P = p'_\xi (B^u y_\eta - B^v x_\eta)}$$

$$V_P = V_P^* + \underline{(C^v x_\xi + C^v y_\xi) p'_\eta} \longrightarrow \boxed{V'_P = p'_\eta (C^v x_\xi - C^u y_\xi)}$$

$$p = p^* + \alpha_p p'$$

6. Starting next iteration with improved velocity and pressure.

11.8 Post-Process and Examples

11.8.1 Data reduction should be conducted in physical plane

11.8.2 Examples

1. Example 1—Natural convection in a circle with hexagon (六边形)
2. Example 2—Forced flow over a bank of tilted (倾斜） plates
3. Example 3—Periodic forced convection in a duct with roughness elements
4. Example 4—Periodic forced convection in a wavy channel

11.8 Post-Process and Examples

11.8.1 Data reduction should be conducted in physical plane

Data reduction (post process, 后处理) should be conducted for the solutions in the physical plane.

The results in the computational plane can not be directly adopted for data reduction by using definition in physical plane.

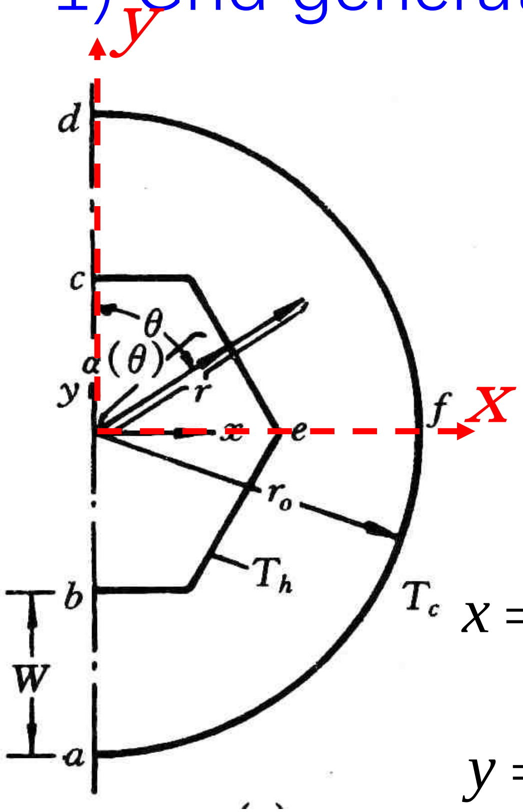
For example，the volume of a control volume is：

$\Delta V = Jd\xi d\eta d\varsigma$ rather than $d\xi d\eta d\varsigma$

11.8.2 Four examples

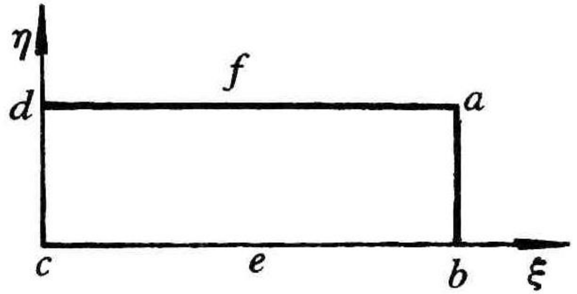
1. Example 1一Natural convection in a circle with an inner hexagon(六边形）

1) Grid generation－algebraic method



$$\xi = \theta$$ (Polar coordinate)

$$\eta = \frac{r - a(\theta)}{r_0 - a(\theta)}$$



(Cartesian coordinate)

$$x = [a(\xi) + \eta[r_0 - a(\xi)]\cos(\frac{\pi}{2} - \xi)$$

$$y = [a(\xi) + \eta[r_0 - a(\xi)]\sin(\frac{\pi}{2} - \xi)$$

2) Local Nusselt on inner surface

$$Nu_i = \frac{h_i W}{\lambda} = \frac{W}{\lambda}[-\lambda(\frac{\partial T}{\partial n})_i \frac{1}{T_h - T_c}] = -[\frac{\partial(\frac{T-T_c}{T_h-T_c})}{\partial(\frac{n}{W})^{(\eta)}}]_i = -[\frac{\partial \Theta}{\partial n^{(\eta)}}]_i = -[\frac{\gamma\Theta_\eta - \beta\Theta_\xi}{J\sqrt{\gamma}}]_i$$

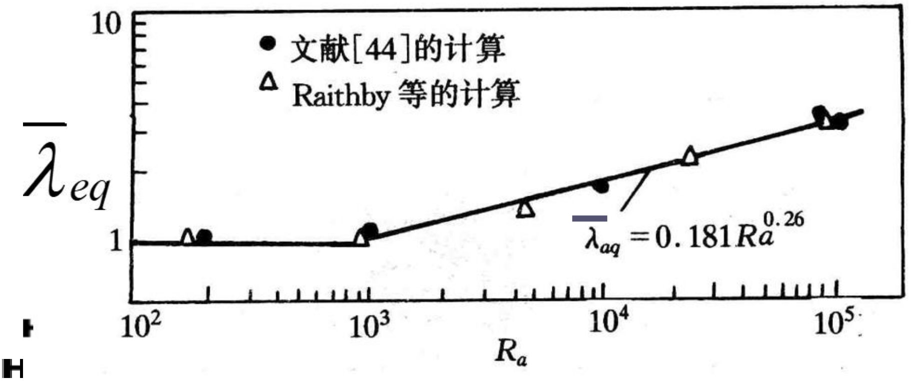
On inner surface $\eta = 0, \Theta)_{\eta=0} \equiv 1$

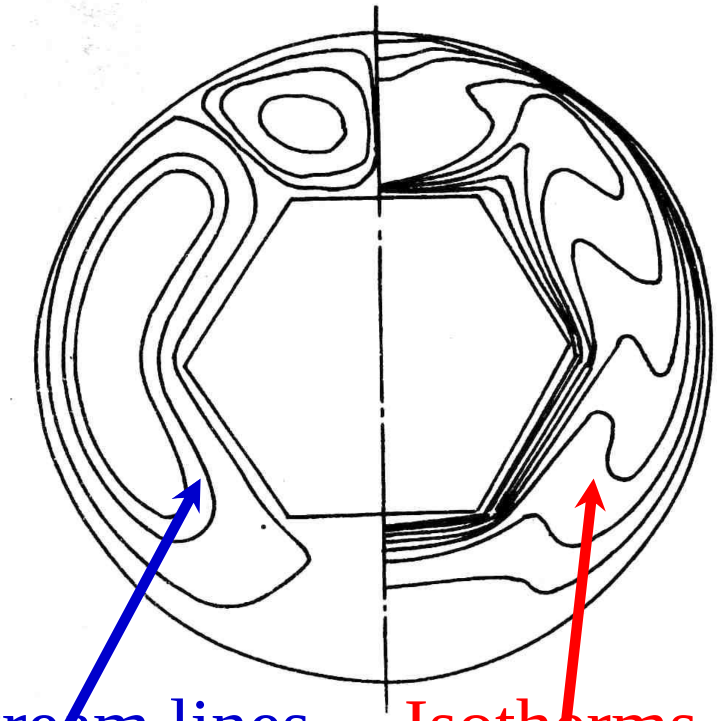
$$\Theta_\xi = \frac{\partial \Theta}{\partial \xi})_{\eta=0} = 0 \longrightarrow \boxed{Nu_i = -(\frac{\gamma\Theta_\eta}{J\sqrt{\gamma}})_i}$$

The averaged Nusselt number can be obtained by integration of Nu_i over the inner surface.

Ra= 9.2×10^4

3) Partial results



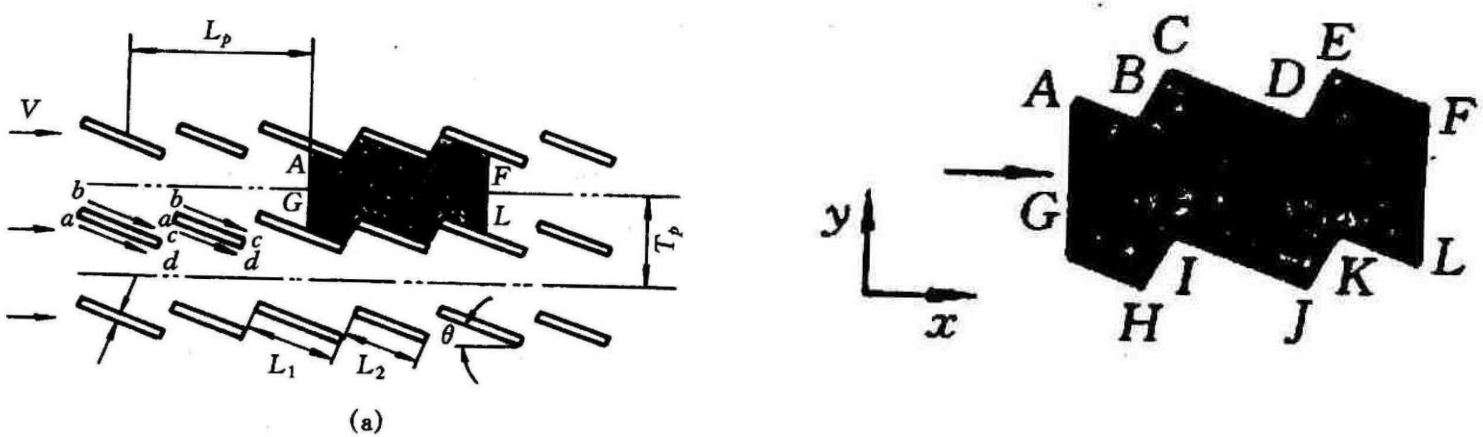


Stream lines   Isotherms

Zhang H L et al. Journal of Thermal Science, 1992, 1(4):249-258

2. Example 2—Forced flow over a bank of tilted plates

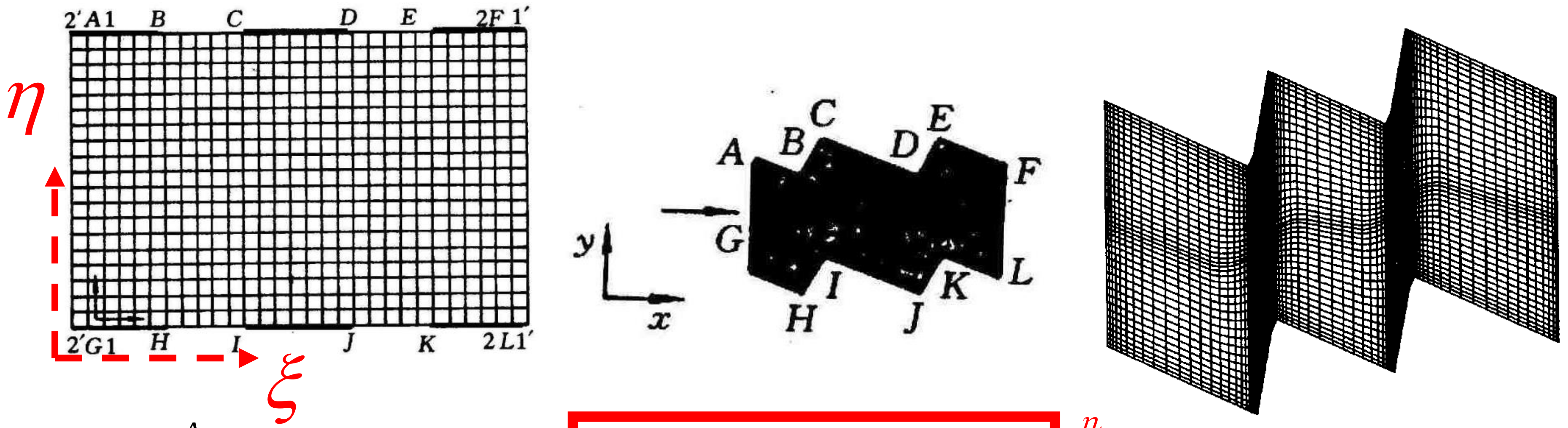
1) Grid generation – algebraic method



Data reduction is conducted for one cycle:

A-G-H-I-J-K-L-F-E-D-C-B-A

2) Data reduction procedure



$$T_b)_{\overline{AG}} = \frac{\int_G^A T(x,y)u(x,y)dy}{\int_G^A u(x,y)dy} \xrightarrow{\boxed{dy = ds^{(\xi)} = d\eta\sqrt{\alpha}}} \frac{\int_{\eta_b}^{\eta_t} T(\xi,\eta)u(\xi,\eta)\sqrt{\alpha}d\eta}{\int_{\eta_b}^{\eta_t} u(\xi,\eta)\sqrt{\alpha}d\eta}$$

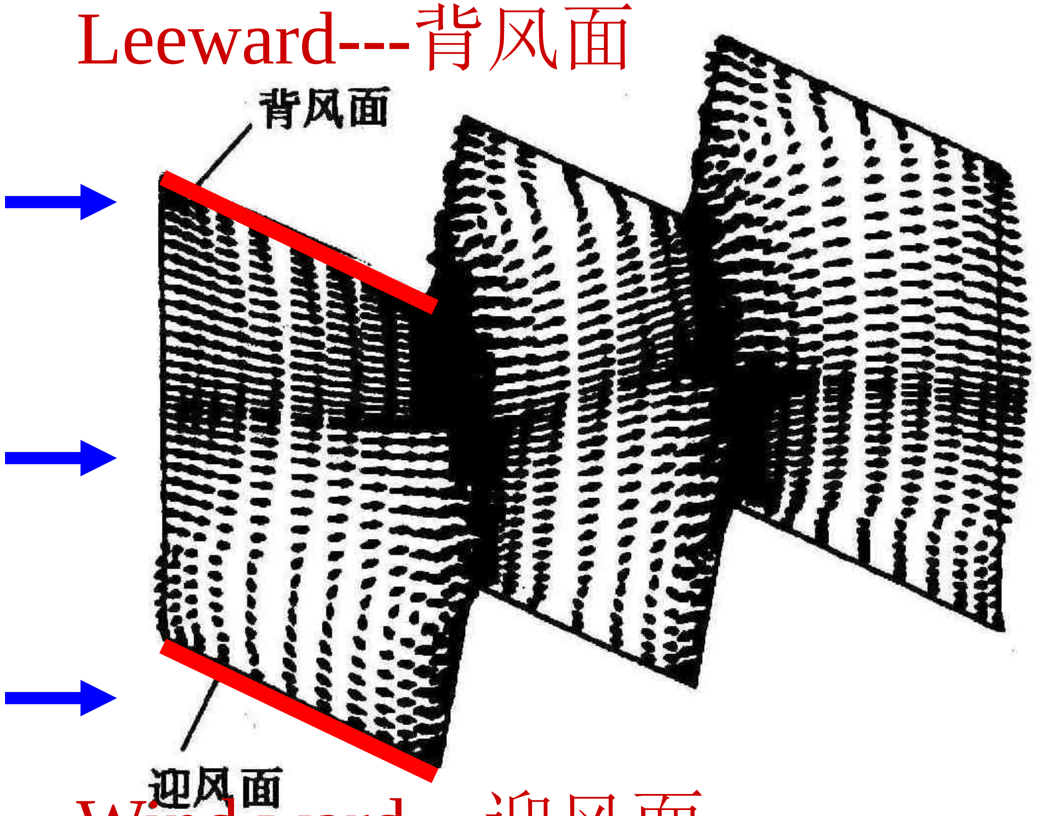
$$q_{A-F} = \frac{\int_{\xi_A}^{\xi_B} q^{(\eta_t)}\sqrt{\gamma}d\xi + \int_{\xi_C}^{\xi_D} q^{(\eta_t)}\sqrt{\gamma}d\xi + \int_{\xi_E}^{\xi_F} q^{(\eta_t)}\sqrt{\gamma}d\xi}{\int_{\xi_A}^{\xi_B} \sqrt{\gamma}d\xi + \int_{\xi_C}^{\xi_D} \sqrt{\gamma}d\xi + \int_{\xi_E}^{\xi_F} \sqrt{\gamma}d\xi}$$

$$\boxed{ds^{(\eta)} = d\xi\sqrt{\gamma}}$$

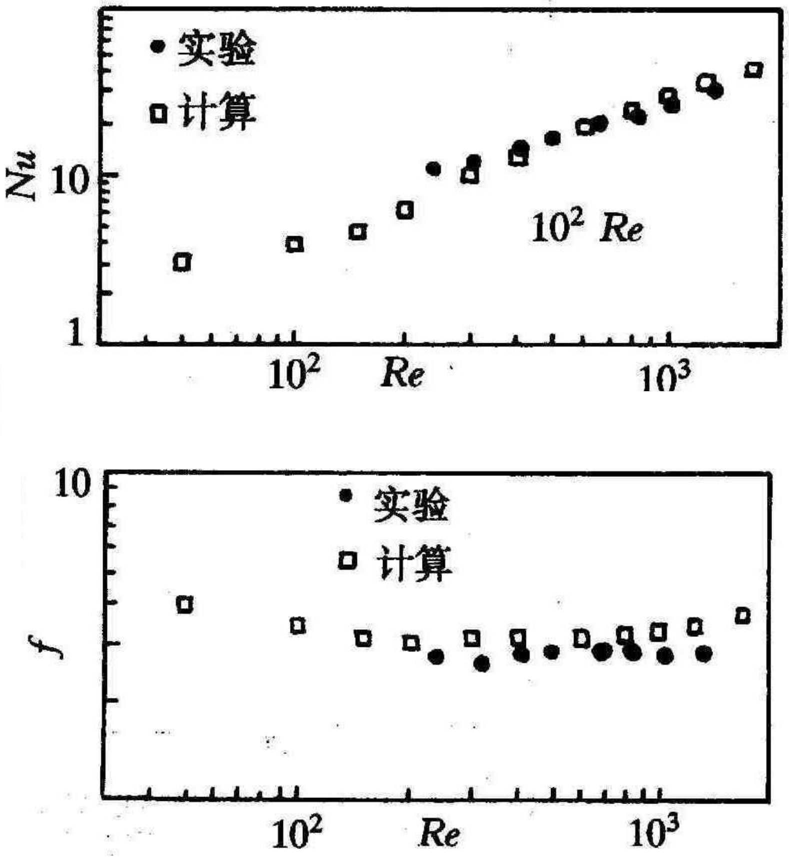
Local heat flux calculation should be conducted as shown in example 1.

3) Partial results

Leeward---背风面



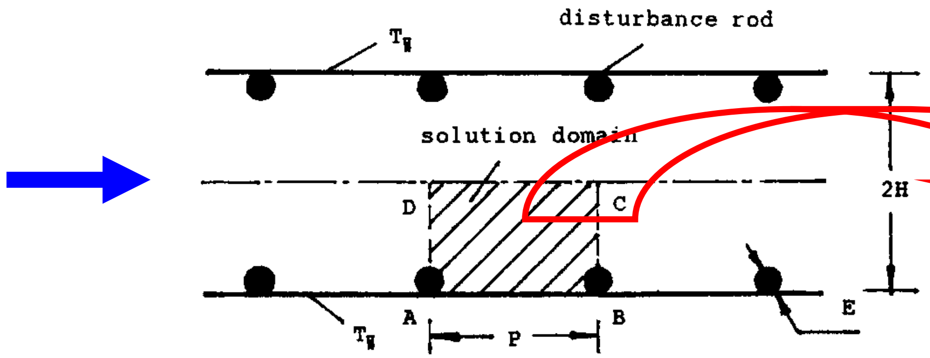
Wind ward---迎风面

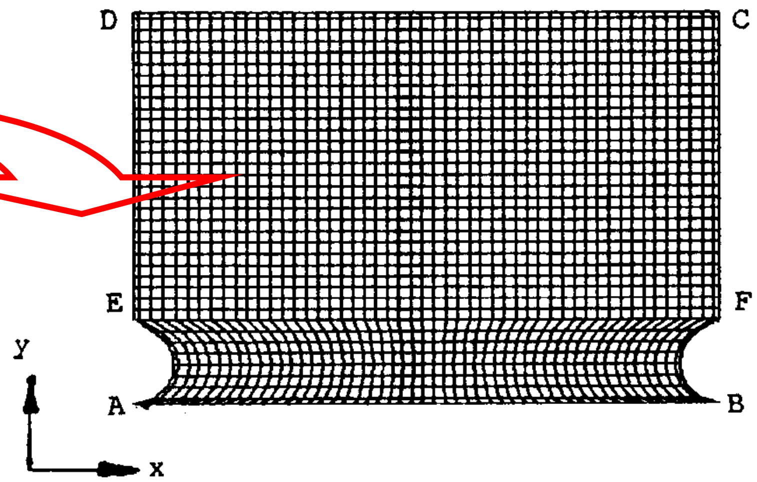


Wang L B， et al. ASME Journal of Heat Transfer， 1998, 120:991-998

3. Example 3—Periodic forced convection in a duct with roughness elements

1) Grid generation－Boundary normalization





2) Numerical methods

(1) Steady vs. unsteady－Unsteady governing equation is used to get a steady solution for the case of (H/E=5, P/E=20,Re = 700). The results are compared with those from steady equation. The differences are small: Nu-3％， f –less than 1％. Thus steady eq. is used.

(1) Scheme of convection term—PLS was used. Reviewer required : it should be shown that false diffusion effect could be neglected. Simulation with CD was conducted and comparison was made.

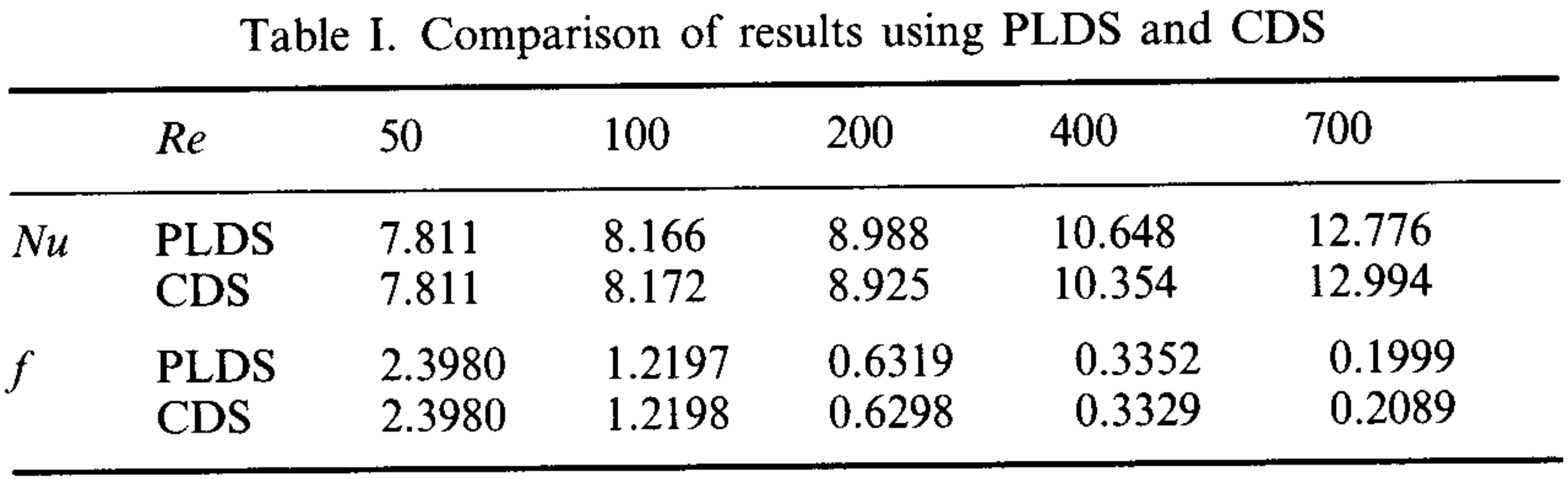


Table I. Comparison of results using PLDS and CDS

	Re	50	100	200	400	700
Nu	PLDS	7.811	8.166	8.988	10.648	12.776
	CDS	7.811	8.172	8.925	10.354	12.994
f	PLDS	2.3980	1.2197	0.6319	0.3352	0.1999
	CDS	2.3980	1.2198	0.6298	0.3329	0.2089

3) Partial results

Yuan Z X， et al. Int Journal Numerical Methods in Fluids， 1998, 28:1371-1378

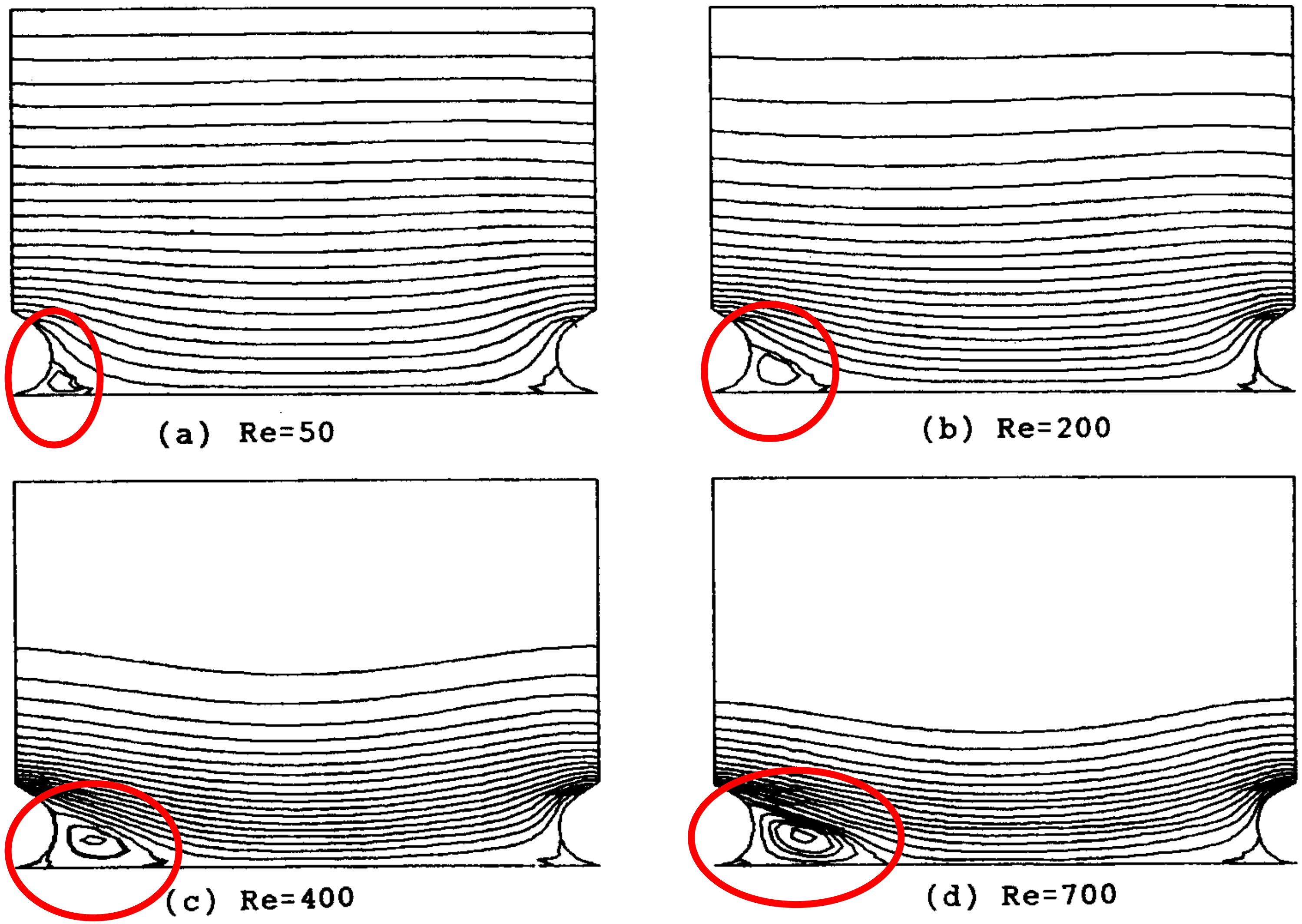
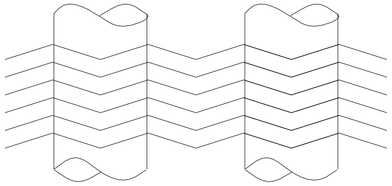
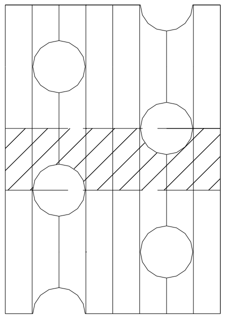


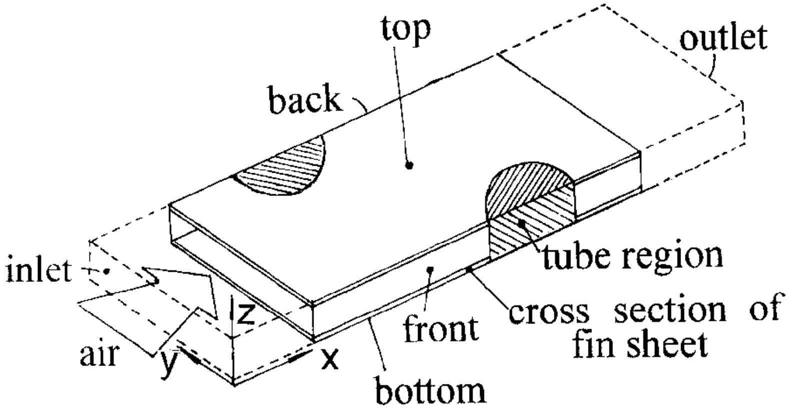
Figure 4. Flow patterns at different Reynolds numbers ($H/E = 5$, $P/E = 7.5$). (a) $Re = 50$; (b) $Re = 200$; (c) $Re = 400$; (d) $Re = 700$.

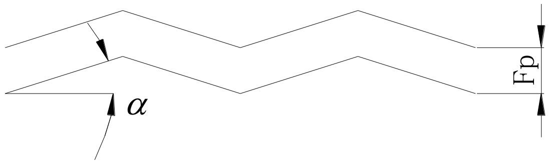
4. Example 4—Periodic forced convection in a wavy channel

1) Grid generation－ （Block structured＋3D Poisson）



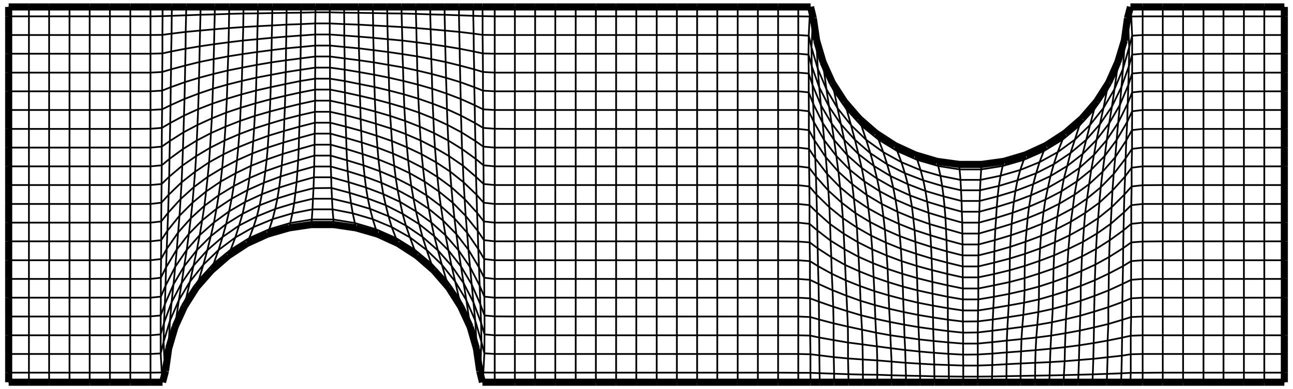






（Taking plain channel as an example）

$$\begin{cases} \alpha_{11}x_{\xi\xi}+\alpha_{22}x_{\eta\eta}+\alpha_{33}x_{\zeta\zeta}+2\alpha_{12}x_{\xi\eta}+2\alpha_{13}x_{\xi\zeta}+2\alpha_{23}x_{\eta\zeta}+J^2(Px_\xi+Qx_\eta+Rx_\zeta)=0 \\ \alpha_{11}y_{\xi\xi}+\alpha_{22}y_{\eta\eta}+\alpha_{33}y_{\zeta\zeta}+2\alpha_{12}y_{\xi\eta}+2\alpha_{13}y_{\xi\zeta}+2\alpha_{23}y_{\eta\zeta}+J^2(Py_\xi+Qy_\eta+Ry_\zeta)=0 \\ \alpha_{11}z_{\xi\xi}+\alpha_{22}z_{\eta\eta}+\alpha_{33}z_{\zeta\zeta}+2\alpha_{12}z_{\xi\eta}+2\alpha_{13}z_{\xi\zeta}+2\alpha_{23}z_{\eta\zeta}+J^2(Pz_\xi+Qz_\eta+Rz_\zeta)=0 \end{cases}$$



2) Grid-independence examination

One row

$102(x)\times 22(y)\times 10(z)$

Two-row

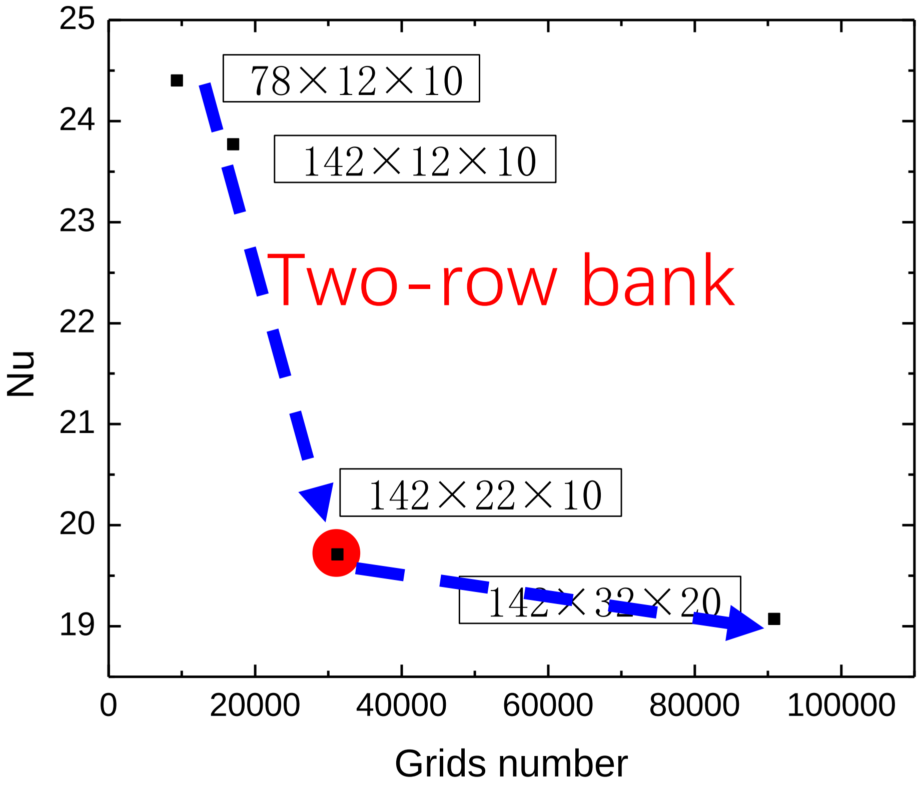
$142\times 22\times 10$

Three-row

$182\times 22\times 10$

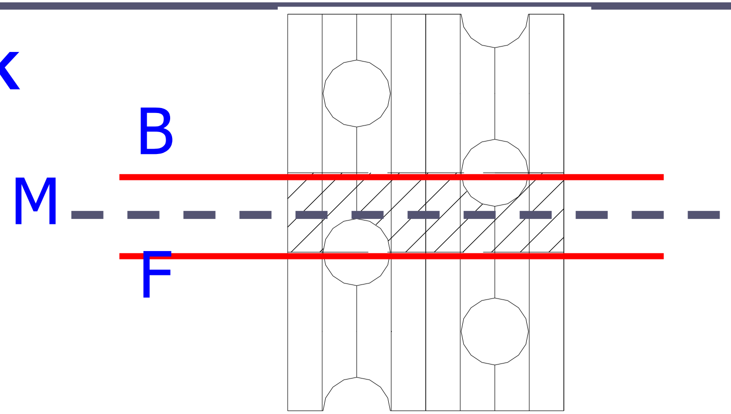
Four-row

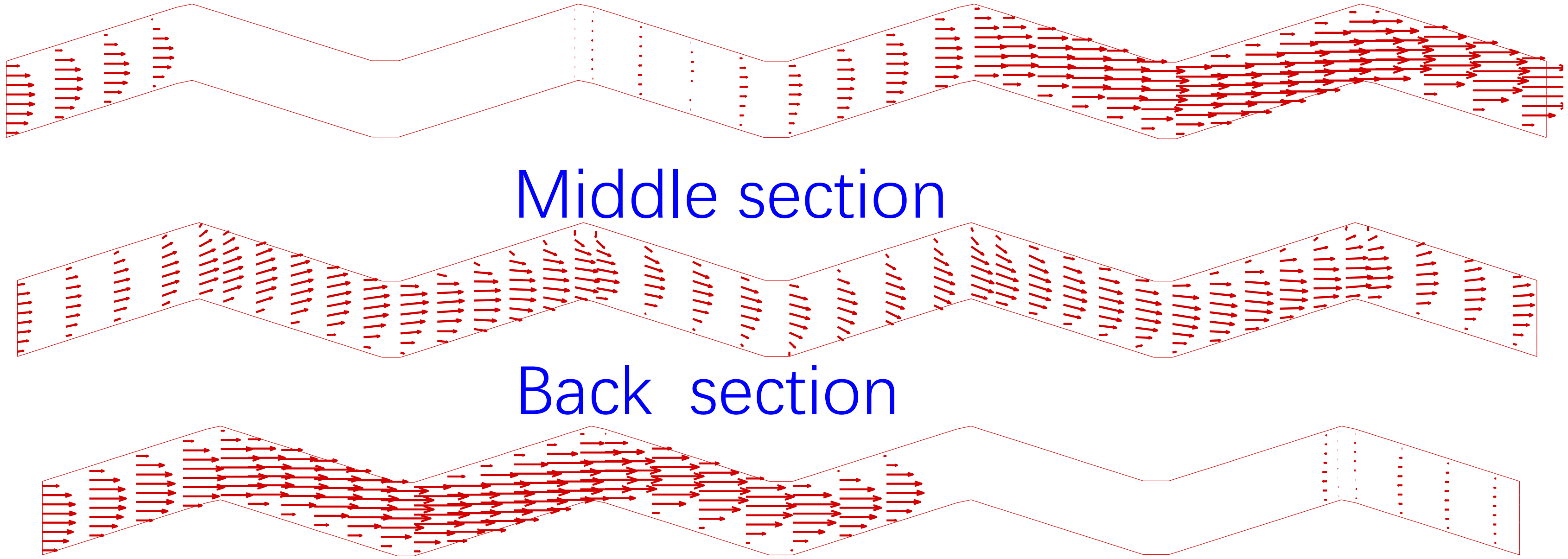
$192\times 22\times 10$



3) Partial results of two-row bank

Velocity distributions of three sections




Front section

Middle section

Back section

Tao Y B， et al. Int Journal Heat Mass Transfer， 2007, 50:1163-1175

End of the 1st part of the course NHT!

Computer-Aided Project of 2019 Numerical Heat Transfe

Xi’an Jiaotong University

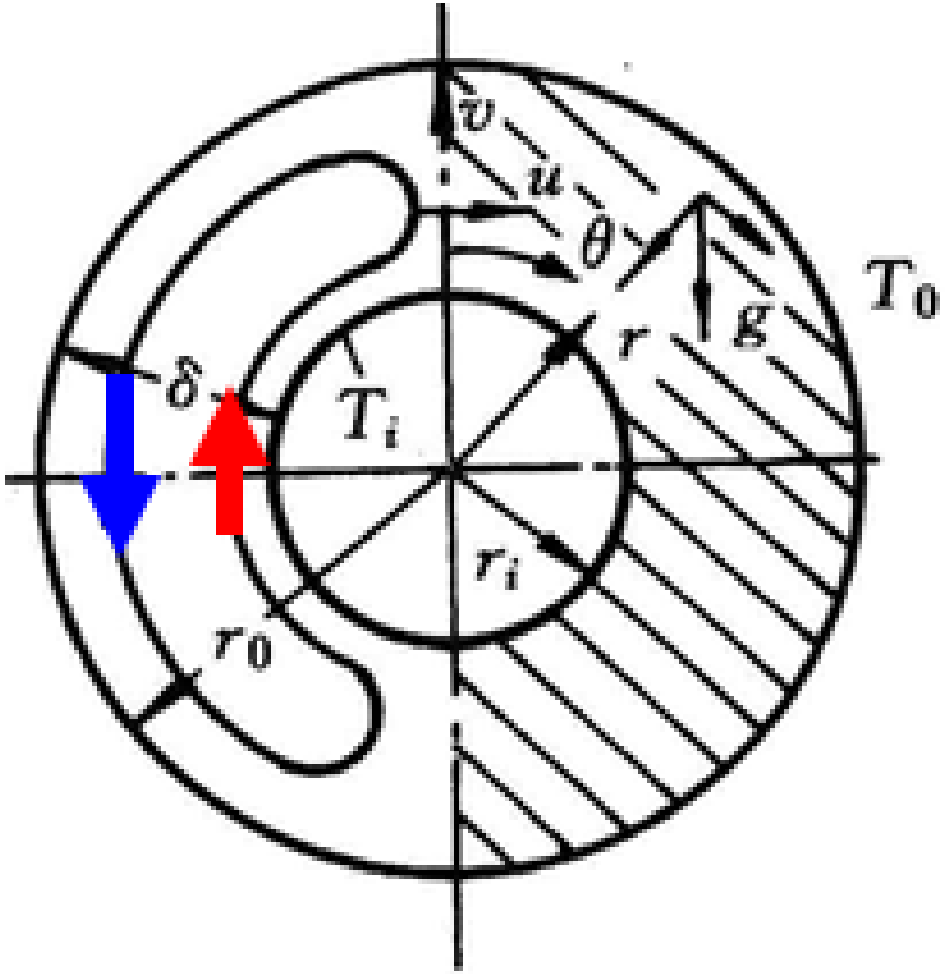
We present three computer-aided projects: one is to be solved by our teaching code (Project 1） , the 2nd and 3rd ones are to be solved by FLUENT (Fundamental , Project 2, Intermediate Project 3) . Every student can choose one project according to your interest and condition.

For the first project the self-developed computer code should attached in your final report.

For the second and third project Class F and Class I will have different projects. The instructors will assign the project at the end of the lecture.

Computer-Aided Project (1) of NHT-2019, Xi’an Jiaotong University

（Laminar natural convection in annular space）



$(T_i > T_0)$

1. Project formulation

For air natural convection within an annular space as shown in Fig. 1 , following conditions are given: $\delta / r_o = 0.4$, flow is laminar and the average air temperature is $50^\circ C$ For $\mathrm{Ra} = g\beta\Delta T\delta^3 \nu / a^2 = 10^2, 10^3, 10^4, 10^5$, determine the relative thermal conductivity: $\lambda_{\mathrm{eq}} / \lambda_{air}$ .The temperature difference between inner wall and outer wall is not large, so the Boussinesq assumption can be adopted. By using Tecplot or other software, display the isotherms and streamlines and the variation of $\lambda_{\mathrm{eq}} / \lambda_{air}$ vs. Ra. Natural convection heat transfer rate between the inner and outer surface is expressed by an effective thermal conductivity λ_{eq} as follows:

$$\phi = \frac{2\pi L \lambda_{eq} \Delta T}{\ln(d_2 / d_1)}$$

λ_{eq} is the equivalent thermal conductivity of the entire annular space.

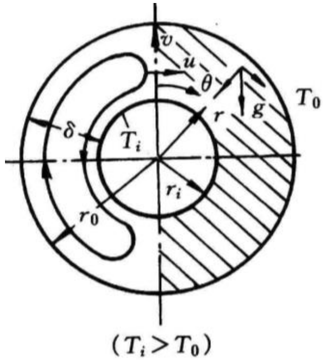
2. Suggestions and Requirements

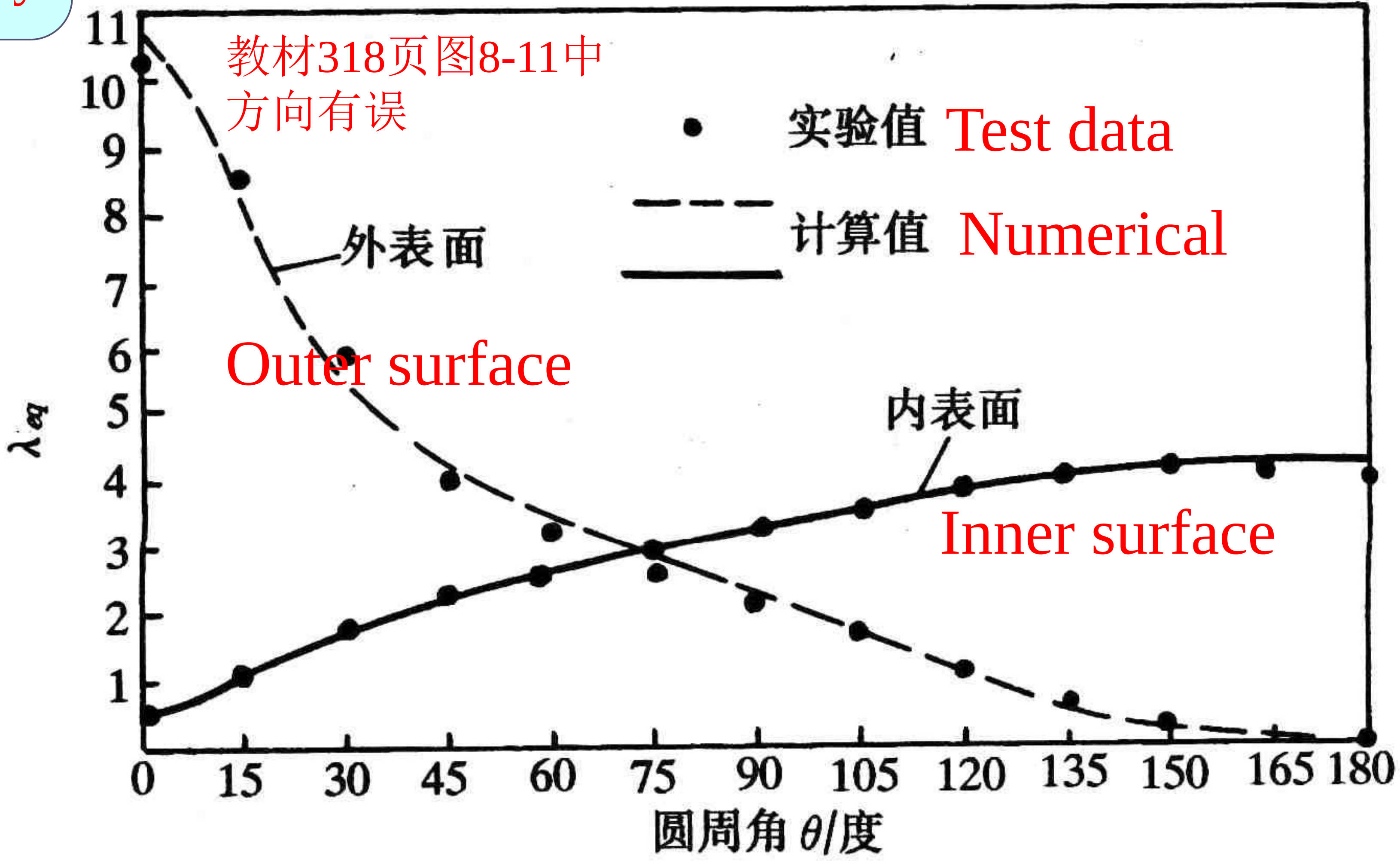
1) Considering the symmetry of the geometry, only half of the structure should be simulated.
2) The solution should be grid-independent.
3) The project report should be written in the format of the Journal of Xi'an Jiaotong University. Both Chinese and English can be accepted.
4) Examples 9-4 (Mode=3) and 9-6（Natural convection） may be consulted.
5) Please submit in the USER part developed by yourself for solving the problem.

The project report should be due in before April 30, 2020 to room 204 of East 3rd Building of Xingqing campus. If you need the course score earlier, please submit your report ealier too and inform me.

Local dimensionless thermal onductivity

Kuehn T H, Goldstein R J. An experimental and theoretical study of natural convection in the annulus between horizontal concentric cylinders. J Fluid Mech, 1971, 74:605-719





教材318页图8-11中方向有误

Test data

Numerical

Outer surface

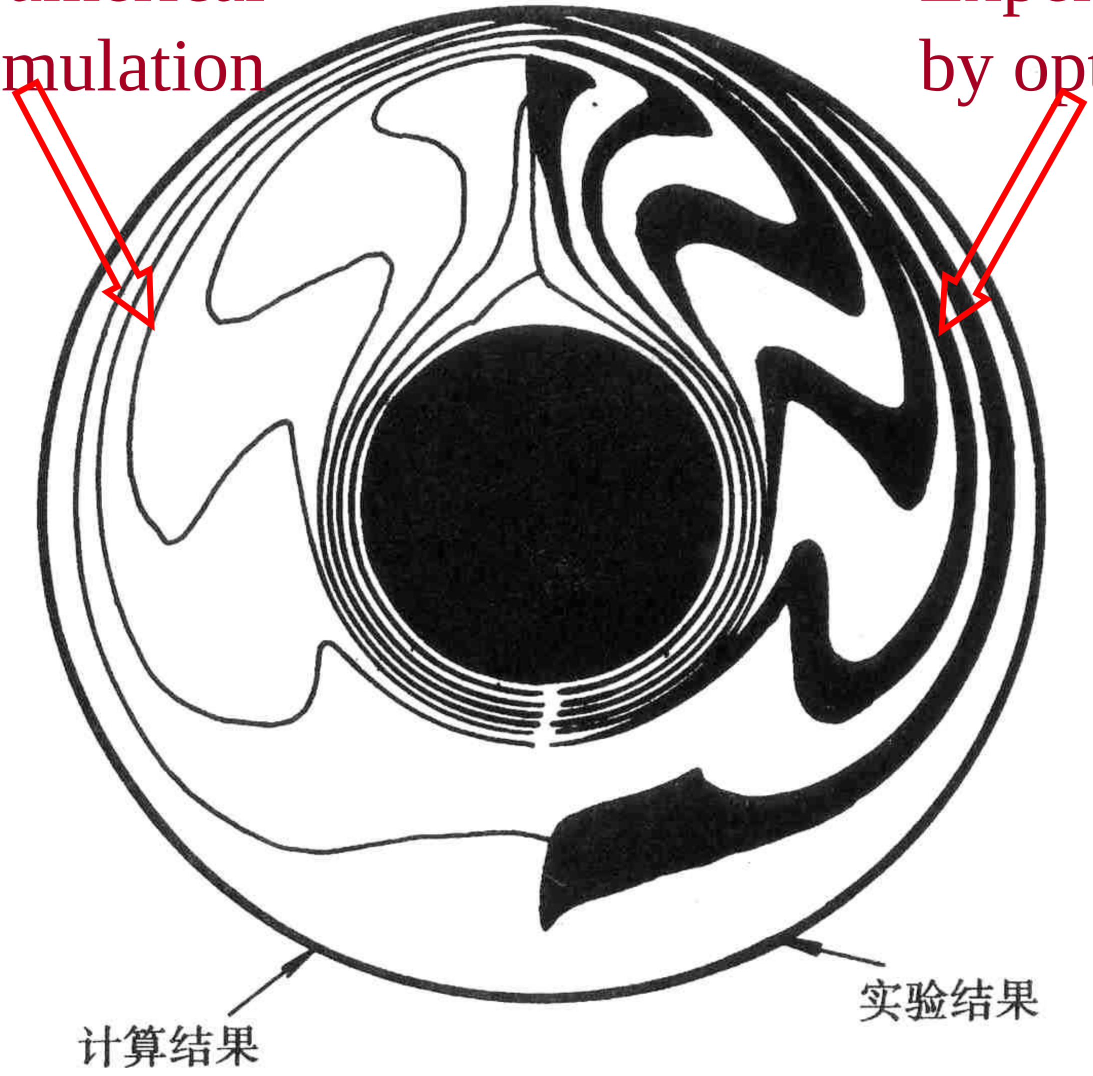
Inner surface

Local relative thermal conductivity

$$\overline{\lambda}_{eq}=\left(\frac{q_{convection}}{q_{conduction}}\right)_{\theta}$$

Numerical simulation

Experiment by optical method



实验与计算条件

	实验	计算
Ra_δ	4.7×10^4	5×10^4
Pr	0.706	0.7
δ/Di	0.8	0.8

Comparison of isotherms (等温线)

本组网页地址：http://nht.xjtu.edu.cn 欢迎访问！

Teaching PPT will be loaded on ou website

同舟共济
渡彼岸！

People in the same boat help each other to cross to the other bank, where….